

# High Precision Reducers for Industrial Robots Driving 4th Industrial Revolution: State of Arts, Analysis, Design, Performance Evaluation and Perspective

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*New industrial revolution - "The 4th industrial revolution" must be a remarkable milestone for the second decade of the twenty-first century. Many countries are competing to innovate their manufacturing chains for eco-friendly and energy-efficient productions. Although this green or sustainable manufacturing system evolves under the support of cyber-physical system (or digital twin) based on ICT technology, industrial robots also play important roles in this speedy, flexible and effective manufacturing chains. Recently, low-cost industrial robots or collaborative robots, are rising in a highly interactive environment with humans. Although an industrial robot consists of many important components such as mechanical parts (kinematic structure and reducer) and electric parts (servo motor, driver, sensors, and controller), precision reducer takes approximately 25% of material-cost and governs important performance indices of industrial robots. This paper presents review of high precision reducers (HPRs) for industrial robots driving 4th industrial revolution. First, we provide HPRs market along with industrial robots. According to previous studies, HPRs for industrial robots can be classified based on their principles: planetary reducer, cycloid reducer, and harmonic drive (HD). Then, principle, characteristics, and three main performances (hysteresis, rotational transmission error (RTE) and efficiency) of HPRs are discussed. In addition, compensation methods overcoming accuracy limits of HPRs are summarized. Finally, other applications of HPRs except industrial robots are presented.*

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## 1. Introduction

The 4th industrial revolution we are experiencing today accelerates the improvement of productivity, flexibility and intelligence, but also provides open challenges to individuals and the development economics, society, culture, and politics. Manufacturing is becoming a hot topic again, undergoing the industry's greatest change.<sup>1,2</sup> The huge demand for intelligent manufacturing and low carbon development is reforming the modes of production and organization.<sup>3,4</sup>

In advance of the 4th industrial revolution, considerable efforts have been made for energy-efficient and environment-friendly manufacturing process and system such as green or sustainable manufacturing.<sup>5,6</sup> In fact, there is significant pressure for effective usage of natural resources and energy due to both increased global population and decreased world's resources. Moreover, green manufacturing also goes towards to

eco-friendly strategies such as reducing emission in manufacturing process, low environmental impact of material or high optimal management system.<sup>7-9</sup>

Although substantial improvements in technology, the rapid growing economy and quickly changing market along with 4th industrial revolution have generated various opportunities in recent years, competent scientific and engineering solutions have been leading intelligent and green manufacturing challenges.<sup>10,11</sup> Advancement of modern equipment and automation not only reduces resources and energy but also supports human efforts in manufacturing process.<sup>11-19</sup> Furthermore, advanced material technology contributes to wide-range advances in industrial processes and manufactured products, saving energy and waste.<sup>20-24</sup>

ICT-powered automation technology plays a crucial role in implementing intelligent and green manufacturing.<sup>25-27</sup> Therefore, conventional factory becomes more flexible, efficient and responsive

Table 1 Performance relationship between industrial robots and precision reducers<sup>36</sup>

		Robot performance				
		Acceleration	Settling time	Payload capacity	Accuracy	Repeatability
Reducer performance	High accuracy				•	•
	Repeatability					•
	Zero backlash	•	•		•	•
	Torsional rigidity	•	•	•	•	•
	Moment load			•		
	Torque density	•	•	•		

based on network technology and are finally transformed into smart factory, which enables not only virtual manufacturing with cyber-physical system, but also customer-centric production and resource efficient supply chains.

As robots are becoming smarter, more mobile, more collaborative and more adaptable nowadays, industrial robots are on the verge of revolutionizing manufacturing and bring major changes to the factory floor, as well as potentially to the global competition.<sup>28</sup> While humans still play a leading role in smart factory, robots and machines will both support people and link them together. Therefore, collaborative robots (also known as cobots) are becoming the solution of choice for smart.<sup>29-31</sup> Cobots have irresistible advantages such as: high efficiency, application flexibility, complex functions capability, continuously 24/7 operation as well as low cost.<sup>32</sup> High performance robots<sup>33</sup> can save energy, reduce scrap and suit with not only tiny products like semiconductors and drugs, but also large products such as LED chain-line, wind turbines and so on.<sup>34</sup> In addition, they may also be used as manufacturing and testing equipment in harmful and harsh environments instead of human.<sup>35</sup>

Although an industrial robot consists of many important components such as mechanical parts (kinematic structure and reducer) and electric parts (servo motor, driver, sensors and controller), precision reducer takes approximately 25% of material cost and governs important performance characteristics of industrial robots, as shown in Table 1.<sup>36,37</sup> An industrial robot can be described with four technical data: geometry, kinematics, load, and accuracy.<sup>38</sup> Geometric parameters such as work space are characterized with link and joint parameters of robots, while kinematic parameters such as acceleration and cycle time are governed by backlash, torsional stiffness and torque density of reducers. Additionally, load parameters of industrial robots are determined by stiffness, moment and torque load of high precision reducers. Obviously, accuracy or repeatability of robots is directly related to that of reducers.

This paper presents review of HPRS for industrial robots driving 4th industrial revolution. In detail, the state-of-the-art on HPRS, their performances and designs are overviewed. First, we provide HPRS market along with industrial robots. Then, three types of HPRS: planetary reducer, cycloid reducer and harmonic drive are presented and compared in terms of principle and characteristics. Three major performances of HPRS: hysteresis, RTE and efficiency are discussed. Moreover, design issues of HPRS are also described. Active control or compensation methods overcoming performance limits of HPRS are presented. Finally, other applications of HPRS except industrial robots are presented.

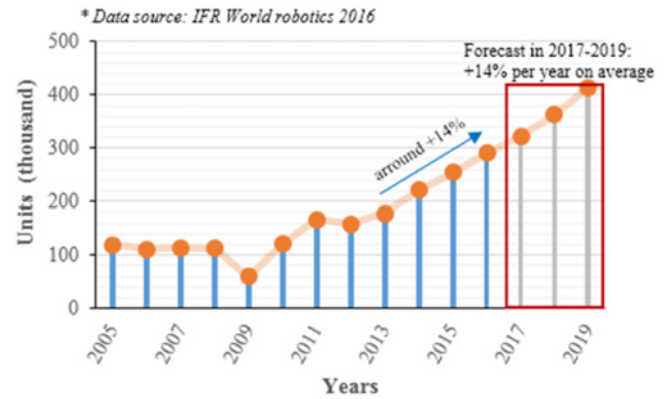


Fig. 1 Worldwide annual supply of industrial robots (2001-2019)

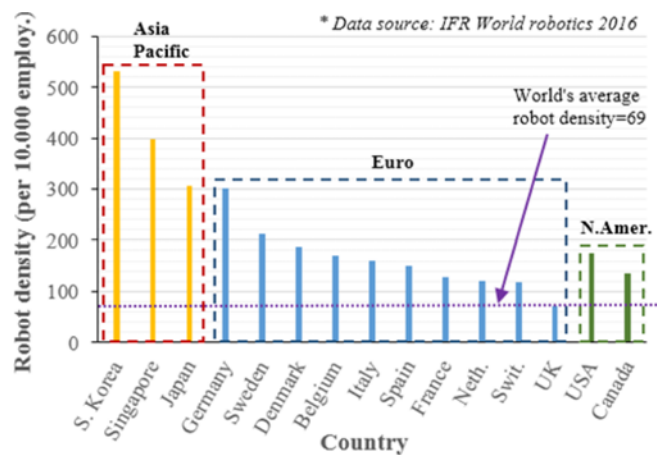


Fig. 2 Number of industrial robots per 10,000 employees (robot density) in the manufacturing system in 2015

## 2. High Precision Reducer

### 2.1 HPRs Market Perspective - An Increment of HPRs

Since major application of HPRs is robotics and the robot market is growing fast due to 4th industrial revolution, we present HPRs market together with robot market perspectives. As mentioned before, HPRs are used in every joint of a robot and play a crucial role in driving the robot. Since joint of a robot transfers an accurate and fast motion in limited installation zone, HPRs should have essential characteristics such as high reduction ratio, considerable torsional rigidity, kinematic accuracy or precision (or low lost motion), low inertia mass and compact size. That is, industrial robots can precisely carry out complex motions with very fast manner because of HPRs embedded in every joint.<sup>36</sup> Therefore, HPRs take large portion of overall cost of industrial robots and we cannot separate markets of HPRs and industrial robots.

International Federation of Robotics (IFR)<sup>39</sup> forecasts that more than 1.4 million new robots will be installed in factories until 2019 and global HPRs market<sup>40</sup> will reach to almost 2.1 billion US dollar. Worldwide annual supply of industrial robots from 2001 to 2019 is shown in Fig. 1. Around a number of 115,000 industrial robots were steadily supplied from 2005 to 2008. After sudden

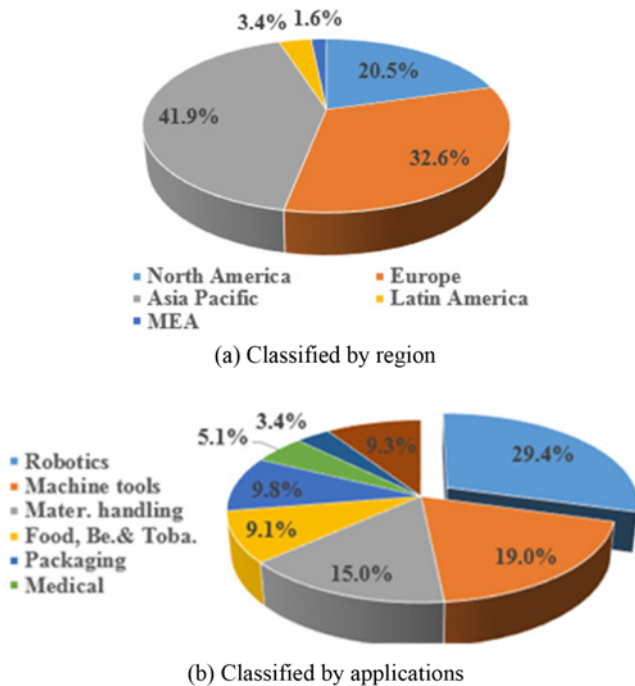


Fig. 3 Global HPRs market in 2013

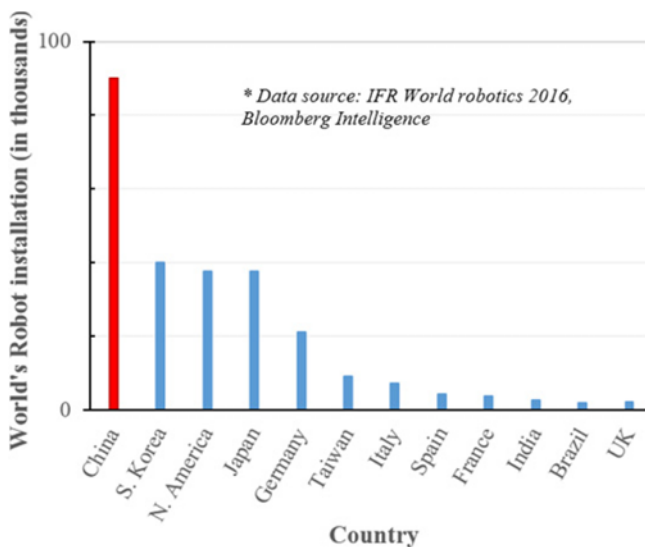


Fig. 4 Number of installing robots around the world

decrease down to 60,000 units in 2009 because of the financial crisis 2008, the annual supply of industrial robots is growing average 13% per year. Especially, north-east Asian countries (Korea, Japan) and Singapore are the world leaders in adopting industrial robots and robot density of these countries is 4-6 times higher than world average number or 69 units per 10,000 employees, as shown in Fig. 2.

Global market of HPRs is increasing according to the robot market growth. As claimed by global market insights in 2013, Asia Pacific, Europe and North America occupied 95% of HPRs world market,<sup>40</sup> as shown in detail in Fig. 3(a). In addition, robotics takes leading

application of HPRs global market based on this report, as shown in Fig. 3(b). Additionally, China is installing more robots than any other nation and requires at least 675,000 HPRs sets until 2018,<sup>41</sup> as shown in Fig. 4.

There is no doubt that Japanese providers are the world leaders of HPRs such as Nabtesco, Harmonic Drive, Sumitomo and so on.<sup>42-47</sup> Nabtesco takes 60% of the global HPRs with RV reducer or combination of planetary and cycloid reducer. In addition, Harmonic Drive Inc. also occupies about 15% of global HPRs market. The other smaller proportion of global HPRs market is shared by Sumitomo, Spinea, Shimpo and so on.

Market of less precise reducer will grow fast for low-cost industrial or co-robot and other industrial applications such as automotive. Recently, low-cost industrial robots are expected to be used as collaborative robots with a lower and more suitable price in the near future. Although they are low-cost robotic arms, the performance of low-cost cobots still meets a demand of manufacturing chains. For cost reduction, low-cost components such as less precise reducer and servo motors will be used in these cobots. Therefore, the market of less precise reducer will expand as fast as the increment of low-cost industrial cobots.<sup>48</sup>

## 2.2 HPRs for Industrial Robots

According to previous studies,<sup>42-47</sup> high precision reducers (HPRs) for industrial robots can be classified into three main categories: planetary reducer, cycloid reducer and harmonic drive. The compositions and principles of the HPRs are different. Planetary reducer mainly comprises a sun gear, a ring gear and a carrier with several planet gears. As the sun gear is used as input and ring gear is fixed, the planet gear is revolved around center of sun gear or ring gear on a carrier. One-stage cycloid reducer consists of four main components: an input shaft, bearing, a cycloid disk and an output mechanism. A pure rotation is actuated from the input shaft to the output mechanism by converting rotating motion to a wobble motion on the cycloid disk. On the other hand, harmonic drive generally comprises an external flexible gear (flexspline or FS), internal rigid gear (circular spline or CS) and a wave generator (WG). The flexible gear is deformed into an oval shape following the elliptic wave generator, and teeth of FS at the major axis of the WG alternatively mesh with teeth of the CS under a rotation of wave generator.

As reducer size, positioning accuracy and efficiency in high reduction ratio are all considered in choosing a transmission mechanism, cycloid reducer and harmonic drive are the best candidates.<sup>36,49</sup> Obviously, cycloid reducer and harmonic drive are being not only manufactured in high quality but also improved continuously. On the contrary, planetary reducer is limited in reduction ratios (usually under 10 : 1) and should have multi-stage for high reduction ratio such as 100 : 1. In addition, backlash and efficiency of the reducer are less advantageous than cycloid reducer and harmonic drive. However, planetary reducer was used occasionally if the price or torque density under low reduction ratio is vital. Finally, a characteristic comparison all types of HPRs is introduced in short-brief as shown in Table 2.

Table 2 Performance characteristics of HPRs

	Planetary reducer <sup>42</sup>	Cycloid reducer <sup>43-46</sup>	Harmonic drive <sup>47</sup>
Features	<ul style="list-style-type: none"> <li>- Low reduction ratio</li> <li>- Low cost</li> <li>- Easy installation &amp; replacement</li> <li>- Spread design and heavy-weight</li> <li>- Medium backlash</li> <li>- Medium torsional rigidity</li> <li>- Low accuracy</li> <li>- Medium rate torque capacity</li> <li>- Low efficiency</li> <li>- Vibration and noise</li> <li>- Medium-weight</li> </ul>	<ul style="list-style-type: none"> <li>- High reduction ratio</li> <li>- Medium price</li> <li>- Complex installation &amp; replacement</li> <li>- Compact design</li> <li>- Low backlash</li> <li>- High torsional rigidity</li> <li>- High accuracy</li> <li>- High rate torque capacity</li> <li>- High efficiency</li> <li>- Smooth operation</li> <li>- Heavy-weight</li> </ul>	<ul style="list-style-type: none"> <li>- High reduction ratio</li> <li>- Expensive</li> <li>- Complex replacement</li> <li>- Compact and thin design</li> <li>- Zero backlash</li> <li>- Low torsional rigidity</li> <li>- High accuracy</li> <li>- Medium rate torque capacity</li> <li>- Medium efficiency</li> <li>- Smooth operation</li> <li>- Light-weight</li> </ul>
Reduction ratio	$\leq 10:1^1, \leq 100:1^2$	$\leq 179:1^S, \leq 203:1^N$	$\leq 160:1$
Weight (kg)	$\leq 120$	$\leq 35.5^S, \leq 102^N$	$\leq 20.9$
Backlash (arc.min)	$\leq 5$	$\leq 1.5$	0
Lost motion (arc.min)	$\leq 5$	$\leq 1.5$	$\leq 3$
Rated torque (Nm)	$\leq 5,300$	$\leq 1,830^S, \leq 7,000^N$	$\leq 951^T, \leq 1,236^H$
Torque density (Nm/Kg)	$\leq 44.16$	$\leq 51.5^S, \leq 76.3^N$	$\leq 45.5^S, \leq 59.14^H$
Moment rigidity (Nm/arc.min)		$\leq 1,600^S, \leq 9,000^N$	$\leq 1,175$
Torsional rigidity (Nm/arc.min)	$\leq 850$	$\leq 1,100^S, \leq 3,334^N$	$\leq 903$
Effi.(%)			
Low out. speed	$\leq 85$	$\leq 86^S, \leq 95^N$	$\leq 86$
High out. speed	$\leq 80$	$\leq 80^S, \leq 90^N$	$\leq 77.5$

<sup>1</sup>: one-stage, <sup>2</sup>: multi-stage, <sup>S</sup>: Sumitomo cyclo series, <sup>N</sup>: Nabtesco RV series, <sup>T</sup>: standard series, <sup>H</sup>: high torque series

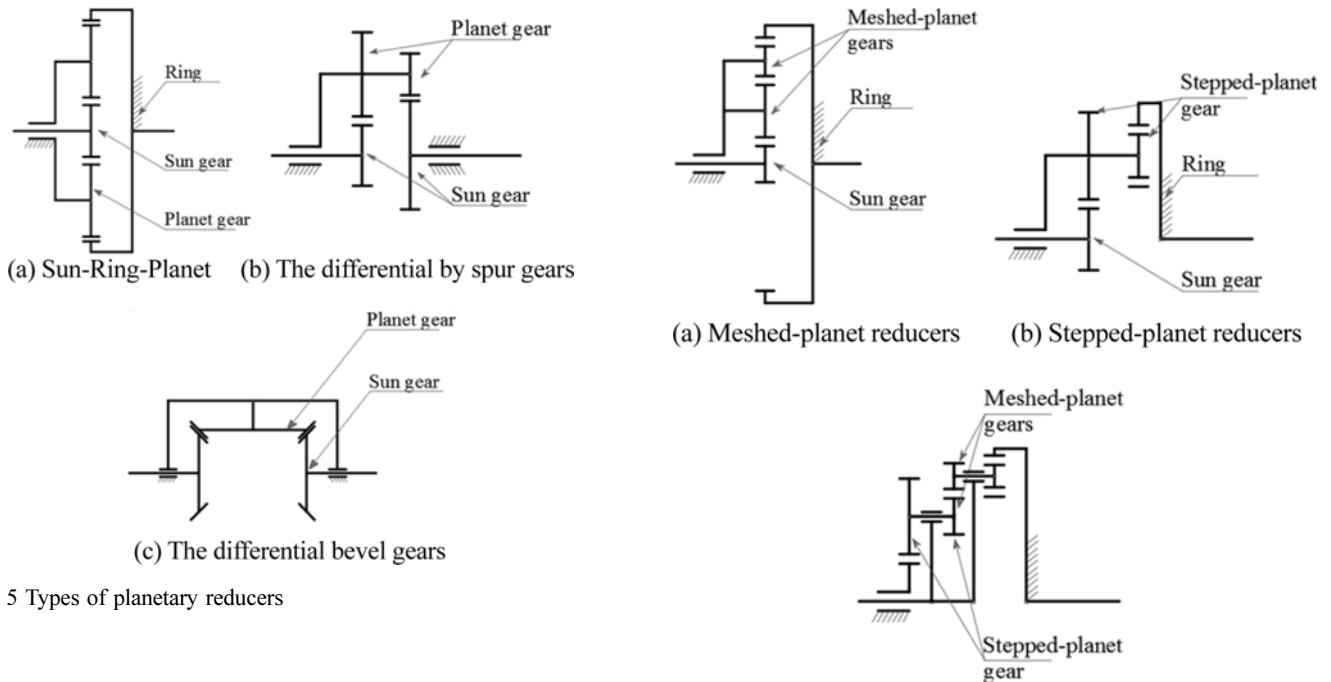


Fig. 5 Types of planetary reducers

### 2.2.1 Planetary Reducer

Planetary reducer consists of a sun gear, planet gears with carrier, and ring gear. The planet gear with a carrier revolves around center of sun gear and inscribes around ring gear. If the planet and sun gears are engaged so that their pitch circles roll without slip, the curve traced by a point on the pitch circle of planet gear is epicycloids curve. On the other hand, the curve traced by point on pitch circle of the planet is hypocycloids if the planet gear rolls on the inside of the pitch circle of a fixed ring gear.<sup>50</sup> Thus, it is sometimes called as Epicyclic gear train.

Fig. 6 Types of compound planetary reducers

We can classify planetary reducers into two main catalogues, as shown in Fig. 5. First, there are only 12 practical models suitable with the Epicyclic motion although Levai<sup>51</sup> illustrated 34 distinguished models of planetary reducers. There are two principal types of planetary reducers such as: sun-ring-planet and the

Table 3 Reduction ratio calculation of planetary reducer<sup>50</sup>

Case	Fixed	Input	Output	Reduction ratio
1	Ring gear	Sun gear	Planet gears (Carrier)	$i_{P1} = \frac{N_s + N_R}{N_S}$
2	Sun gear	Planet gears (Carrier)	Ring gear	$i_{P2} = \frac{N_R}{N_R + N_S}$
3	Planet gears (Carrier)	Sun gear	Ring gear	$i_{P3} = \frac{N_R}{N_S}$

$N_S, N_R$ : Number of teeth on sun gear, ring gear, respectively,

$i_{P1}, i_{P2}, i_{P3}$ : reduction ratio of planetary reducer

differential (by using spur gears or bevel gear). As shown in Fig. 5(a), input or output can be one among carrier, ring and sun gears in case of in sun-ring-planet type reducer. In case of the differential by spur gears, input is or output is one of two sun gears. Although this type of reducer is suitable for high speed operation, it may have low efficiency without careful design. In last, the differential type reducer with bevel gears is suitable for perpendicular-axis transmission and the planet gear does not undergo epicyclic motion around sun gears. Obviously, sun-ring-planet is usually used in robot system; whereas the differential is often applied in automotive drivetrains to overcome the speed discrepancy of driving wheels. Reduction ratios of planetary reducers are shown as in Table 3. Furthermore, torques and efficiencies of planetary reducer are provided in detail in.<sup>50</sup>

To increase the reduction ratios of a gearbox, compound planetary reducers are occasionally used.<sup>52</sup> The compound planetary reducers consisting of at least two or more planet gear sets, can be arranged in three primary categories such as meshed-planet, stepped-planet, and multi-stage structure shown in Fig. 6. The mesh-planet of Fig. 6(a) has two or more parallel planet gears engaging with each other. The carrier-shaft of the stepped-planet has two planet gears in series, as shown in Fig. 6(b). On the other hand, multi-stage of Fig. 6(c) is a combination of mesh-planet and stepped-planet. Although variety of compound planetary reducers can be applied according to applications, the multi-stage structure type is the most popular product for both robotics and vehicle applications. While the compound planetary reducers boost up reduction ratio and torque capacity dramatically, it makes the gear box complex and inefficient.

### 2.2.2 Cycloid Reducer

Cycloid reducer includes four principal elements such as high-speed input eccentric shaft with a bearing, one or two cycloid disk, housing with pin-rollers and slow-speed output mechanism, as shown in Fig. 7. The high-speed input eccentric shaft with bearing generates a wobble motion of the cycloid disk supported with pin-rollers in the housing. Then, output mechanism interacts with the cycloid disk and the wobble motion of the cycloid disk is converted into pure rotation of the low-speed output shaft.

Various types of cycloid reducers were developed, as shown in Fig. 7: one-stage (Figs. 7(a) and 7(b)) and two-stage (Fig. 7(c)) cycloid reducers. For improvement of moment stiffness, Spinea proposed another type of one-stage of cycloid reducer as in Fig. 7(b).

Table 4 Reduction ratio calculation of cycloid reducer<sup>44,45</sup>

Case	Fixed	Input	Output	Reduction ratio
1	Housing	Input shaft	Output mechanism	$i_{C1} = \frac{1}{SR}$
2	Output mechanism	Input shaft	Housing	$i_{C2} = \frac{1}{SR-1}$
3	Input shaft	Housing	Output mechanism	$i_{C3} = \frac{SR-1}{SR}$

$i_{C1}, i_{C2}, i_{C3}$ : reduction ratio of cycloid reducer

\*  $SR$ : speed ratio of cycloid reducer (equal to number of pin-rollers  $N_{rol}$  on housing). However, the speed ratio of RV drive is calculated by tooth number of sun gear  $N_S$ , planet gear  $N_P$  and  $N_{rol}$

$$SR_{RV} = 1 + \frac{N_P}{N_S} N_{rol}$$

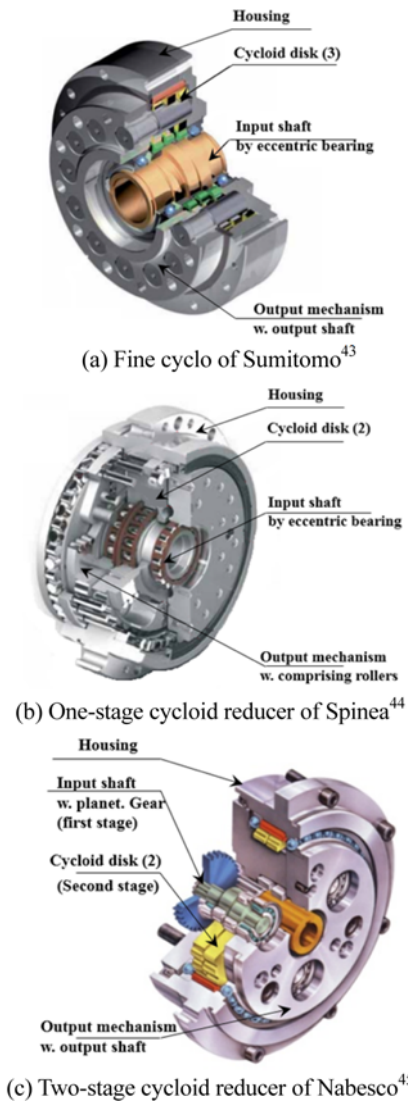


Fig. 7 Types of cycloid reducers (Adapted from Refs. 43-45 on the basis of open access)

A key of difference between cycloid reducers of Sumitomo and Spinea is their output mechanisms. While the pins of the output shaft transmit the rotation of the cycloid disk by rolling internally around



the bores of each cycloid disc in case of Sumitomo, two sliding plates with rollers transmits only pure rotation of cycloid disk to the output shaft in case of Spinea. On the other hand, RV reducer is a two-stage cycloid reducer including first planetary mechanism to drive cycloid reducer of the second stage. The reduction ratio of RV reducer can be easily modified by changing the reduction ratio of the first planetary mechanism.<sup>45</sup> Reduction ratio of cycloid reducer in various cases is summarized in Table 4.

Bearings of cycloid reducer are so important to determine not only size of the reducer but also many key performance indices. Nabtesco recently developed RV-N series that has an integrated bearing inner race. Thus, RV-N series has more compact size by 8-20% and lighter weight by 16-36%.<sup>46</sup> In similar principle, the RV drive may have lighter weight by changing material of some parts such as output shaft or housing.

### 2.2.3 Harmonic Drive

Invented by W. Musser in 1955,<sup>53,54</sup> the harmonic drive was used in defense applications at first, before widely applied in industrial robots and machine tools around the 90s. The first principal space application of harmonic drive was introduced during 1971 as a transmission mechanism of wheel drives in the Lunar roving vehicle. As hydraulically-driven robots were replaced by electrically-driven robots from the early 1970s, harmonic drive is continuously growing along with the development of industrial robots.

Harmonic drive (HD) is a compact transmission mechanism with a few components and characterized by no backlash, great efficiency and high reduction ratio. One prototypical HD comprises three main concentric components: a WG, FS and CS, as shown in Fig. 8.<sup>47</sup> The WG (input), which is normally a thin-race ball bearing, rotates inside FS (output) and deform it to make a tooth engagement with CS (fixed) longitudinal the major axis of elliptical shape. FS is rotated in an opposite direction with input rotation by a different number of teeth between FS and CS. Nevertheless, as any rotational angle of input, there are always two teeth-pairs in a full contact between FS and CS, thus this type of reducer is also called by name of “double wave reducer”. In addition, a reduction ratio of HD is summarized for various cases, as shown in Table 5.

HD is becoming smaller and lighter. An ultra-flat type CSD or SHD was developed for limited-space applications in 1999.<sup>55</sup> Furthermore, dedicate design and surface coating improved torque capacity and life span of CSG or SHG.<sup>47</sup> In addition, materials of some parts were replaced so that 30% lighter weight (LW) HD was introduced.<sup>36</sup> Furthermore, special types of HDs were developed such as Flat-wheel HD,<sup>56</sup> Plastic HD<sup>57</sup> or magnetic HD.<sup>58</sup>

In near future, HD may generally have two-stages which are a combination of planetary in the first stage and HD in the second stage. A planetary mechanism is installed inside FS instead of WG by a thin-race bearing. This planetary mechanism with two planet gears and one sun gear plays a role as a new WG. A rotational input from a motor is directly connected to the sun gear and transfer to planet gears. This design will help users to easily change the reduction ratio of HD by changing a gear ratio of planetary reducers. Certainly, the most benefit is a flexible option of reduction without replacing HD.

Table 5 Reduction ratio calculation of HD

Case	Fixed	Input	Output	Reduction ratio
1	CS	WG	FS	$i_{H1} = \frac{N_F}{N_F - N_C}$
2	FS	WG	CS	$i_{H2} = \frac{N_C}{N_C - N_F}$
3	WG	FS	CS	$i_{H3} = \frac{N_C}{N_F}$

$N_C$ : Number of teeth on CS;  $N_F$ : Number of teeth on FS;

$i_{H1}, i_{H2}, i_{H3}$ : reduction ratio of HD

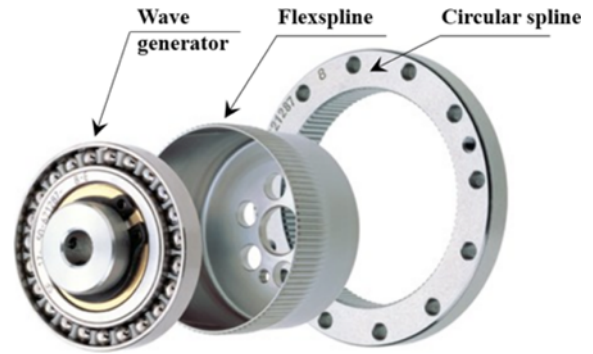


Fig. 8 Harmonic drive<sup>47</sup> (Adapted from Ref. 47 on the basis of open access)

## 3. Performance of HPRs

High precision reducers are characterized with various criteria such as torsional and moment stiffness, silent operation (or low vibration), small backlash and lost motion, and a wide range of reduction ratios.<sup>44</sup> Undoubtedly, the transmission error and the lost motion of HPRs usually do not exceed 1 arc.min. Although some reducers of small size or low reduction ratio may have larger than 1 arc.min.. In particular, harmonic drive has zero backlash. In addition, HPRs has superior torque characteristics such as high rated output torque (up to 4,500 Nm), high allowable torque during acceleration/deceleration (250% of rated torque), considerable torsional rigidity ( $\leq 1,200$  Nm/arc.min.) and great moment rigidity ( $\leq 7,500$  Nm/arc.min.). Furthermore, HPRs also keep demand of high efficiency (75-90%).

There are three keys performance indices for evaluating the performance of HPRs such as: hysteresis curve, rotational transmission errors (RTE), efficiency.<sup>42,46,47</sup> Especially, many important characteristics such as backlash, lost motion and torsional rigidity of reducers are determined from the hysteresis curve.

Depend on structure of HPRs, each HPRs has a distinctive method to analyze its performance. All details about key performances of HPRs will be discussed in following sub sections.

### 3.1 Rotational Transmission Error

The rotational transmission error (RTE)  $\theta_{RTE}$  is defined as the difference between a theoretical output angle of rotation  $\theta_{out\_theory}$  and an actual output angle of rotation  $\theta_{out}$  as shown in Eq. (1). Moreover, the theoretical output angle can be determined by dividing input angle  $\theta_m$  and reduction ratio R.

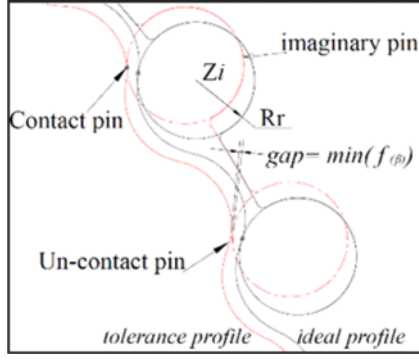


Fig. 9 Difference between ideal and tolerance profiles of cycloid disk<sup>61</sup> (Adapted from Ref. 61 with permission)

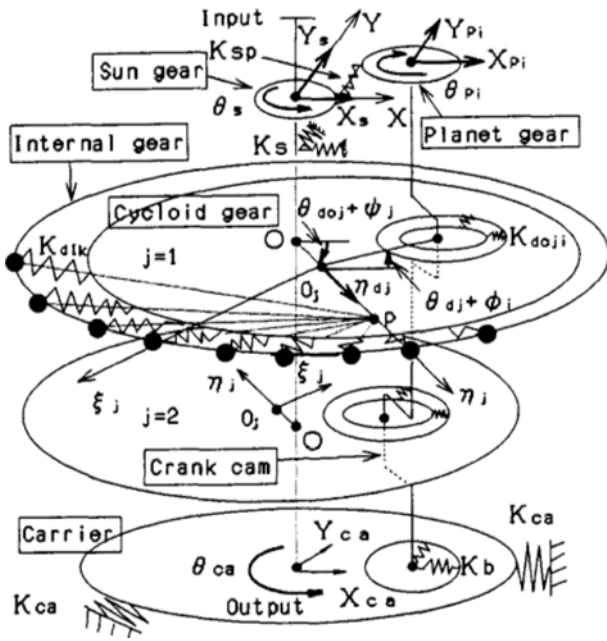
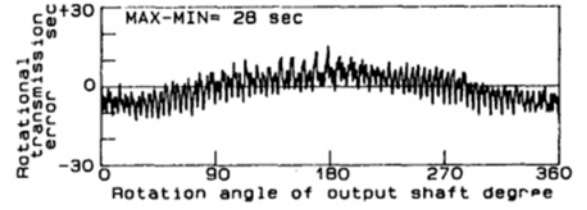


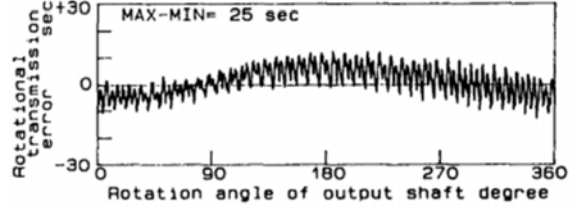
Fig. 10 RTE analysis of RV reducer<sup>63</sup> (Adapted from Ref. 63 on the basis of open access)

$$\theta_{RTE} = \theta_{out\_theory} - \theta_{out} = \frac{\theta_{in}}{R} - \theta_{out} \quad (1)$$

In few papers, RTE of cycloid reducer and planetary reducers is illustrated based on geometrical errors and machining tolerances. Blanche and Yang<sup>59,60</sup> introduced a kinematic analysis of backlash and torque ripple of a one stage cycloid reducer using difference between ideal and non-ideal reducer profiles (with machining tolerances), as shown in Fig. 9.<sup>61</sup> The tolerance of the cycloid reducer makes RTE fluctuate in a sinusoidal form and Zen et al. confirmed this result considering dynamic performance and tooth modification of a cycloid reducer.<sup>62</sup> The most completed and complex analysis RTE of a cycloid reducer was presented by Hishida et al.,<sup>63-65</sup> as shown in Fig. 10.<sup>63</sup> Effects of various errors such as position error between holes and crankshafts, geometric error of cam shaft, pitch error of cycloid disk, etc. on RTE were illustrated in an iterative way using 20-DOF static equations of force, stiffness, and displacement of a K-H-V type cycloid reducer (or RV drive). Particularly, RTEs of theoretical analyses and experiments have good agreements, as shown in Fig. 11.<sup>64</sup>

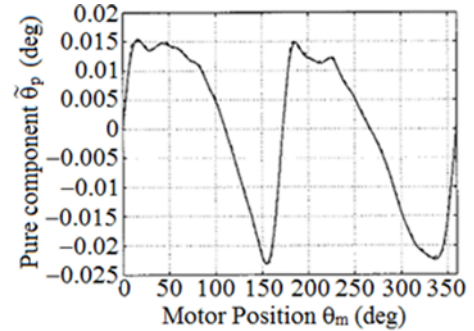


(a) Experimental results

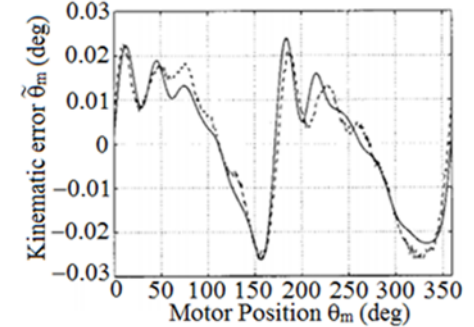


(b) Theoretical results from Hishida's method

Fig. 11 A comparison of experimental an theoretical RTE<sup>64</sup> (Adapted from Ref. 64 on the basis of open access)



(a) Kinematic error



(b) Transmission error considering dynamic component

Fig. 12 RTE of HD considering dynamic component<sup>67</sup> (solid line for theoretical and dash-line for experimental results) (Adapted from Ref. 67 on the basis of open access)

Although there is no theoretical method to calculate RTE for a HD, experimentally-measured RTEs of HD were approximated with a sinusoid function<sup>66</sup> and Fourier transform,<sup>67</sup> respectively. In addition, some researchers developed mathematical methods to investigate effect of flexibility and friction of components on RTE, as shown in Fig. 12.<sup>67</sup> Most studies used a “black-box” model for RTEs of HPRs since sophisticated modeling procedure can be skipped.<sup>68,69</sup>

Some researchers proposed an experimental model for RTE analysis by comparing signals of input and output.<sup>70,71</sup> Sensors and encoders are

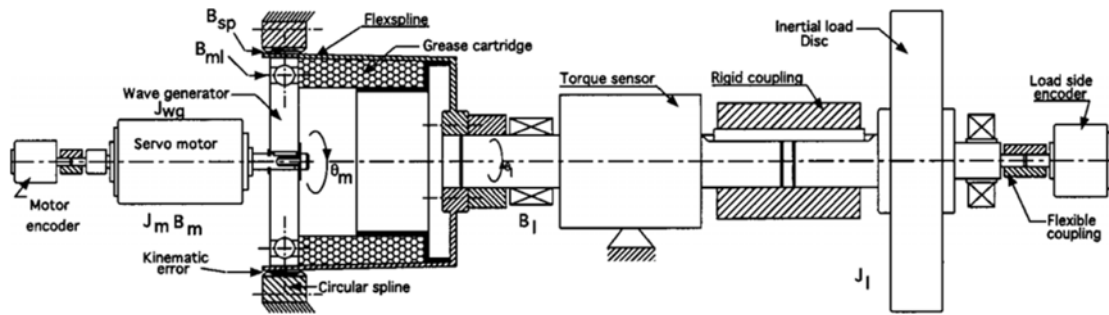


Fig. 13 Test-rig for RTE analysis of harmonic drive<sup>67</sup> (Adapted from Ref. 67 on the basis of open access)

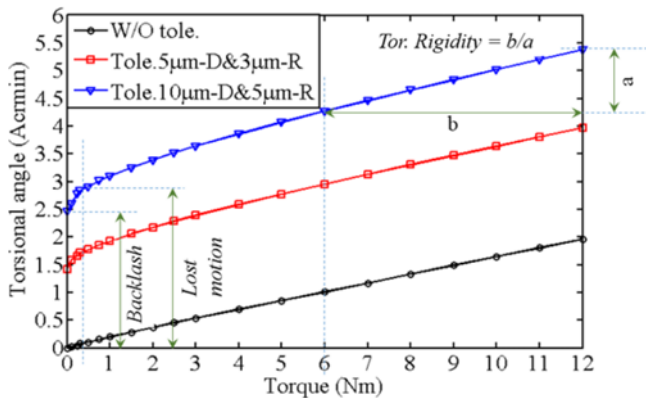


Fig. 14 Hysteresis curve analysis of cycloid reducer by using nonlinear springs with dead zone<sup>80</sup> (Adapted from Ref. 80 with permission)

placed both in front and back positions of reducer, and results of RTE is compared with theoretical calculation. A typical example of experimental set-up is shown as in Fig. 13.<sup>67</sup> Moreover, RTE can be used to detect fault of HPRs. In case of planetary reducer, a novel method was developed to diagnose a fault of a sun-planet-ring planetary system using RTE.<sup>72</sup>

Although RTE is originated from both kinematic errors (manufacturing errors and tolerances) and dynamic errors due to various torque. So far, RTE was investigated as the kinematic error neglecting torque variation. Therefore, further study on RTE analysis and compensation considering load variation is necessary.

### 3.2 Hysteresis Curve

Information of many principal features such as backlash, lost motion, and torsional rigidity can be verified on hysteresis curve.<sup>43,61</sup> If torque is applied to the output while the input is fixed, the output of reducer moves in a small angle following to the applied torque. The hysteresis curve represents output angle variation due to applied torque under the fixed input and there is small difference between loading and unloading states. From the hysteresis curve as shown in Fig. 14, the backlash of HPRs is defined as a moving angle with zero output torque. In addition, lost motion is known by an angle of output within  $\pm 3\%$  of rated torque in the middle line of hysteresis curve. Finally, torsional rigidity indicates a proportional slope of output angle according to the applied torque with a range from 50% to 100% rated torque.

FE methods were normally used to analyze hysteresis curve of

HPRs. In case of planetary reducer, meshing stiffness analysis between two teeth is illustrated by a contact spring.<sup>73</sup> Since cycloid reducer has complex structure and multi-contact,<sup>74,75</sup> torsional rigidity was not investigated up to recently. Researchers usually introduced springs at contact points between gear teeth.<sup>76,77</sup> Direct FE modeling of multi contact point did not result in reliable results so that an iterative combination method of kinematic and FE analysis is used.<sup>78</sup> Not only contact stiffness but also bearing and structure stiffness have great effect on torsional rigidity.<sup>47,79</sup> However, this iterative procedure is a time-consuming and complicate process.

An efficient FE analysis procedure for hysteresis curve of cycloid reducer using a Hertzian nonlinear spring was presented.<sup>80</sup> The hysteresis characteristic (backlash, lost motion and torsional rigidity) of a cycloid reducer was directly evaluated at one time based on the FE model so that its analysis effort and time are significantly saved. These nonlinear springs with dead zone are also used for other connecting elements of the cycloid reducer such as the input bearing, pin-roller and output roller. Results showed that tolerance had a great effect on torque and the torsional angle relationship of the cycloid reducer. In the future, it is necessary to validate the analysis results experimentally.

Hysteresis characteristic of a HD can be measured experimentally and modelled empirically using a polynomial of torsional angle.<sup>81,82</sup> In addition, FE model with contact element was introduced to analyze hysteresis curve of HD.<sup>83,84</sup> The FS of HD is firstly modeled as a shell body with fine mesh, and balls of bearing on WG is approximated with uniaxial elements of constant stiffness. Since it is very difficult to find out exact contact points on initially-deformed FS due to WG<sup>85-87</sup> contact elements are introduced between FS and CS<sup>83,84</sup> Therefore, two-step procedure is necessary for FE analysis of HD under torque load: First, WG is inserted into FS and FS is deformed uniformly. Second, the deformed FS is connected to CS using contact elements and torque is applied to CS. Not only tooth profile of HD but also parameters of elliptical WG significantly affects to torsional stiffness of HD.<sup>84</sup>

### 3.3 Efficiency

Global energy issues force efficiency to be one of the most important criteria for choosing a HPR. Efficiency is defined by a relationship between input and output power and depends on an amount of energy loss inside of HPRs. Power loss in HPRs consists of load independent and dependent losses. While load independent loss includes losses of oil churning, friction in seals, and bearing loss (a function of speed), load-dependent loss is due to some friction-dominated components such as bearing and tooth contacts.



Several methods of efficiency analysis were proposed considering lubrication type, design parameters for various types of HPRs: planetary reducer, cycloid reducer and harmonic drive. Applied torque, speed, a temperature of lubrication and reduction ratio have the same effect on the efficiency of all types of HPRs (planetary,<sup>88,89</sup> cycloid drive,<sup>90</sup> harmonic drive<sup>91</sup>). Power loss of HPRs decreases as applied torque and operating temperature increase, and speed decreases. Moreover, power losses by tooth engagement and bearing have large portion in total power loss of HPRs.<sup>91,92</sup> An optimal design parameters,<sup>93</sup> lubricated method<sup>94</sup> and high-efficiency bearings<sup>95</sup> can improve efficiency of HPRs.

Occasionally, static and dynamic efficiencies are defined in different ways. Static efficiency of a cycloid reducer was evaluated from hysteresis curve and dynamic efficiency provided by manufacturer is higher than static efficiency evaluated from hysteresis curve.<sup>96</sup> In addition, tolerance has a significant effect on not only hysteresis characteristic but also dynamic efficiency of a cycloid reducer.<sup>97</sup>

There are some differences in loss characteristics among planetary reducer, cycloid reducer, and harmonic drive. Power loss by tooth engagement is higher than that by bearing in cases of cycloid reducer<sup>90,98</sup> and planetary reducer.<sup>88</sup> On the other hand, power loss by bearing is significant in case of HD because contact force of bearing for WG is higher than that of tooth engagement.<sup>91</sup> Furthermore, cycloid reducer has higher efficiency than planetary reducer<sup>99</sup> and harmonic drive,<sup>100</sup> especially, for high reduction ratio. However, it is necessary to investigate efficiency analysis of HD since few papers on efficiency of HD compared with planetary and cycloid reducers.

#### 4. Error Compensation of HPRs Based on Control Theory

Many researchers tried to improve accuracy of an actuator considering static and dynamic errors of HPRs. Static errors (or RTE) were studied with planetary reducer,<sup>72</sup> cycloid drive<sup>59-62</sup> and HD.<sup>67-82</sup> In addition, dynamic errors come from non-linear stiffness and friction loss of a hysteresis curve of a reducer.<sup>81,82,101</sup> Particularly, the hysteresis of a HD was modeled mathematically considering transmission compliance based on a damped structure of FS.<sup>68,69</sup> On the other hand, Hertz-contact stiffness was used to determine dynamic errors in control systems with planetary<sup>71,73</sup> and cycloid reducer.<sup>80</sup>

Both feedback and feedforward control improved accuracy of an actuator with HPRs.<sup>102-104</sup> Even though a feedback control based on nonlinear control theory was applied to compensate kinematic and hysteresis errors of HD, static errors have not improved much.<sup>102,103</sup> In addition, a model-based feedforward compensator was proposed to reduce synchronous and nonlinear elastic components of RTE, as shown in Fig. 15.<sup>104</sup> Significant improvement of settling accuracy (40%) in the position system was reported with the model-based feedforward compensation comparing to control system without compensation.

Recently, a compensation for nonlinear friction of HD in a gimbal servo-system was presented.<sup>105</sup> Both a torque observer and a coulomb-viscous friction model are added to the feedforward

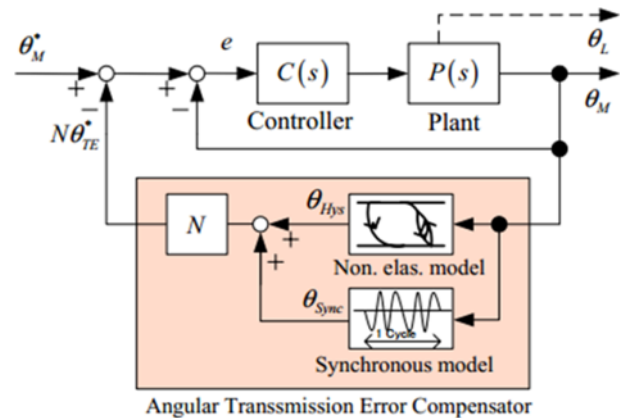


Fig. 15 Control system diagram of angular transmission error compensator<sup>104</sup> (Adapted from Ref. 104 with permission)

compensation for supervising nonlinear friction. As the result, a peak-peak value of tracking error of the gimbal servo-system was reduced about 11%.

#### 5. Other Application Using HPRs

Besides a significant expansion of robotic fields, HPRs are widely applied to other fields such as aerospace, automotive, medical device, machine tool and so on. HD is one of main components for aerospace lander applications on Mars pathfinder or positioning control mechanisms of a solar array.<sup>106</sup> Planetary reducer was used as a critical transmission part from an engine to main rotor blades of helicopter or wind turbine.<sup>107-109</sup> In addition, cycloid reducer is applied to an electrically-controlled variable valve timing (ECCVVT) system to improve the fuel efficiency of cars.<sup>110</sup> Furthermore, HPRs play an important role in other industrial fields such as automotive, machine tool and medical instruments.<sup>111-114</sup> They can appear from a simple actuator of welding positioner to a complex one as precision surgical arm for ear. Some examples of various applications are shown in Fig. 16.

#### 6. Future Perspectives

The growth of industrial robot and their applications (estimated 2.6 million units in 2019) may result in a huge demand of HPRs for their actuators. However, the ability of manufacturer for providing products to market nowadays is limited. Although planetary reducer can be easily manufactured in worldwide, RV drive and HD are monopolized by Nabtesco and Harmonic drive system, respectively. Thus, many researchers and factories from China and Korea are going to take parts in developing these types of HPRs.

Despite a significant improvement of HPRs nowadays, there are still many aspects to be considered in points of research, design and development. Becoming compact and having light weight are two leading trends for HPRs although HPRs have highest ever performance characteristics with extremely compact size by optimizing design

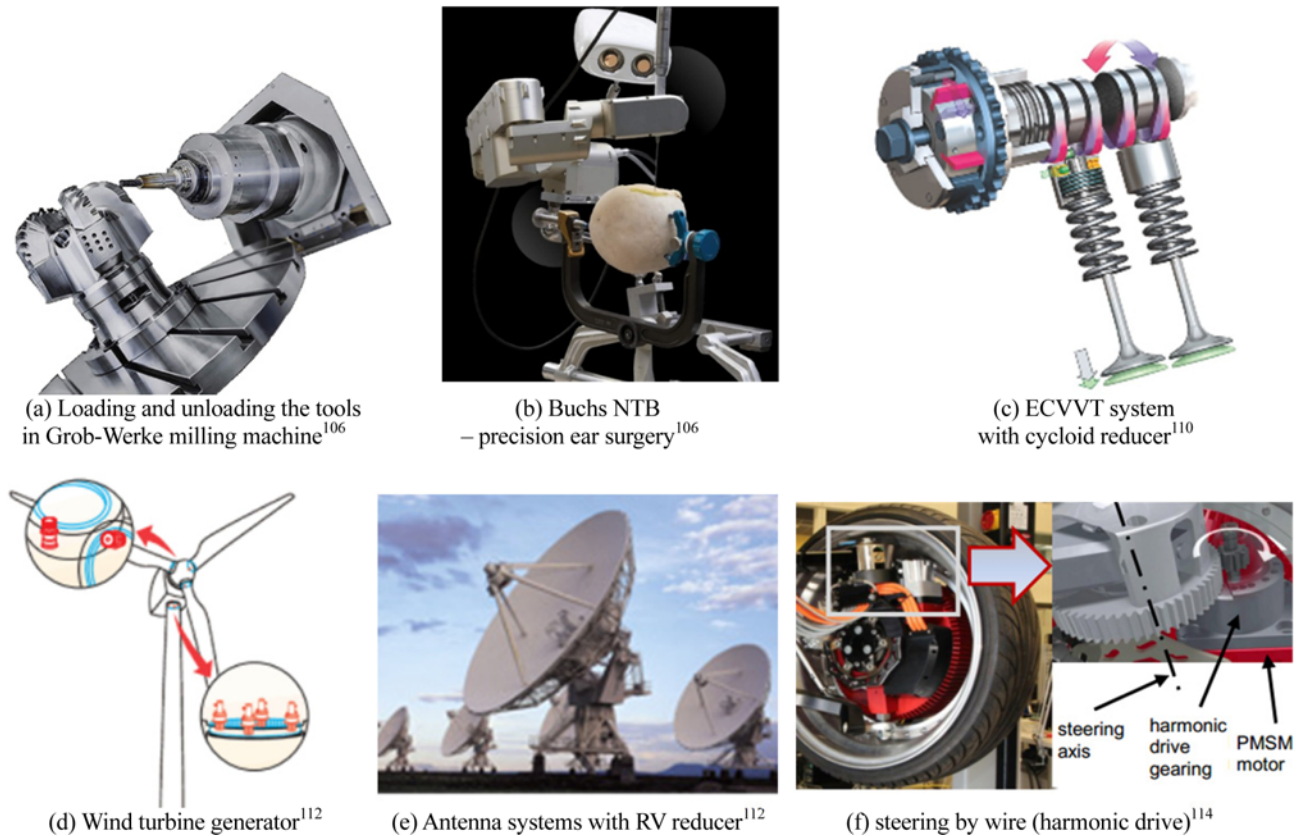


Fig. 16 Applications of HPRs (Adapted from Refs. 106, 110, 112, and 114 on the basis of open access)

parameters and adopting light materials and new types of bearing. In addition, a commercial product of non-contact HPR with magnetic principle may be introduced in near future.

## 7. Summary

There is no doubt about important roles of HPRs in robotics for the 4th industrial revolution. Due to a pressing demand of HPRs from not only industrial robots but also automation applications, global commercial market for HPRs has very high potential of growth. Moreover, market of less precise reducer will also expand for both low-cost collaborative robots (Cobots) and automotive. Due to these reasons, more and more research on HPRs is necessary.

Performance of HPRs can be determined based on three criteria: hysteresis, RTE and efficiency. Hysteresis curve defines backlash, lost motion and torsional rigidity of reducers. Efficiency relates to dynamic performance control and torque density parameters. In addition, RTE is used to determine accuracy, positioning errors and repeatability. Lastly, efficiency becomes one of the most important criteria due to sustainable manufacturing.

Control theory was used to improve accuracy of the HPRs. Particularly, model-based feedforward compensator improved the accuracy of positioning system with HPRs.

Besides robotic applications, HPRs are also widely used in many other fields such as aerospace, automotive, medical device, machine tool and so on.

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