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Gear geometry of cycloid drives

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According to differential geometry and gear geometry, the equation of meshing for small teeth difference planetary gearing and a universal equation of conjugated profile are established based on cylindrical pin tooth and given motion. The correct meshing condition, contact line, contact ratio, calculating method for pin tooth's maximum contact point are developed. Investigation on the theory of conjugated meshing is carried out when the tooth difference numbers between pin wheel and cycloidal gear are 1, 2, 3 and −1, respectively. A general method called enveloping method to generate hypocycloid and epicycloid is put forward. The correct meshing condition for cycloid pin wheel gearing is provided, and the contact line and the contact ratio are also discussed.

pin wheel, cycloid, conjugated profile, equation of meshing, enveloping method, contact ratio, contact line

Cycloid drives are widely used in many industries, such as machinery, mine, metallurgy, chemical, textile, national defense, etc., due to their large gear ratio, compact size, high load capacity and high efficiency. More attentions are paid to this drive in precision transmission because half of its teeth are meshing simultaneously, because the outstanding error average effect leads to high precision, and because it has high torsion stiffness for there is no flexible element. Additionally, cycloidal gear pumps based on the principle of cycloidal pin drives are attached importance in many countries due to their smooth transmission, low pulsation and low noise.

Generally, the gear geometry of cycloidal drives is described as: curtate cycloid by outer and inner rolling method; curtate epicycloid and pin teeth satisfying the Willis law; terms of continuous transmission[1,2]. Compared with involute gears, the meshing principle of cycloid drives has the following defects. 1) Lack of close math deriving. There are no descriptions for meshing equation, meshing line, etc., which greatly interrelate with the transmission traits. 2) Meshing theory being not systematic. For example, planetary transmissions for one tooth difference and two teeth difference are discussed separately[3]; the essence of planetary gear conjugated profile is not revealed when the inner gear is given as pin wheel. 3) Existing contradictory conclusions. For example, the term of continuous transmission is stated as that pin wheel should have one more tooth than cyc-

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loidal gear, but actually two teeth and three teeth difference transmissions can mesh correctly. 4) Vague concepts. No clear definition and calculating method are established for correct meshing condition, contact-ratio, etc.

Many researchers have devoted their efforts to the field of cycloidal gear geometry in recent years. For example, Li[4] established the universal equation of cycloidal gear, which synthetically considers shape correcting by moving cutter, changing cutter's radius, and rotating a tiny angle of the workbench. Litvin et al.[5−10] developed the equation of meshing and the formation of envelope by multi-branches of cycloidal gear pumps, Root's Blowers and the like, based on the fundamental gearing kinematics and enveloping theory. Shin[11,12] used the principle of the instant velocity center in the general contact mechanism and the homogeneous coordinate transmission to the lobe profile design of a cycloidal gear. Lai[13,14] adopted the enveloping theory for one parameter of curves to derive the equation of meshing. However, these researches are limited to establishment of one tooth difference cycloid pin wheel gearing's meshing equation and its computerized methods. Profound analyses about general gear geometry and meshing characteristics have not been carried out. In this paper, the universal equation of planetary gear's profile based on cylindrical pin tooth and given motion will be established, and the cycloid pin wheel gearing's meshing characteristics will be analyzed in detail according to gear geometry.

1 Conjugated profile of pin tooth

1.1 Coordinate systems

The coordinate systems are shown in Figure 1, where 1 is the pin wheel and 2 is the planetary gear. The moving coordinate systems $o_b x_1 y_1$ and $o_g x_2 y_2$ are rigidly connected to the centers of pin wheel and the planetary gear, respectively. The fixed coordinate system OXY is connected to the center of pin wheel. The initial positions of the axes X and x_1 are coincident,

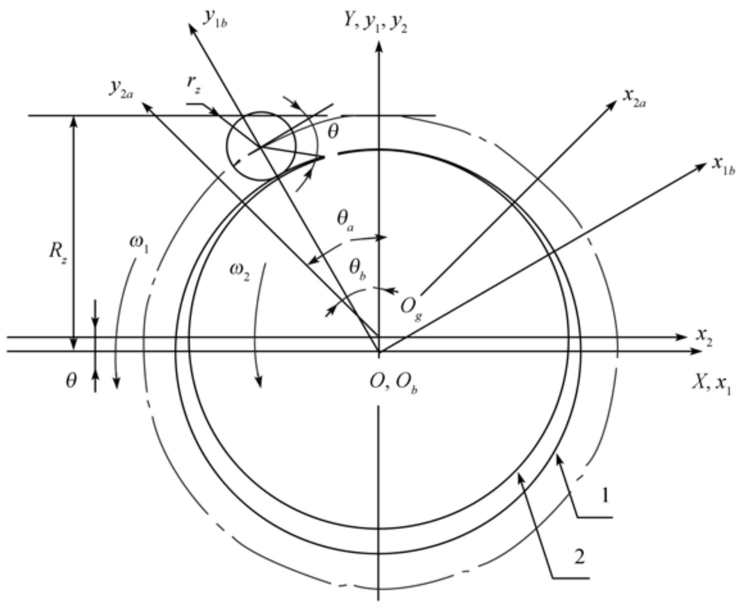

Figure 1 Coordinate systems.

x_2 is parallel with X. The radius of pin teeth distributed circle is R_Z, and the radius of pin tooth is r_Z. The tooth numbers of pin wheel and planetary gear are Z_b and Z_g, and the gear center distance (eccentricity of input arm bearing) is e. The method in case of the arm (crank) O_bO_g being fixed is adopted to simplify the discussion. When the planetary gear rotates by angle θ_a with counterclockwise about the z_2 axis, the pin wheel will rotate by angle θ_b about the z_1 axis with same direction according to the motion relation.

1.2 Equation of meshing

The pin tooth in coordinate system $o_bx_1y_1$ is given as follows:

$$\boldsymbol{\Sigma}^{(1)} = x_1 i_1 + y_1 \boldsymbol{j}_1 = r_Z \cos\theta \boldsymbol{i}_1 + \left(r_Z \sin\theta + R_Z\right)\boldsymbol{j}_1, \tag{1}$$

where θ is angle parameter of pin tooth.

According to the kinematics of gear geometry, the equation of meshing is given as

$$\phi\left(\theta,\theta_b\right) = \boldsymbol{n}_1 \cdot \boldsymbol{v}_1^{(12)} = 0, \tag{2}$$

where $\boldsymbol{n}_1$ represents the normal of pin tooth profile, its projections on coordinate axes x_1 and y_1 are

$$n_{x1} = dy_1 / d\theta = r_Z \cos\theta, \quad n_{y1} = -dx_1 / d\theta = r_Z \sin\theta;$$

$\boldsymbol{v}_1^{(12)}$ represents the relative velocity at the conjugate points between the pin wheel and the planetary gear.

$$\boldsymbol{v}_1^{(12)} = \boldsymbol{v}_1^{(1)} - \boldsymbol{v}_1^{(2)} = \left(\boldsymbol{\omega}^{(1)} - \boldsymbol{\omega}_1^{(2)}\right) \times \boldsymbol{\Sigma}^{(1)} + \boldsymbol{\omega}^{(2)} \times \boldsymbol{e},$$

where $\boldsymbol{v}_1^{(1)} = \boldsymbol{\omega}^{(1)} \times \boldsymbol{\Sigma}^{(1)}$, $\boldsymbol{v}_1^{(2)} = \boldsymbol{\omega}_1^{(2)} \times \boldsymbol{\Sigma}^{(1)} + \boldsymbol{e} \times \boldsymbol{\omega}^{(2)}$, $\boldsymbol{\omega}^{(1)} = \omega_1 \boldsymbol{k}_1$, $\boldsymbol{\omega}_1^{(2)} = \boldsymbol{\omega}^{(2)} = \omega_2 \boldsymbol{k}_1$, $\boldsymbol{i}_1$, $\boldsymbol{j}_1$ and k_1 are the unit vectors of axes x_1, y_1 and z_1, respectively.

Substituting the corresponding expressions into eq. (2), the equation of meshing is obtained as

$$\phi\left(\theta,\theta_b\right) = \lambda \cos\left(\theta + \theta_b\right) - \cos\theta = 0, \tag{3}$$

where λ is a coefficient, and

$$\lambda = e i_{gb}^H \Big/ \left[R_Z \left(i_{gb}^H - 1\right)\right]. \tag{4}$$

1.3 Profile equation of planetary gear $\boldsymbol{\Sigma}^{(2)}$

In coordinate system $o_g x_{2a} y_{2a}$, the profile $\boldsymbol{\Sigma}^{(2)}$ of planetary gear conjugated to pin tooth $\boldsymbol{\Sigma}^{(1)}$ is determined by the set of equations:

$$\begin{cases} \boldsymbol{\Sigma}^{(2)} = M_{21} \boldsymbol{\Sigma}^{(1)}, \\ \phi(\theta,\theta_b) = 0, \end{cases} \tag{5}$$

where $M_{21} = M_{20} M_{01}$, which is the transformation matrix from $o_b x_{1b} y_{1b}$ to $o_g x_{2a} y_{2a}$.

The transformation matrix from $o_b x_{1b} y_{1b}$ to OXY can be expressed as

$$M_{01}=\begin{bmatrix}\cos\theta_b & -\sin\theta_b & 0\\ \sin\theta_b & \cos\theta_b & 0\\ 0 & 0 & 1\end{bmatrix}. \tag{6}$$

The transformation matrix from *OXY* to $o_g x_{2a} y_{2a}$ can be expressed as

$$M_{20}=\begin{bmatrix}\cos\theta_a & \sin\theta_a & -e\sin\theta_a\\ -\sin\theta_a & \cos\theta_a & -e\cos\theta_a\\ 0 & 0 & 1\end{bmatrix}. \tag{7}$$

Here, in case of $\theta_a-\theta_b=\varphi_b'$, according to $i_{gb}^H=\theta_a/\theta_b=Z_b/Z_g$, we have $\theta_a=Z_b\varphi_b'/(Z_b-Z_g)$, $\theta_b=Z_g\varphi_b'/\left(Z_b-Z_g\right)$, so the transformation matrix M_{21} can be written as

$$M_{21}=\begin{bmatrix}\cos\varphi_b' & \sin\varphi_b' & -e\sin\left[Z_b\varphi_b'/\left(Z_b-Z_g\right)\right]\\ -\sin\varphi_b' & \cos\varphi_b' & -e\cos\left[Z_b\varphi_b'/\left(Z_b-Z_g\right)\right]\\ 0 & 0 & 1\end{bmatrix}. \tag{8}$$

According to the formula of trigonometric function, the solution to eq. (3) is

$$\sin\theta=\pm\left(\lambda\cos\theta_b-1\right)\Big/\sqrt{1+\lambda^2-2\lambda\cos\theta_b},\cos\theta=\pm\lambda\sin\theta_b\Big/\sqrt{1+\lambda^2-2\lambda\cos\theta_b}. \tag{9}$$

Substituting eqs. (1), (8) and (9) into eq. (5) yields the general profile equation of the planetary gear $\boldsymbol{\Sigma}^{(2)}$:

$$\begin{cases}x_2=R_Z\sin\varphi_b'-e\sin\left[Z_b\varphi_b'/\left(Z_b-Z_g\right)\right]+r_Z\cos\beta,\\ y_2=R_Z\cos\varphi_b'-e\cos\left[Z_b\varphi_b'/\left(Z_b-Z_g\right)\right]-r_Z\sin\beta,\end{cases} \tag{10}$$

where

$$\begin{aligned}\cos\beta&=\pm\left\{\lambda\sin\left[Z_b\varphi_b'/\left(Z_b-Z_g\right)\right]-\sin\varphi_b'\right\}\Big/\sqrt{1+\lambda^2-2\lambda\cos\left[Z_g\varphi_b'/\left(Z_b-Z_g\right)\right]},\\ \sin\beta&=\pm\left\{-\lambda\cos\left[Z_b\varphi_b'/\left(Z_b-Z_g\right)\right]+\cos\varphi_b'\right\}\Big/\sqrt{1+\lambda^2-2\lambda\cos\left[Z_g\varphi_b'/\left(Z_b-Z_g\right)\right]}.\end{aligned} \tag{11}$$

1.4 Enveloping method for curtate cycloid

Because eq. (10) is similar to that of equidistant curve of curtate epicycloids in form, the conception of equivalent gear is introduced here. If the equivalent cycloidal gear tooth number is $Z_d=Z_g\big/\left(Z_b-Z_g\right)$, then the tooth number of equivalent pin wheel conjugated to cycloidal gear is $Z_e=i_{gb}^HZ_d=i_{gb}^HZ_g\big/\left(Z_b-Z_g\right)=Z_b\big/\left(Z_b-Z_g\right)$. Defining the curtate coefficient of equivalent cycloidal gear as $K_1=\lambda$, from eq. (4) we have

$$\lambda=ei_{gb}^H\big/R_z(i_{gb}^H-1)=eZ_b\big/R_z(Z_b-Z_g)=eZ_e\big/R_z=r_b'\big/R_z=e'Z_e\big/R_z=K_1, \tag{12}$$

where e' is the eccentricity of equivalent cycloidal gear, and r_b' is the radius of pin wheel pitch circle.

Therefore, we have

$$\begin{cases} x_2 = R_Z \sin\varphi_b' - e\sin\left(Z_e\varphi_b'\right) + r_Z \cos\beta, \\ y_2 = R_Z \cos\varphi_b' - e\cos\left(Z_e\varphi_b'\right) - r_Z \sin\beta. \end{cases} \tag{13}$$

where

$$\begin{aligned} \cos\beta &= \pm\left[K_1 \sin\left(Z_e\varphi_b'\right) - \sin\varphi_b'\right] \Big/ \sqrt{1 + K_1^2 - 2K_1 \cos\left(Z_d\varphi_b'\right)}, \\ \sin\beta &= \pm\left[-K_1 \cos\left(Z_e\varphi_b'\right) + \cos\varphi_b'\right] \Big/ \sqrt{1 + K_1^2 - 2K_1 \cos\left(Z_d\varphi_b'\right)}. \end{aligned} \tag{14}$$

Eq. (13) is the same as that of the ordinary equidistant curve of curtate cycloid, so the previous method can produce the curve. If $r_Z = 0$, the theoretical cycloid is obtained. When the tooth number of pin wheel is greater than that of cycloidal gear, eq. (14) takes “positive sign”, curtate cycloid makes an inner equidistance, the lobe is equidistant curve of curtate epicycloids, and the ordinary cycloid pin wheel gearing is gained. When the tooth number is smaller than that of cycloidal gear, eq. (14) takes “negative sign”, curtate cycloid makes an outer equidistance, the lobe is equidistant curve of curtate hypocycloid, and the inner cycloid pin wheel gearing can be given.

The above method to obtain the equation of curtate cycloid conjugated with pin tooth is called “enveloping method”. It is a general method to generate curtate cycloid by giving corresponding motion to pin tooth, for either curtate epicycloid or curtate hypocycloid.

2 Characteristics of cycloid pin wheel gearing

2.1 Condition of correct meshing

From the process to develop the equation of pin tooth conjugated profile and eq. (12), we can draw a conclusion: for a given pin wheel and center distance between cycloidal gear and pin wheel, the necessary condition of correct meshing is $e = e'$, i.e., the center distance is equal to eccentricity of curtate cycloid. In fact, it is also the sufficient condition of correct meshing for cycloid pin wheel gearing.

It is known that the pitch of pin wheel is $p_{tb} = 2\pi r_b' / Z_e$. According to $K_1 = r_b' / R_Z$, we have $p_{tb} = 2\pi r_b' / Z_e = 2\pi K_1 R_z / Z_e$. The pitch of cycloidal gear is $p_{tg} = 2\pi e' = 2\pi e = 2\pi K_1 R_z / Z_e$. Therefore, $p_{tg} = p_{tb}$, the pitches of two gears are equivalent, cycloidal gear and pin wheel can mesh correctly and continuously.

When $Z_b - Z_g = 1$, we have $Z_b = 2\pi r_b' / p_{tb} = 2\pi(r_g + e) / 2\pi e = r_g / e + 1$. Apparently, r_g / e is an integer, the cycloidal profile is continuous and integrated. From eq. (12), we can easily get

$$e = K_1 R_z / Z_e = K_1 R_z (Z_b - Z_g) / Z_b. \tag{15}$$

According to the previous discussions, the equations to determine relations between fundamental geometrical parameters of small teeth difference cycloid pin wheel gearing are given in Table 1.

2.2 Determination of φ_b' in equation of cycloidal profile

The two sides of cycloidal gear’s profile should be symmetrical to guarantee uniform transmission on both directions. The angle between symmetry axis and the starting point of one cycloidal tooth is $\varphi_0 = \pi / Z_g$. The equation of symmetry axis is set to be $y = kx$, its slope is determined

Table 1 Relations between fundamental geometrical parameters

Names	Symbols	Equations
Curtate coefficient	K_1	$K_1 = \frac{r'_b}{R_Z} = \frac{eZ_b}{R_Z(Z_b - Z_g)}$
Radius of pin wheel's pitch circle	r'_b	$r'_b = K_1 R_Z = \frac{eZ_b}{Z_b - Z_g}$
Radius of cycloidal gear's pitch circle	r'_g	$r'_g = \frac{Z_g}{Z_b} r'_b = \frac{eZ_g}{Z_b - Z_g} = K_1 R_Z \cdot \frac{Z_g}{Z_b}$
Eccentric distance	e	$e = r_b - r_g = \frac{K_1 R_Z \left(Z_b - Z_g\right)}{Z_b}$
Rolling circle's radius of outer engaging method	r	$r = \frac{e}{K_1} = \frac{R_Z \left(Z_b - Z_g\right)}{Z_b}$

by $k = \mathrm{tg}(\pi/2 - \varphi_0) = \mathrm{ctg}\varphi_0$, and then we have $y = x\mathrm{ctg}(\pi/Z_g)$.

Substituting eq. (10) into the equation of symmetrical axis, we obtain

$$R_Z \cos\varphi'_b - e\cos\left[Z_b\varphi'_b \big/ \left(Z_b - Z_g\right)\right] - \left\{ \pm r_Z \frac{-K_1 \cos\left[Z_b\varphi'_b \big/ \left(Z_b - Z_g\right)\right] + \cos\varphi'_b}{\sqrt{1 + K_1^2 - 2K_1\cos\left[Z_g\varphi'_b \big/ \left(Z_b - Z_g\right)\right]}} \right\}$$

$$= \left\{ R_Z \sin\varphi_b' - e\sin\left[Z_b\varphi_b' \big/ \left(Z_b - Z_g\right)\right] \pm r_Z \frac{K_1 \sin\left[Z_b\varphi'_b \big/ \left(Z_b - Z_g\right)\right] - \sin\varphi'_b}{\sqrt{1 + K_1^2 - 2K_1\cos\left[Z_g\varphi'_b \big/ \left(Z_b - Z_g\right)\right]}} \right\} \cdot \mathrm{ctg}\left(\pi / Z_g\right). \tag{16}$$

The φ'_b corresponding to the intersection point of symmetry axis and cycloidal profile (i.e., addendum of cycloidal gear) can be obtained by numerical computing method, which is represented by $\varphi_{\max}$, and one side of a cycloidal tooth's profile can be obtained just by making $\varphi'_b \in \left[0, \varphi_{\max}\right]$ in the equation of cycloidal profile. Then according to its symmetry, the profile on both sides of a cycloidal lobe can be produced. It should be noted that when the tooth difference number between pin wheel and cycloidal gear is 1, the cycloidal profile is a continuous and integrated curtate cycloid, and when the difference number is 2 or others the cycloidal profile is just part of a curtate cycloid. Considering the addendum cannot be a single point, the practical $\varphi_{\max}$ should be determined by addendum circle.

2.3 Double contacting of cycloid pin wheel gearing

Making transformation of meshing equation $\phi\left(\theta, \theta_b\right) = K_1 \cos\left(\theta + \theta_b\right) - \cos\theta = 0,$ we obtain the function of pin tooth's contact angle as

$$\theta = \arctan\left[\left(K_1 \cos\theta_b - 1\right) \big/ K_1 \sin\theta_b\right], \theta_b \neq n\pi (n = 0, 1, 2, \cdots). \tag{17}$$

The first derivative of eq. (17) with respect to θ_b is

$$d\theta \big/ d\theta_b = \left(-K_1^2 + K_1 \cos\theta_b\right) \big/ \left(K_1^2 - 2K_1 \cos\theta_b + 1\right). \tag{18}$$

The second derivative of eq. (17) with respect to θ_b is

$$d^2\theta \big/ d\theta_b^2 = K_1 \sin\theta_b \left(K_1^2 - 1\right) \big/ \left(K_1^2 - 2K_1 \cos\theta_b + 1\right)^2. \tag{19}$$

When $K_1 = 0.75$, the graphs of eqs. (17)—(19) are shown as Figures 2—4, respectively.

Combining Figures 2, 3 and 4, it is revealed that 2π is the period of eq. (17), and this function is symmetrical to point π. When $\theta_b \in (0,\pi)$, the value of $f''(\theta_b)$ is negative, and the graph of original function is convex; when $\theta_b \in (\pi, 2\pi)$, the value of $f''(\theta_b)$ is positive, and the graph of original function is concave; the maximum value $|\theta_{\max}|$ can be reached when $f'(\theta_b)=0$ (points *A*, *B*). $\theta_{\max}$ is the pin tooth's maximum contact point, i.e., the maximum angle of pin tooth rotating about its own central axis. It is proved that only part of the pin tooth takes part in meshing. $\theta_{\max}$ can be given when $f'(\theta_b)=0$, so the $\theta_{\max}$ is determined by the following equations:

$$\begin{cases} \theta_{\max} = \arctan\left[\left(K_1 \cos\theta_b - 1\right) / K_1 \sin\theta_b\right], \\ \left(-K_1^{\,2} + K_1 \cos\theta_b\right) / \left(1 + K_1^{\,2} - 2K_1 \cos\theta_b\right) = 0. \end{cases}$$

Solving the equations above, $\theta_{\max}$ can be expressed as

$$\theta_{\max} = -\arctan\sqrt{\left(1-K_1^2\right)/K_1^2}. \tag{20}$$

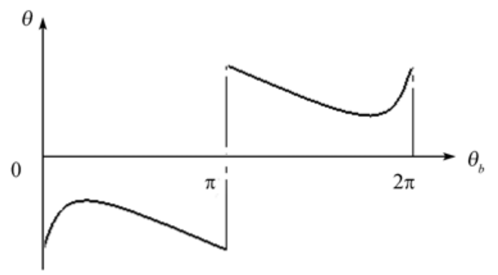

Figure 2 Pin tooth's contact angle *θ*.

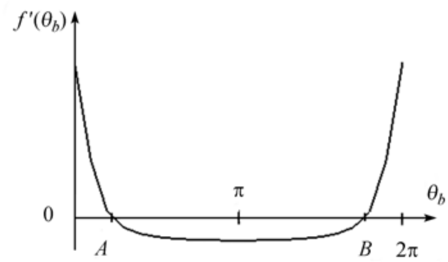

Figure 3 First derivative of pin tooth's contact angle *θ* with respect to *β*.

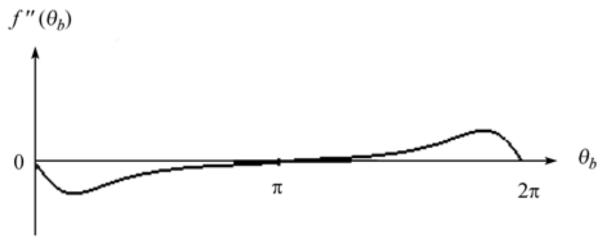

Figure 4 Second derivative of pin tooth's contact angle *θ* with respect to *β*.

According to Figure 2, when $\theta_b \in (0,\pi)$ or $\theta_b \in (\pi, 2\pi)$, there are always two different values of θ_b corresponding to any discretionary values of θ if only $\theta \neq \theta_{\max}$, i.e., one point on the pin tooth's profile contacts with two points (convex point and concave point) of the cycloidal profile during transmission. The pin tooth repeats the movement rotating from 0° to $\theta_{\max}$, then reverses to 0° all the time because just part of its arc takes part in meshing. At the point of zero, the pin tooth contacts with the cycloidal gear convex point or concave point, respectively. So double contacting of pin tooth during the transmission is testified[15].

2.4 Equation of contacting line

2.4.1 Equation of contacting line in coordinate system *OXY*. In coordinate system *OXY*, the equation of contacting line is determined by the following equations:

$$\begin{cases} \boldsymbol{\Sigma} = M_{01}\boldsymbol{\Sigma}^{(1)}, \\ \phi(\theta,\theta_b) = 0. \end{cases} \tag{21}$$

Substituting eqs. (1), (6) and (9) into eq. (21), we have

$$\begin{cases} x = -R_Z \sin\left[Z_g\varphi_b'\big/\left(Z_b - Z_g\right)\right] + r_Z \cos\delta, \\ y = R_Z \cos\left[Z_g\varphi_b\big/\left(Z_b - Z_g\right)\right] + r_Z \sin\delta, \end{cases} \tag{22}$$

where

$$\begin{aligned} \cos\delta &= \pm\sin\left[Z_g\varphi_b'\big/\left(Z_b - Z_g\right)\right]\Big/\sqrt{1 + K_1^2 - 2K_1\cos\left[Z_g\varphi_b'\big/\left(Z_b - Z_g\right)\right]}, \\ \sin\delta &= \pm\left\{K_1 - \cos\left[Z_g\varphi_b'\big/\left(Z_b - Z_g\right)\right]\right\}\Big/\sqrt{1 + K_1^2 - 2K_1\cos\left[Z_g\varphi_b'\big/\left(Z_b - Z_g\right)\right]}. \end{aligned} \tag{23}$$

If $r_Z = 0$, the equation of contacting line can be simplified to a circle.

2.4.2 Equation of contacting line in coordinate system $o_b x_1 y_1$. In coordinate system $o_b x_1 y_1$, the contacting line is the set of points satisfying the equation of meshing on pin tooth. So, considering the equations of pin tooth and meshing simultaneously, the equation of meshing line can be obtained as

$$\begin{cases} x_1 = r_Z \cos\theta, \\ y_1 = r_Z \sin\theta + R_Z, \end{cases} \tag{24}$$

where

$$\begin{aligned} \sin\theta &= \pm\left\{K_1\cos\left[Z_g\varphi_b'\big/\left(Z_b - Z_g\right)\right] - 1\right\}\Big/\sqrt{1 + K_1^2 - 2K_1\cos\left[Z_g\varphi_b'\big/\left(Z_b - Z_g\right)\right]}, \\ \cos\theta &= \pm K_1\sin\left[Z_g\varphi_b'\big/\left(Z_b - Z_g\right)\right]\Big/\sqrt{1 + K_1^2 - 2K_1\cos\left[Z_g\varphi_b'\big/\left(Z_b - Z_g\right)\right]}. \end{aligned} \tag{25}$$

The choices of the signs for eqs. (23) and (25) are the same with eq. (14). Apparently, the contacting line in coordinate system $o_b x_1 y_1$ is part of the pin tooth.

2.5 Contact ratio

According to the theory of gearing, the contact ratio of cycloid pin wheel gearing can be defined as: the number of teeth simultaneously taking part in meshing when one side of cycloidal lobe contacts from addendum to dedendum. The corresponding angle of contacting line is used to calculate the contact ratio ε because the contacting line is a curve which makes it difficult to calculate ε directly.

$$\varepsilon = \theta_b / A, \tag{26}$$

where $\theta_b = Z_g\varphi_{\max}\big/\left(Z_b - Z_g\right)$, the pin wheel rotating angle corresponding to contacting line; $\varphi_{\max}$ is determined by eq. (16).

$A = 2\pi/Z_b$, the angle between two adjacent pin teeth.

So, the equation of contact ratio can be written as

$$\varepsilon = \theta_b / A = Z_b Z_g \varphi_{\max} / 2\pi \left(Z_b - Z_g \right). \tag{27}$$

When the tooth difference number between pin wheel and cycloidal gear is 1, the contact ratio is $\varepsilon = \theta_b / A = Z_b / 2$ because of $\varphi_{\max} = \pi / Z_g$. When that number is 2, $\varphi_{\max}$ can be obtained from eq. (16), then the contact ratio is given by substituting $\varphi_{\max}$ into $\varepsilon = Z_b Z_g \varphi_{\max} / 4\pi$. The method to calculate that of 3 teeth difference gearing is alike, the equation of contact ratio is $\varepsilon = Z_b Z_g \varphi_{\max} / 6\pi$.

3 Gear geometry for typical small teeth difference cycloid drives

3.1 One tooth difference cycloid drives

3.1.1 Equation of cycloidal profile. In case of $Z_b - Z_g = 1$ in eqs. (10) and (11), we have the equation of cycloidal profile:

$$\begin{cases} x_2 = R_Z \sin \varphi_b' - e \sin Z_b \varphi_b' + r_Z \cos \beta, \\ y_2 = R_Z \cos \varphi_b' - e \cos Z_b \varphi_b' - r_Z \sin \beta, \end{cases} \quad \varphi_b' \in \left[0, \varphi_{\max} \right], \tag{28}$$

where

$$\cos \beta = \frac{K_1 \sin Z_b \varphi_b' - \sin \varphi_b'}{\sqrt{1 + K_1^2 - 2K_1 \cos Z_g \varphi_b'}}, \quad \sin \beta = \frac{-K_1 \cos Z_b \varphi_b' + \cos \varphi_b'}{\sqrt{1 + K_1^2 - 2K_1 \cos Z_g \varphi_b'}}. \tag{29}$$

In case of $Z_g = 11$, $R_Z = 90$, $r_Z = 7$, $e = 4$, $\varphi_{\max} = 16.36°$ can be obtained by eq. (16). Substituting $\varphi_b' \in \left[0, 16.36° \right]$ into eqs. (28) and (29), and according to periodicity of cycloidal profile, the profile of cycloidal gear and the meshing scheme of one tooth difference cycloid pin wheel gearing are shown in Figures 5 and 6, respectively.

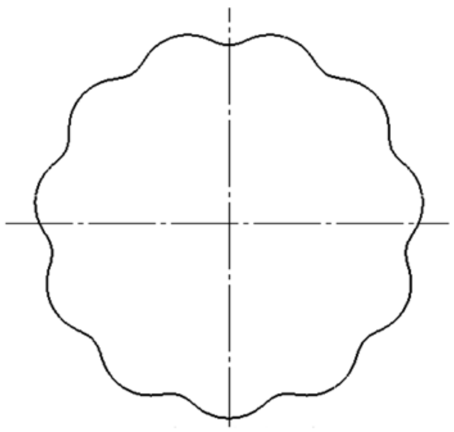

Figure 5 One tooth difference cycloidal profile.

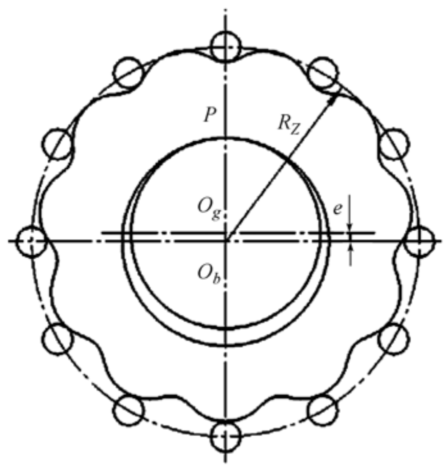

Figure 6 Meshing scheme of one tooth difference cycloid gearing.

3.1.2 Equation of contacting line. In case of $Z_b - Z_g = 1$ in eqs. (22) and (23), the equation of contacting line in coordinate system OXY can be reached, i.e.,

$$\begin{cases} x = -R_Z \sin Z_g \varphi_b' + r_Z \cos\delta, \\ y = R_Z \cos Z_g \varphi_b' + r_Z \sin\delta, \end{cases} \quad \varphi_b' \in [0, \varphi_{\max}], \tag{30}$$

where

$$\cos\delta = \frac{\sin Z_g \varphi_b'}{\sqrt{1+K_1^2 - 2K_1 \cos Z_g \varphi_b'}}, \quad \sin\delta = \frac{K_1 - \cos Z_g \varphi_b'}{\sqrt{1+K_1^2 - 2K_1 \cos Z_g \varphi_b'}}. \tag{31}$$

In case of $Z_b - Z_g = 1$ in eqs. (24) and (25), the equation of contacting line in coordinate system $o_b x_1 y_1$ can be reached, i.e.,

$$\begin{cases} x_1 = r_Z \cos\theta, \\ y_1 = r_Z \sin\theta + R_Z, \end{cases} \tag{32}$$

where

$$\sin\theta = \frac{K_1 \cos Z_g \varphi_b' - 1}{\sqrt{1+K_1^2 - 2K_1 \cos Z_g \varphi_b'}}, \quad \cos\theta = \frac{K_1 \sin Z_g \varphi_b'}{\sqrt{1+K_1^2 - 2K_1 \cos Z_g \varphi_b'}} \quad \varphi_b' \in [0, \varphi_{\max}]. \tag{33}$$

Choosing the same parameters as in section 3.1.1, the contacting lines in coordinate systems OXY and $o_b x_1 y_1$ are shown as Figures 7 and 8, respectively. The bold line in Figure 8 represents the contacting line which reveals the only part of pin tooth's profile taking part in meshing, and the maximum contact angle $\theta_{\max}$ can be obtained by eq. (20).

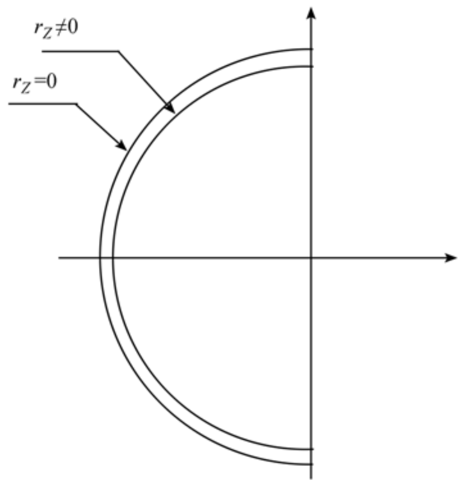

Figure 7 Meshing line in coordinate system OXY.

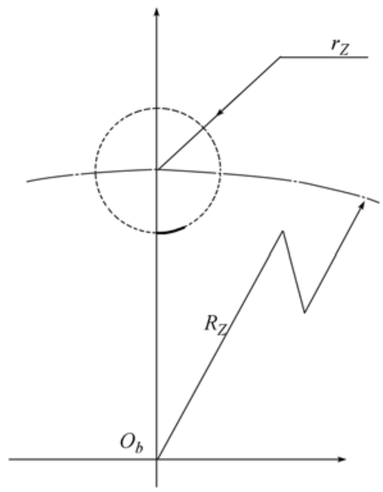

Figure 8 Meshing line in coordinate system $o_1 x_1 y_1$.

3.2 Two teeth difference cycloid drives

The method to establish the equations of cycloidal profile and contacting line is similar to that of one tooth difference cycloid pin wheel gearing, except that we just replace $(Z_b - Z_g)$ with 2 in universal equations. Note that the parameter $\varphi_{\max}$ in general equations must be restricted by eq. (16) no matter what teeth difference of the cycloid drives is. Here, we need not recount the process any more, but draw the meshing scheme and contacting line.

In case of $R_Z = 90$, $r_Z = 7$, $e = 4$, $Z_g = 22$, $\varphi_{\max} = 8.77°$ can be obtained by eq. (16). The meshing scheme and contacting line in fixed coordinate system OXY are shown in Figures 9 and 10, respectively. Also, the contact ratio $\varepsilon = 6.43$ can be obtained from eq. (27). According to eq. (20), because any kinds of small teeth difference cycloid pin wheel gearings have the common characteristic that only part of the pin tooth's profile take part in meshing, the contacting line in moving coordinate system $o_b x_1 y_1$ is similar to Figure 8.

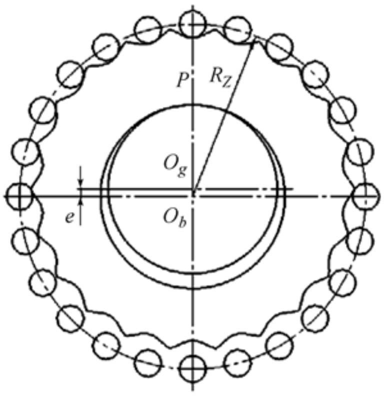

Figure 9 Meshing scheme of two teeth difference cycloid gearing.

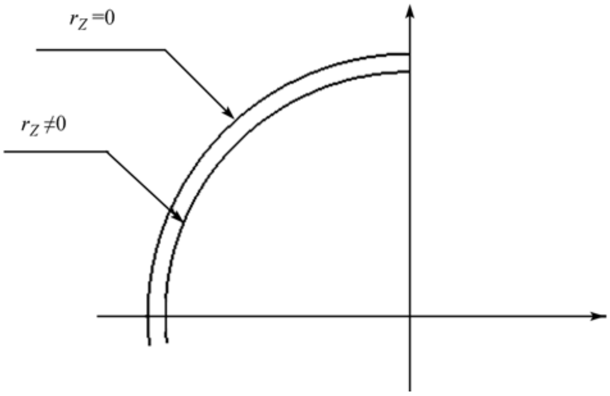

Figure 10 Meshing line in coordinate system *OXY*.

3.3 Three teeth difference cycloid drives

Using the same method as in the previous section, the equations of cycloidal profile and contacting line can be obtained. Figures 11 and 12 show the three teeth difference cycloid pin wheel gearing's meshing scheme and the contacting line in fixed coordinate system OXY, respectively. For $R_Z = 90,\ r_Z = 7,\ e = 4,\ Z_g = 33$, $\varphi_{\max} = 5.22°$ is determined by eq. (16). Also, contact ratio $\varepsilon = 5.74$ can be obtained from eq. (27).

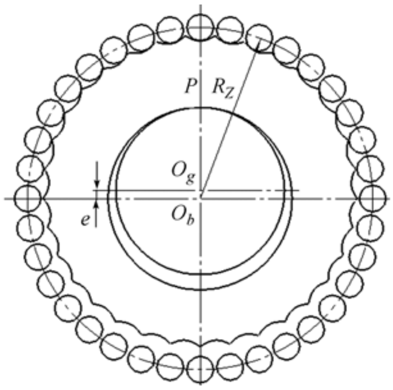

Figure 11 Meshing scheme of three teeth difference cycloid gearing.

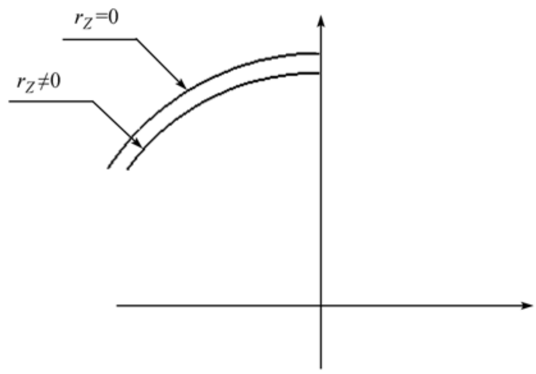

Figure 12 Meshing line in coordinate system *OXY*.

3.4 Minus one tooth difference cycloid drives

In case of $Z_b - Z_g = -1$ in eqs. (10) and (11), eq. (14) takes "negative sign" because the cycloidal profile is curtate hypocycloid outer equidistant curve in this case.

Figures 13 and 14 show the minus one teeth difference cycloid pin wheel gearing's cycloidal profile and meshing scheme. Figures 15 and 16 show the contacting lines in two different coor-

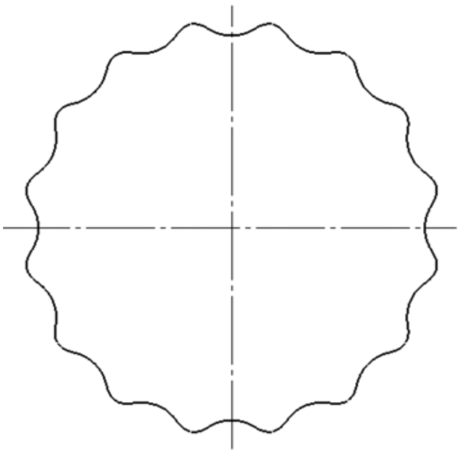

Figure 13 Cycloidal profile of minus one tooth difference cycloid gearing.

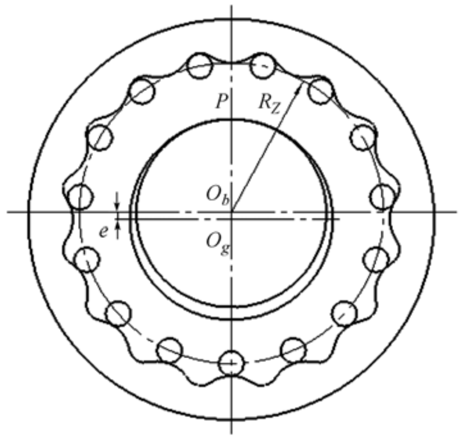

Figure 14 Meshing scheme of minus one tooth difference cycloid gearing.

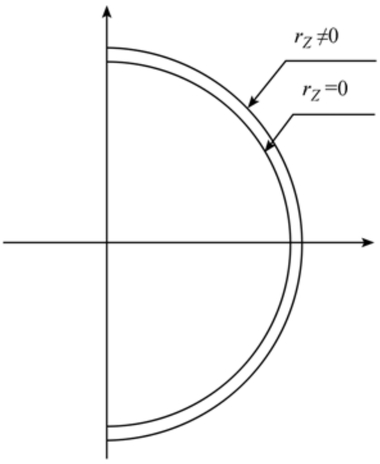

Figure 15 Meshing line in coordinate system *OXY*.

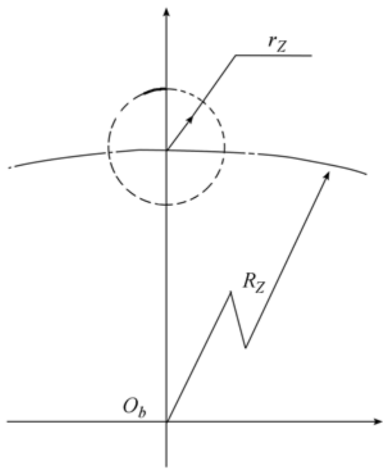

Figure 16 Meshing line in coordinate system $o_1x_1y_1$.

dinate systems, respectively, for $R_Z = 120$, $r_Z = 10$, $e = 5$, $Z_g = 16$, $\varphi_{\max} = 11.25°$ is determined by eq. (16).

In Figure 16, the bold line represents the contacting line. From Figures 15 and 16, we find that the practical profile of minus tooth difference cycloidal gear is a curtate hypocycloid outer equidistant curve, which is different from positive teeth difference cycloidal gear. Similarly, only part of pin tooth's arc taking part in meshing.

The research above revealed the essential points of gear geometry of cycloid pin wheel gearing. The results are of significance for common cycloid drives, especially for two teeth and larger teeth difference cycloid gearing, which include the design and calculation of geometry parameters, analysis of the meshing characteristics of multi-teeth difference cycloid gearing, and calculating the forces as well as machining of cycloidal gear.

4 Conclusions

(1) According to gear geometry kinematics, the equation of meshing for one tooth and small

teeth difference cycloid pin wheel gearing and universal equation of cycloidal profile are established based on cylindrical pin tooth and given motion. The method to determine the parameters in the equation of cycloidal profile is also presented.

(2) Enveloping method which is a general method to generate cycloid is developed according to the universal equation of cycloidal profile. The theoretical cycloid, curtate epicycloids and curtate hypocycloid can be obtained respectively with different parameters.

(3) There is a double contacting phenomenon in cycloid pin wheel gearing; the maximum point contact of pin tooth is only related to the curtate coefficient, and the length of contacting arc on pin tooth increases with the increasing of curtate coefficient. The sufficient and necessary condition of correct meshing for cycloid pin wheel gearing is that the center distance between pin wheel and cycloidal gear equals the eccentric distance of curtate cycloid. The contact ratio is the ratio between pin wheel's rotating angle corresponding to contacting line and the angle of two adjacent pin teeth. The contact ratio is one half of pin teeth when the tooth number difference is 1; if the difference is greater than 1, the ratio is decided by the curtate coefficient, pin tooth radius and so on.

(4) The theories established in this paper are useful for designing and machining of small teeth difference cycloid pin wheel gearing, as well as the gear geometry of other kind of small teeth difference planetary transmission.

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