

## A General Analytical Algorithm for Collaborative Robot (cobot) with 6 Degree of Freedom (DOF)

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### Abstract

The Forward and inverse kinematics of the robot is the connection of mechanical system motion and control system. There is no closed form solution for inverse kinematics of 6 *DOF* robots currently. The method of numerical iterative algorithm is used usually, but which is hard to ensure the requirement of real-time and accuracy. Considering the application and characteristic of cobot, we designed the mechanical structure and distributed the 6 *DOF* of it. This paper uses the analytical method to get the consequence of the forward and inverse kinematics of the robot. It ensures the most important characteristics of real-time and accuracy. It can also get the singularity of the robot easily. At last, we verified the method by simulation in *MATLAB*.

**Key words:** cobot, forward and inverse kinematics, analytical method, matrix theory

### Introduction

Choosing appropriate kinematics algorithm what is fast and accuracy is the base of trajectory planning and control for the cobot. Iteration method is the frequently-used method. Iteration method is feasible in most cases, but it can't get all solutions. Geometric method just can be used to some special robot which structure is simple. Regnier proposed an algorithm at 1997 what bases on iteration method and distributed method [1].this method can calculate the inverse kinematics of many robots with 6 *DOF* (degree of freedom). But it need more time to get the solution. Jun proposed a method at 2009 what transforms the input speed of workspace to inverse kinematics [2].it can get the jacobian matrix quickly. Rolland proposed an optimization genetic algorithm at 2009 about parallel manipulator[3]. He transformed the non-linear equations solution to an optimization process separately.

For the inverse kinematics, there are many new methods, such as artificial neural network[4], groebner basis and so on. These methods can achieve the target in theory[5]. But it need more time to calculate and can't meet with require of rapidity.

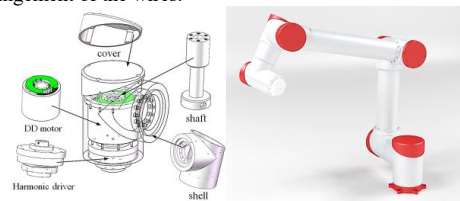
This cobot uses a general analytical algorithm to get the forward and inverse kinematics. It can ensure the requirement of real-time, accuracy and can be used to any other similar robot.

### 1. The structure and kinematic model of the robot.

Considering the application and characteristic of cobot, the distribution of the degrees of freedom (*DOF*), we designed the

structure of the robot and then made lightweight design through topology optimization.

The Fig.1 shows the model of the robot. The robot mainly consists of DD motor, harmonic driver, feedback components (photoelectric encoder and magnetic encoder), and brake mechanism and so on. The robot adopts series structure. All the internal structure of the joint is hollow. This can ensure the arrangement of the wires.



(1) Single joint (2) Whole robot

Fig.1 The structure of the robot

In the aspect of *DOF* configuration, 6 *DOF* can reach any position of the work space. So, this robot uses 6 *DOF* to realize the flexible movement and enhance the ability of avoiding obstacles. All the 6 joints are rotational *DOF*, and adopt series structure.

As shown in the Fig.2, the kinematics model is built on the basis of the *D-H* theory. The reference coordinate system is set in the base joint. The  $Z_0$  axis is vertical. The  $X_0$  axis is perpendicular to  $Z_0$  axis and follows outward direction of the paper. The direction of  $X_0$  axis is determined by the right-hand screw rule. Another 6 coordinate systems  $O_i-X_iY_iZ_i$  ( $i=1\sim6$ ) build at each rotary joint. The  $Z_i$  axis is heading in the forward direction of the motor. The directions of  $X_i$  and  $Y_i$  are determined by *D-H* theory.

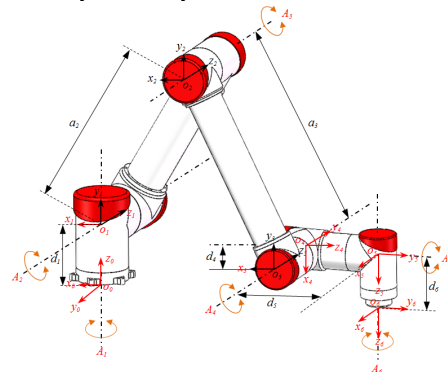


Fig.2 The kinematics model

The Fig.2 shows the 6 axes  $A_i$  ( $i=1\sim6$ ) of robot. The 7 frames  $O_i-X_iY_iZ_i$  ( $i=0\sim6$ ) connect with each basic part.

$\theta_i, a_i, d_i, \alpha_i$  represent the joint angle, link length, link offset and link twist respectively. The table 1 gives the  $D-H$  parameters of the robot.

TABLE 1  
D-H PARAMETERS

|         | $\theta_i(rad)$ | $a_i(m)$ | $d_i(m)$ | $\alpha_i(rad)$ |
|---------|-----------------|----------|----------|-----------------|
| joint 1 | $\theta_1$      | 0        | $d_1$    | $\pi/2$         |
| joint 2 | $\theta_2$      | $a_2$    | 0        | 0               |
| joint 3 | $\theta_3$      | $a_3$    | 0        | 0               |
| joint 4 | $\theta_4$      | 0        | $d_4$    | $\pi/2$         |
| joint 5 | $\theta_5$      | 0        | $d_5$    | $-\pi/2$        |
| joint 6 | $\theta_6$      | 0        | $d_6$    | 0               |

### The kinematics research of the robot

The analysis of kinematic can solve the problem of mapping relation with the joint angle  $\theta_i$  and the pose of end effector. This is the theoretical basis of motion control and one of the most important steps to realize close-loop control accurately[6].

#### A Forward Kinematics

The joint angle  $\theta_i$  ( $i=1\sim6$ ) is known, and then need to calculate the position and orientation of TCP (terminal center point). This process is described as forward kinematics.  ${}^{i-1}_iT$  is the  $D-H$  transformation matrix and what transforms from the coordinate frame  $i$  to  $i-1$ . Every coordinate frame  $i$  can transform to the coordinate frame  $i-1$  through 4 relative transformations orderly.

The  $D-H$  transformation matrix  ${}^{i-1}_iT$  can be got as follows:

$${}^{i-1}_iT = T_{RZ-1}(\theta_i) \cdot T_Z(d_i) \cdot T_X(a_i) T_{RX}(\alpha_i)$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.1)$$

The homogeneous transformation matrix of each joint can be got from (1.1) and table 1.

$${}^0_6T = {}^0_1T_1 {}^1_2T_2 {}^2_3T_3 {}^3_4T_4 {}^4_5T_5 {}^5_6T_6 \quad (1.2)$$

${}^0_6T$  represents the position and orientation of TCP what is based on the base coordinate frame. It can be also written as this:

$${}^0_6T = \begin{bmatrix} x_6 & y_6 & z_6 & p_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^0R_6 & {}^0P_6 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \bar{n} & \bar{s} & \bar{a} & \bar{p} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.3)$$

$n, s, a$  represent the coordinates of  $X_6$  axis,  $Y_6$  axis and  $Z_6$  axis what are based on the base coordinate frame  $O_0-X_0Y_0Z_0$ .  $P$  represents the coordinate of TCP what is also based on the base coordinate frame  $O_0-X_0Y_0Z_0$ . So,  ${}^0_6T$  can be got as this:

$${}^0_6T = {}^0_1T_1 {}^1_2T_2 {}^2_3T_3 {}^3_4T_4 {}^4_5T_5 {}^5_6T_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.4)$$

The position and orientation of TCP can be calculated when the initial  $\theta_i$  ( $i=1\sim6$ ),  $a_2, a_3, d_1, d_4, d_5, d_6$  are given.

#### B Inverse Kinematics

The position and orientation of TCP ( $n_x, n_y, \dots, p_y, p_z$ ) is known. Then we need to calculate the joint variate  $\theta_i$  ( $i=1\sim6$ ). This process is called inverse kinematics. In the field of engineering application, inverse kinematics is more important than forward kinematics. It's the base of motion planning and trajectory control. There are two sufficient conditions we should follow if we want to get the closed-form solution[7-8].

- (1) There are 3 adjacent axes of the joints intersect at one point.
- (2) There are 3 adjacent axes of the joints parallel to each other.

This robot meets the second condition. So, we can get the closed-form solution.

#### 1、Calculate joint angle $q_1$

Multiplying each side of (1.4) by  ${}^0_1T_1^{-1}$ .

$${}^0_1T_1^{-1} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^1_2T_2 {}^2_3T_3 {}^3_4T_4 {}^4_5T_5 {}^5_6T_6 \quad (1.5)$$

The left side of (1.5) is:

$$\begin{bmatrix} n_x C_1 + n_y S_1 & o_x C_1 + o_y S_1 & a_x C_1 + a_y S_1 & p_x C_1 + p_y S_1 \\ n_z & o_z & a_z & p_z - d_1 \\ n_x S_1 - n_y C_1 & o_x S_1 - o_y C_1 & a_x S_1 - a_y C_1 & p_x S_1 - p_y C_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.6)$$

The right side of (1.5) is:

$$\begin{bmatrix} C_{234}C_5C_6 - S_{234}S_6 & -S_{234}C_6 - C_{234}C_5S_6 & -C_{234}S_5 \\ C_{234}S_6 + S_{234}C_5C_6 & C_{234}C_6 - S_{234}C_5S_6 & -S_{234}S_5 \\ C_6S_5 & -S_5S_6 & C_5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} d_5 S_{234} + a_3 C_{23} + a_2 C_2 + d_6 S_5 C_{234} \\ d_5 C_{234} + a_3 S_{23} + a_2 S_2 - d_6 S_5 S_{234} \\ d_4 + d_6 C_5 \\ 1 \end{bmatrix} \quad (1.7)$$

There is one linear equation about  $q_1$  can be got, comparing (1.6) and (1.7) in Row 3/Column 3 and 4.

$$\begin{cases} a_x S_1 - a_y C_1 = C_5 \\ p_x S_1 - p_y C_1 = d_4 + d_6 C_5 \end{cases} \quad (1.8)$$

It can be derived out from (1.8),

$$(p_x - d_6 a_x) S_1 + (d_6 a_y - p_y) C_1 = d_4 \quad (1.9)$$

Using trigonometric substitution:

$$\begin{cases} \rho \sin \phi = d_6 a_y - p_y \\ \rho \cos \phi = p_x - d_6 a_x \end{cases} \quad (1.10)$$

$$\phi = a \tan 2(d_6 a_y - p_y, p_x - d_6 a_x) \quad (1.11)$$

Putting (1.10) and (1.11) into (1.9), we can get:

$$\sin(\phi + q_1) = \frac{d_4}{\rho}$$

$$\text{So, } \phi + q_1 = a \tan 2\left(\frac{d_4}{\rho}, \pm \sqrt{1 - \frac{d_4^2}{\rho^2}}\right)$$

Combining with (1.10), we can get 2 solutions of  $q_1$ .

$$\begin{aligned} q_1 &= a \tan 2(d_4, \pm \sqrt{(d_6 a_y - p_y)^2 + (p_x - d_6 a_x)^2 - d_4^2}) \\ &- a \tan 2(d_6 a_y - p_y, p_x - d_6 a_x) \end{aligned} \quad (1.12)$$

## 2、 Calculate joint angle $q_5$

There is one linear equation about  $q_5$  can be got, comparing (1.6) and (1.7) in Row 3/Column 1 and 2.

$$\begin{cases} n_x S_1 - n_y C_1 = C_5 S_5 \\ o_x S_1 - o_y C_1 = -S_5 S_6 \end{cases} \quad (1.13)$$

Squaring the two equations in (1.13), the variate  $q_6$  can be eliminated.

$$S_5 = \pm \sqrt{(n_x S_1 - n_y C_1)^2 + (o_x S_1 - o_y C_1)^2} \quad (1.14)$$

Because  $a_x S_1 - a_y C_1 = C_5$  in equation (1.8), so

$$q_5 = a \tan 2(\pm \sqrt{(n_x S_1 - n_y C_1)^2 + (o_x S_1 - o_y C_1)^2}, a_x S_1 - a_y C_1) \quad (1.15)$$

There are two solutions of  $q_5$  from the symbol '±' in (1.15).

## 3、 Calculate joint angle $q_6$

As we can see in (1.13), the robot has singular value if  $S_5=0$  ( $q_5=0$  or  $q_5=\pi$ ). So, we can get  $q_6$  just when  $S_5 \neq 0$ .

We can get equation (1.16) from (1.13).

$$\begin{cases} \frac{n_x S_1 - n_y C_1}{S_5} = C_6 \\ -\frac{o_x S_1 - o_y C_1}{S_5} = S_6 \end{cases} \quad (1.16)$$

$$\text{Then, } q_6 = a \tan 2\left(-\frac{o_x S_1 - o_y C_1}{S_5}, \frac{n_x S_1 - n_y C_1}{S_5}\right) \quad (1.17)$$

## 4、 Calculate joint angle $q_{234}$

There is one linear equation about  $q_2+q_3+q_4$  can be got, comparing (1.6) and (1.7) in Row 1 and 2/Column 3.

$$\begin{cases} a_x C_1 + a_y S_1 = -C_{234} S_5 \\ a_z = -S_{234} S_5 \end{cases} \quad (1.18)$$

As the same in  $q_6$ , the robot has singular value if  $S_5=0$  ( $q_5=0$  or  $q_5=\pi$ ). So, we can get  $q_2+q_3+q_4$  just when  $S_5 \neq 0$ .

We can get equation (1.19) from (1.18).

$$\begin{cases} -\frac{a_x C_1 + a_y S_1}{S_5} = C_{234} \\ -\frac{a_z}{S_5} = S_{234} \end{cases} \quad (1.19)$$

Then,

$$q_{234} = q_2 + q_3 + q_4 = a \tan 2\left(-\frac{a_z}{S_5}, -\frac{a_x C_1 + a_y S_1}{S_5}\right) \quad (1.20)$$

## 5、 Calculate joint angle $q_2$

There is one linear equation about  $q_2$  can be got, comparing (1.6) and (1.7) in Row 1 and 2/Column 4.

$$\begin{cases} p_x C_1 + p_y S_1 = d_5 S_{234} + a_3 C_{23} + a_2 C_2 - d_6 S_5 C_{234} \\ p_z - d_1 = -d_5 C_{234} + a_3 S_{23} + a_2 S_2 - d_6 S_5 S_{234} \end{cases} \quad (1.21)$$

Squaring the two equations in (1.13), the variate  $q_{23}$  can be eliminated.

$$\text{Then, } 2a_2 A C_2 + 2a_2 B S_2 = A^2 + B^2 + a_2^2 - a_3^2 \quad (1.22)$$

We can assume that

$$A = p_x C_1 + p_y S_1 - d_5 S_{234} + d_6 S_5 C_{234},$$

$$B = p_z - d_1 + d_5 C_{234} + d_6 S_5 S_{234} \text{ in the (1.22), and order}$$

$$\begin{cases} \eta \sin \varphi = A \\ \eta \cos \varphi = B \end{cases} \quad (1.23)$$

$$\text{Then, } \eta = \sqrt{A^2 + B^2} \text{ and } \varphi = a \tan 2(A, B).$$

We can get (1.24) when putting (1.23) into (1.22).

$$2a_2 \eta \sin \varphi C_2 + 2a_2 \eta \cos \varphi S_2 = A^2 + B^2 + a_2^2 - a_3^2 \quad (1.24)$$

$$\text{And then, } \sin(\varphi + q_2) = \frac{A^2 + B^2 + a_2^2 - a_3^2}{2a_2 \eta}$$

$$\cos(\varphi + q_2) = \pm \sqrt{1 - \left(\frac{A^2 + B^2 + a_2^2 - a_3^2}{2a_2 \eta}\right)^2}$$

So,

$$\varphi + q_2 = a \tan 2\left(\frac{A^2 + B^2 + a_2^2 - a_3^2}{2a_2 \eta}, \pm \sqrt{1 - \left(\frac{A^2 + B^2 + a_2^2 - a_3^2}{2a_2 \eta}\right)^2}\right)$$

we can get 2 solutions of  $q_2$ .

$$\begin{aligned} q_2 &= a \tan 2(A^2 + B^2 + a_2^2 - a_3^2, \\ &\pm \sqrt{(4a_2^2 - 1)(A^2 + B^2 - a_2^2 + a_3^2)}) - a \tan 2(A, B) \end{aligned} \quad (1.25)$$

## 6、 Calculate joint angle $q_3$

We can get the solution of  $q_{23}$  after  $q_2$  was calculated.

$$\begin{cases} p_x C_1 + p_y S_1 - d_5 S_{234} - a_2 C_2 + d_6 S_5 C_{234} = a_3 C_{23} \\ p_z - d_1 + d_5 C_{234} - a_2 S_2 + d_6 S_5 S_{234} = a_3 S_{23} \end{cases} \quad (1.26)$$

So,

$$q_{23} = q_2 + q_3 = a \tan 2 \left( \frac{p_z - d_1 + d_5 C_{234} - a_2 S_2 + d_6 S_5 S_{234}}{a_3}, \frac{p_x C_1 + p_y S_1 - d_5 S_{234} - a_2 C_2 + d_6 S_5 C_{234}}{a_3} \right) \quad (1.27)$$

Thereby, the remaining joints  $q_3 = q_{23} - q_2$ ,

$$q_4 = q_{234} - q_{23}.$$

At this moment, all the joints ( $q_1 \sim q_6$ ) of inverse kinematics were calculated.

### The Simulation with Matlab

#### A Simulation of Forward Kinematics

The length of link  $a_2 = -0.5$ ,  $a_3 = -0.4$  and the setover of link  $d_1 = 0.1$ ,  $d_4 = 0.1$ ,  $d_5 = 0.1$ ,  $d_6 = 0.08$  is known. Putting these initial values into the equation (1.4). The terminal position and orientation can be got as follows, when angle  $\theta_1 = \theta_4 = \theta_6 = 0$ ,  $\theta_2 = \theta_3 = \theta_5 = \pi/2$  of the robot was given.

$${}^0T = \begin{bmatrix} 0 & 0 & 1 & 0.48 \\ -1 & 0 & 0 & -0.1 \\ 0 & -1 & 0 & -0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The software Matlab Robotics Toolbox was used to make forward kinematics simulation. The terminal position and orientation can be got as Fig.3.

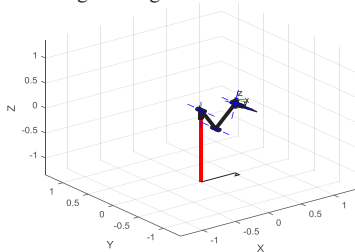


Fig.3 terminal position and orientation of the robot

#### B Simulation of Inverse Kinematics

In order to prove the correctness of the algorithm, we put the result we calculated from chapter 3.1 as the known parameter. Then, the terminal position and orientation was known and used it to calculate the 8 solutions of joint angle  $\theta_i$  ( $i=1 \sim 6$ ). We used the software Matlab to make numerical and matrix operations. The table 2 gives the 8 solutions of inverse kinematics. We can see from the table 2. The NO 1 is one solution of the 8 solutions. it shows that  $\theta_1 = \theta_4 = \theta_6 = 0$ ,  $\theta_2 = \theta_3 = \theta_5 = \pi/2$  and correspond to the chapter 3.1. so, the algorithm is correct.

TABLE 2  
EIGHT SOLUTIONS OF INVERSE KINEMATICS

| No | joint | $q(\text{rad})$ | joint | $q(\text{rad})$ |
|----|-------|-----------------|-------|-----------------|
|----|-------|-----------------|-------|-----------------|

|   |   |         |   |         |
|---|---|---------|---|---------|
| 1 | 1 | 0       | 2 | 1.5708  |
|   | 3 | 1.5708  | 4 | 0       |
|   | 5 | 1.5708  | 6 | 0       |
| 2 | 1 | 0       | 2 | -0.2213 |
|   | 3 | -0.3805 | 4 | -2.5398 |
|   | 5 | 1.5708  | 6 | 0       |
| 3 | 1 | 0       | 2 | -1.4665 |
|   | 3 | 0.3213  | 4 | 1.055   |
|   | 5 | -1.5708 | 6 | 3.1416  |
| 4 | 1 | 0       | 2 | 0.1795  |
|   | 3 | -0.6111 | 4 | 0.232   |
|   | 5 | -1.5708 | 6 | 3.1416  |
| 5 | 1 | 2.6516  | 2 | 2.9621  |
|   | 3 | -4.7531 | 4 | 2.9096  |
|   | 5 | 1.0808  | 6 | -3.1416 |
| 6 | 1 | 2.6516  | 2 | 4.6081  |
|   | 3 | -5.8432 | 4 | 2.0866  |
|   | 5 | 1.0808  | 6 | -3.1416 |
| 7 | 1 | 2.6516  | 2 | 2.9468  |
|   | 3 | -4.7385 | 4 | 2.9471  |
|   | 5 | -1.0808 | 6 | -3.1416 |
| 8 | 1 | 2.6516  | 2 | 4.3487  |
|   | 3 | -5.6938 | 4 | 2.3237  |
|   | 5 | -1.0808 | 6 | -3.1416 |

### Conclusion

This paper proposed an analytical algorithm to get the consequence of the forward and inverse kinematics of the robot with 6 DOF. It ensured the most important characteristics of real-time and accuracy of the robot, and also can get the singularity of the robot easily. The detailed derivation process given by this paper can be used in any similar robot. At the last of this paper, we given the simulation of the forward and inverse kinematics. The algorithm was efficient proved by the simulation of matlab.

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