Mathematical Modeling and Trajectory Planning of a 5 Axis Robotic Arm for Welding Applications

Joji Augustine[1], Anand Kumar Mishra[2], Karali Patra[1]
[1]Department of Mechanical Engineering, Indian Institute of Technology Patna, Patna-80013, India
(kpatra@iitp.ac.in)
[2]Masters Programme in Mechatronics, Indian Institute of Technology Patna, India

Abstract-- **Industrial Robot is one of the most crucial components of modern industries. This is because it is flexible and intelligent machine with the potential to perform tasks in a repetitive manner at acceptable cost and quality levels. Robotic welding is one of the most successful applications of industrial robot manipulators. At present the welding robots are most commonly used in automobile industries where a large amount of the welding tasks are repetitive. This paper aims at studying the design of a five axis robotic manipulator for welding application. A mathematical model of the manipulator is formulated first and the model is constructed in computer and simulation is done. Trajectory planning algorithms of the robot is determined for efficient welding where the end effector carrying the welding torch traces a circular path. The simulation of joint angle will help to plan the control strategy of the proposed manipulator for welding applications.**

Keywords— Industrial Robots, Manipulator, Welding robot, Trajectory planning, Kinematics, Denavit Hartenberg parameters.

I. INTRODUCTION

Industrial robots are essential components of today's factory and even more of the factory of the future [1]. Most of the industrial robots are precise automation devices that can repeat a desired task any number of times without loss of precision. The robots must be flexible enough to reach the required workspace but at the same time should be easy to control. As we increase the reach and flexibility of a robotic arm the complexity of control increases. When a requirement can be achieved using a particular number of links, increasing the link number will lead to redundancy and also make planning and control difficult. Most of the industrial robots have 4/5 degrees of freedom (4/5 axis). The first 3 axes help the robot to reach a particular point in 3D space and the rest of the axes help in the orientation of the end effector [2]. The axes used for orientation are concurrent so that there is only rotational transformation using these axes.

In competitive modern industries, manual welding must be limited to shorter periods of time because of the required setup time, operator discomfort, safety considerations and cost. Thus, robotic welding is critical to welding automation in many industries. It is estimated that as much as 25% of all industrial robots are applied for welding tasks [1]. Now advances in automation techniques are pushing forward the fast and wide applications of industrial robots in welding operations in automobile, aerospace and shipbuilder industries [3]. High quality weld seam can be obtained using modern automatic control technology, such as open loop and closed loop control [4]. One major difference between a pick and place robot and a welding robot is the path of motion. For a pick and place robot the path is not significant. For welding robots the motion is continuous path motion where the end effector has to move in a particular path. End effector path motion can be designed and controlled by motion controller [5].

Most of the robot in applied in welding operation are still on-line teaching and playback type [6], which cannot often meet the efficiency requirement to various products since on-line teaching spends lots of time. There are attempts to make the robotic welding process more effective, and there are two routines for the same. One routine is online vision based system which recognizes the welding seam by vision sensors [7,8]. The other routine is off-line computer simulation which manipulates the trajectory of the end effector to generate numerical control (NC) code for real time controlling of welding operations [9,10]. This routine can be more effective by saving the online teaching time. Furthermore, it is also more convenient CAD/CAM/CAPP integration [10].

Many industrial robots designed for jobs like wire EDM, polishing, grinding, arc welding, etc, have only four axis or five axis. This is due to the tools or objects for such jobs are symmetrical. The tool can be treated as line segment as the rotation of the tool about its own axis does not change the attitude of the tool. A four axis robot can guide a line in 3-dimension; A robot with five-axis can guide a line segment for a specified pose [2]. This paper will be discussing about a 5 axis manipulator following a continuous path motion. CAD modeling the proposed 5-axis articulated robot configuration is developed and Denavit–Hartenberg (DH) parameters are estimated. Forward and inverse kinematics problems of this manipulator are formulated. In this work, a trajectory planning approach is devised to generate the trajectory of the end effector maintaining the constant velocity of the welding torch to maintain the weld quality. Finally, the trajectory planning is simulated through Simulink, MatLab.

II. PROBLEM FORMULATION

As discussed earlier for proper placing and orientation of the end effector a minimum of 5 DOF is required for the manipulator. One more DOF will increase the dexterity but we will limit the discussion to 5 axis robotic manipulator to reduce the complexity. The 3D diagram of the proposed design is shown in Fig 1.

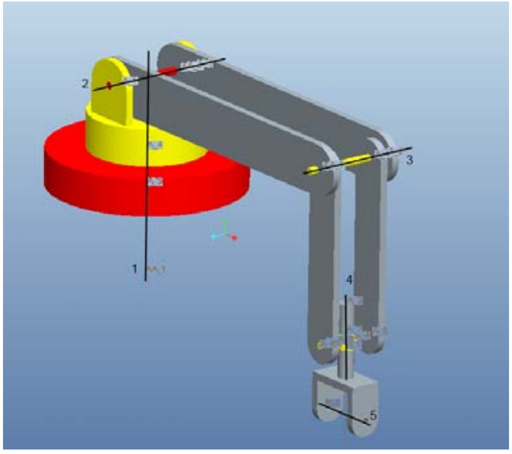

Fig.1: 3D diagram of proposed Robotic Arm

The projection views with dimensions are shown in Fig 2. As can be seen from the figures the manipulator has 5 links and 5 joints. All are rotary joints. The welding torch or the tool will form the 5^{th} link (not shown in diagram).In order to plan and control the motion of the robot; we need to mathematically model the system. This is done using forward kinematics using transformation matrices.

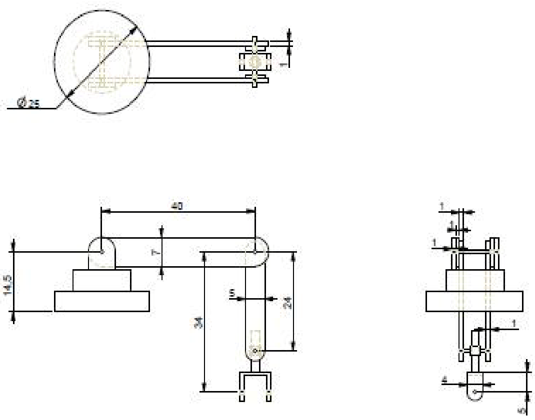

Fig 2. Projection view of robot with dimensions (in mm)

III. FORWARD KINEMATICS

The various parameters of the manipulator can be expressed as Denavit – Hartenberg (DH) parameters [4] as shown in Table 1.

TABLE 1
DH PARAMETER

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-π/2	0	0	θ_2
3	0	L1	D1	θ_3
4	-π/2	D2	L2	θ_4
5	π/2	0	0	θ_5

The transformation matrix for each links frame can be written as

$$^{0}[T]_{1} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$^{1}[T]_{2} = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$^{2}[T]_{3} = \begin{bmatrix} c_3 & -s_3 & 0 & L_1 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & D_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$^{3}[T]_{4} = \begin{bmatrix} c_4 & -s_4 & 0 & D_2 \\ 0 & 0 & 1 & L_2 \\ -c_4 & -s_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$^{4}[T]_{5} = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$ {}^{5}[T]_E = \begin{bmatrix} 1 & 0 & 0 & R \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6} $$

$$ {}^{0}[T]_3 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & -s_1 & L_1c_1c_2 - D_1s_1 \\ s_1c_{23} & -s_1s_{23} & c_1 & L_1s_1c_2 + D_1c_1 \\ -s_{23} & c_{23} & 0 & L_1c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7} $$

$$ {}^{3}[T]_5 = \begin{bmatrix} c_4c_5 - s_4s_5 & -c_4s_5 - s_4c_5 & 0 & D_2 \\ 0 & 0 & 1 & L_2 \\ -s_4c_5 - c_4s_5 & s_4s_5 - c_4c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8} $$

$$ {}^{0}[T]_E = {}^{0}[T]_3 * {}^{3}[T]_5 * {}^{5}[T]_E \tag{9} $$

IV. INVERSE KINEMATICS

In order to find the 5 joint angles we need inverse kinematics. The first 3 joint angles determine the position of the wrist point. The last 2 joint angles determine the orientation of the welding torch. The Cartesian coordinates of the end of the welding electrode is known. Also a new term called Approach vector is defined to specify the direction of approach f or welding. Depending on the weld surface or joint the approach vector is defined. It defines the orientation of the weld torch.

If **A** is the unit approach vector and **X**(x,y,z) is the desired position vector of the end effector, then the following can be done to find joint angles θ_1, θ_2, θ_3, θ_4, θ_5

Wrist point vector = **W, r** is the length of the electrode/tool

$$ \mathbf{W} = \mathbf{X} - r * \mathbf{A} \tag{10} $$

Thus we get the wrist point coordinates which we use to find $\boldsymbol{\theta_1}$, $\boldsymbol{\theta_2}$, $\boldsymbol{\theta_3}$

$$ \theta_1 = 2\tan^{-1}\left(\frac{-x \pm \sqrt{x^2 + y^2 - D_1^2}}{y + D_1} \right) \tag{11} $$

$$ K = \frac{\left(x^2 + y^2 + z^2 - D_1^2 - L_1^2 - D_2^2 - L_2^2 \right)}{2L_1} \tag{12} $$

$$ \theta_3 = 2\tan^{-1}\left(\frac{-L_2 \pm \sqrt{L_2^2 + D_2^2 - K^2}}{k + D_2} \right) \tag{13} $$

$$ \theta_2 = 2\tan^{-1}\left(\frac{-L_1 - D_2c_3 + L_2s_3 \pm F}{z - \left(D_2s_3 + L_2c_3 \right)} \right) \tag{14} $$

$$ F = \sqrt{L_1^2 + D_2^2 + L_2^2 + 2L_1(D_2c_3 - L_2s_3) - z^2} \tag{15} $$

For finding θ_4, θ_5 we use the approach vector, **A.**
The Approach vector wrt 3rd frame is

$$ \mathbf{A_3} = {}^{0}\mathbf{R_3}^{T}\mathbf{A} \tag{16} $$

$$ \mathbf{A_3} = [A_3^x \; A_3^y \; A_3^z] \tag{17} $$

$$ \theta_5 = \operatorname{a\,tan2}\left(\sqrt{\left(\left(A_3^x\right)^2 + \left(A_3^z\right)^2 \right)}, A_3^y \right); \tag{18} $$

$$ \theta_4 = \operatorname{a\,tan2}\left(A_3^z / \sin(\theta_5), A_3^x / \cos(\theta_5) \right); \tag{19} $$

V. TRAJECTORY PLANNING

As our focus is on welding operations the movement of the torch will be at a constant velocity, decided by the type of welding, weld materials, atmospheric conditions etc. We have to move the torch at the velocity V in order to maintain the weld quality. The approach is to devise a method to generate the trajectory maintaining the velocity.

As we know the position of the end effector we can find the joint angles. But we need to find the position of the end effector as a function of time to move and control the end effector along the required path. For this we use trajectory planning. Both Cartesian and Joint space trajectory planning is combined to get the required trajectory.

The equation of the path on which the torch moves is known, f(x,y,z)=0. The speed of motion is also known V=const.

We can find the total arc length of the curve by

$$ \text{Arc length} = \int_a^b \sqrt{1 + \left(\frac{dy}{dx} \right)^2}\, dx \tag{20} $$

The total time for the operation will be

$$ T = \frac{Arclength}{V} \tag{21} $$

Depending on the accuracy of the path we want to achieve we split the whole path into sections of equal time ie, the time required to complete each section will be same. Let the number of time divisions be N. Each section's time required for completion is given as **T'=T/N.**

We can find the initial and final position of the end effector for each time step and then Joint Space trajectory planning can be done considering each section as point to point motion.

The joint space trajectory [9] planning is done taking each joint angle as function of time.

$$\theta(t) = a+bt+ct^2+dt^3 \quad (22)$$

$\theta(t=0)$ and $\theta(t=T')$ can be found using inverse kinematics. The derivatives of θ can be kept according to the type of joint movement desired. Using these equations the joint angles can be found as a function of time.

VI. MATLAB SIMULATION

To verify the trajectory planning method proposed simulation was carried out in MATLAB using SimuLink and SimMechanics. The five link mechanism was generated using SimMechanics and the mechanism was driven by joint actuators which were in turn controlled by the algorithms explained earlier. The algorithm was tested for a variety of curves including straight line, circle and parabolic curves and simulation has shown in Fig 3

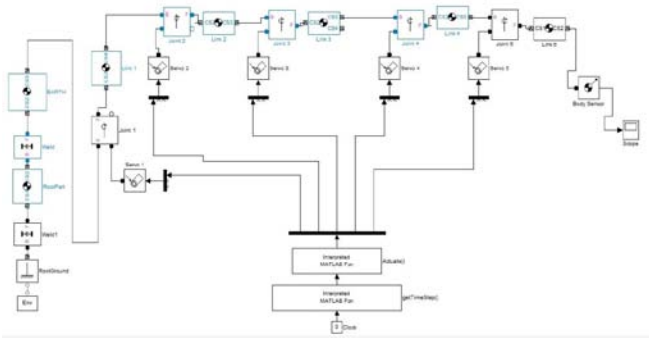

Fig 3 Simulink block Diagram

VII. SIMULATION RESULT

For different input trajectories the simulation was done. The following are the results.

For equation of path

$(x-X)^2+(y-Y)^2=R^2$, X=50cm, Y=0, R=15cm, Velocity=1cm/s and N=200;

The trajectory output in simulation was for work space in x-y plane is shown in Fig. 4.

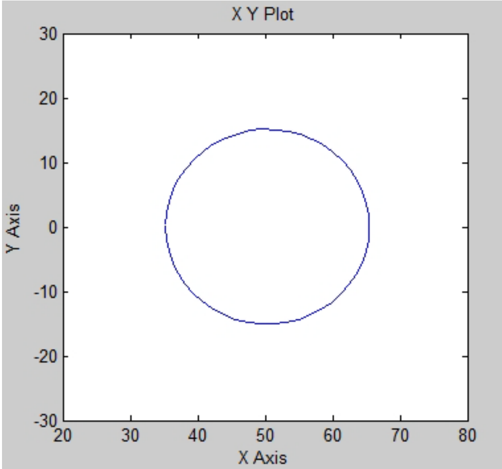

Fig 4. Workspace of welding robot in x-y plane

The Joint angles as a function of time are given in Fig. 5 to Fig. 9.

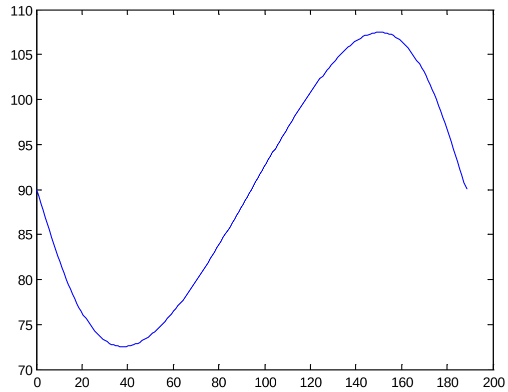

Fig 5. θ_1 vs Time

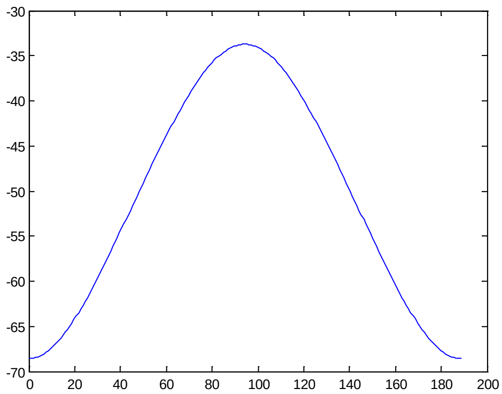

Fig 6. θ_2 vs Time

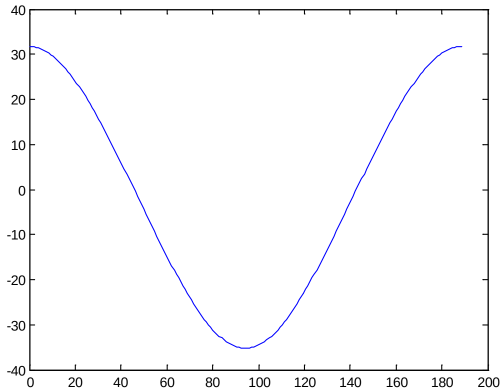

Fig 7. θ_3 vs Time

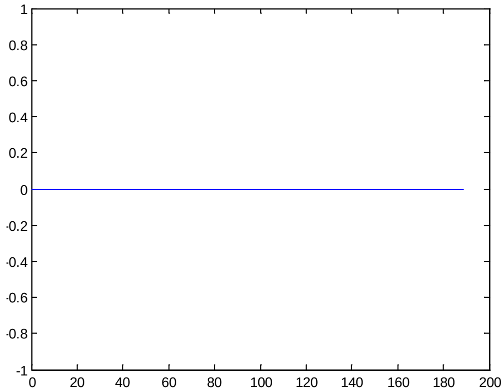

Fig 8. θ_4 vs Time

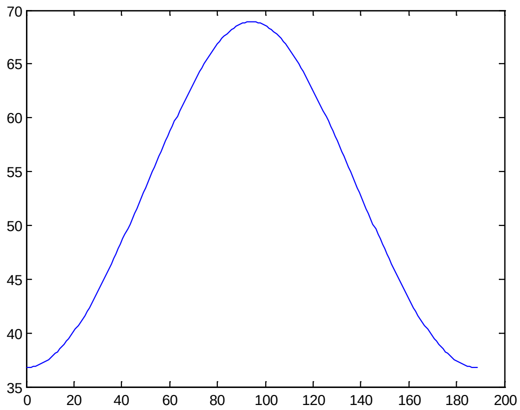

Fig 9. θ_5 vs Time

VIII. CONCLUSION

The proposed method is good enough to follow a particular path as was proved in simulation. As the number of time divisions decrease the error increases as the point to point motion within a time step is not along the curve. The drawback of having higher number of time divisions is the computational time required to plan the trajectory. So for highly precise motion this method cannot be adopted but real time control using sensors should be used. But as the whole planning is done before the operation starts, there won't be any delay or pause during the motion of the end effector.

REFERENCES

[1] J. Norberto Pires, Altino Loureiro and Gunnar Bölmsjo, ''Welding Robots Technology, System Issues and Applications'' *springer* 2005

[2] Ming J.Tsai, Duen-Jyh Stone ''Inverse velocity analysis for line guidance five-axis robot'' *Robotics and Computer-Integrated Manufacturing* 25 (2009) 736–745.

[3] Donghun Lee, TaeWon Seo, Jongwon Kim, Optimal design and workspace analysis of a mobile welding robot with a 3P3R serial manipulator, *Robotics and Autonomous Systems* 59 (2011) 813–826.

[4] Ashitava Ghoshal, *Robotics:Fundamental Concepts and Analysis*, Oxford University Press, 2006.

[5] LI Ru-xiong, ''Design and Realization of 3-DOF Welding Manipulator Control System Based on Motion Controller,'' *Energy Procedia* 14 (2012) 931 – 936.

[6] John J. Craig, Chen SB, Qiu T, Lin T, Wu L, Tian JS, Lv WX, et al. Intelligent technologies for robotic welding. Robotic Welding, *Lecture Notes in Control and Information Sciences* 2004;299: 123–43.

[7] Zhu Zh-Y, Lin T, Piao YJ, Chen SB. Recognition of the initial position of weld based on the image pattern match technology for welding robot. *International Journal of Advanced Manufacturing Technology* 2005; 26(7–8): 784–8.

[8] Ming J.Tsai, Hong-WenLee, Nai-JunAnn, Machine vision based path planning for a robotic golf club, *Robotics and Computer-Integrated Manufacturing* 27 (2011) 843– 849.

[9] Mitsi S, Bouzakis K-D, Mansour G, Sagris D, Maliaris G. Off-line programming of an industrial robot for manufacturing. *International Journal of Advanced Manufacturing Technology* 2005;26(3):262-7.

[10] Zhenyu Liu, Wanghui Bu,Jianrong Tan, Motion navigation for arc welding robots based on feature mapping in a simulation environment, *Robotics and Computer-Integrated Manufacturing* 26 (2010) 137–144.