Lemma. *In SRLC1 (Section 5.3 and Algorithm 7), the chosen* m *fulfills* $m = O(\kappa \log^2(\kappa) \log(\epsilon_F^{-1}))$.

Since $0 < \epsilon_F < 1$ and $1 < \kappa$, we have:

$$\begin{aligned}
&1 - \prod_{i=1}^{\kappa}(1 - (1 - \frac{3}{\kappa})^{m-i+1}) \\
&\leq 1 - \prod_{i=1}^{\kappa}(1 - (1 - \frac{3}{\kappa})^{m-\kappa+1}) \\
&\leq 1 - (1 - \kappa(1 - \frac{3}{\kappa})^{m-\kappa+1})) \\
&= 1 - (1 - (1 - \frac{3}{\kappa})^{m-\kappa+1})^{\kappa} \\
&\leq \kappa(1 - \frac{3}{\kappa})^{m-\kappa}
\end{aligned}$$

Therefore, if $\kappa(1 - \frac{3}{\kappa})^{m-\kappa} \leq \epsilon_F$, we have $\epsilon_F \geq \kappa(1 - \frac{3}{\kappa})^{m-\kappa} \geq 1 - \prod_{i=1}^{\kappa}(1 - (1 - \frac{3}{\kappa})^{m-i+1})$, which satisfies the requirement in algorithm 7.

We also have:

$$\begin{aligned}
&\kappa(1 - \frac{3}{\kappa})^{m-\kappa} \leq \epsilon_F \\
\Leftrightarrow\ &\log(\kappa(1 - \frac{3}{\kappa})^{m-\kappa}) \leq \log(\epsilon_F) \\
\Leftrightarrow\ &\log(\kappa) + (m - \kappa)\log(1 - \frac{3}{\kappa}) \leq \log(\epsilon_F) \\
\Leftrightarrow\ &m \log(1 - \frac{3}{\kappa}) \leq \log(\epsilon_F) - \log(\kappa) + \kappa \log(1 - \frac{3}{\kappa}) \\
\Leftrightarrow\ &m \geq \frac{\log(\epsilon_F/\kappa)}{\log(1 - \frac{3}{\kappa})} + \kappa \\
\Leftrightarrow\ &m \geq \frac{-\log(\epsilon_F/\kappa)}{-\log(1 - \frac{3}{\kappa})} + \kappa \\
\Leftrightarrow\ &m \geq \frac{\log(\epsilon_F^{-1}\kappa)}{-\log(1 - \frac{3}{\kappa})} + \kappa
\end{aligned}$$

Since $\frac{\log(\epsilon_F^{-1}\kappa)}{-\log(1-\frac{3}{\kappa})} = O(\kappa \log^2(\kappa) \log(\epsilon_F^{-1}))$ and we choose the smallest m possible for algorithm 7, we get $m = O(\kappa \log^2(\kappa) \log(\epsilon_F^{-1}))$.