# Unfoldings of 4D-Hypercube Unfoldings that tile R2

Presentation

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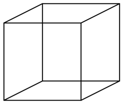
# Section

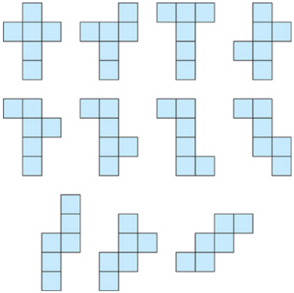
1 Introduction

2 Results

# Background (3D)

This 3D-cube has the following properties:





- Tiles $R^3$
- 11 edge-unfoldings each of which tiles $R^2$
- Therefore it is a Dimension-descending Tiler (DDT)

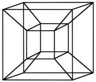
# Result

## DDT

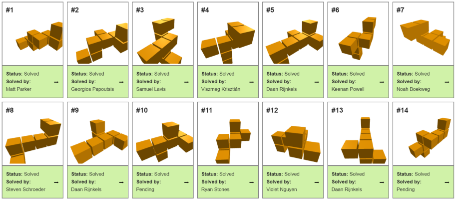
A polytope that monohedrally tiles $R^d$ is a DDT if all of its facet-unfoldings tile $R^{d-1}$, and every $R^{d-1}$ polytope has a facet-unfolding that tiles $R^{d-2}$ and so on till an edge-unfolding that tiles $R^2$.

# Background (4D)

This 4D-cube has the following properties:



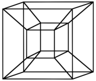
- Tiles $R^4$
- 261 face-unfoldings each of which tiles $R^3$ [Tur84][DR15][Wh21]
- 2 (of 261) face-unfoldings have edge-unfoldings that tile $R^2$ [DR15][AS16]




Screenshot of Whuts.org [Wh21]

# Background (4D)

This 4D-cube has the following properties:



- Tiles $R^4$
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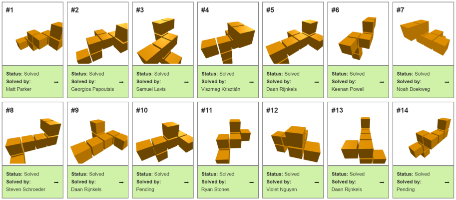



Screenshot of Whuts.org [Wh21]

- X? (of 261) face-unfoldings have edge-unfoldings that tile $R^2$ [S. Langerman]

# Question

Question posed by Stefan Langerman:

**Question 1**

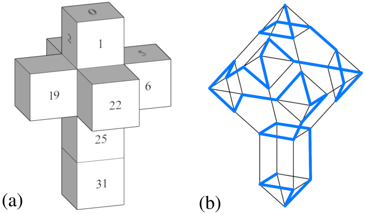
Does every face-unfolding of the hypercube have an edge-unfolding that tiles space?

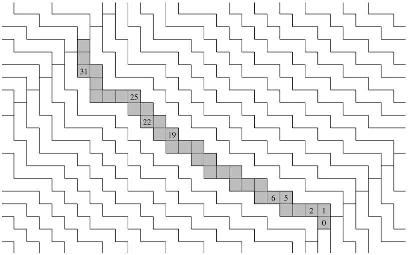
# Linear Unfoldings

## Question 1

Let T be a hamiltonian path on the dual graph of the polycube. The unfolding corresponding to T is linear if direction x is used and direction −x is not.

(a) Dali Cross
(b) A hamiltonian path on the dual graph of the Dali Graph



# Result

## Theorem 1

Every face-unfolding of the hypercube has an edge-unfolding that tiles $R^2$

## Tesseract is c-DDT

A polytope that monohedrally tiles $R^d$ is a c-DDT if all of its facet-unfoldings tile $R^{d-1}$, and every $R^{d-1}$ polytope has a facet-unfolding that tiles $R^{d-2}$ . The $R^{d-2}$ polytope must then have a facet-unfolding that tiles $R^{d-3}$ and so on till an edge-unfolding that tiles $R^2$.

# References

[Tur84] P. Turney. Unfolding the tesseract. Journal of Recreational Mathematics, Vol. 17(1), 1984-85

[DR15] G. Diaz and J. O' Rourke. Hypercube unfoldings that tile and . Technical report, arXiv, 2015. http://arxiv.org/abs/1512.02086.

[Wh21] Whuts: Which Hypercube Unfoldings Tile Space, 2021. https://whuts.org

[AS16] S. Langerman and A. Winslow. Polycube Unfoldings Satisfying Conway's Criterion, 2016. https://andrewwinslow.com/papers/polyunfold-jcdcggg16.pdf