COMPARISON BETWEEN FOC AND DTC STRATEGIES FOR PERMANENT MAGNET SYNCHRONOUS MOTORS

X. T. Garcia[1], B. Zigmund[2], A. Terlizzi[3], R. Pavlanin[2], L. Salvatore[3]

[1]*University of Glamorgan, School of Electronics, CF37 1DL Pontypridd, Wales, UK, xdeltoro@glam.ac.uk*
[2]*University of Zilina, Faculty of Electrical Engineering, Department of Mechatronics and Electronics, Univerzitna 1, 010 26 Zilina, SK, brano.zigmund@gmail.com, pavlanin@fel.utc.sk*
[3]*Politecnico di Bari, Dipartimento di Elettrotecnica ed Elettronica, V. Orabona 4, 70125 Bari, Italy, salvatore@poliba.it*

Summary This paper presents a comparison between two control strategies for Permanent Magnet Synchronous Motors (PMSM): Field Oriented Control (FOC) and Direct Torque Control (DTC). These two strategies can be considered among the family of Vector Control (VC) methods and provide a solution for high-performance drives. This paper presents the implementation of both strategies in PMSM drives. Advantages and disadvantages of both methods are discussed and different simulation tests are performed to illustrate the features of both methods. The criteria followed to establish a fair comparison between both control methods is also presented.

1. INTRODUCTION

The main feature of Synchronous Motors (SM) is the fact that the rotating speed of the rotor is equal to the frequency of the supply voltage divided by the number of pole pairs. The rotor of the synchronous motor can be built using an electrically excited winding or alternatively a Permanent Magnet (PM). In the second case, the resulting motor is called Permanent Magnet Synchronous Motor (PMSM).

Induction Motors (IM) are nowadays dominating the drive market and have substituted the DC motor in high performance applications where variable speed and torque control is needed. Nevertheless, PMSM are gaining market since the introduction of new materials like neodymium-iron-boron (NdFeB) in 1983. The main advantages of PMSM are [1, 2]:

- Absence of brushes and slip rings, lower maintenance is required.
- Lower inertia and better dynamic performance.
- Higher efficiency, there are no rotor losses.
- Higher power/weight ratio.

These merits are counterbalanced by the higher cost and the variation of the PM properties during time and with temperature. PMSM are currently employed in applications where high acceleration and precise control is required such as robotics and machine tools.

The control of the IM and the PMSM is not a trivial matter when compared to the DC motor. Only after the introduction of the Vector Control (VC) concept in the early 70's a precise control method for both steady-state and transients was available. The first and most popular VC method was Field Oriented Control (FOC) [3]. Later in mid 80's a new VC method appeared to become an alternative to FOC. This method was born in parallel with two different names: Direct Torque Control (DTC) [4] and Direct Self control (DSC) [5]. The main feature of DTC is the high performance achieved with a simpler structure and control diagram. Both methods, FOC and DTC, achieve decoupled control of torque and flux and they were first implemented in the control of IM drives. More recently and due to their success FOC and DTC were also applied to PMSM drives [6, 7].

The aim of this paper is to compare the performance of both FOC and DTC when applied to PMSM drives and point out the strengths and weaknesses that can help to make a choice between them for a particular application. This kind of comparison has already been made for IM drives [8]. When analysing both methods it is very important to establish the conditions to have a fair comparison between them.

The paper starts by presenting the PMSM model and the operating principles of FOC and DTC. This is followed by an explanation of the comparative analysis and simulation tests performed. Finally the main characteristics, advantages and disadvantages of both methods are shown and discussed in the results of the simulation tests.

2. PMSM MODEL

The mathematical model of the PMSM is generally presented in a rotating *d-q* frame fixed to the rotor. The resulting model is described by the following equations:

$$\psi_{sd} = L_{sd} i_{sd} + \Psi \tag{1}$$

$$\psi_{sq} = L_{sq} i_{sq} \tag{2}$$

$$v_{sd} = R_s i_{sd} + L_{sd}\frac{di_{sd}}{dt} - \omega_r L_{sq} i_{sq} \tag{3}$$

$$v_{sq} = R_s i_{sq} + L_{sq}\frac{di_{sq}}{dt} + \omega_r \left(L_{sd} i_{sd} + \Psi \right) \tag{4}$$

$$\Gamma_e = \frac{3}{2} P\left(\psi_{sd} i_{sq} - \psi_{sq} i_{sd} \right) \tag{5}$$

where ψ_{sd}, ψ_{sq}, v_{sd}, v_{sq}, i_{sd} and i_{sq} are respectively the motor fluxes, voltages and currents in *d-q* coordinates; ω_r is the electrical angular speed and Γ_e is the electromagnetic torque. Regarding the motor parameters, Ψ is the flux of the permanent magnet, P is the number of pole pairs, R_s is the stator resistance and the stator inductance can be divided into two different components L_{sd} and L_{sq} due to the particularities of the PMSM. If the motor has Surface Mounted (SM)

PM, both inductances have similar values and for simplification they can be considered equal. The model is completed with the mechanical equation, which is defined as:

$$J\frac{d\omega_m}{dt} = \Gamma_e - \Gamma_l - B\omega_m \tag{6}$$

$$\omega_r = P\omega_m \tag{7}$$

where J is the inertia of the motor and coupled load, Γ_l is the load torque, B is the friction coefficient and ω_m is the mechanical angular speed.

In order to understand the torque production in the PMSM (5) can be rewritten to obtain an expression of the torque as a function of the stator flux and the PM flux:

$$\Gamma_e = \frac{3}{2}P\frac{\psi_s \Psi}{L_s}\sin\delta \tag{8}$$

It can be seen from (8) that the torque produced depends on the amplitude of the stator flux, the PM flux and the angle between both fluxes. It can also be concluded that maximum torque is produced when the angle between both fluxes is 90 degrees. Fig. 1 shows a vector diagram of the fluxes in the cross section of the motor.

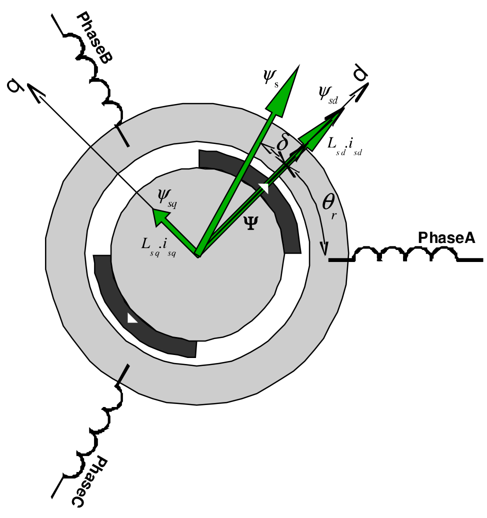

Fig. 1. Vector diagram of the PMSM cross section

3. FIELD ORIENTED CONTROL

Similarly to the IM, in the PMSM a decoupled control of the torque and flux magnitudes can be achieved, emulating a DC motor, by means of the FOC strategy [9]. This is done using the *d-q* transformation that separates the components *d* and *q* of the stator current responsible for flux and torque production respectively. Due to the presence of the constant flux of the permanent magnet, there is no need to generate flux by means of the i_{sd} current, and this current can be kept to a zero value, which in turns decreases the stator current and increases the efficiency of the drive. The control scheme of the FOC strategy is shown in Fig. 2:

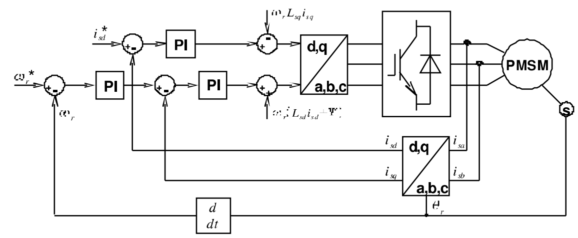

Fig. 2. FOC control scheme for PMSM

The control system is divided into three different loops: the *d* loop, which controls the flux; and the *q* loops, which control the speed and torque. The *d* loop performs the control of i_{sd} with a current PI regulator. The reference value for this loop can be set to 0. The *q* loops are connected in cascade. The inner loop controls the torque by means of controlling i_{sq} with a current PI regulator. The fact that the torque can be controlled by means of i_{sq} comes from the following simplification of (5), valid for Surface Mounted (SM) PMSM:

$$\Gamma_e = \frac{3}{2}P\Psi i_{sq} \tag{9}$$

The reference for this inner loop is given by the speed PI regulator of the outer loop. From the voltage equations of the PMSM model (3) and (4) it can be seen that *d* and *q* axis are not completely independent and there are coupling terms which depend on the current from the other axis. To achieve completely independent regulation it is necessary to cancel the effect of these coupling terms at the output of the current PI regulator (see Fig.2). The use of decoupling achieves the linearization of the control system as well as higher dynamics.

4. DIRECT TORQUE CONTROL

The DTC principle can be explained by means of equation (8). Considering the modulus of the stator flux constant, torque can be controlled by changing the relative angle between the stator flux and the PM flux vectors. Stator flux can be adjusted by means of the stator voltage according to the stator voltage equation in stator fixed coordinates:

$$\vec{u}_s = R_s\vec{i}_s + \frac{d\vec{\psi}_s}{dt} \tag{10}$$

If the voltage drop in the stator resistance is neglected the variation of the stator flux is directly proportional to the stator voltage applied:

$$\vec{u}_s \approx \frac{d\vec{\psi}_s}{dt} \qquad \Delta\vec{\psi}_s \approx \vec{u}_s\Delta t \tag{11}$$

Thus torque can be controlled by quickly varying the

stator flux position by means of the stator voltage applied to the motor. The desired decoupled control of the stator flux modulus and torque is achieved by acting on the radial (x) and tangential (y) components respectively of the stator flux vector. According to (11) these two components will depend on the components of the stator voltage vector applied in the same directions. The tangential component of the stator voltage will affect the relative angle between the PM flux and the stator flux vectors and in turns will control the torque variation according to (8). The radial component will affect the amplitude of the stator flux vector.
Fig. 3 shows the stator flux in the α-β plane, and the effect of the different states of a two-level VSI regarding torque and stator flux modulus variation. The α-β plane is divided into six different sectors (K=1,…,6). As an example, for sector 1 (K=1), V2 can increase both stator flux and torque.

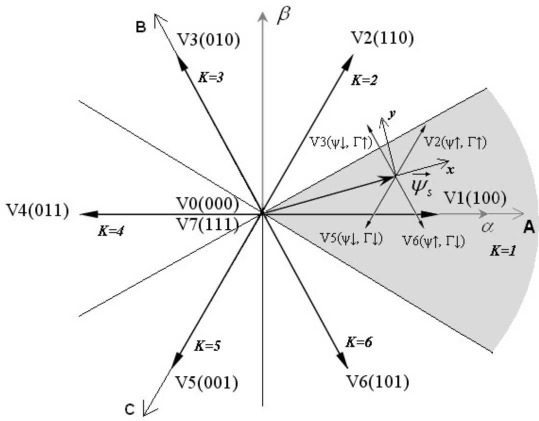

Fig. 3. Influence of the voltage vector selected on the variation of stator flux modulus and torque

Following all the considerations made, the control scheme of DTC for PMSM is developed as shown in Fig. 4. As it can be seen, there are two different loops corresponding to the magnitudes of the stator flux modulus and torque. The reference values for the stator flux modulus and the torque are compared with the estimated values, the resulting error values are fed into two and three-level hysteresis blocks respectively. In [7] the hysteresis block for torque error has only two levels and zero vectors are never selected. This choice produces a considerable torque ripple that can be reduced using the classical approach of [4]. The outputs of the stator flux error and torque error hysteresis blocks, together with the sector position of the stator flux are used as inputs to the look-up table (Tab. 1). The sector position is found according to Fig. 3 and defining the stator flux vector in a polar way as follows:

$$\vec{\psi}_s = \psi_s e^{j\gamma_s} \tag{12}$$

The output of the look-up table is the VSI state that will be applied during a sampling period. The stator flux modulus and torque errors tend to be restricted within their respective hysteresis bands.

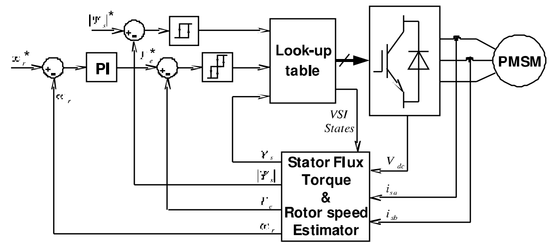

Fig. 4. DTC control scheme for PMSM

Tab. 1. Classical DTC look-up table

$K(\gamma_s)$		1	2	3	4	5	6
$d_\psi = 1$	$d_\Gamma = 1$	V2	V3	V4	V5	V6	V1
	$d_\Gamma = 0$	V7	V0	V7	V0	V7	V0
	$d_\Gamma = -1$	V6	V1	V2	V3	V4	V5
$d_\psi = -1$	$d_\Gamma = 1$	V3	V4	V5	V6	V1	V2
	$d_\Gamma = 0$	V0	V7	V0	V7	V0	V7
	$d_\Gamma = -1$	V5	V6	V1	V2	V3	V4

Regarding the stator flux reference value, it can be fixed to the nominal value or be variable in order to make i_{sd}=0 using the following expression:

$$\psi_s = \sqrt{\Psi^2 + L_{sq}^2 i_{sq}^2} \tag{13}$$

Additionally, DTC requires the estimation of stator flux and torque. In a sensorless implementation without position sensor this estimation can be performed by means of two stator phase currents, the state of the VSI and the voltage level in the DC-link. In the classical DTC this estimation is based on the integration of the stator voltage equation:

$$\vec{\psi}_s = \int \left(\vec{u}_s - R_s \vec{i}_s \right) dt \tag{14}$$

A different possibility when the position sensor is available is to use two phase currents and the rotor position. This estimator will use equations (1) and (2) to estimate the stator flux. These equations are expressed in d-q coordinates and some transformations are therefore necessary. First the current has to be transformed from three-axes fixed coordinates to d-q coordinates. Once the stator flux is calculated in d-q coordinates it has to be transformed to α-β coordinates. Finally the modulus and argument of the stator flux can be calculated from the α-β components. Once the stator flux is obtained, equation (5) can be used to estimate the torque value.

It can be said to conclude that the lower inductance of the PMSM when compared to the IM has as a result higher torque ripples due to the quicker variation of current when DTC is employed. In order to compensate this problem lower sampling time has to be used to reduce the torque ripple to an acceptable level.

5. COMPARISON BETWEEN FOC AND DTC

In order to carry out a comparative analysis of FOC and DTC, the behaviour of both methods during transients and steady-state operation must be studied. Regarding transients the main characteristic to be analysed is the time response to a torque step. This test can be performed at different speeds. For the steady-state performance, two different characteristics can be analysed regarding flux and torque responses: the average error as defined in (15) and the oscillation or ripple in the torque and stator flux that can be calculated by means of the standard deviation as defined in (16) (being n the number of samples):

$$\bar{e} = \frac{1}{n}\sum_{i=1}^{n} e_i \tag{15}$$

$$\sigma = \left(\frac{1}{n-1}\sum_{i=1}^{n}\left(e_i - \bar{e}\right)^2\right)^{1/2} \tag{16}$$

Another interesting feature in steady-state operation is the distortion of the stator phase currents. This can be evaluated by means of the current spectrum and the Total Harmonic Distortion (THD). The presence of harmonics in the audible noise band can be noticed from this analysis.

In [8], where both control methods are compared for the IM, it is discussed how FOC and DTC can be fairly compared. The authors claim that a fair comparison can only be made if the average switching frequency of the inverter is approximately the same. In FOC the switching frequency is adjusted by the PWM period. DTC, however, has variable frequency due to the hysteresis blocks, which depends on the operating point. The variation of the switching frequency depending on the speed and load torque is illustrated in Fig. 5.

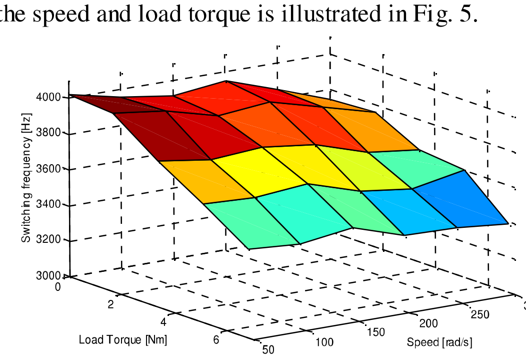

Fig. 5. Variation of switching frequency in DTC of PMSM for steady-state depending on the operating point

Fig. 6 presents the variation of the switching frequency during a transient, when a step change of the load torque at 0 rad/s speed occurs. During this transient torque error is bigger and active VSI vectors are applied for longer time, reducing the switching frequency.

In order to have similar switching frequency in the DTC and FOC systems the hysteresis bands of the DTC scheme must be adjusted.

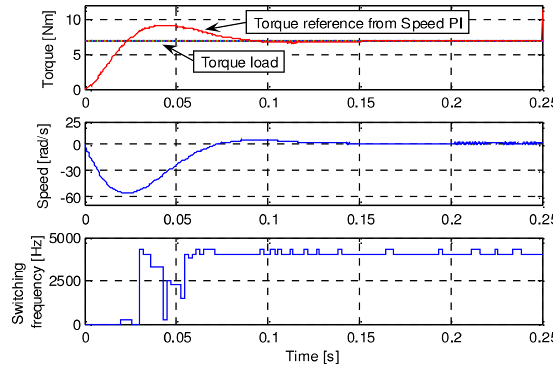

Fig. 6. Variation of switching frequency during a transient in DTC

In this case the values employed for the hysteresis bands are 2.5% for flux and 7.5% for torque.

In [8] it is also said that the sampling time of the control loop cannot be equal in both systems. First of all equal sampling times would imply different switching frequencies. Secondly the main advantages of DTC are the simplicity and lower calculation requirements when compared to FOC, and therefore these advantages must be exploited to have a fair comparison.

6. SIMULATION RESULTS

Some simulation tests have been designed and executed to obtain comparative results of both systems. The parameters shown in Table 2 corresponding to a real motor have been used in the simulation model. The sampling time used in the control loop is 25µs for DTC and 100µs for FOC. In the FOC system the current PI regulators have been tuned using the Absolute Value Optimum (AVO) criterion (K_p=8.86, K_i=778.6), and the Symmetric Optimum (SO) criterion has been used for the speed PI (K_p=0.0934, K_i=3.18) [10].

Tab. 2. PMSM characteristics (Siemens 1KF7)

Nominal Output Power (P_n)	2135W
Nominal Speed (ω_n)	3000rpm
Nominal Torque (M_n)	6.8N·m
Nominal Current (I_n)	4.4A
Number of pole pairs (P)	4
Stator resistance (R_s)	1.09Ohm
Stator inductance (L_{sd} and L_{sq})	0.0124H
Inertia (J)	$4.15e^{-4}$ Kg·m^2
Permanent Magnet Flux (Ψ)	0.1821Wb

The first test performed is shown in Fig. 7. It consists on a torque step change from 0 to nominal torque at 3 different speeds. It can be seen how the response time is considerably smaller for DTC when compared to FOC. Table 3 contains the settling time of both systems. It can also be observed a higher torque ripple for DTC.

Tab. 3. Torque settling time at different speeds

Electrical speed	**DTC**	**FOC**
0 rad/s	0.22ms	6ms
300 rad/s	0.32ms	5ms
1200 rad/s	1ms	15ms

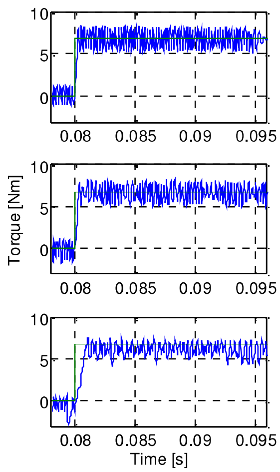

Fig. 7. Torque transients for DTC (left) and FOC (right) at 0 rad/s (up), 300 rad/s (middle) and 1200 rad/s (down)

The second test performed for transient conditions consists on the response to a speed step change. Fig. 8 shows the stator flux path in the α-β plane during the test. Fig. 9 shows the speed, torque, stator flux modulus, i_{sd} and i_{sq} responses. It can also be seen how the torque response for DTC performs a better tracking of its reference and as a result the rise time of the speed response is slightly smaller. It can also be noticed the higher ripple of the DTC system regarding the flux and torque responses.

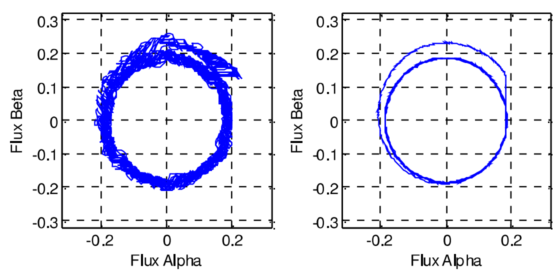

Fig. 8. Stator flux circular path in α-β coordinates during the speed step response for DTC (left) and FOC (right)

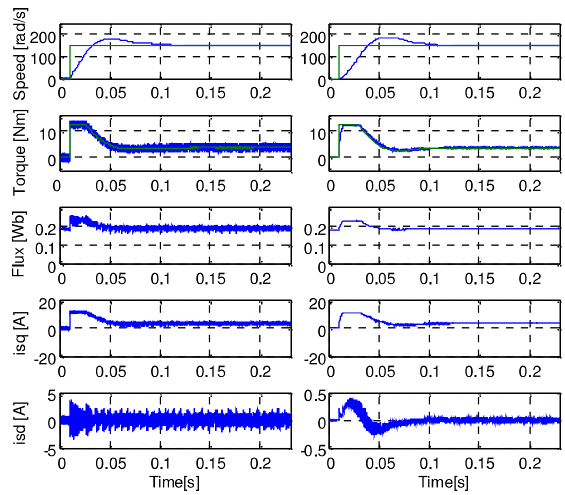

Fig. 9. Torque and stator flux behaviour during a speed step change for DTC (left) and FOC (right)

It can be seen in Fig. 9 how the higher torque ripple is not affecting the speed response due to the inertia of the motor.

Some additional tests have been carried out to assess the behaviour of both systems in steady-state conditions. The torque and stator flux average errors and standard deviation have been calculated for both systems at 9 different operation points obtained combining three different speeds (0, 150 and 300 rad/s) with three different levels of load torque (0, 3.4 and 6.8 Nm). The resulting values have been averaged and are presented in Table 4. They are presented as a percentage of the nominal values of the stator flux modulus and torque.

Tab. 4. Steady-state performance indexes

	DTC	FOC
Stator Flux Average Error	0.3%	0.1%
Stator Flux Standard Deviation	4.85%	0.15%
Torque Average Error	1.8%	0.08%
Torque Standard Deviation	12.8%	2.81%

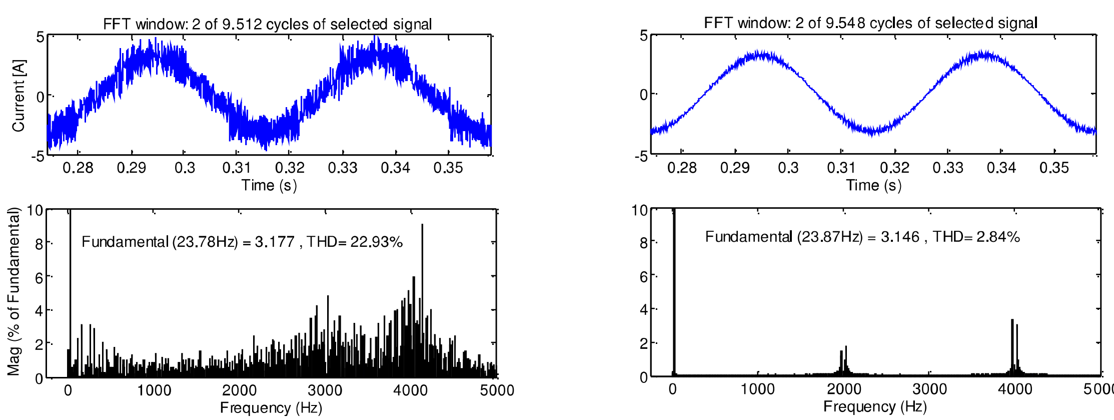

Fig. 10. Steady-state stator phase currents and harmonic spectrum at 150 rad/s and 3.4 Nm for DTC (left) and FOC (right)

Tab. 5. Summary of the comparison between FOC and DTC

	DTC	FOC
Dynamic response for torque	Quicker	Slower
Steady-state behaviour for torque, stator flux and currents	High ripple and distortion	Low ripple and distortion
Parameter sensitivity	• For a sensorless estimator: R_s • For a non-sensorless estimator: L_{sd}, L_{sq} and Ψ	Decoupling depends on L_{sd}, L_{sq} and Ψ
Requirement of rotor position	No	Yes
Current control	No	Yes
PWM modulator	No	Yes
Coordinate transformation	No	Yes
Switching frequency	Variable, depending on the operating point and during transients	Constant
Audible noise	Spread spectrum, high noise especially at low speed	Low noise at a fixed frequency
Control tuning	Hysteresis bands	PI gains
Complexity and processing requirements	Lower	Higher

It can be seen how the FOC performance in steady-state is considerable superior to DTC. The current distortion has also been analysed and Fig. 10 shows the stator phase currents in steady-state at 150 rad/s and 3.4 Nm load. It can be seen that current distortion is considerably bigger for DTC and it is calculated the THD value, which is almost 10 times higher for DTC.

Finally Table 5 presents a summary of the comparison between DTC and FOC not only regarding the performance but also considering the control structure and requirements of both systems.

7. CONCLUSION

This paper has presented a comparison between two vector control methods for PMSM drives: FOC and DTC. Both methods provide a decoupled control of torque and flux during transients and steady-state. The description of both control schemes and their principle of operation has been presented. The criterions for a fair comparison between FOC and DTC have been established and the results of simulation tests have been presented to show the performance of both methods in various conditions. Summarising, it can be said that both methods provide a high performance response with quicker torque dynamics in the case of DTC and better steady-state behaviour for FOC. Depending on the requirements of a particular application one method can be more convenient than the other.

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