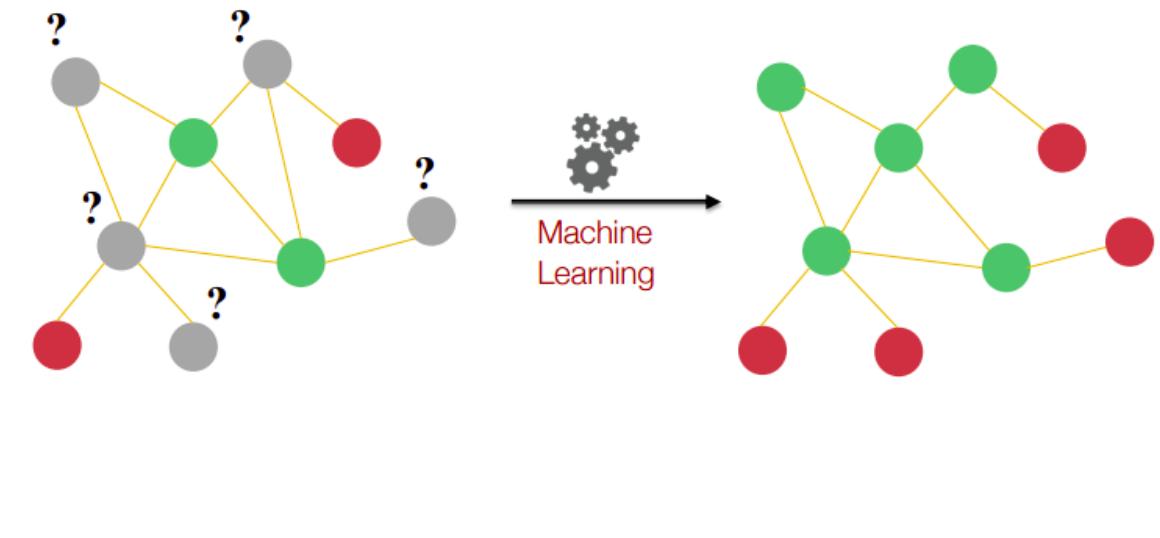
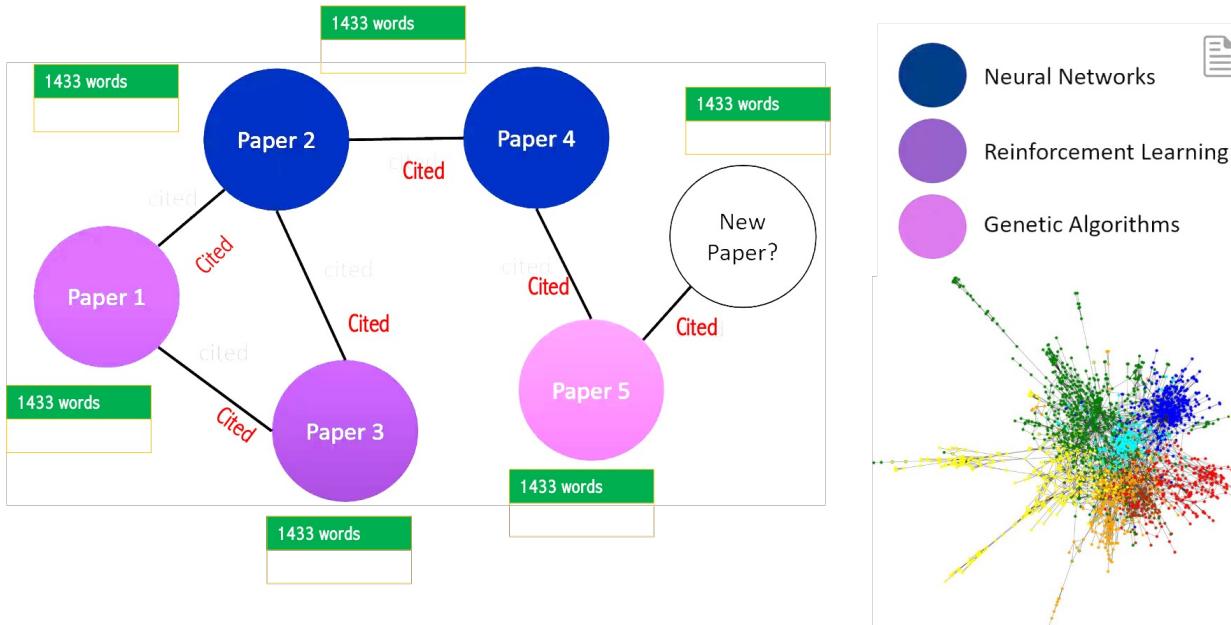


# Introduction to Graph Neural Network

## (Node Classification: Cora Citation Dataset)



Vinh Dinh Nguyen  
PhD in Computer Science

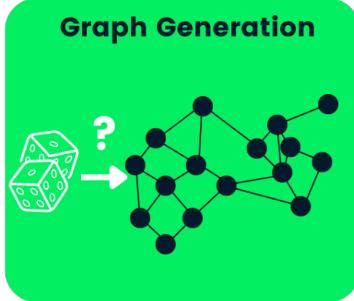
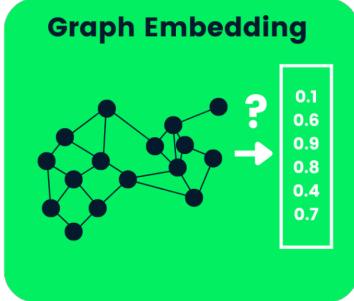
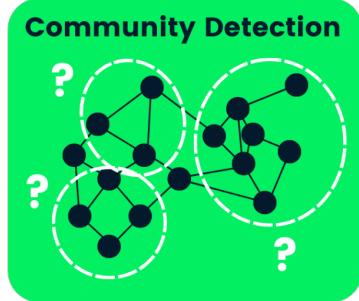
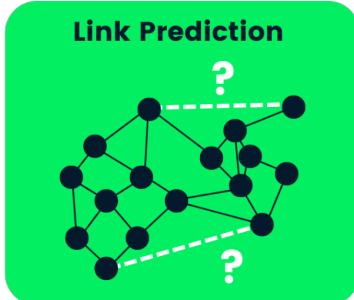
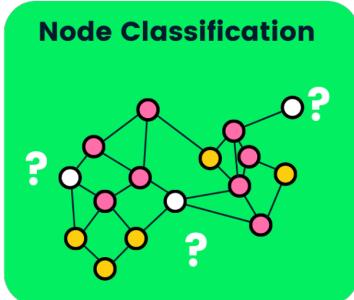
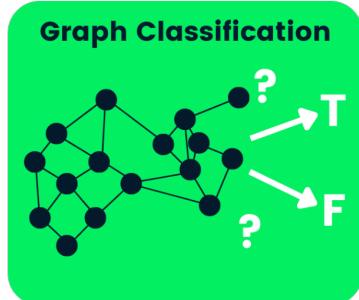
# Outline

- **Objective**
- **Introduction to Graph Data**
- **Graph Data with Neural Network**
- **Node Classification Problem: Cora Citation Dataset**
- **Summary**

# Outline

- **Objective**
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# Objective

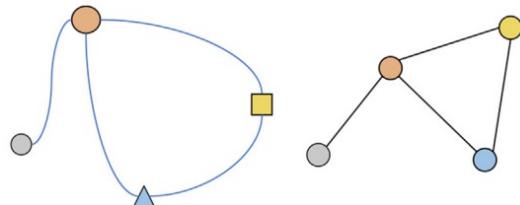


- What is Graph Data Around Us
- Understand Graph Neural Network
- Understand Graph Convolutional Neural Network
- Node Classification with Cora Citation Dataset

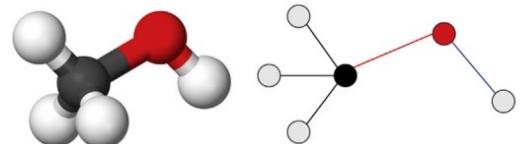
# Outline

- **Objective**
- **Introduction to Graph Data**
- **Graph Data with Neural Network**
- **Node Classification Problem: Cora Citation Dataset**
- **Summary**

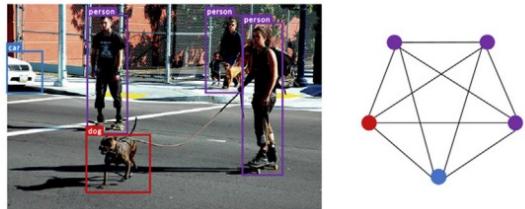
# Applications of GNNs



(a) Physics



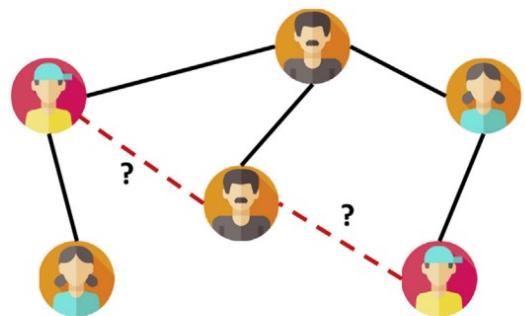
(b) Molecule



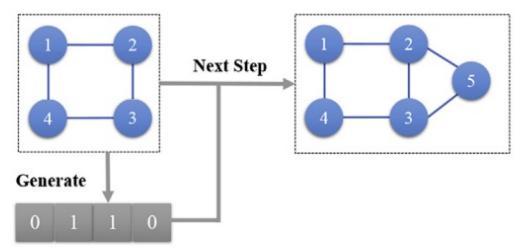
(c) Image

A sequence of words: The, quick, brown, fox, jumped, over, the, lazy, dog. Arrows show dependencies between words like "quick" → "brown" and "fox" → "jumped". Below the sequence is a graph where each word is a node and dependencies are edges.

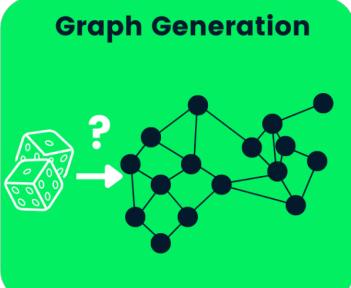
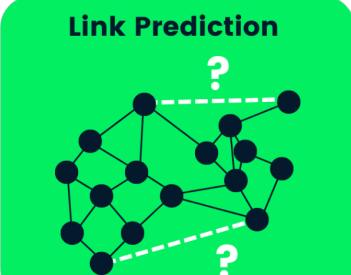
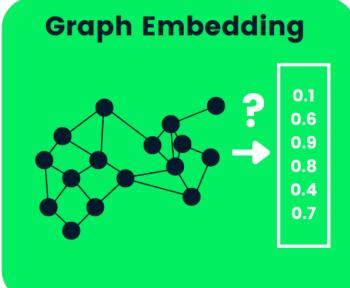
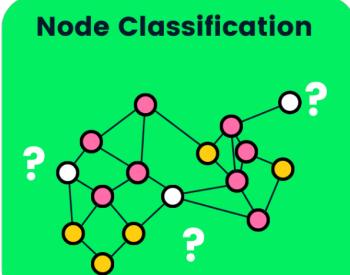
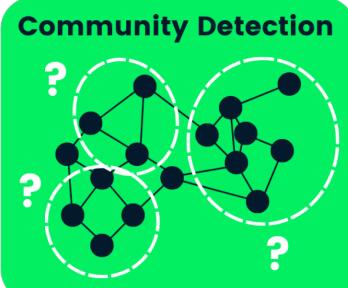
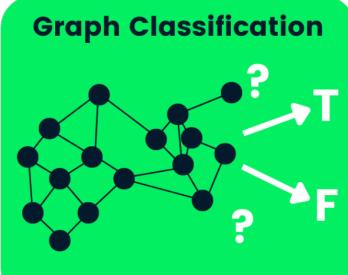
(d) Text



(e) Social Network

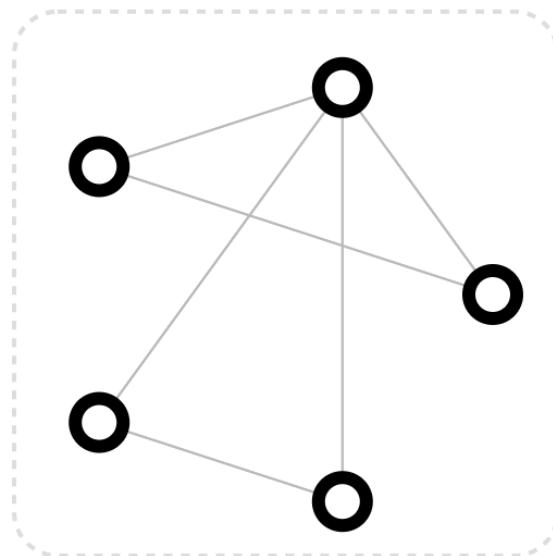


(f) Generation

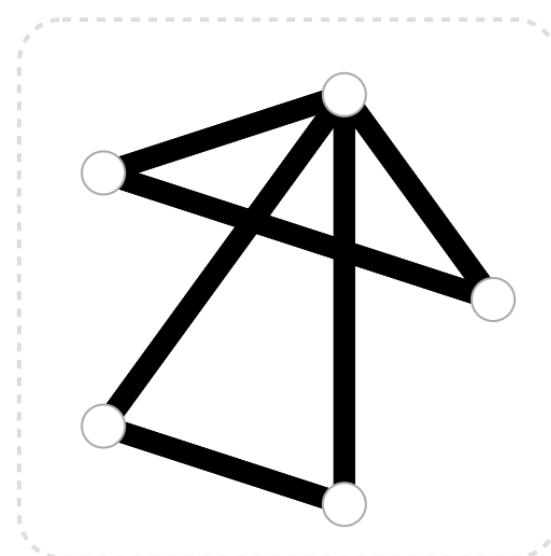


# Graph Definition

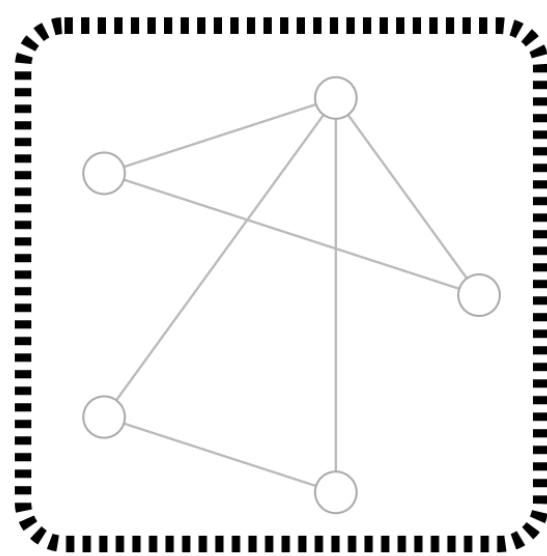
A graph represents the relations (edges) between a collection of entities (nodes).



**V** Vertex (or node) attributes  
e.g., node identity, number of neighbors

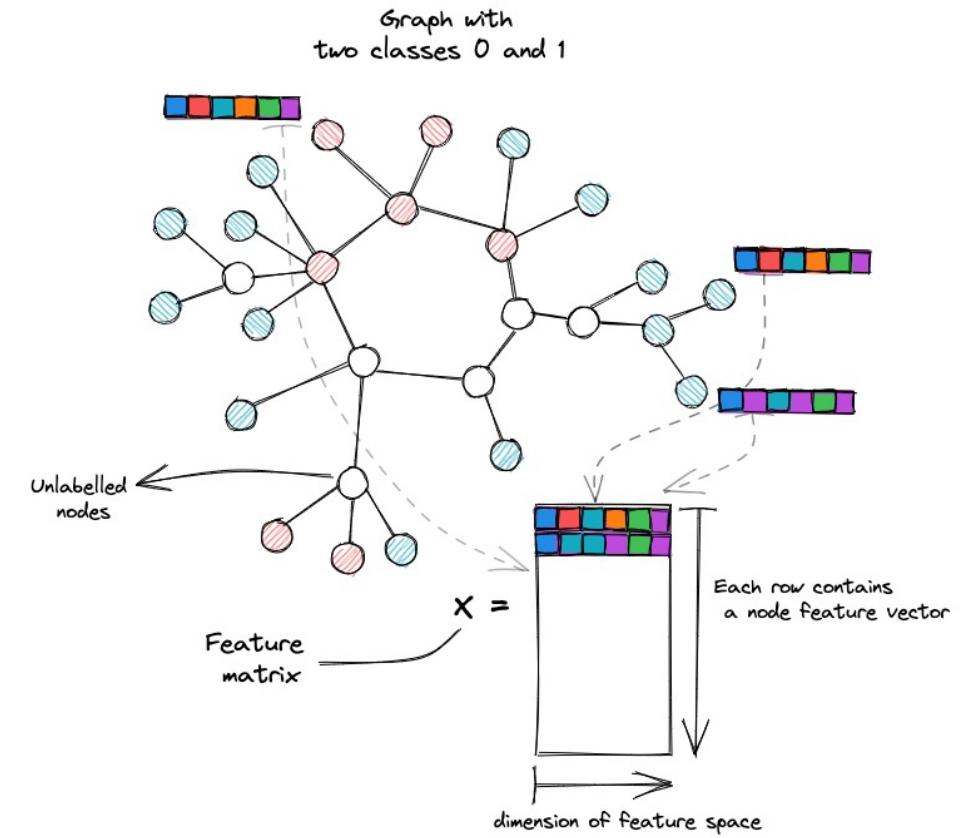
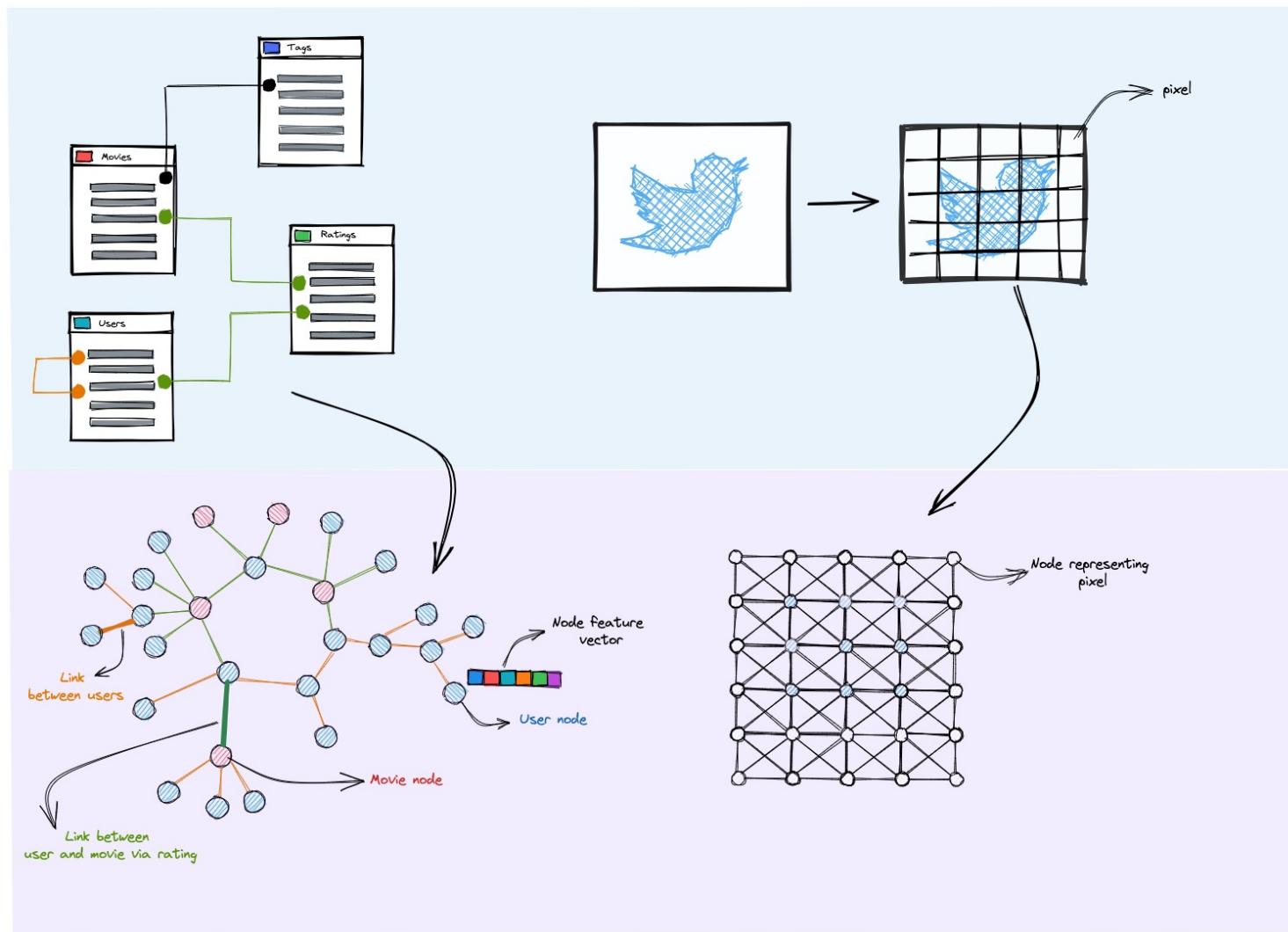


**E** Edge (or link) attributes and directions  
e.g., edge identity, edge weight



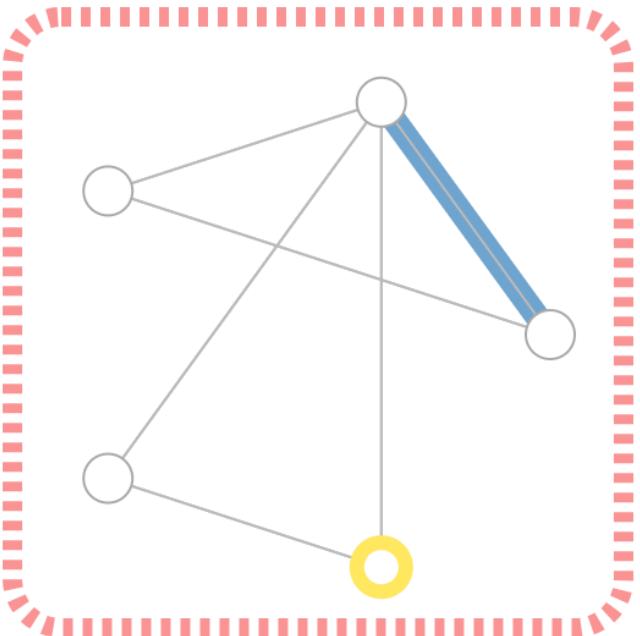
**U** Global (or master node) attributes  
e.g., number of nodes, longest path

# Graphs are everywhere



# Graph Definition

To further describe each node, edge or the entire graph, we can store information in each of these pieces of the graph



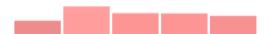
Vertex (or node) embedding



Edge (or link) attributes and embedding



Global (or master node) embedding



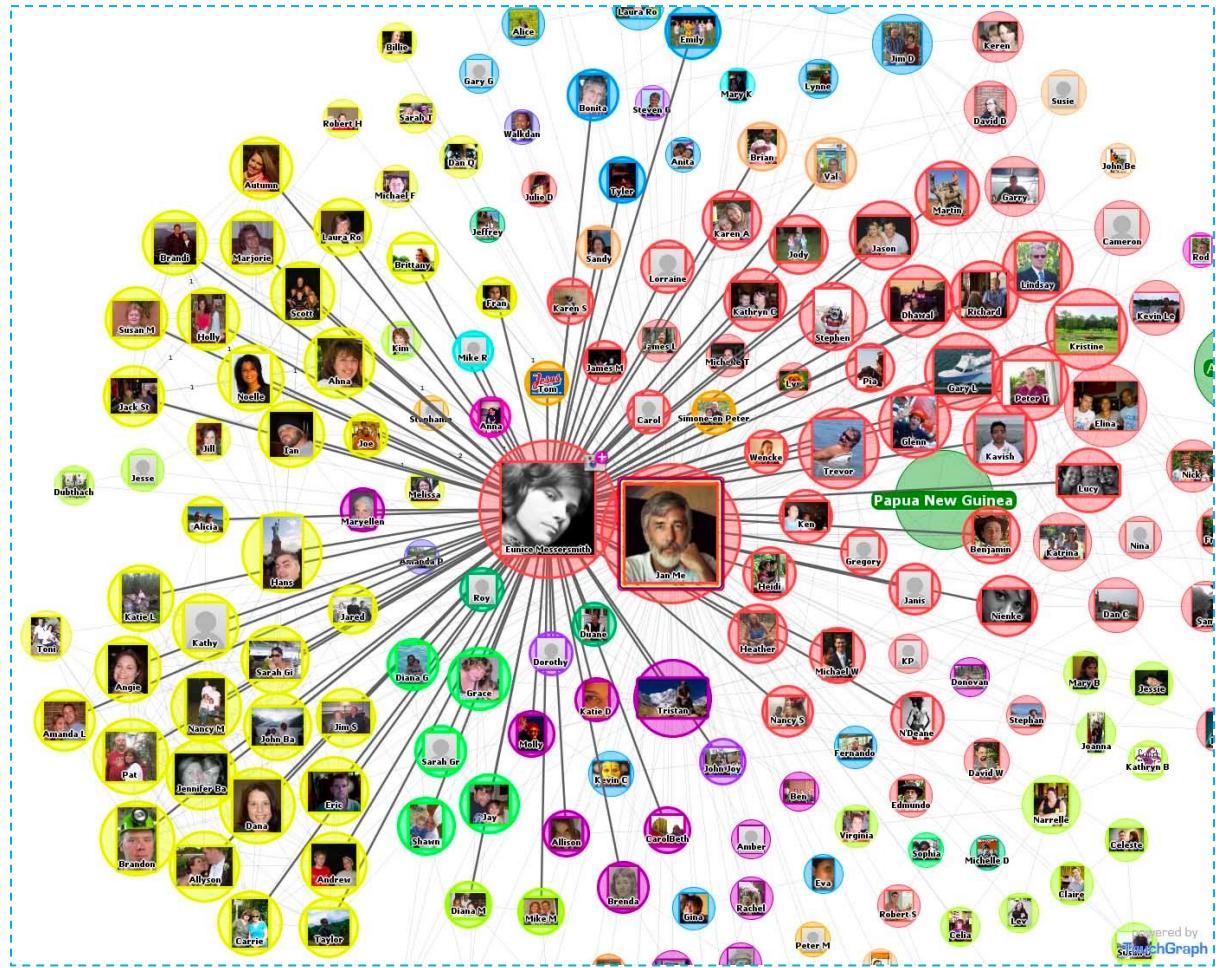
Undirected edge



Directed edge



# Graphs and where to find them



Social networks

Two types of data that you might not think could be modeled as graphs:



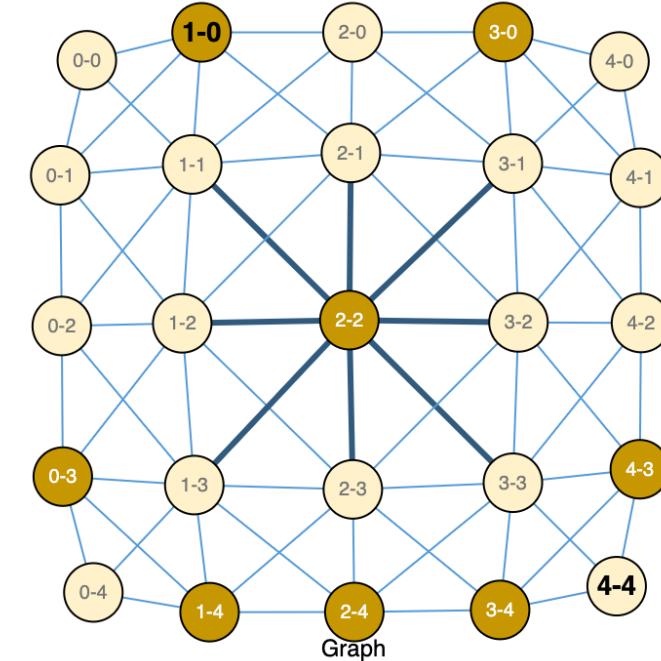
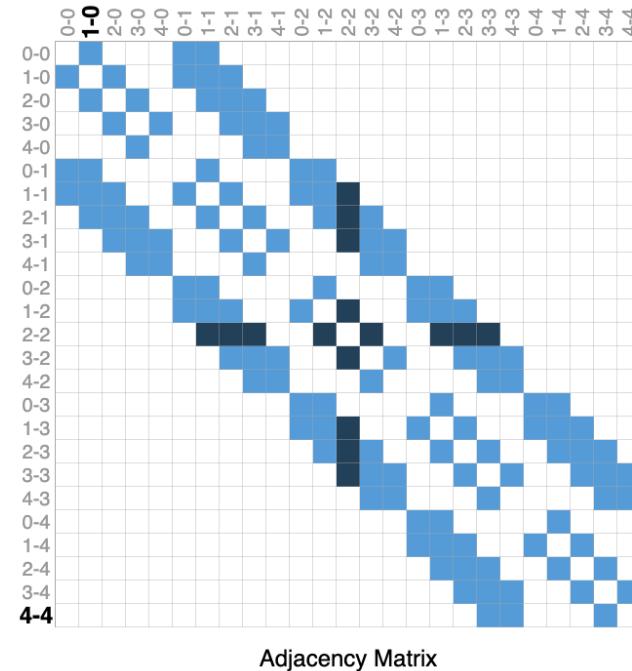
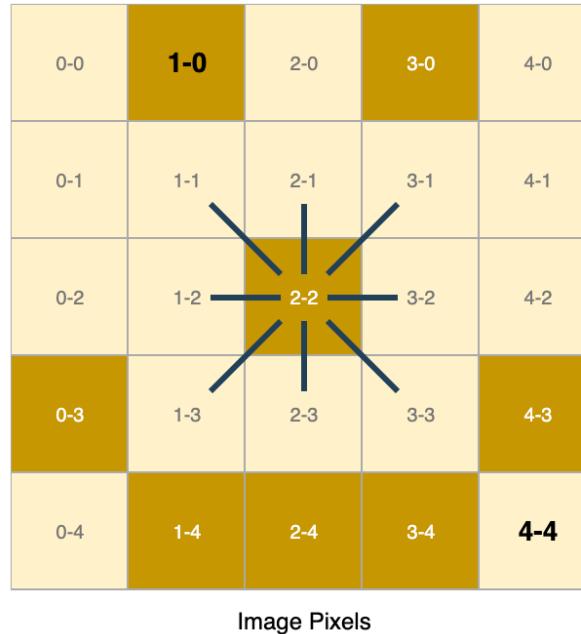
Image



Graphs are all around us

Text

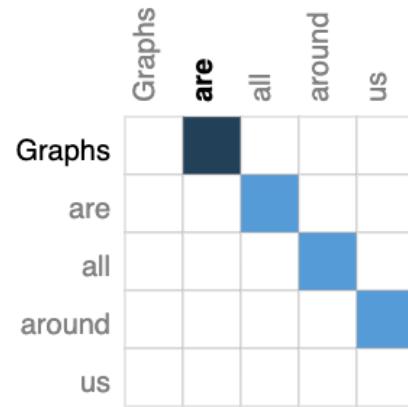
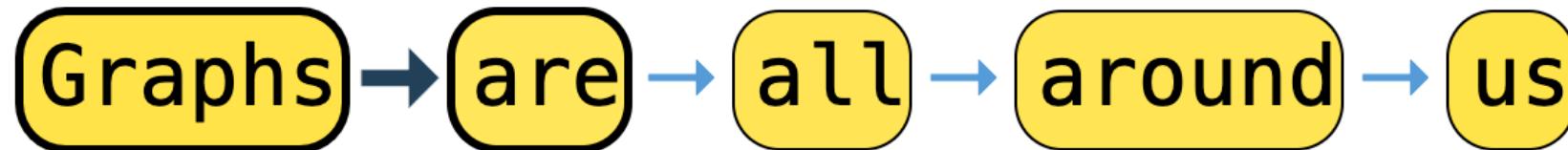
# Image as Graphs



Another way to think of images is as graphs with regular structure, where each pixel represents a node and is connected via an edge to adjacent pixels. Each non-border pixel has exactly 8 neighbors, and the information stored at each node is a 3-dimensional vector representing the RGB value of the pixel.

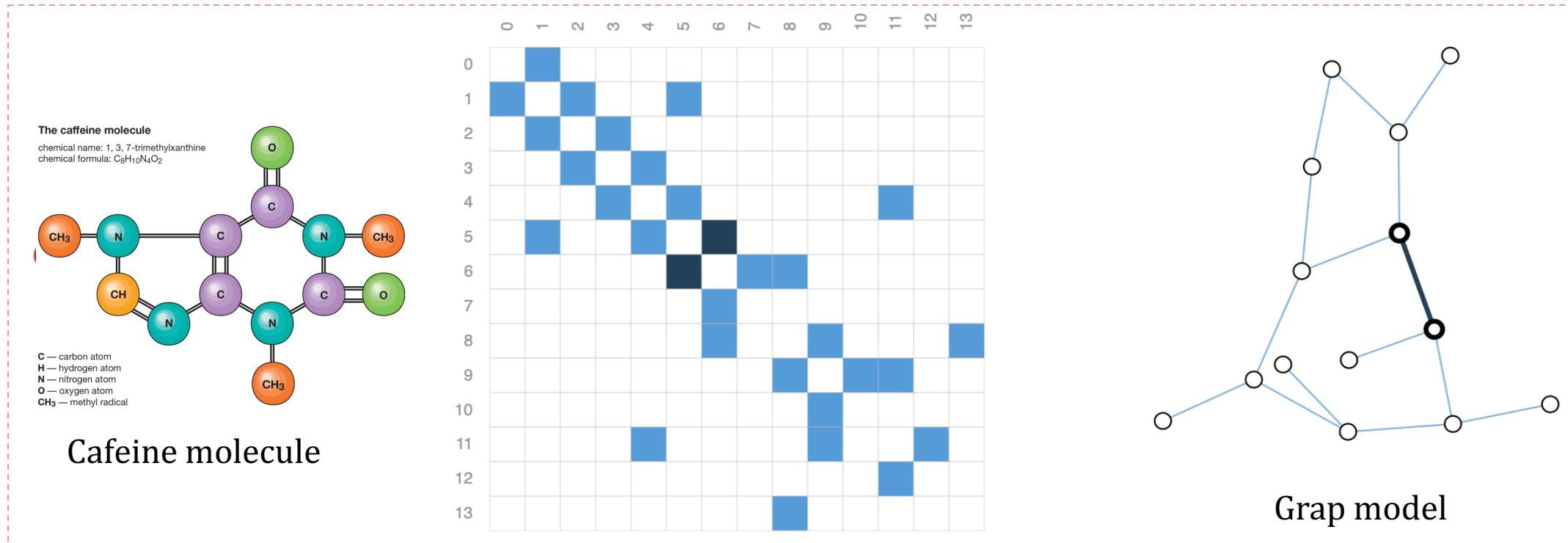
Source: <https://distill.pub/2021/gnn-intro/>

# Text as Graphs



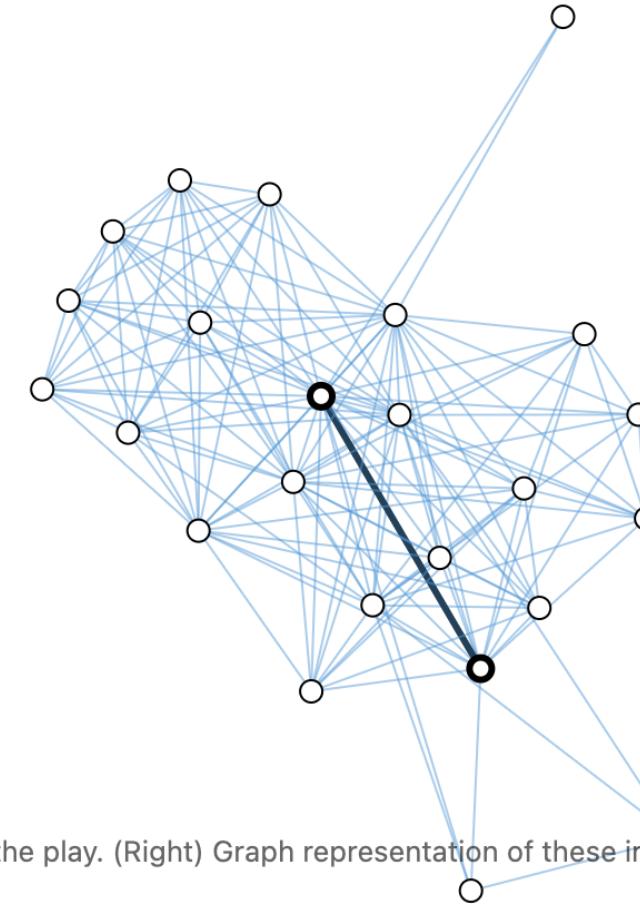
We can digitize text by associating indices to each character, word, or token, and representing text as a sequence of these indices. This creates a simple directed graph, where each character or index is a node and is connected via an edge to the node that follows it.

# Other Graph Data



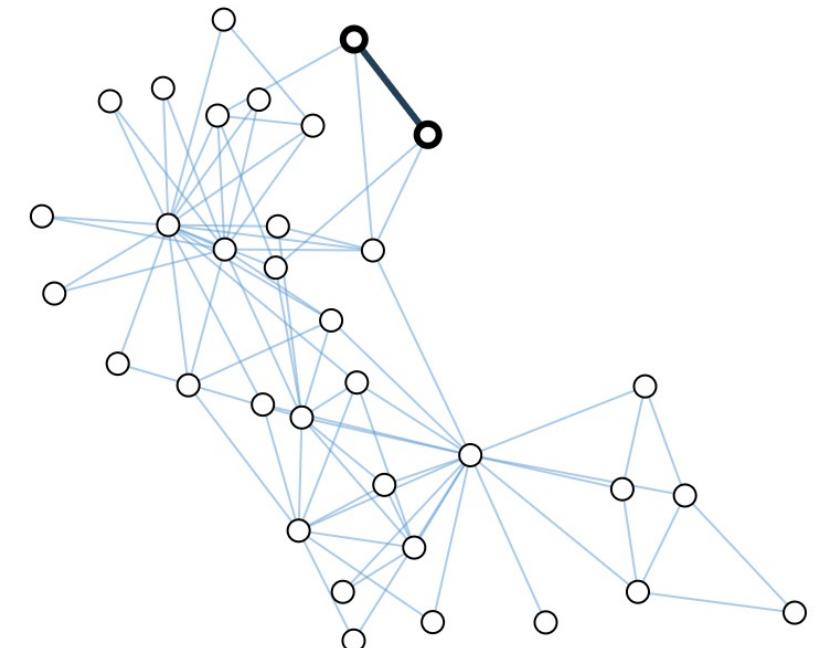
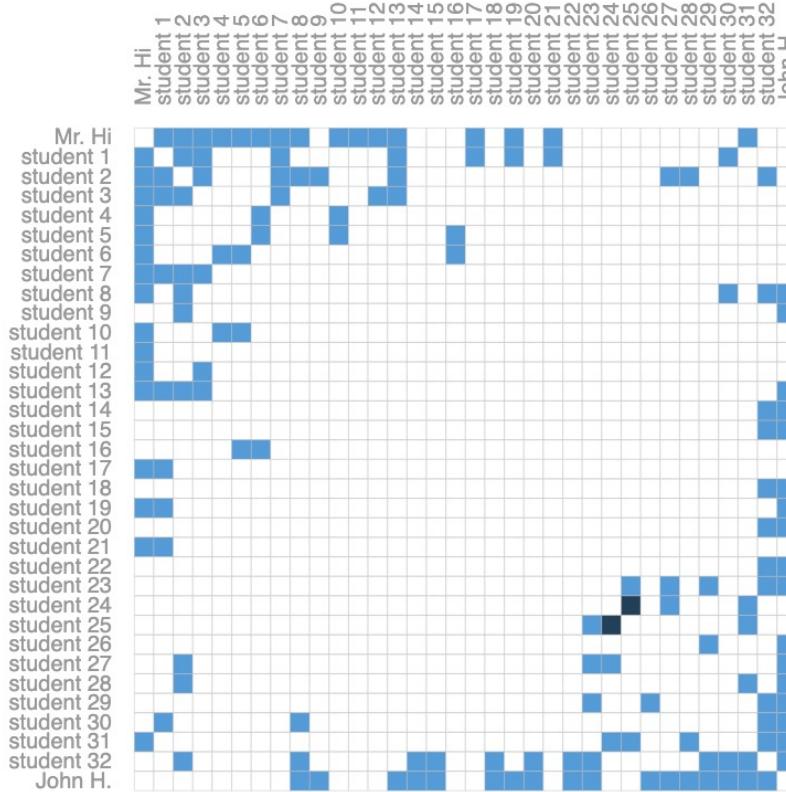
Molecules as graphs

# Other Graph Data



(Left) Image of a scene from the play "Othello". (Center) Adjacency matrix of the interaction between characters in the play. (Right) Graph representation of these interactions.

# Other Graph Data



(Left) Image of karate tournament. (Center) Adjacency matrix of the interaction between people in a karate club. (Right) Graph representation of these interactions.

# Other Graph Data

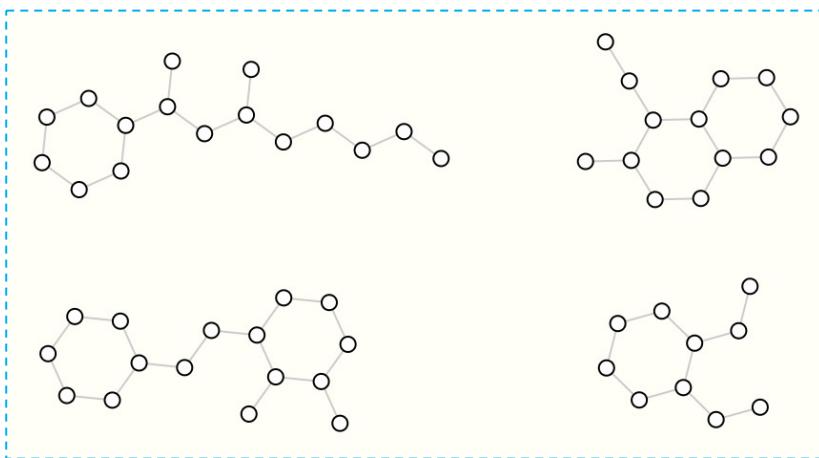
Dataset	Domain	Edges per node (degree)					
		graphs	nodes	edges	min	mean	max
karate club	Social network	1	34	78		4.5	17
qm9	Small molecules	134k	$\leq 9$	$\leq 26$	1	2	5
Cora	Citation network	1	23,166	91,500	1	7.8	379
Wikipedia links, English	Knowledge graph	1	12M	378M		62.24	1M



Summary statistics on graphs found in the real world. Numbers are dependent on featurization decisions.

# Tasks on Graph Data

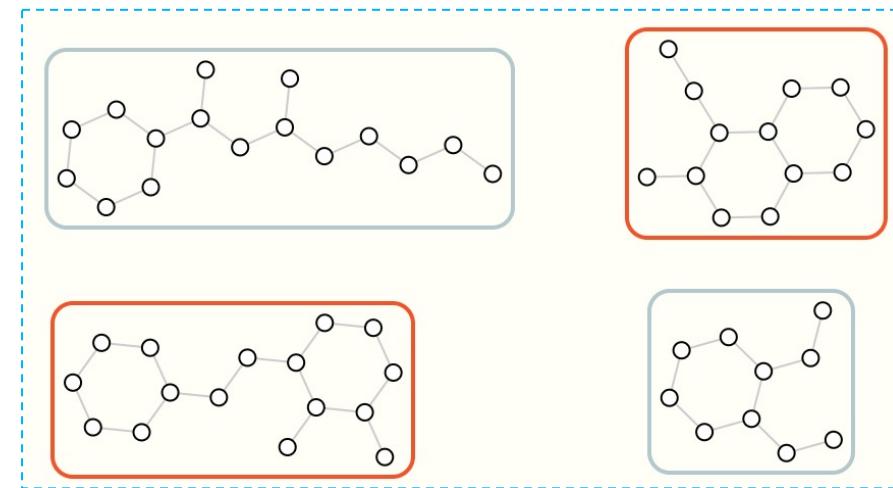
## Graph-level task



Input: graphs



Similar to image classification problems



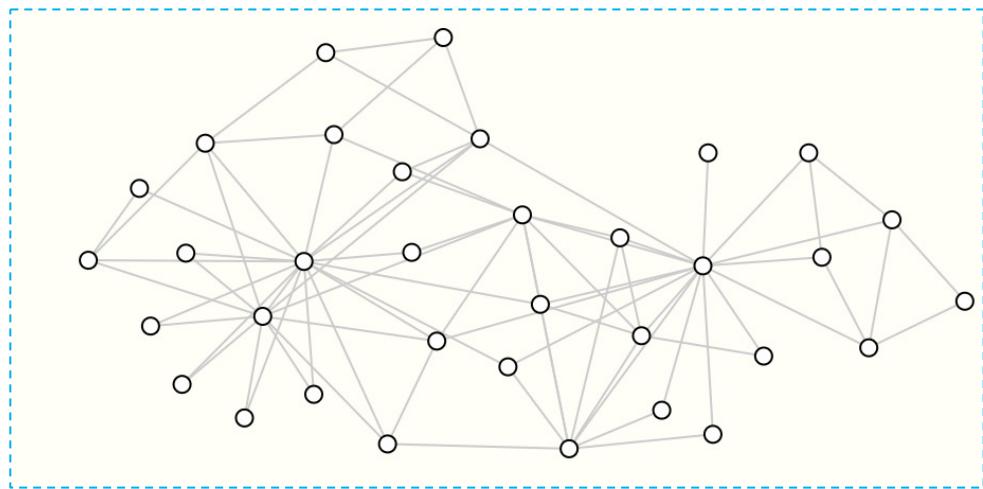
Output: labels for each graph, (e.g., "does the graph contain two rings?")



In a graph-level task, our goal is to predict the property of an entire graph. For example, for a molecule represented as a graph, we might want to predict what the molecule smells like, or whether it will bind to a receptor implicated in a disease.

# Tasks on Graph Data

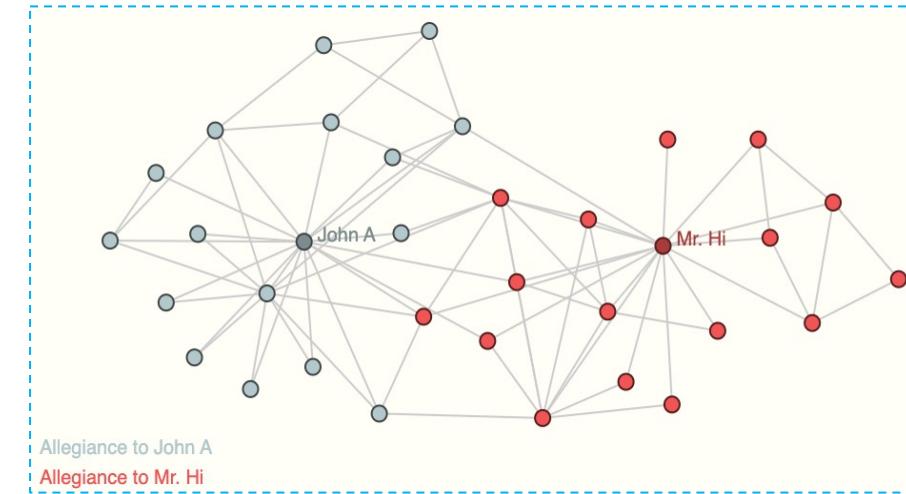
## Node-level task



Input: graph with unlabeled nodes



Similar to image segmentation problems



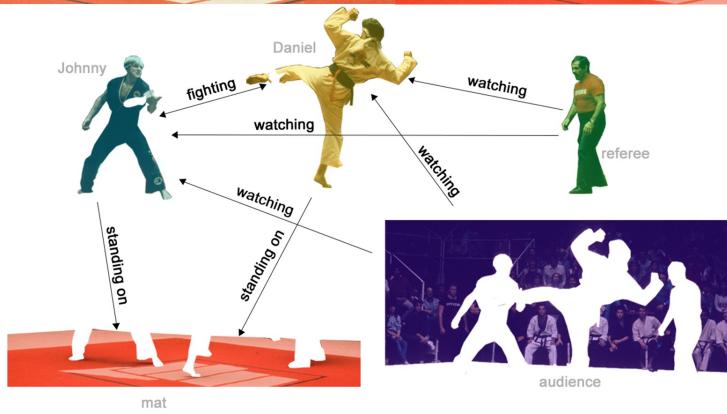
Output: graph node labels



Node-level tasks are concerned with predicting the identity or role of each node within a graph.

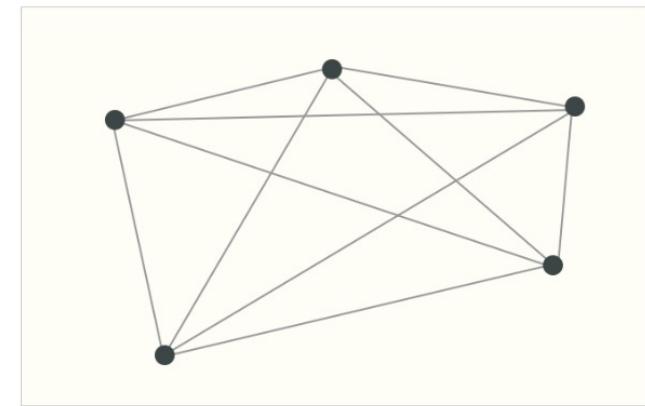
# Tasks on Graph Data

## Edge-level task

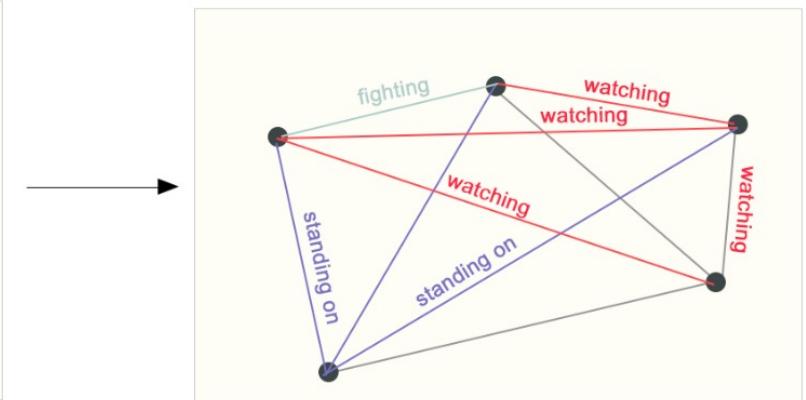


Similar to image scene understanding

Node-level tasks are concerned with predicting the identity or role of each node within a graph.



Input: fully connected graph, unlabeled edges



Output: labels for edges

# A Simple Graph

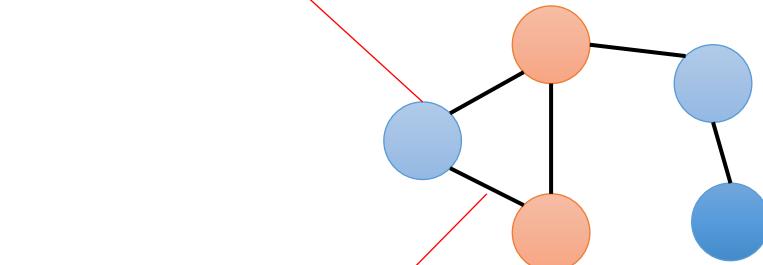
Node information

x1		x2		...		xn
----	--	----	--	-----	--	----

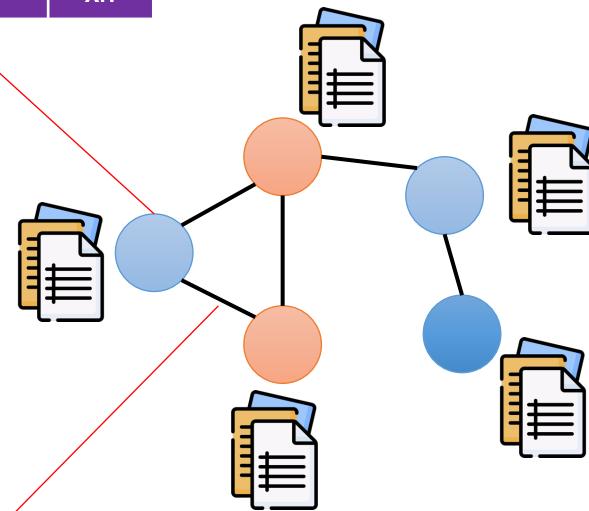
Paper Content

x1		x2		...		xn
----	--	----	--	-----	--	----

Edge information

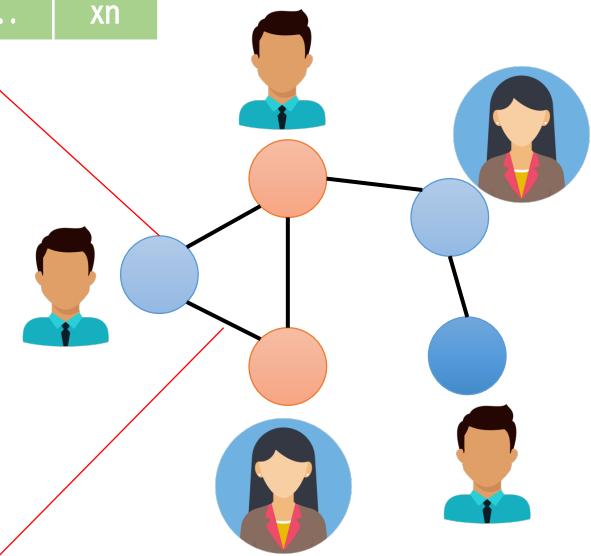


Paper citation



Personal Information

x1		x2		...		xn
----	--	----	--	-----	--	----

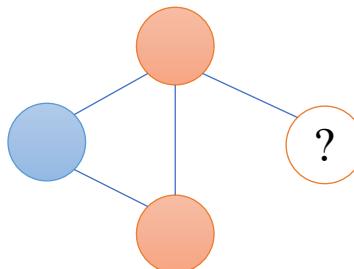


Relationship

x1		x2		...		xn
----	--	----	--	-----	--	----

# Example

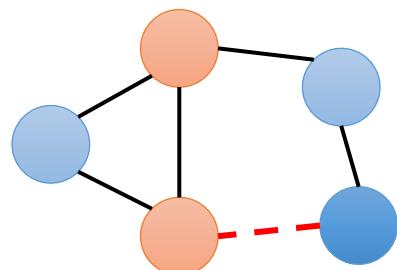
## Nodel level prediction



Does this student smoke?  
(unlablable node)



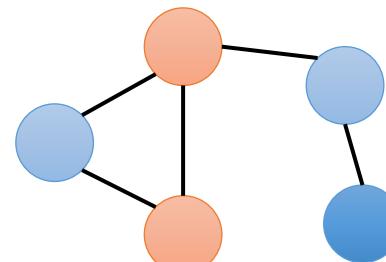
## Edge level predictions (Link prediction)



# Next Youtube Video?



# Graph Level Predictions



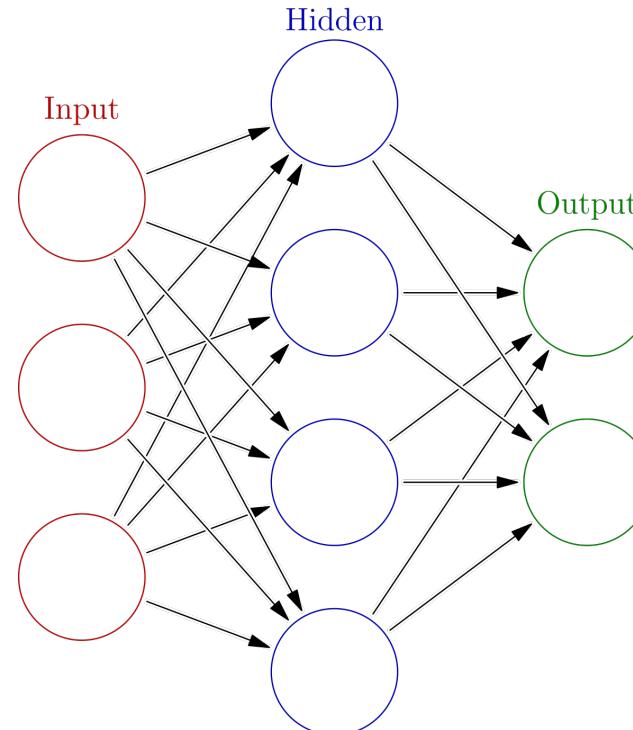
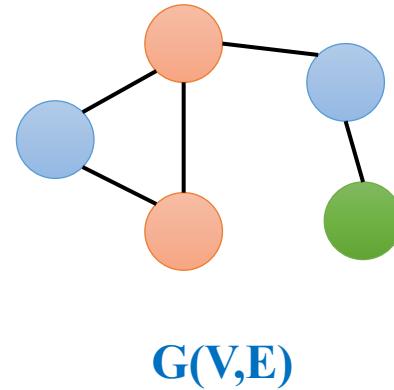
# Is this molecule a suitable drug?



# Outline

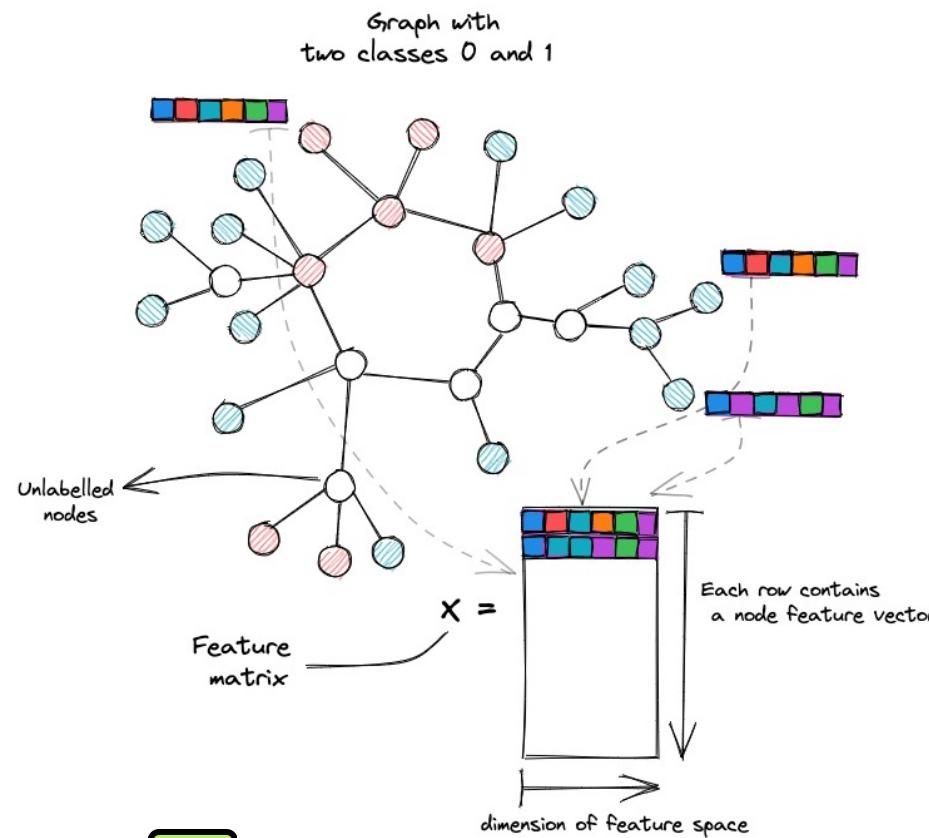
- **Objective**
- **Introduction to Graph Data**
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- **Node Classification Problem: Cora Citation Dataset**
- **Summary**

# Graph Data with Neural Network

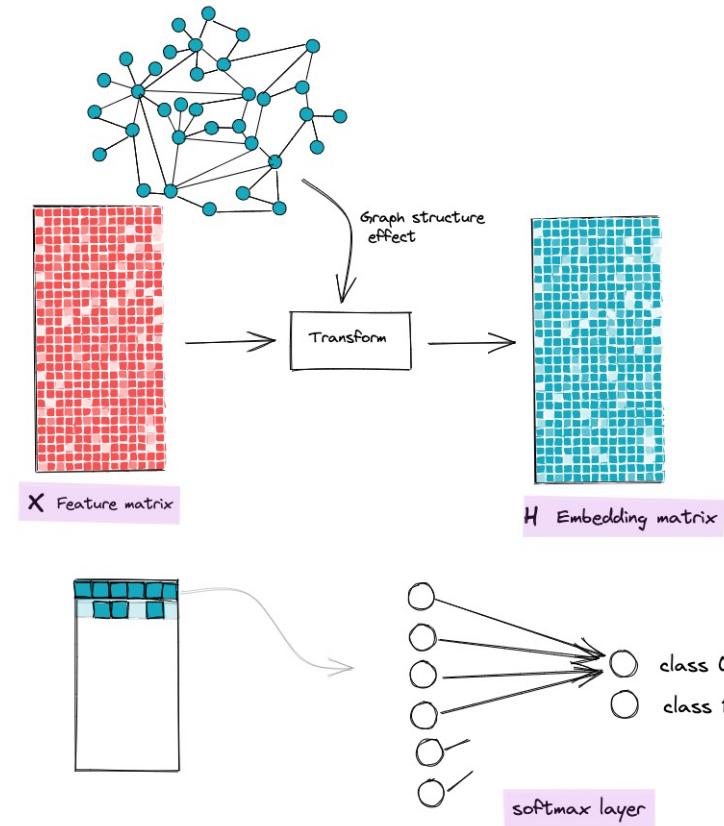


So, how do we go about solving these different graph tasks with neural networks? The first step is to think about how we will represent graphs to be compatible with neural networks.

# Graph Data with Neural Network

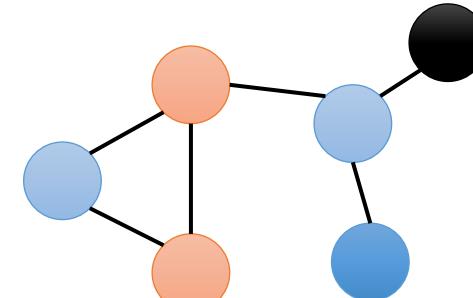
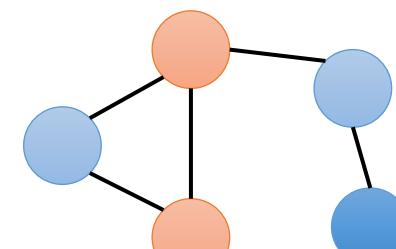


The goal of GNN is to transform node features to features that are aware of the graph structure



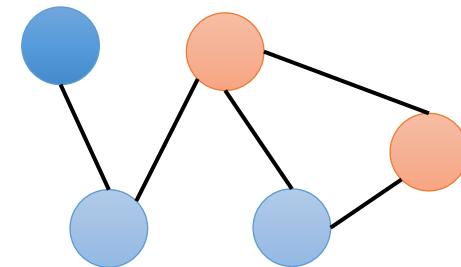
# Problem with Graph Data

## Difference in Size and Shape

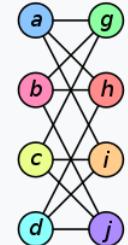
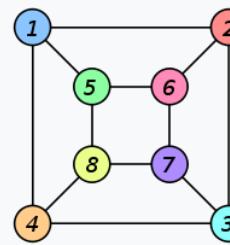


How to handle arbitrary input graph shape?

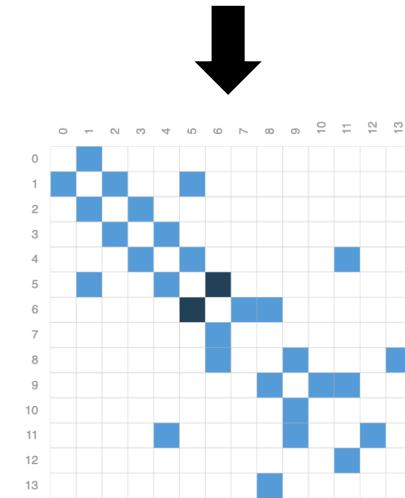
## Isomorphism



Permutation invariant

Graph G	Graph H	An isomorphism between G and H
		$f(a) = 1$ $f(b) = 6$ $f(c) = 8$ $f(d) = 3$ $f(g) = 5$ $f(h) = 2$ $f(i) = 4$ $f(j) = 7$

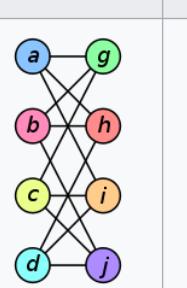
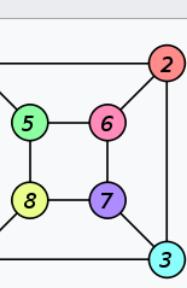
Cannot use adjacency matrix as an input



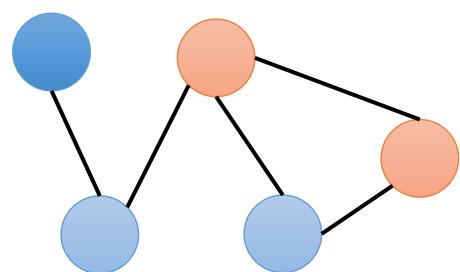
# Problem with Graph Data



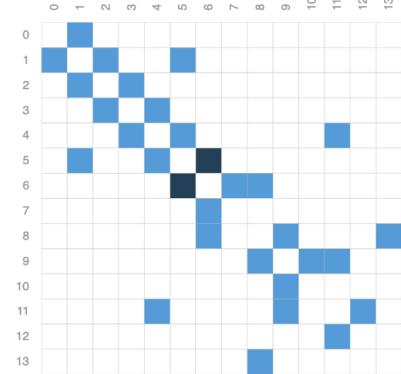
Isomorphism

Graph G	Graph H	An isomorphism between G and H
		$\begin{aligned}f(a) &= 1 \\f(b) &= 6 \\f(c) &= 8 \\f(d) &= 3 \\f(g) &= 5 \\f(h) &= 2 \\f(i) &= 4 \\f(j) &= 7\end{aligned}$

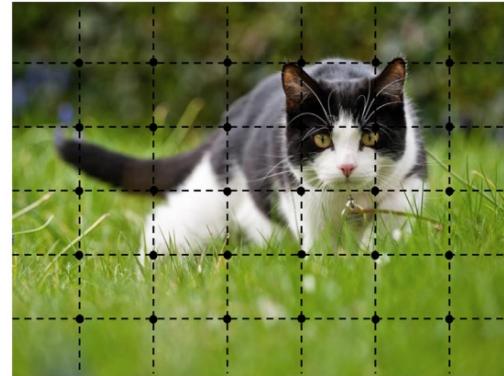
Cannot use adjacency matrix as an input



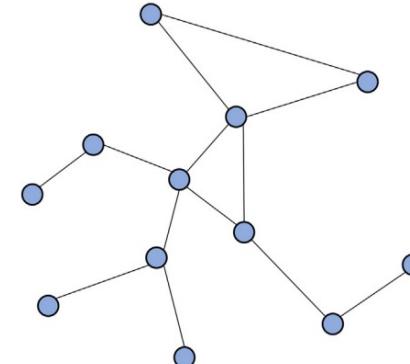
Permutation invariant



Grid Structure



Information between nodes

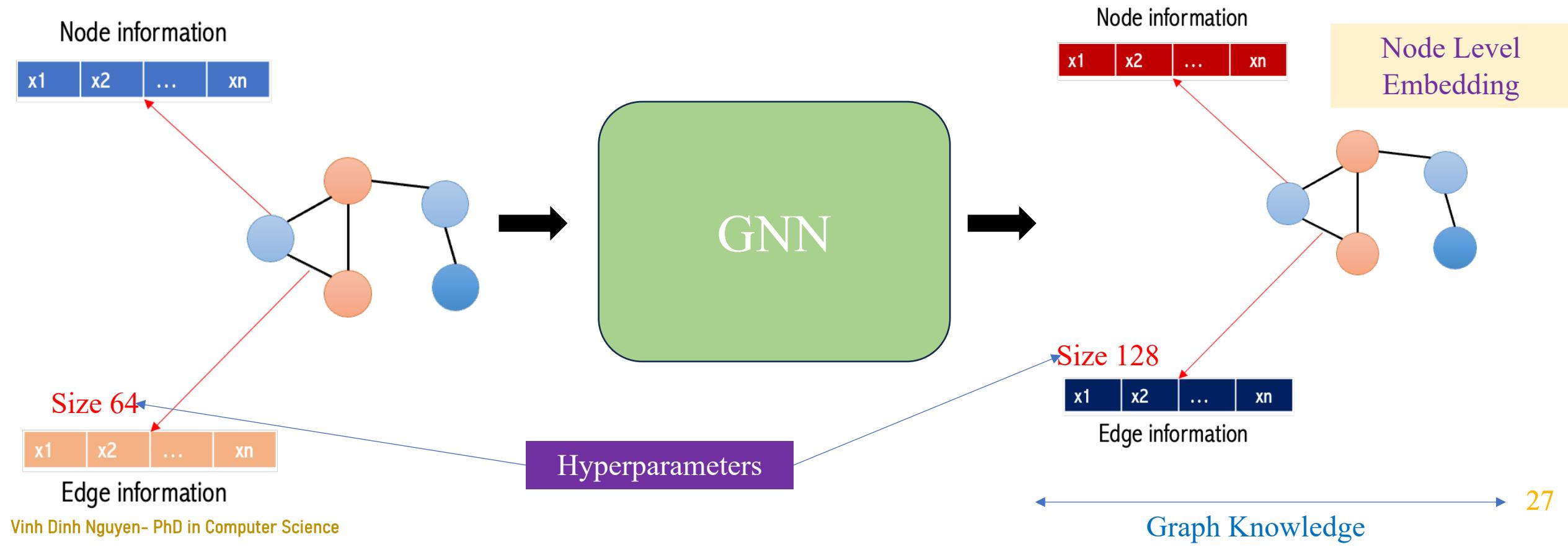


Eulidean space

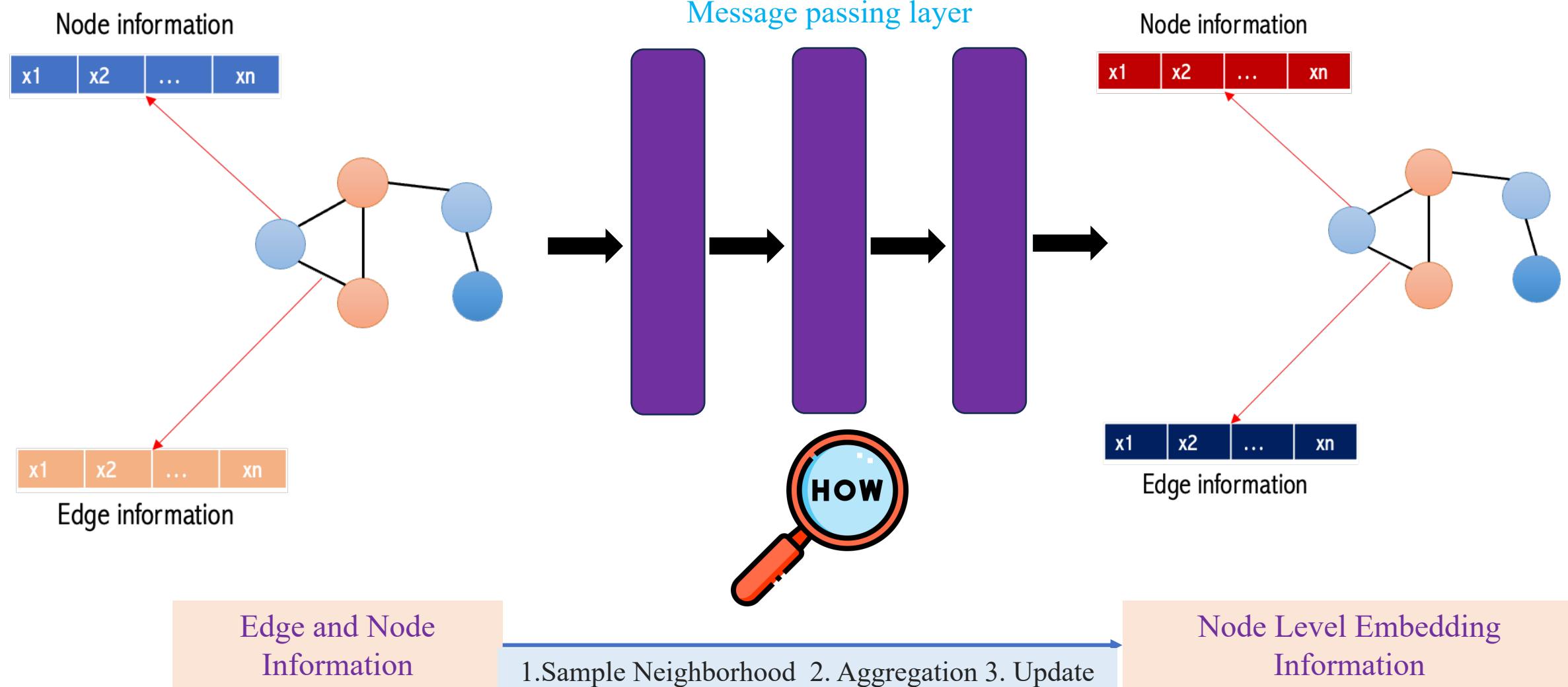
Non-Euclidean space

# Fundamental Idea of GNNs

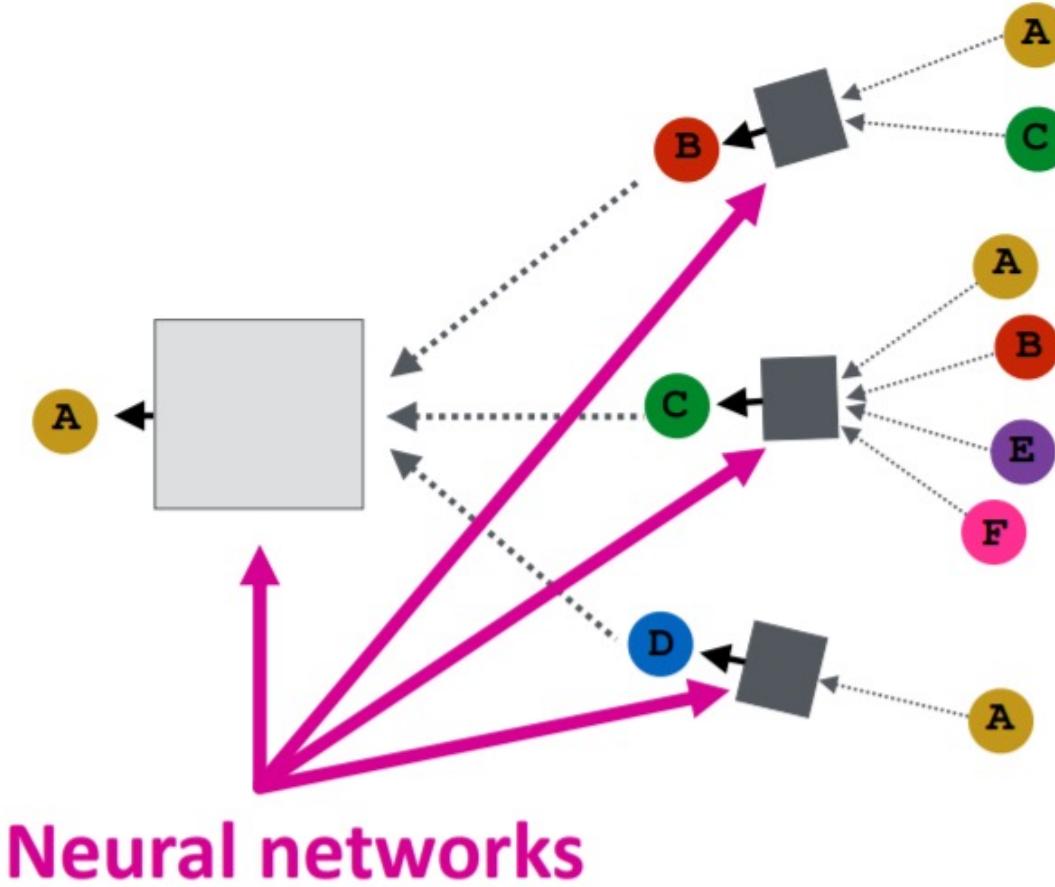
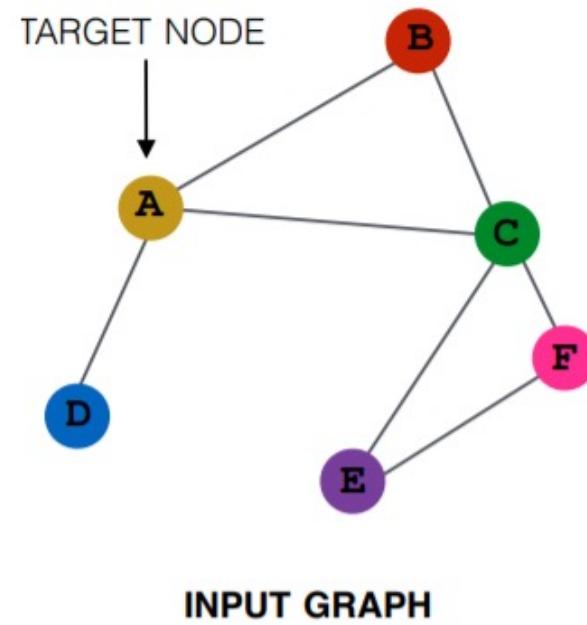
Learning a neural network suitable representation of graph data  
**<Representation Learning>>**



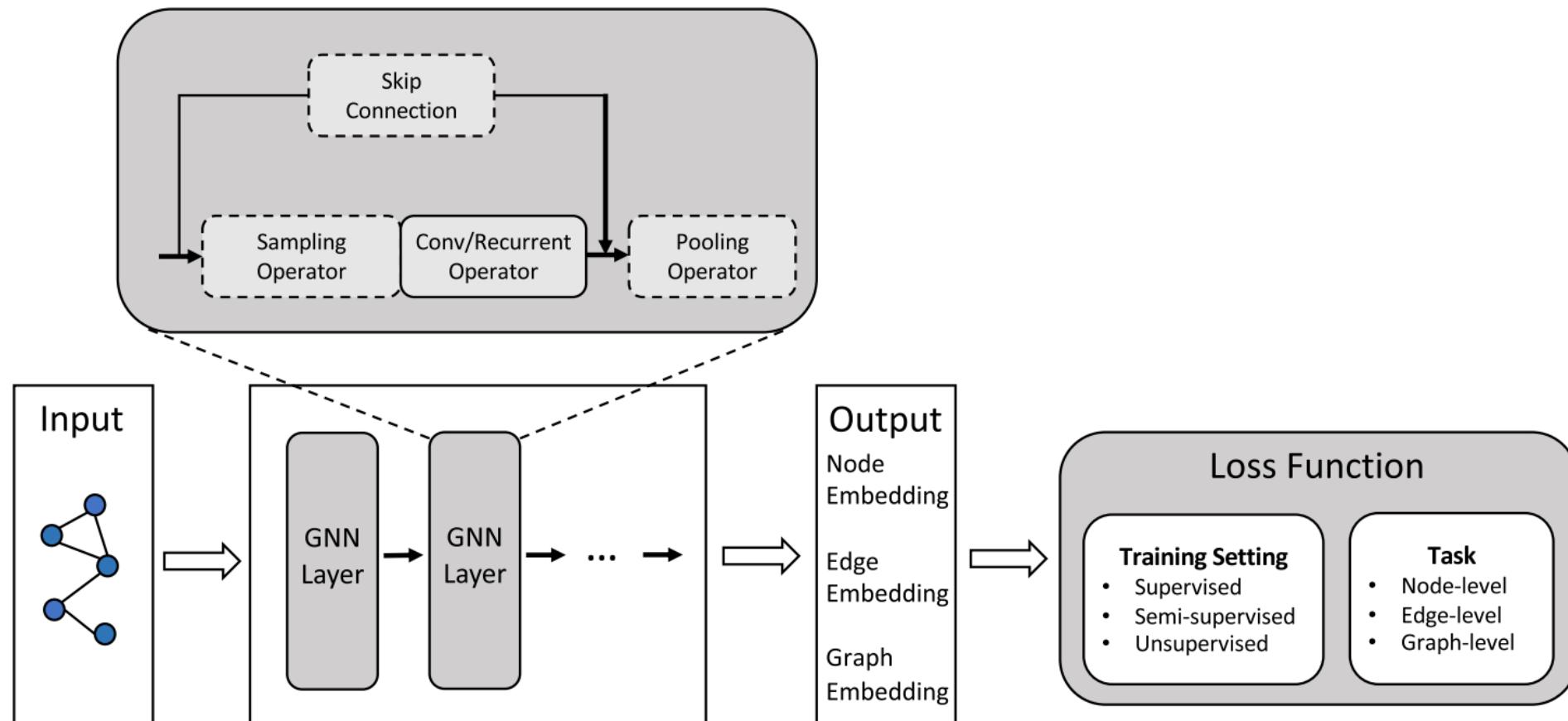
# How do Graph Neural Network Work?



# How do Graph Neural Network Work?



# How do Graph Neural Network Work?



1. Find graph structure.

2. Specify graph type and scale.

4. Build model using computational modules.

3. Design loss function.

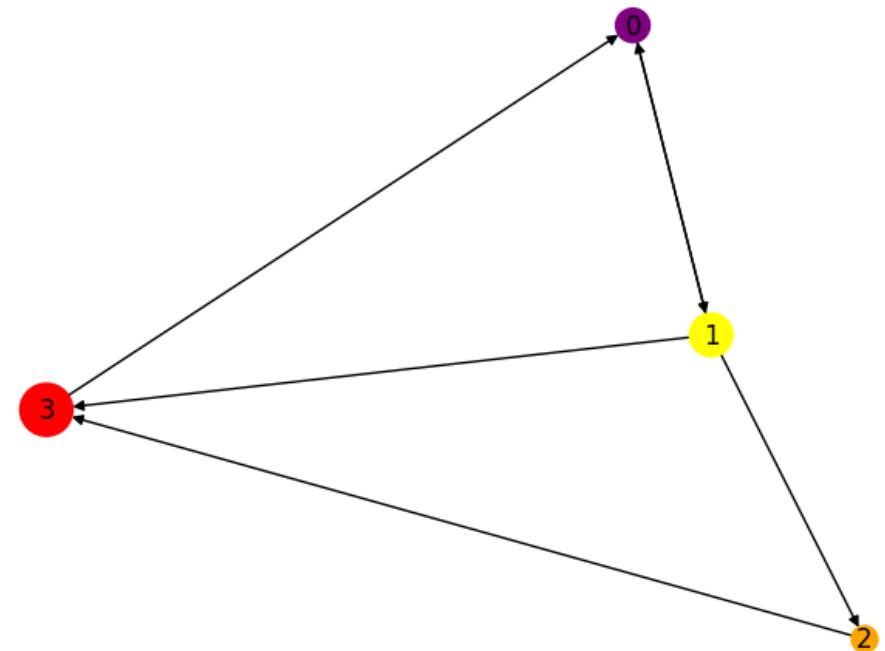
# Create a graph using NetworkX

```
import networkx as nx
H = nx.DiGraph()

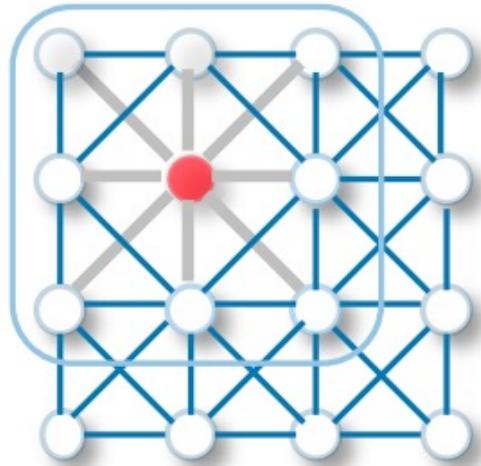
#adding nodes
H.add_nodes_from([
    (0, {"color": "purple", "size": 250}),
    (1, {"color": "yellow", "size": 400}),
    (2, {"color": "orange", "size": 150}),
    (3, {"color": "red", "size": 600})
])

#adding edges
H.add_edges_from([
    (0, 1), (1, 2), (1, 0), (1, 3), (2, 3), (3,0)
])
|
node_colors = nx.get_node_attributes(H, "color").values()
colors = list(node_colors)
node_sizes = nx.get_node_attributes(H, "size").values()
sizes = list(node_sizes)

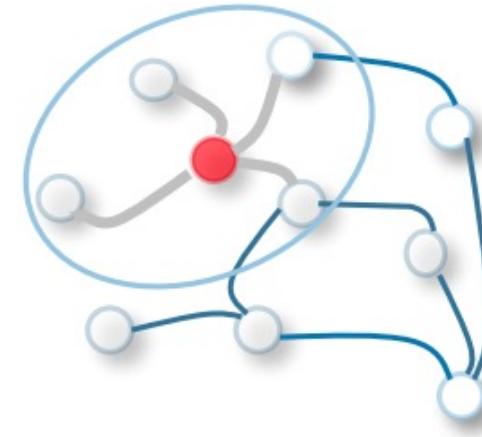
#Plotting Graph
nx.draw(H, with_labels=True, node_color=colors, node_size=sizes)
```



# CNN Vs. GNN: Message Passing

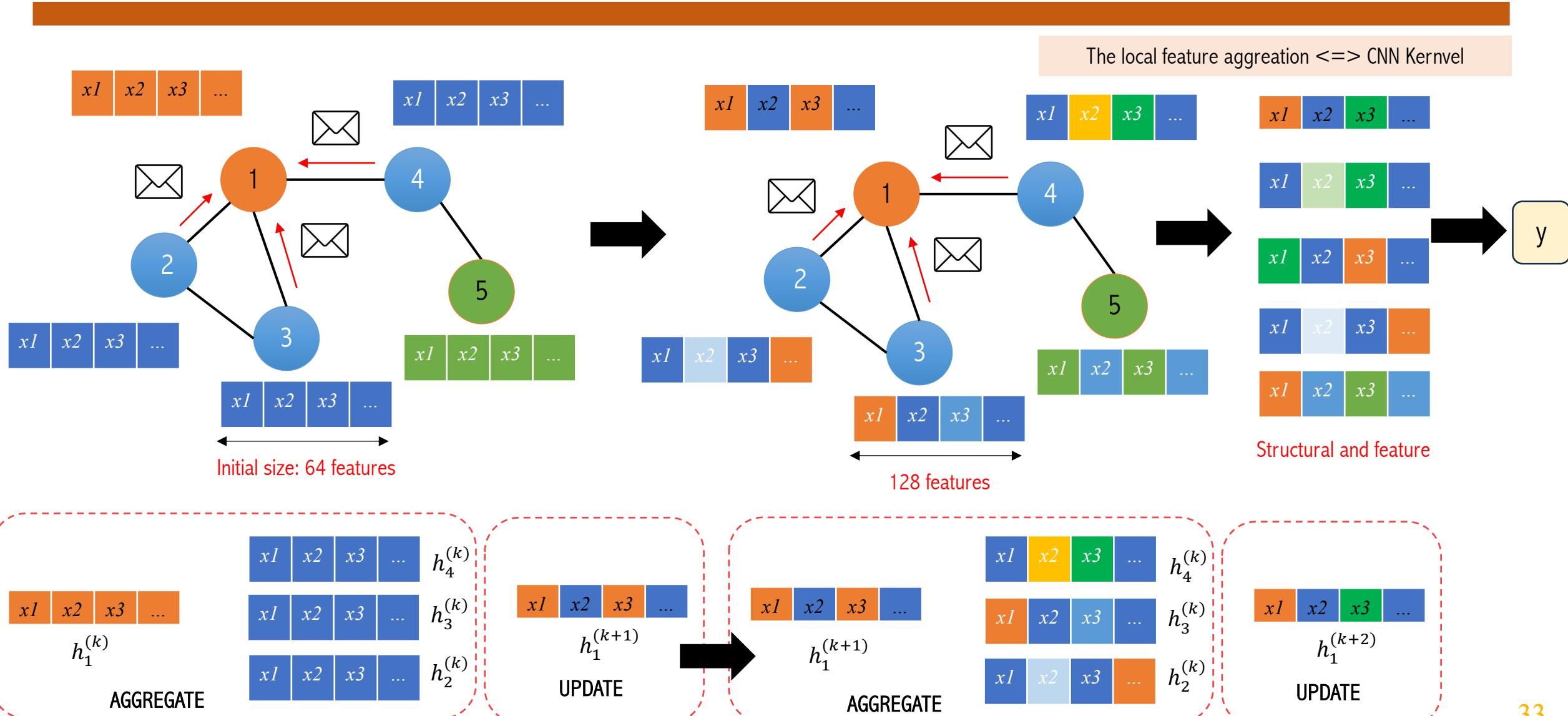


2D Convolution. Analogous to a graph, each pixel in an image is taken as a node where neighbors are determined by the filter size. The 2D convolution takes the weighted average of pixel values of the red node along with its neighbors. The neighbors of a node are ordered and have a fixed size

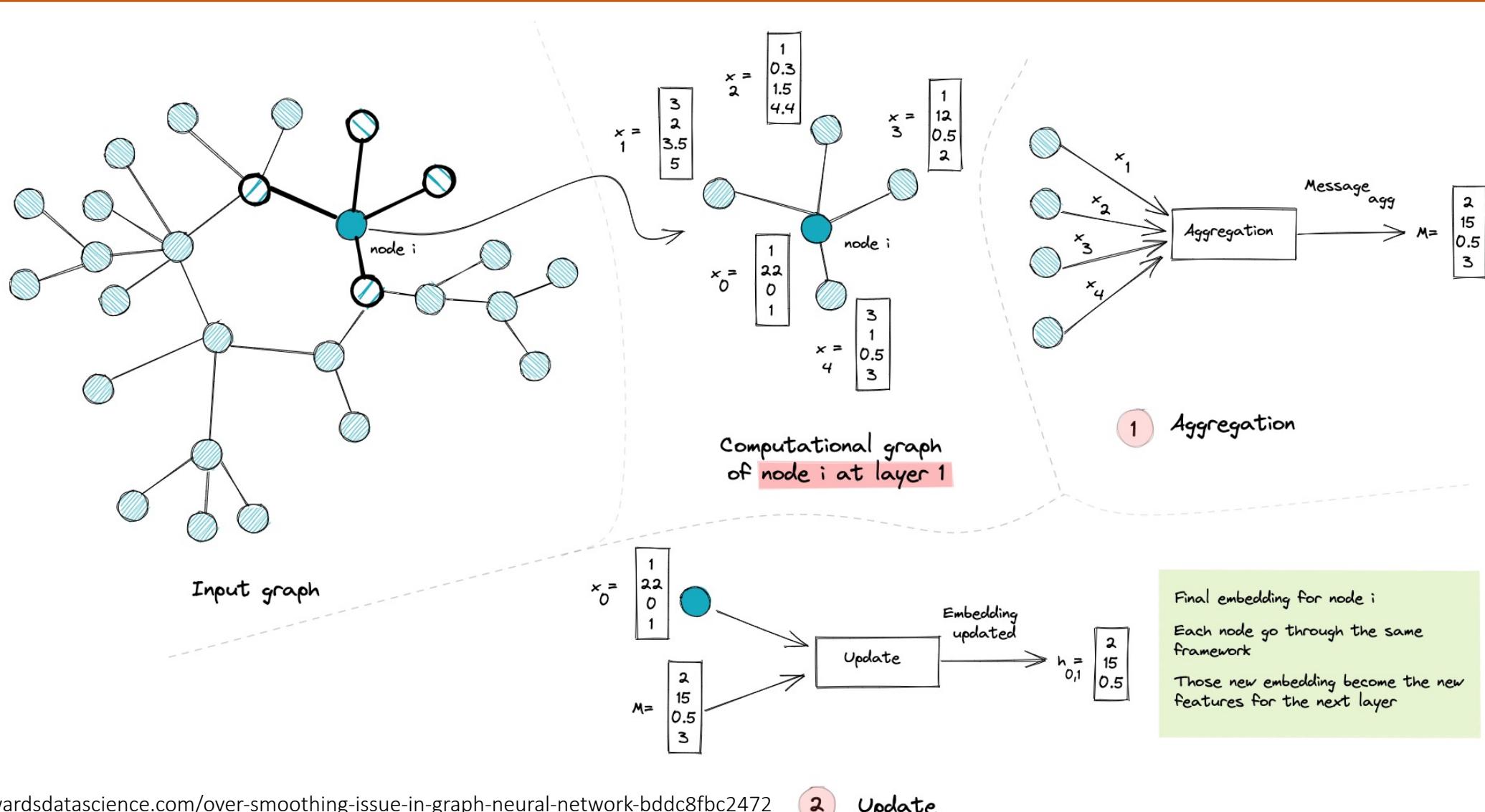


Graph Convolution. To get a hidden representation of the red node, one simple solution of the graph convolutional operation is to take the average value of the node features of the red node along with its neighbors. Different from image data, the neighbors of a node are unordered and variable in size

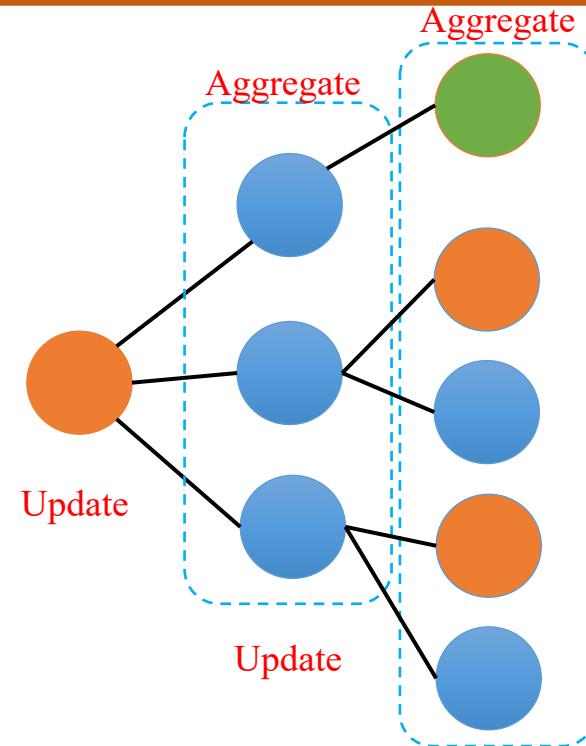
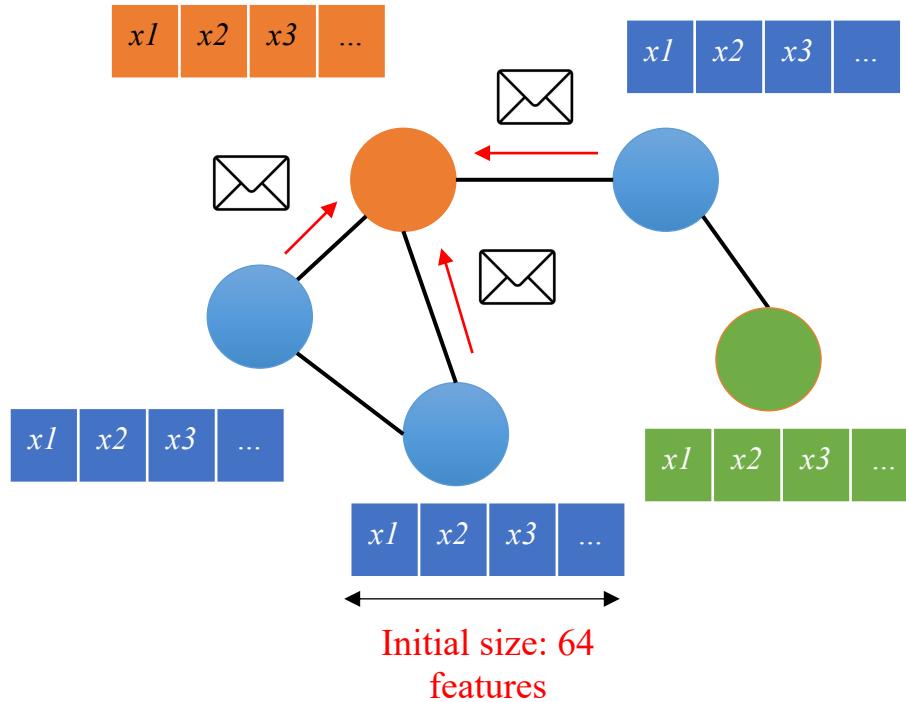
# Message Passing: Behind the Scene



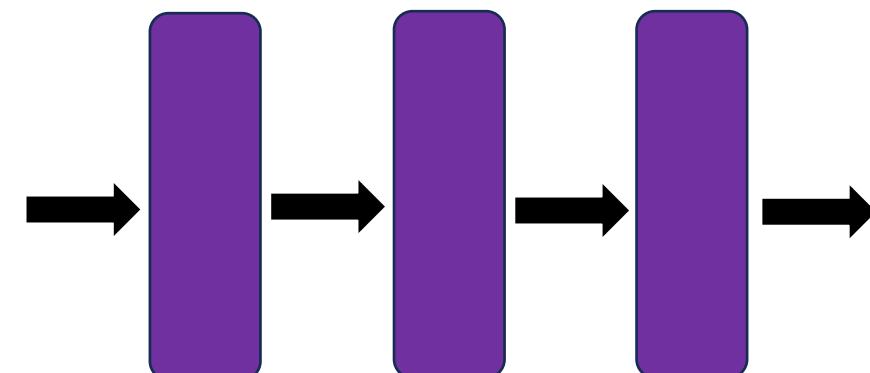
# Graph: Example



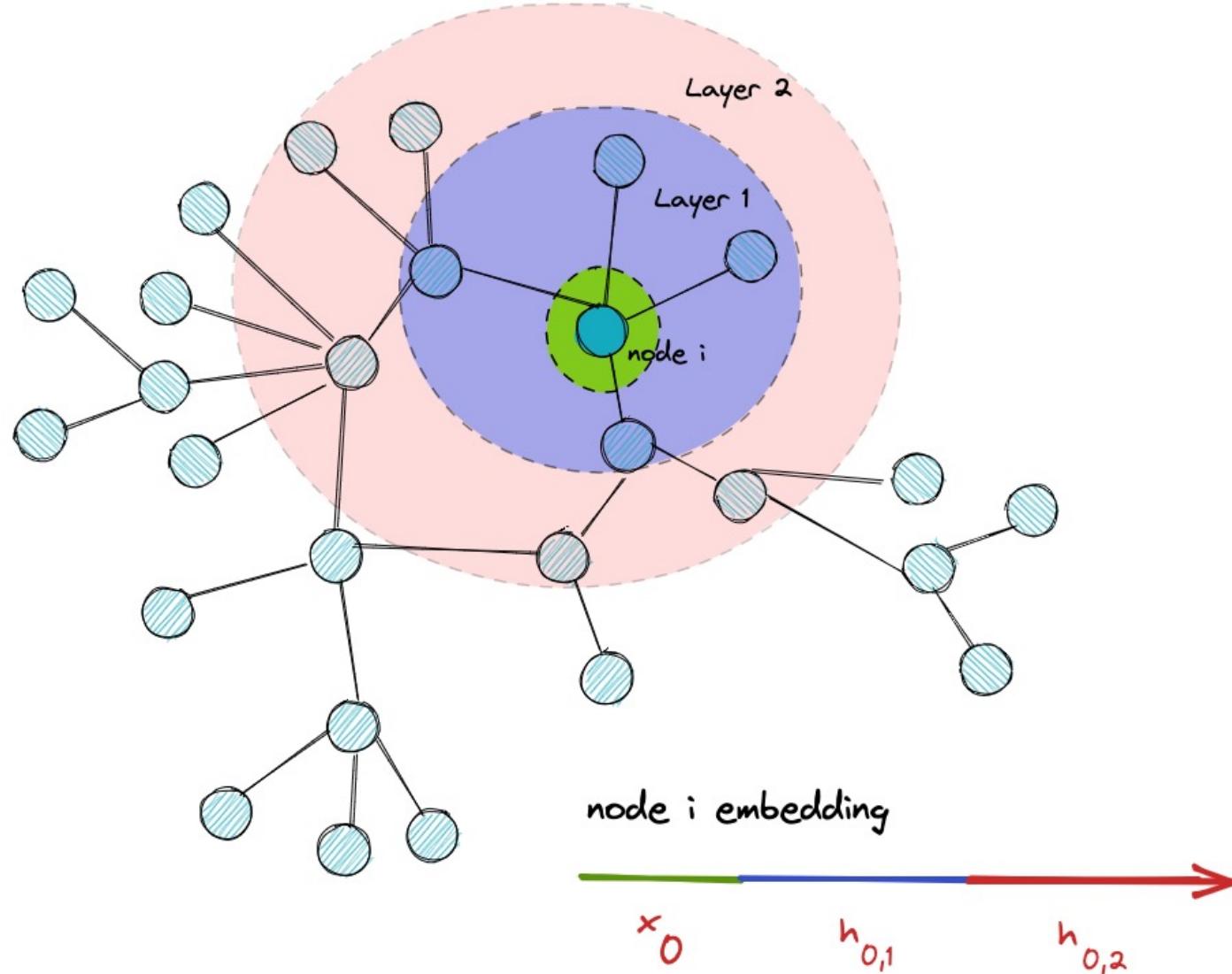
# Computation Graph Representation



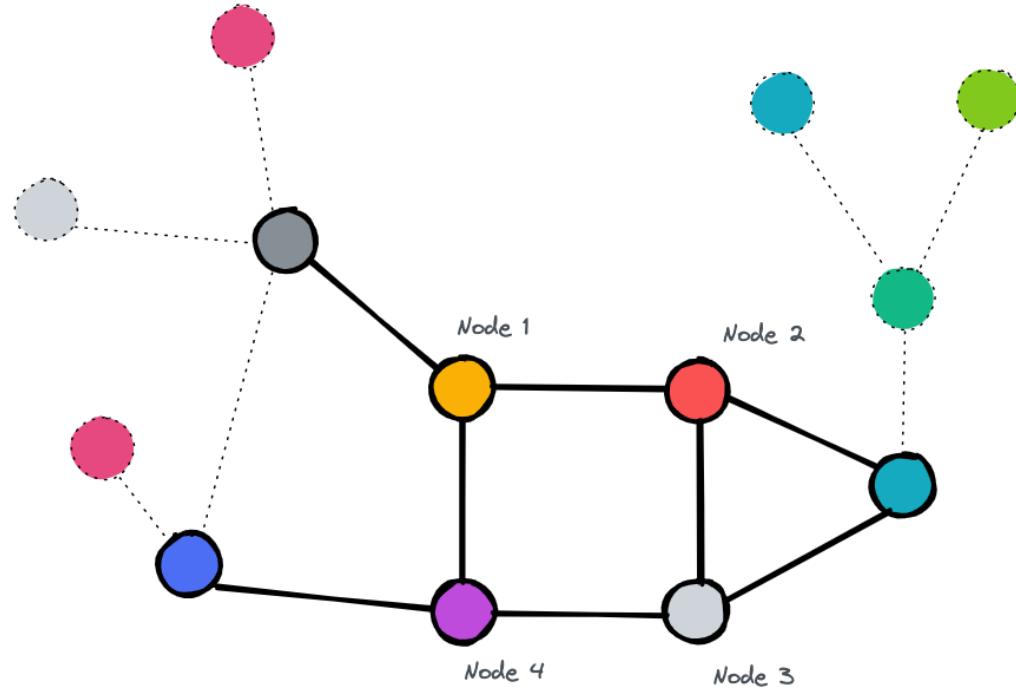
Computational Graph  
for Node v



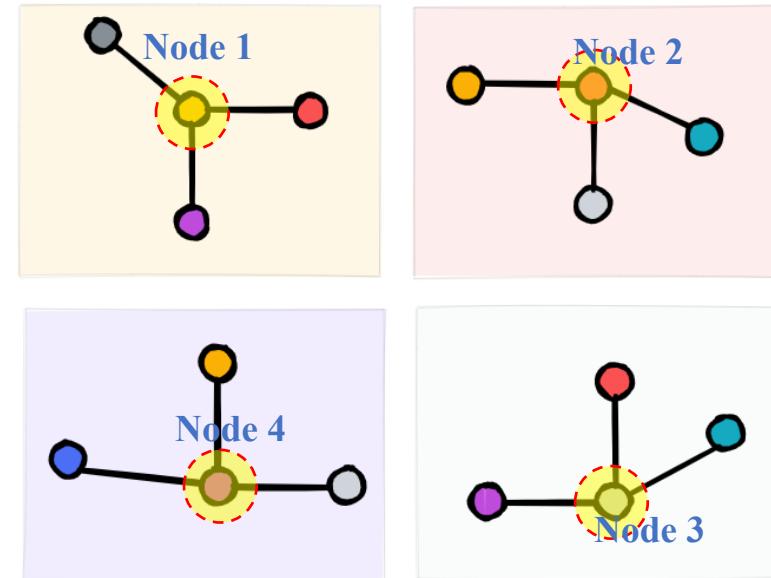
# Graph: Example



# Over-smoothing in GNN



Input graph



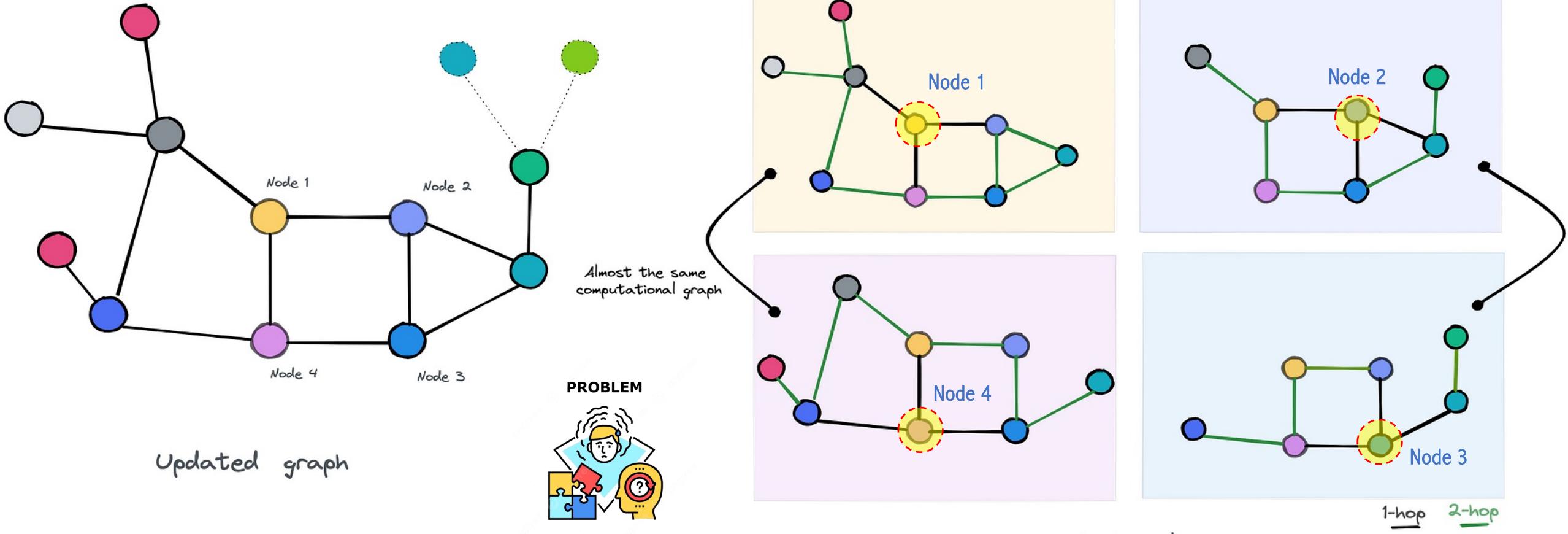
Computational graphs  
at layer 1



Node 2 and node 3 have almost access to the same information -> We can predict that their embeddings will be slightly similar.

Node 1 and Node 4, they interact with each other but have different neighbors -> We may predict that their new embeddings will be different.

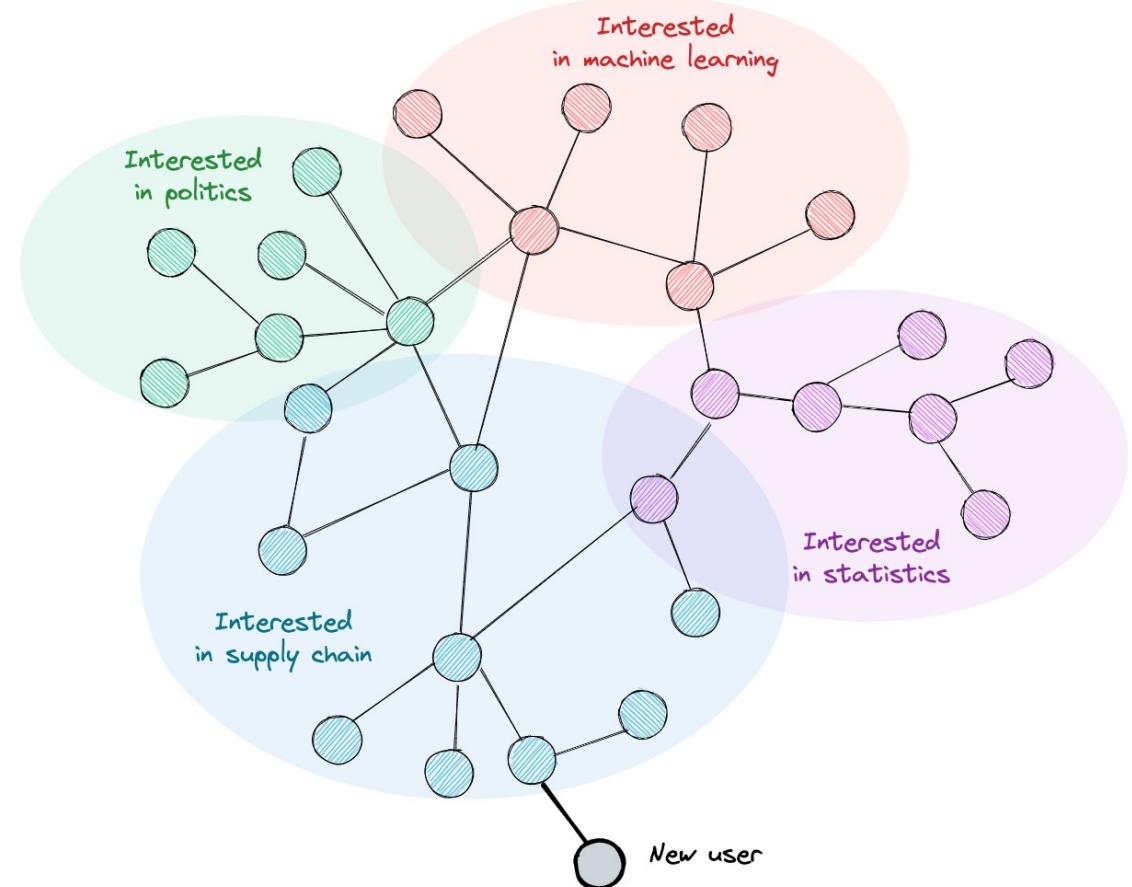
# Over-smoothing in GNN



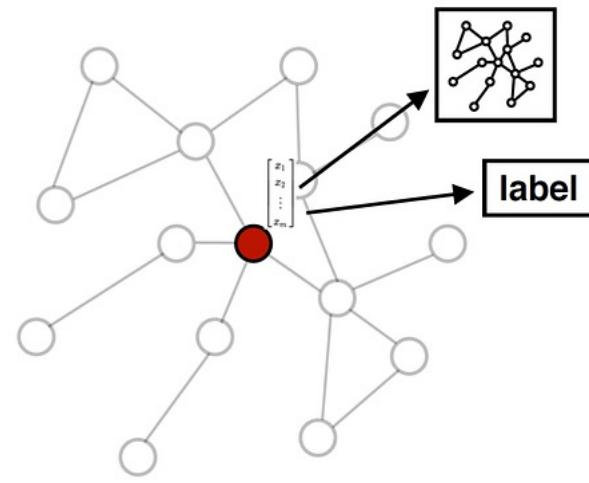
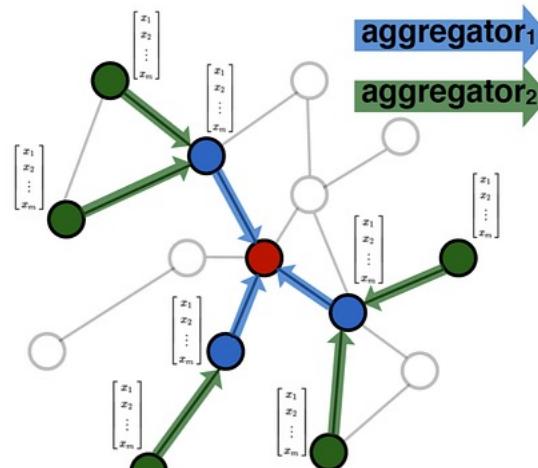
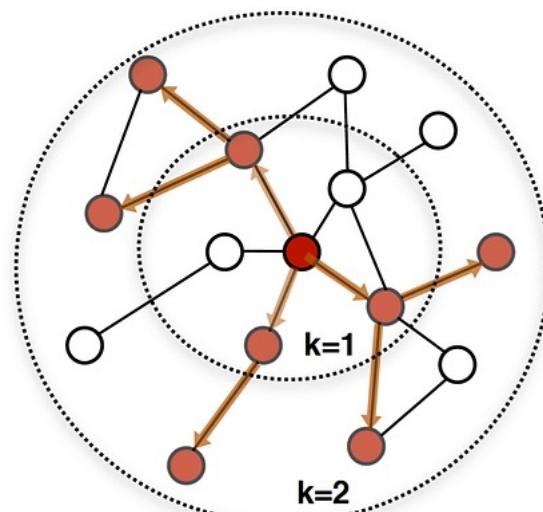
The computational graphs of nodes 1,4, and 2,3 are almost the same respectively

# Over-smoothing in GNN

Graph Neural Networks, or GNNs, are really good at working with data that is organized in a graph structure. But sometimes, when we add more layers to a GNN architecture, it doesn't work as well as we would like. This is called over-smoothing.



# Why over-smoothing happens?

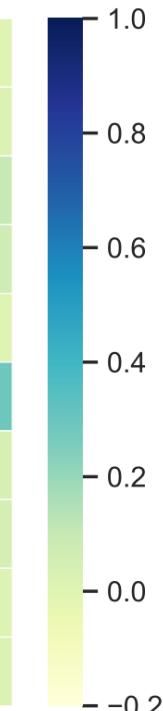


Reason 1 → More number of layers (depth)  
Reason 2 → Default nature of GNNs

# How to detect over-smoothing?

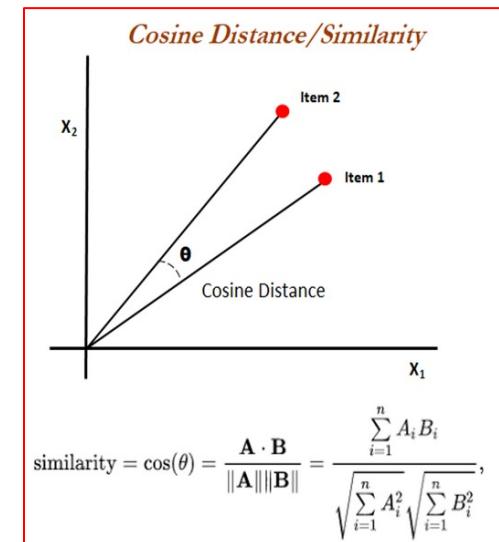
## Metric 1: MAD(Mean Average Distance)

Model	2	3	4	5	6
ARMA	0.629	0.860	0.608	0.305	0.004
ChebGCN	0.557	0.756	0.138	0.024	0.018
DNA	0.665	0.352	0.347	0.172	0.096
FeaSt	0.778	0.770	0.677	0.182	0.072
GAT	0.794	0.704	0.232	0.047	0.005
GCN	0.796	0.765	0.714	0.602	0.289
GGNN	0.661	0.078	0.021	0.033	0.039
GraphSAGE	0.925	0.816	0.632	0.303	0.053
HighOrder	0.629	0.145	0.023	0.004	0.012
HyperGraph	0.828	0.742	0.493	0.046	0.023



MAD computes the Mean Average Distance (MAD) between node representations (embeddings) in the graph

$$D_{ij} = 1 - \frac{\mathbf{H}_{i,:} \cdot \mathbf{H}_{j,:}}{|\mathbf{H}_{i,:}| \cdot |\mathbf{H}_{j,:}|} \quad i, j \in [1, 2, \dots, n],$$



## Measuring and Relieving the Over-smoothing Problem for Graph Neural Networks from the Topological View

Deli Chen,<sup>1</sup> Yankai Lin,<sup>2</sup> Wei Li,<sup>1</sup> Peng Li,<sup>2</sup> Jie Zhou,<sup>2</sup> Xu Sun<sup>1</sup>

<sup>1</sup>MOE Key Lab of Computational Linguistics, School of EECS, Peking University

<sup>2</sup>Pattern Recognition Center, WeChat AI, Tencent Inc., China

{chendeli, liweitj47, xusun}@pku.edu.cn, {yankailin, patrickpli, withtomzhou}@tencent.com,

# How to detect over-smoothing?

## Metric 2 : MADGap

$$\text{MADGap} = \text{MAD}^{\text{rmt}} - \text{MAD}^{\text{neb}}, \quad (5)$$

where  $\text{MAD}^{\text{rmt}}$  is the MAD value of the remote nodes in the graph topology and  $\text{MAD}^{\text{neb}}$  is the MAD value of the neighbouring nodes.

It is based on the main hypothesis that when nodes interact, they have access to either important information from nodes of the same class or noise from nodes of other classes.

### Measuring and Relieving the Over-smoothing Problem for Graph Neural Networks from the Topological View

Deli Chen,<sup>1</sup> Yankai Lin,<sup>2</sup> Wei Li,<sup>1</sup> Peng Li,<sup>2</sup> Jie Zhou,<sup>2</sup> Xu Sun<sup>1</sup>

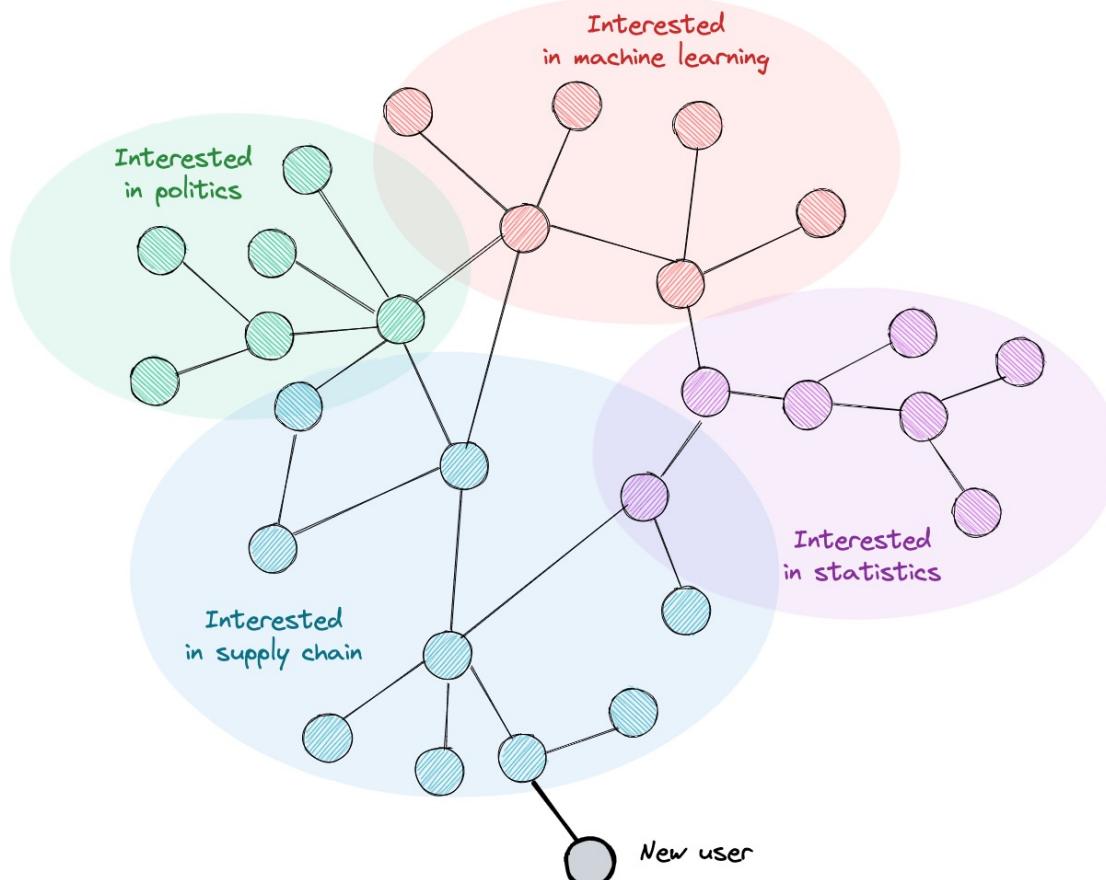
<sup>1</sup>MOE Key Lab of Computational Linguistics, School of EECS, Peking University

<sup>2</sup>Pattern Recognition Center, WeChat AI, Tencent Inc., China

{chendeli,liweitj47,xusun}@pku.edu.cn, {yankailin,patrickpli,withtomzhou}@tencent.com,



# Over-smoothing in GNN



Imagine that we're dealing with a social network graph with thousands of nodes. Some new users just signed in to the platform and subscribed to their friend's profiles. Our goal is to find topic suggestions to fill their feed.

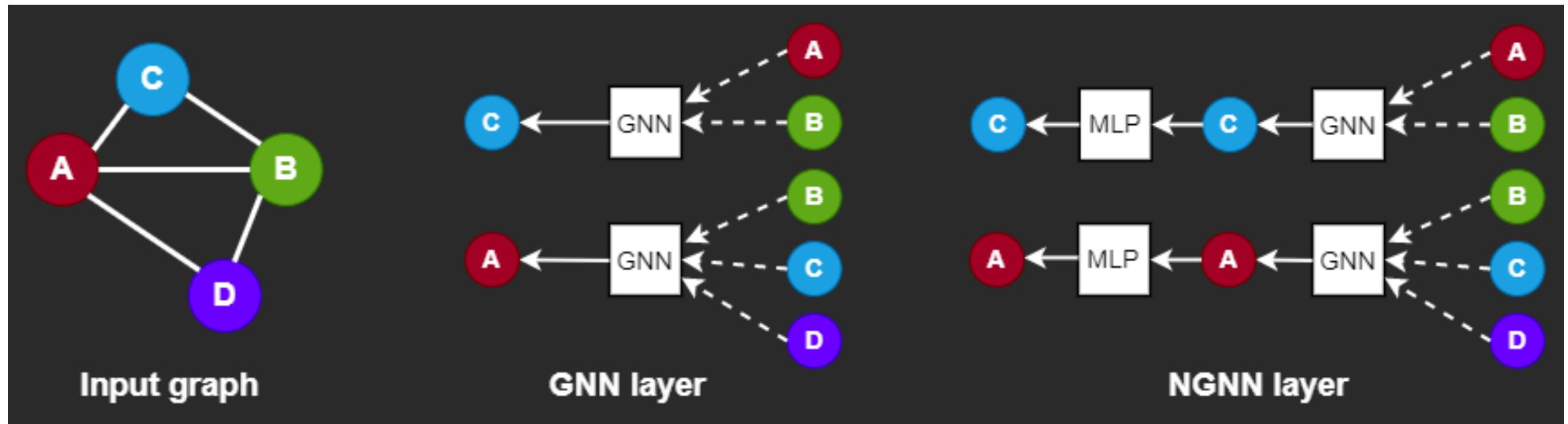


## How to reduce the effect of over-smoothing.

We encounter a trade-off between a low-efficiency model and a model with more depth but less expressivity in terms of node representations

# Over-smoothing in GNN

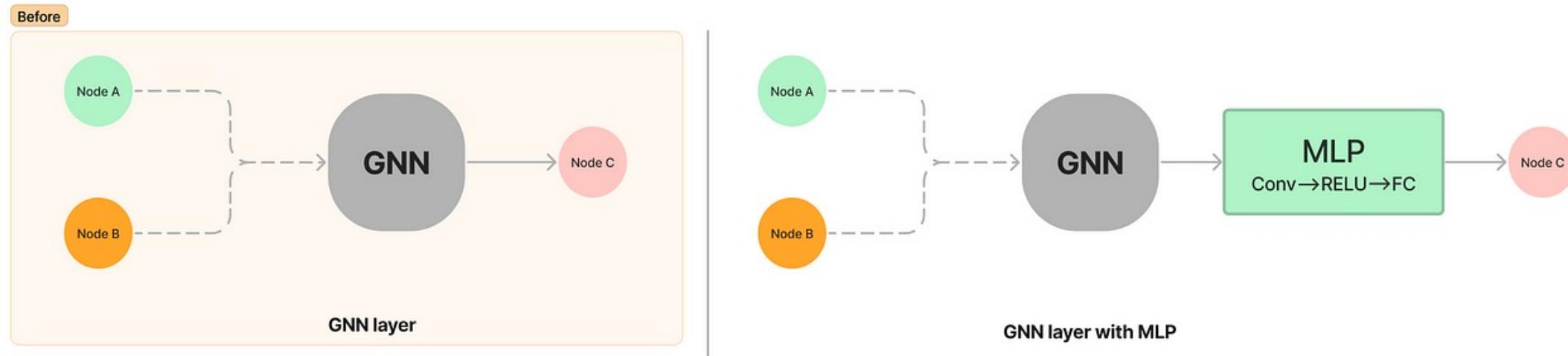
Solution → Inserting nonlinear feedforward neural network layer(s) within each GNN layer.



<https://www.dgl.ai/blog/2022/11/28/ngnn.html>

# Over-smoothing in GNN

Solution → Inserting nonlinear feedforward neural network layer(s) within each GNN layer.



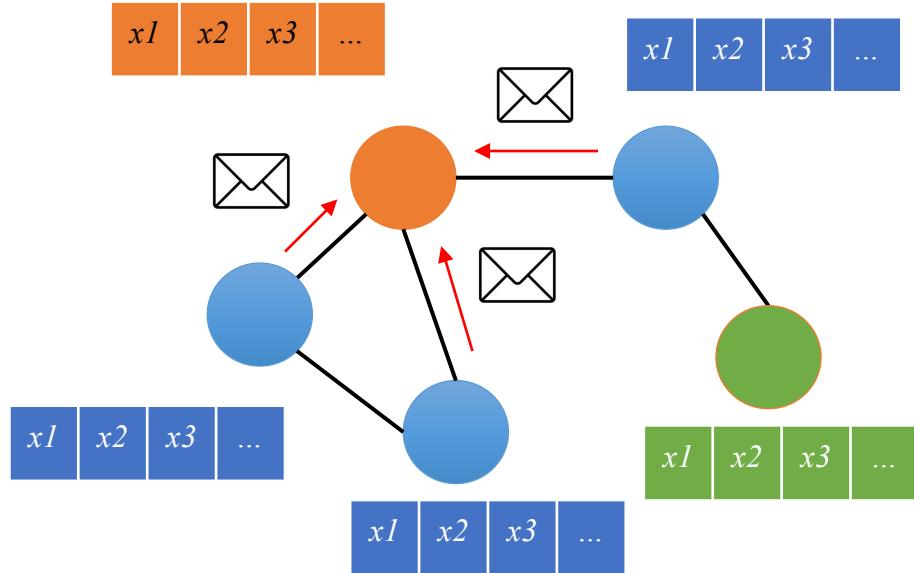
<https://www.dgl.ai/blog/2022/11/28/ngnn.html>

# Over-smoothing in GNN

Solution → Inserting nonlinear feedforward neural network layer(s) within each GNN layer.

Dataset	Metric	Model		Performance
ogbn-proteins	ROC-AUC(%)	GraphSage+Cluster Sampling	Vanilla	$67.45 \pm 1.21$
			+NGNN	<b><math>68.12 \pm 0.96</math></b>
ogbn-products	Accuracy(%)	GraphSage	Vanilla	$78.27 \pm 0.45$
			+NGNN	<b><math>79.88 \pm 0.34</math></b>
		GAT+Neighbor Sampling	Vanilla	$79.23 \pm 0.16$
			+NGNN	<b><math>79.67 \pm 0.09</math></b>
ogbl-collab	hit@50(%)	GCN	Vanilla	$49.52 \pm 0.70$
			+NGNN	<b><math>53.48 \pm 0.40</math></b>
		GraphSage	Vanilla	$51.66 \pm 0.35$
			+NGNN	<b><math>53.59 \pm 0.56</math></b>
ogbl-ppa	hit@100(%)	SEAL-DGCNN	Vanilla	$48.80 \pm 3.16$
			+NGNN	<b><math>59.71 \pm 2.45</math></b>
		GCN	Vanilla	$18.67 \pm 1.32$
			+NGNN	<b><math>36.83 \pm 0.99</math></b>

# Message Passing: Math is Fun



Mean  
Max  
Neural Network  
Recurrent Neural Network

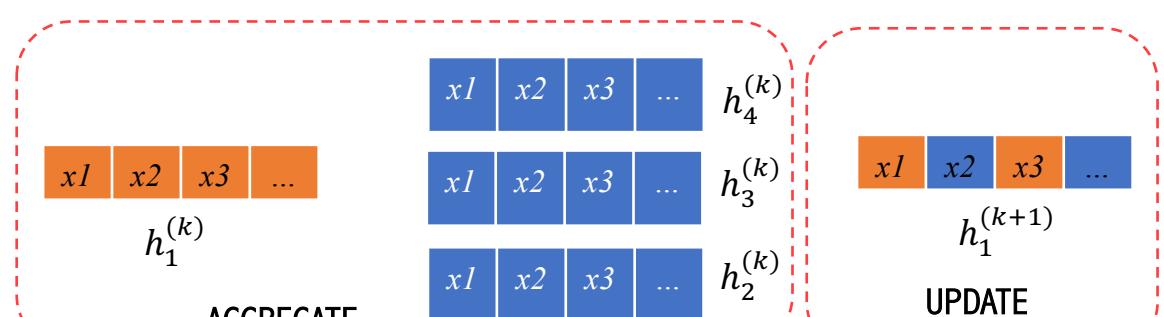
$$h_u^{k+1} = \text{UPDATE}^{(k)} \left( h_u^{k+1}, \text{AGGREGATE}^{(k)} \left( \{h_v^k\}, \forall v \in N(u) \right) \right)$$

$x_1 \ x_2 \ x_3 \ \dots$

$x_1 \ x_2 \ x_3 \ \dots$

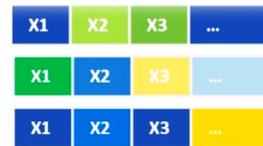
$x_1 \ x_2 \ x_3 \ \dots$   
 $x_1 \ x_2 \ x_3 \ \dots$   
 $x_1 \ x_2 \ x_3 \ \dots$

Mean  
Max  
Normalization Sum  
Neural Network



# Message Passing: Variants

*AGGREGATE*  
(permutation invariant)



*UPDATE*



Graph Convolutional Networks,  
Kipf and Welling [2016]

$$\mathbf{h}_v^{(k)} = \sigma \left( \mathbf{W}^{(k)} \sum_{v \in \mathcal{N}(u) \cup \{u\}} \frac{\mathbf{h}_v}{\sqrt{|\mathcal{N}(u)||\mathcal{N}(v)|}} \right)$$

Sum of normalized neighbor embeddings

Multi-Layer-Perceptron as  
Aggregator, Zaheer et al. [2017]

Aggregated message

$$\mathbf{m}_{\mathcal{N}(u)} = \boxed{\text{MLP}_\theta \text{ trainable!}} \left( \sum_{v \in \mathcal{N}(u)} \text{MLP}_\phi(\mathbf{h}_v) \right)$$

Send states through a MLP

Graph Attention Networks,  
Veličković et al. [2017]

$$\mathbf{m}_{\mathcal{N}(u)} = \sum_{v \in \mathcal{N}(u)} \alpha_{u,v} \mathbf{h}_v$$

Attention weights

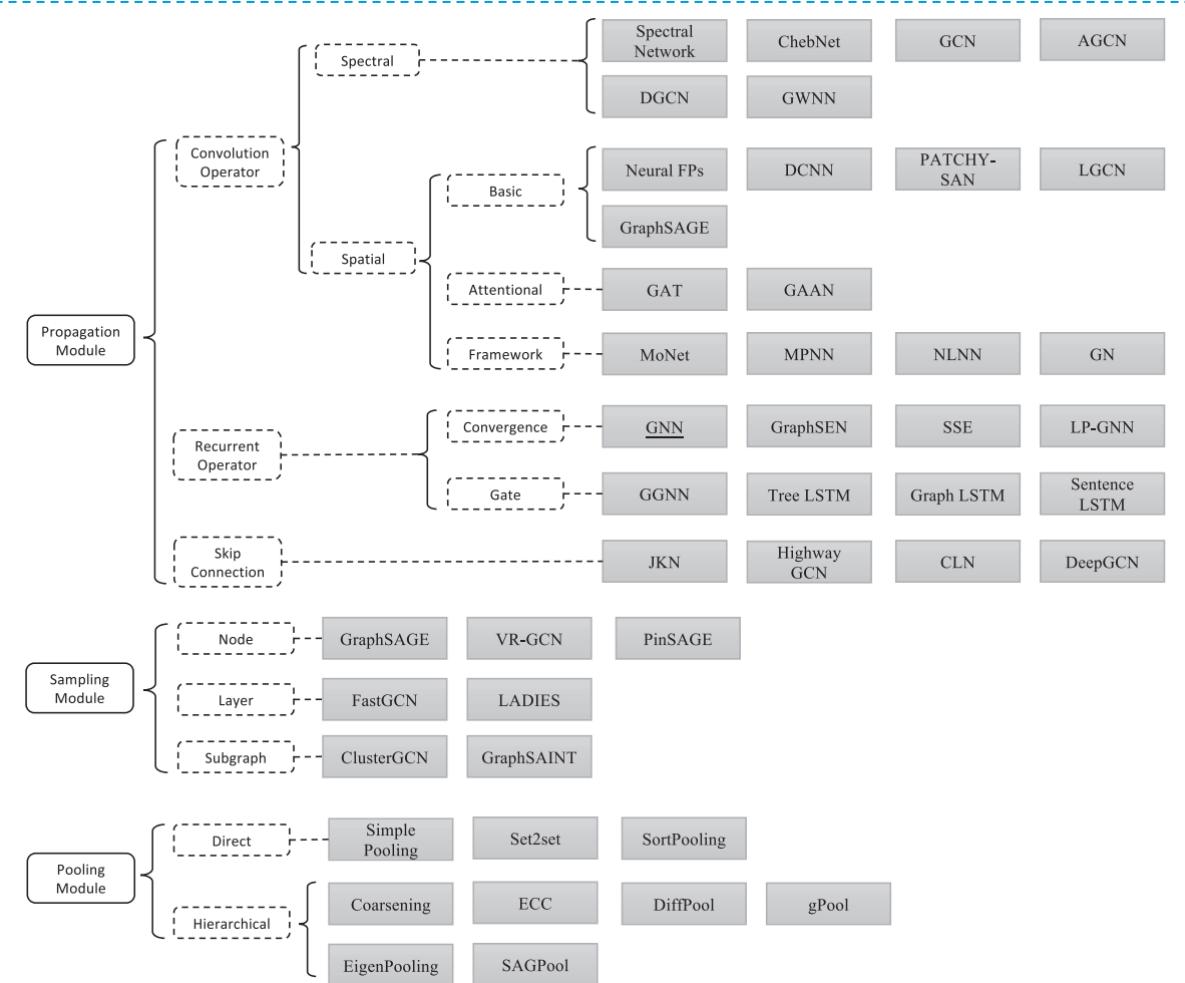
$$\alpha_{u,v} = \frac{\exp(\mathbf{a}^\top [\mathbf{W}\mathbf{h}_u \oplus \mathbf{W}\mathbf{h}_v])}{\sum_{v' \in \mathcal{N}(u)} \exp(\mathbf{a}^\top [\mathbf{W}\mathbf{h}_u \oplus \mathbf{W}\mathbf{h}_{v'}])}$$

Gated Graph Neural Networks,  
Li et al. [2015]

$$\mathbf{h}_u^{(k)} = \text{GRU}(\mathbf{h}_u^{(k-1)}, \mathbf{m}_{\mathcal{N}(u)}^{(k)})$$

Recurrent update of the state

# Message Passing: Variants



Different variants of recurrent operators.

Variant	Aggregator	Updater
GGNN	$\mathbf{h}_{\mathcal{N}_v}^t = \sum_{k \in \mathcal{N}_v} \mathbf{h}_k^{t-1} + \mathbf{b}$	$\mathbf{z}_v^t = \sigma(\mathbf{W}^z \mathbf{h}_{\mathcal{N}_v}^t + \mathbf{U}^z \mathbf{h}_v^{t-1})$ $\mathbf{r}_v^t = \sigma(\mathbf{W}^r \mathbf{h}_{\mathcal{N}_v}^t + \mathbf{U}^r \mathbf{h}_v^{t-1})$ $\tilde{\mathbf{h}}_v^t = \tanh(\mathbf{W}^h \mathbf{h}_{\mathcal{N}_v}^t + \mathbf{U}(\mathbf{r}_v^t \odot \mathbf{h}_v^{t-1}))$ $\mathbf{h}_v^t = (1 - \mathbf{z}_v^t) \odot \mathbf{h}_v^{t-1} + \mathbf{z}_v^t \odot \tilde{\mathbf{h}}_v^t$
Tree LSTM (Child sum)	$\mathbf{h}_{\mathcal{N}_v}^t = \sum_{k \in \mathcal{N}_v} \mathbf{U}^i \mathbf{h}_k^{t-1}$ $\mathbf{h}_{\mathcal{N}_v,k}^f = \mathbf{U}^f \mathbf{h}_k^{t-1}$ $\mathbf{h}_{\mathcal{N}_v}^{to} = \sum_{k \in \mathcal{N}_v} \mathbf{U}^o \mathbf{h}_k^{t-1}$ $\mathbf{h}_{\mathcal{N}_v}^u = \sum_{k \in \mathcal{N}_v} \mathbf{U}^u \mathbf{h}_k^{t-1}$	$\mathbf{i}_v^t = \sigma(\mathbf{W}^i \mathbf{x}_v^t + \mathbf{h}_{\mathcal{N}_v}^t + \mathbf{b}^i)$ $\mathbf{f}_v^t = \sigma(\mathbf{W}^f \mathbf{x}_v^t + \mathbf{h}_{\mathcal{N}_v,k}^f + \mathbf{b}^f)$ $\mathbf{o}_v^t = \sigma(\mathbf{W}^o \mathbf{x}_v^t + \mathbf{h}_{\mathcal{N}_v}^{to} + \mathbf{b}^o)$ $\mathbf{u}_v^t = \tanh(\mathbf{W}^u \mathbf{x}_v^t + \mathbf{h}_{\mathcal{N}_v}^u + \mathbf{b}^u)$ $\mathbf{c}_v^t = \mathbf{i}_v^t \odot \mathbf{u}_v^t + \sum_{k \in \mathcal{N}_v} \mathbf{f}_v^t \odot \mathbf{c}_k^{t-1}$ $\mathbf{h}_v^t = \mathbf{o}_v^t \odot \tanh(\mathbf{c}_v^t)$
Tree LSTM (N-ary)	$\mathbf{h}_{\mathcal{N}_v}^t = \sum_{l=1}^K \mathbf{U}_l^i \mathbf{h}_{vl}^{t-1}$ $\mathbf{h}_{\mathcal{N}_v,k}^f = \sum_{l=1}^K \mathbf{U}_{kl}^f \mathbf{h}_{vl}^{t-1}$ $\mathbf{h}_{\mathcal{N}_v}^{to} = \sum_{l=1}^K \mathbf{U}_l^o \mathbf{h}_{vl}^{t-1}$ $\mathbf{h}_{\mathcal{N}_v}^u = \sum_{l=1}^K \mathbf{U}_l^u \mathbf{h}_{vl}^{t-1}$	
Graph LSTM in (Peng et al., 2017)	$\mathbf{h}_{\mathcal{N}_v}^t = \sum_{k \in \mathcal{N}_v} \mathbf{U}_{m(v,k)}^i \mathbf{h}_k^{t-1}$ $\mathbf{h}_{\mathcal{N}_v,k}^f = \mathbf{U}_{m(v,k)}^f \mathbf{h}_k^{t-1}$ $\mathbf{h}_{\mathcal{N}_v}^{to} = \sum_{k \in \mathcal{N}_v} \mathbf{U}_{m(v,k)}^o \mathbf{h}_k^{t-1}$ $\mathbf{h}_{\mathcal{N}_v}^u = \sum_{k \in \mathcal{N}_v} \mathbf{U}_{m(v,k)}^u \mathbf{h}_k^{t-1}$	

## Graph neural networks: A review of methods and applications

Jie Zhou <sup>a,1</sup>, Ganqu Cui <sup>a,1</sup>, Shengding Hu <sup>a</sup>, Zhengyan Zhang <sup>a</sup>, Cheng Yang <sup>b</sup>, Zhiyuan Liu <sup>a,\*</sup>, Lifeng Wang <sup>c</sup>, Changcheng Li <sup>c</sup>, Maosong Sun <sup>a</sup>

<sup>a</sup> Department of Computer Science and Technology, Tsinghua University, Beijing, China

<sup>b</sup> School of Computer Science, Beijing University of Posts and Telecommunications, China

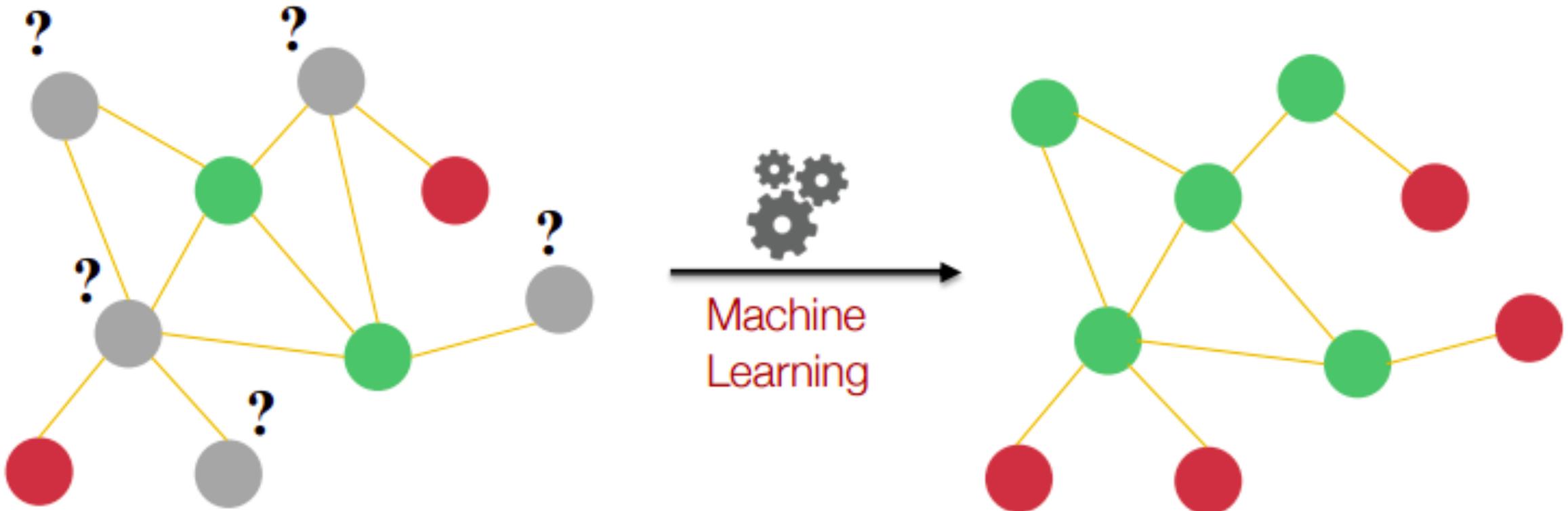
<sup>c</sup> Tencent Incorporation, Shenzhen, China

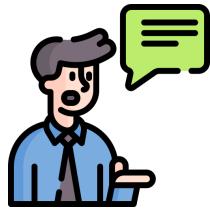
QUIZ TIME

# Outline

- **Objective**
- **Introduction to Graph Data**
- **Graph Data with Neural Network**
- **Node Classification Problem: Cora Citation Dataset**
- **Summary**

# Node Classification Problem





## Knowledge Graphs and Node Classification

We have one large graph and not many individual graphs (like molecules)

We infer on unlabeled nodes in this large graph and hence perform node-level predictions

--> We have to use different nodes of the graph depending on what we want to do



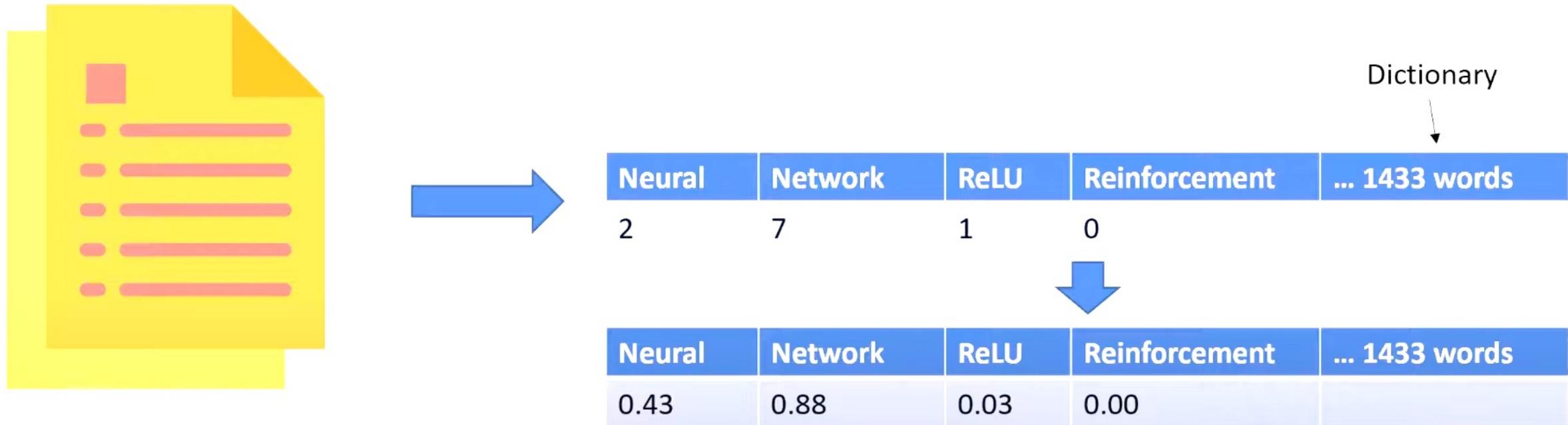
## Dataset Introduction: Cora Citation Dataset in PyTorch Geometric

The Cora dataset consists of 2708 scientific publications classified into one of seven classes.

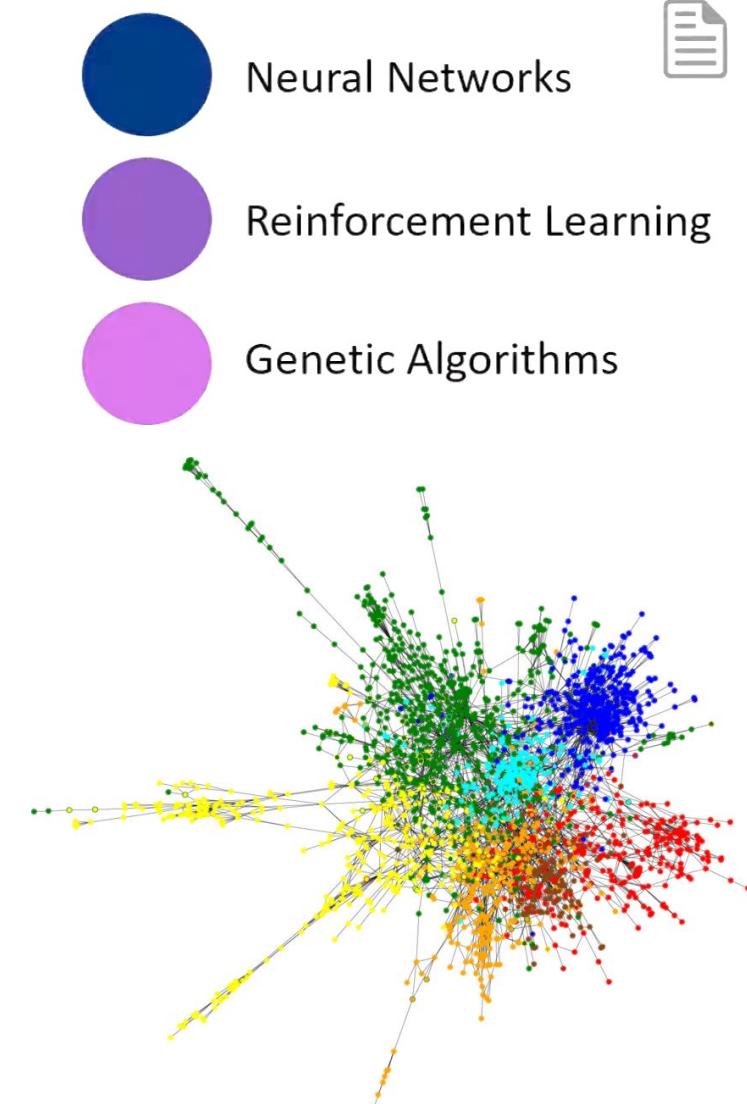
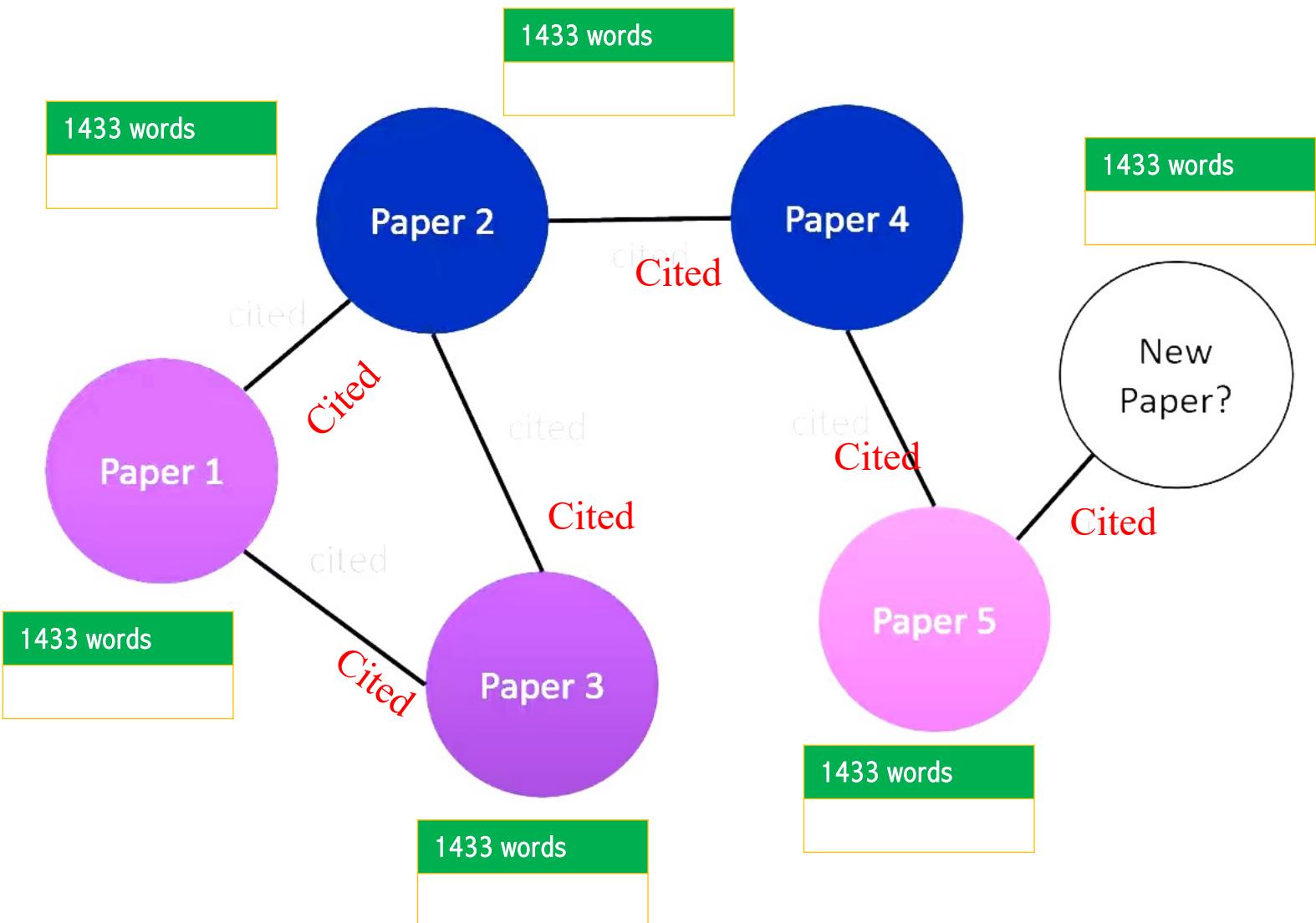
Each publication in the dataset is described by a 0/1-valued word vector indicating the absence/presence of the corresponding word from the dictionary.

- The dictionary consists of 1433 unique words.
- Nodes = Publications (Papers, Books ...)
- Edges = Citations Node Features = word vectors
- 7 Labels = Publication
- type e.g. Neural\_Networks, Rule\_Learning, Reinforcement\_Learning, Probabilistic\_Methods...

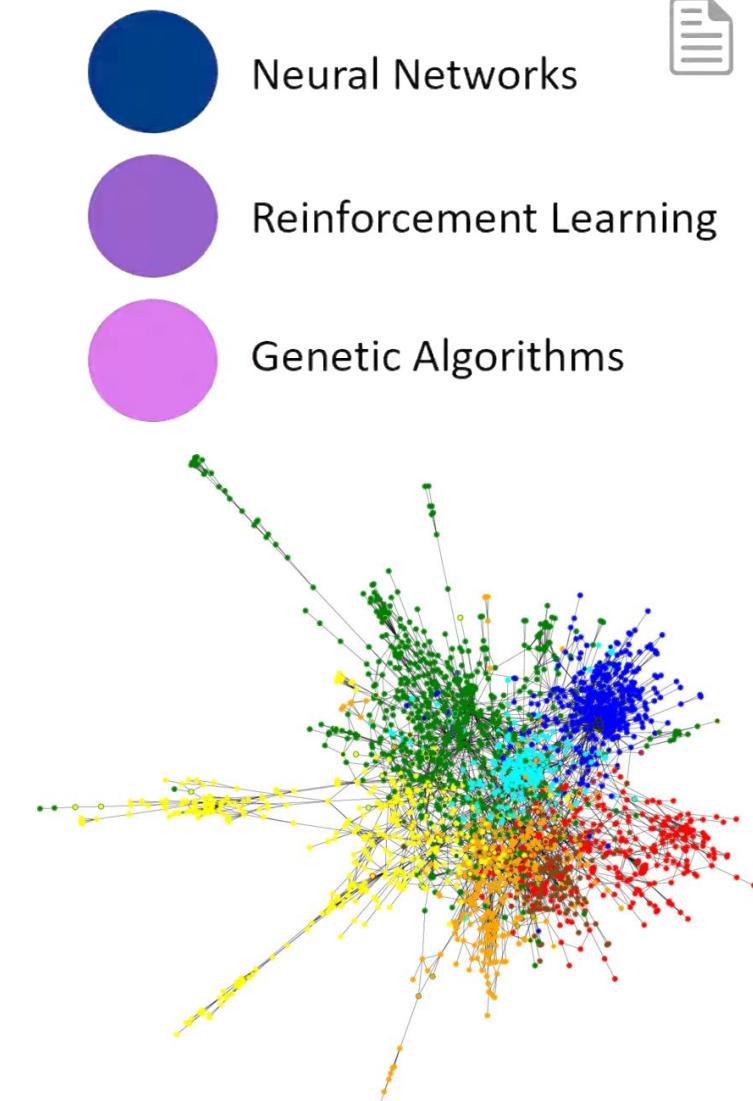
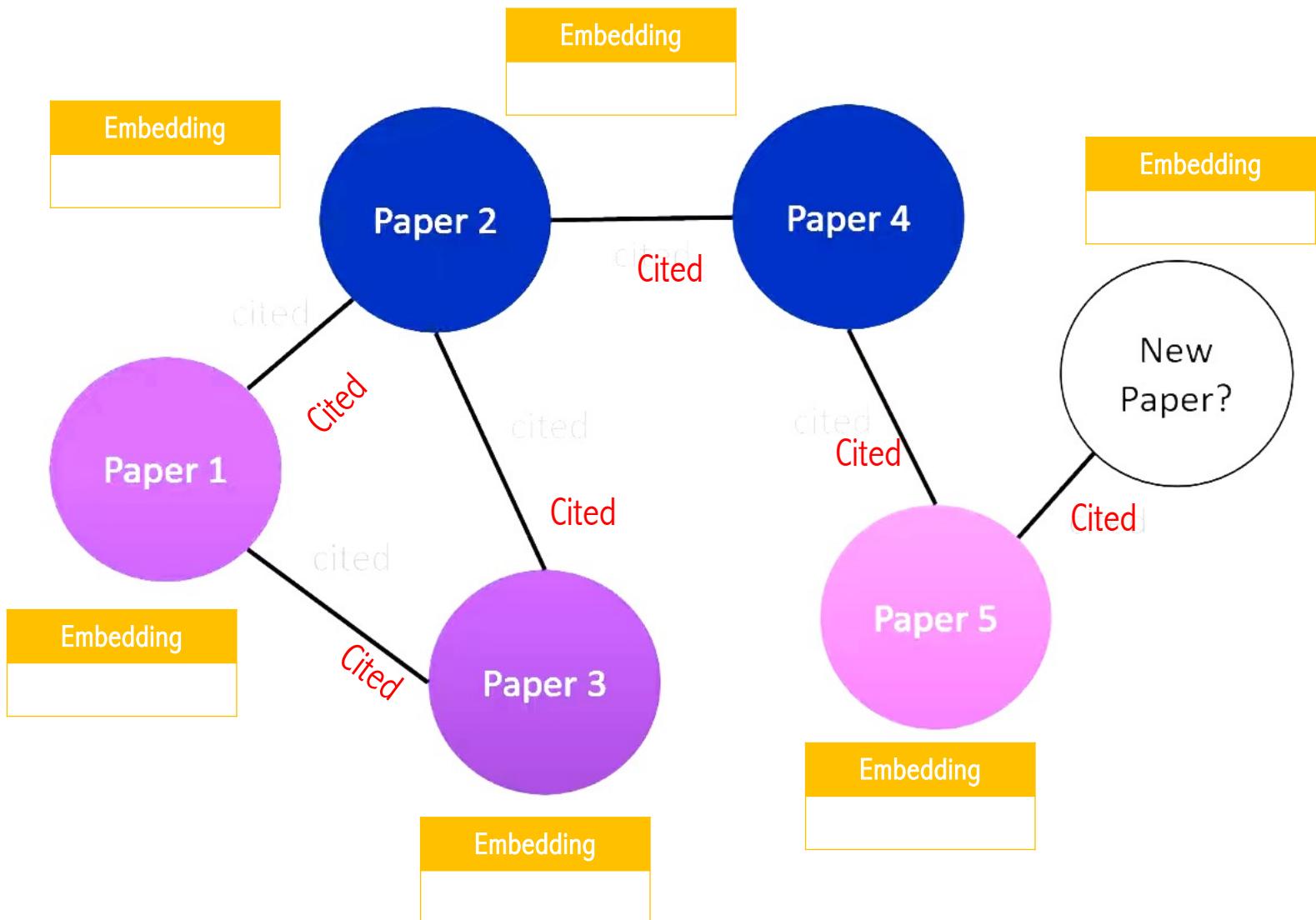
# BoW Representation



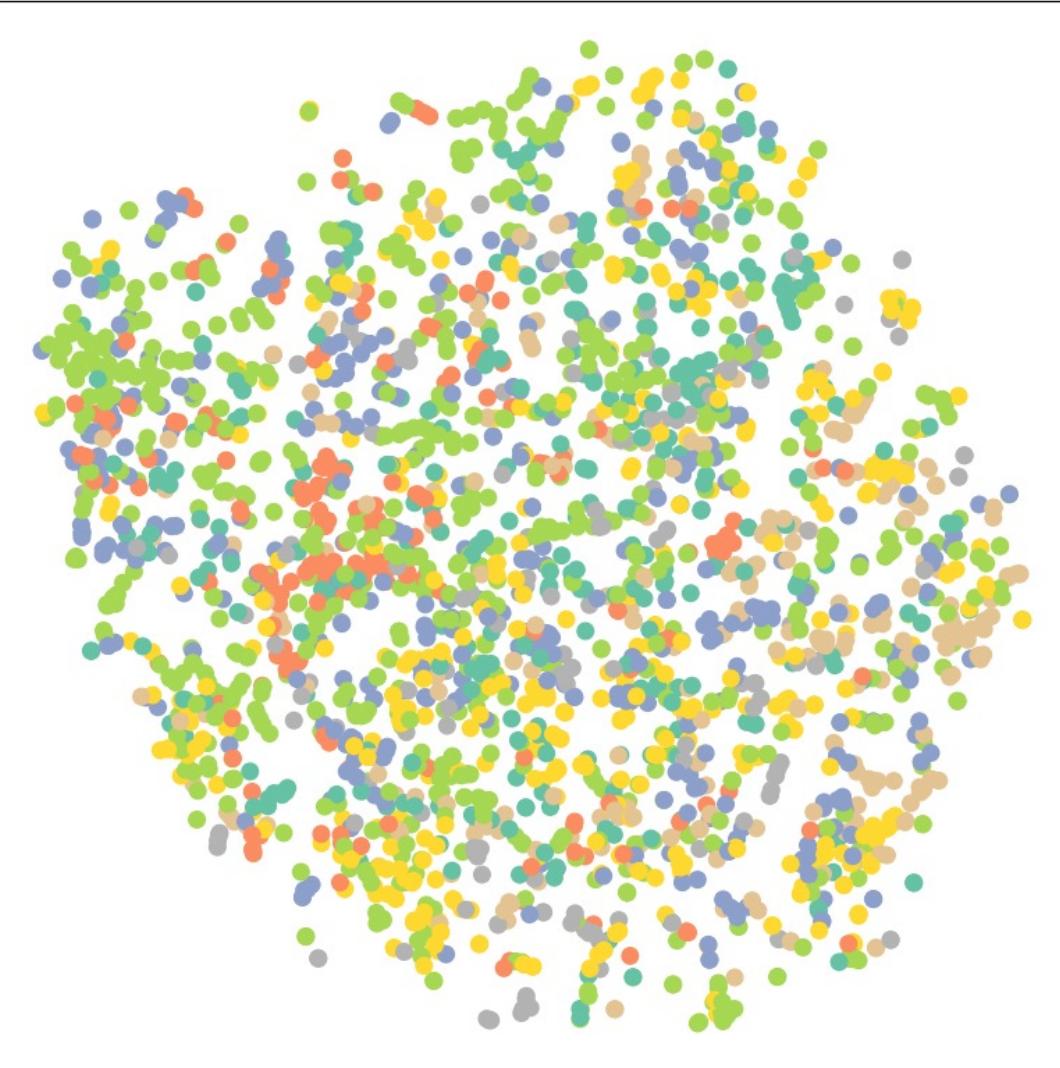
# Cora Citation Dataset



# Cora Citation Dataset



# Visualize the node embeddings of the untrained GCN network



```
model = GCN(hidden_channels=16)  
model.eval()  
  
out = model(data.x, data.edge_index)  
visualize(out, color=data.y)
```



## Install Pytorch Geometric

```
# Check CUDA Version
!python -c "import torch; print(torch.version.cuda)"

# Add this in a Google Colab cell to install the correct version of Pytorch Geometric.
import torch

def format_pytorch_version(version):
    return version.split('+')[0]

TORCH_version = torch.__version__
TORCH = format_pytorch_version(TORCH_version)

def format_cuda_version(version):
    return 'cu' + version.replace('.', '')

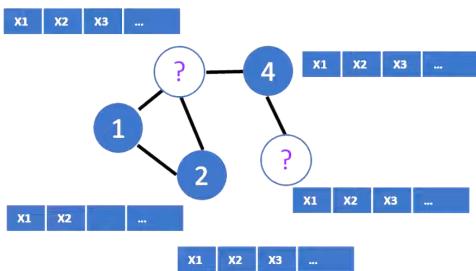
CUDA_version = torch.version.cuda
CUDA = format_cuda_version(CUDA_version)

!pip install torch-scatter      -f https://pytorch-geometric.com/whl/torch-{TORCH}+{CUDA}.html
!pip install torch-sparse       -f https://pytorch-geometric.com/whl/torch-{TORCH}+{CUDA}.html
!pip install torch-cluster      -f https://pytorch-geometric.com/whl/torch-{TORCH}+{CUDA}.html
!pip install torch-spline-conv  -f https://pytorch-geometric.com/whl/torch-{TORCH}+{CUDA}.html
!pip install torch-geometric
```

# GNN: Node Classification



## Load Cora Dataset



```
from torch_geometric.datasets import Planetoid
from torch_geometric.transforms import NormalizeFeatures

dataset = Planetoid(root='data/Planetoid', name='Cora', transform=NormalizeFeatures())

# Get some basic info about the dataset
print(f'Number of graphs: {len(dataset)}')
print(f'Number of features: {dataset.num_features}')
print(f'Number of classes: {dataset.num_classes}')
print(50*'=')

# There is only one graph in the dataset, use it as new data object
data = dataset[0]

# Gather some statistics about the graph.
print(data)
print(f'Number of nodes: {data.num_nodes}')
print(f'Number of edges: {data.num_edges}')
print(f'Number of training nodes: {data.train_mask.sum()}')
print(f'Training node label rate: {int(data.train_mask.sum()) / data.num_nodes:.2f}')
print(f'Is undirected: {data.is_undirected()}'
```

```
Number of graphs: 1
Number of features: 1433
Number of classes: 7
=====
Data(x=[2708, 1433], edge_index=[2, 10556], y=[2708], train_mask=[2708], val_mask=[2708], test_mask=[2708])
Number of nodes: 2708
Number of edges: 10556
Number of training nodes: 140
Training node label rate: 0.05
Is undirected: True
```

train\_mask=[2708], val\_mask=[2708], test\_mask=[2708])

# GNN: Node Classification MLP



## Define MLP

Here, we first reduce the 1433-dimensional feature vector to a low-dimensional embedding (hidden\_channels=16), while the second linear layer acts as a classifier that should map each low-dimensional node embedding to one of the 7 classes.

```
import torch
from torch.nn import Linear
import torch.nn.functional as F

class MLP(torch.nn.Module):
    def __init__(self, hidden_channels):
        super().__init__()
        torch.manual_seed(12345)
        self.lin1 = Linear(dataset.num_features, hidden_channels)
        self.lin2 = Linear(hidden_channels, dataset.num_classes)

    def forward(self, x):
        x = self.lin1(x)
        x = x.relu()
        x = F.dropout(x, p=0.5, training=self.training)
        x = self.lin2(x)
        return x

model = MLP(hidden_channels=16)
print(model)
```

```
test_acc = test()
print(f'Test Accuracy: {test_acc:.4f}')
```

Test Accuracy: 0.5900

# GNN: Node Classification MLP



## How to Train

Here, we first reduce the 1433-dimensional feature vector to a low-dimensional embedding (hidden\_channels=16), while the second linear layer acts as a classifier that should map each low-dimensional node embedding to one of the 7 classes.

```
test_acc = test()  
print(f'Test Accuracy: {test_acc:.4f}')
```

Test Accuracy: 0.5900

```
from IPython.display import Javascript # Restrict height of output cell.  
display(Javascript("google.colab.output.setIframeHeight(0, true, {maxHeight: 300})"))  
  
model = MLP(hidden_channels=16)  
criterion = torch.nn.CrossEntropyLoss() # Define loss criterion.  
optimizer = torch.optim.Adam(model.parameters(), lr=0.01, weight_decay=5e-4) # Define optimizer.  
  
def train():  
    model.train()  
    optimizer.zero_grad() # Clear gradients.  
    out = model(data.x) # Perform a single forward pass.  
    loss = criterion(out[data.train_mask], data.y[data.train_mask]) # Compute the loss solely based on the  
    loss.backward() # Derive gradients.  
    optimizer.step() # Update parameters based on gradients.  
    return loss  
  
def test():  
    model.eval()  
    out = model(data.x)  
    pred = out.argmax(dim=1) # Use the class with highest probability.  
    test_correct = pred[data.test_mask] == data.y[data.test_mask] # Check against ground-truth labels.  
    test_acc = int(test_correct.sum()) / int(data.test_mask.sum()) # Derive ratio of correct predictions.  
    return test_acc
```

# GNN: Node Classification GCN



Define GCN

```
class GCN(torch.nn.Module):
    def __init__(self, hidden_channels):
        super().__init__()
        torch.manual_seed(1234567)
        self.conv1 = GCNConv(dataset.num_features, hidden_channels)
        self.conv2 = GCNConv(hidden_channels, dataset.num_classes)

    def forward(self, x, edge_index):
        x = self.conv1(x, edge_index)
        x = x.relu()
        x = F.dropout(x, p=0.5, training=self.training)
        x = self.conv2(x, edge_index)
        return x

model = GCN(hidden_channels=16)
print(model)
```

GCN(
 (conv1): GCNConv(1433, 16)
 (conv2): GCNConv(16, 7)
)

we will use one of the most simple GNN operators, the **GCN layer** ([Kipf et al. \(2017\)](#)), which is defined as

$$\mathbf{x}_v^{(\ell+1)} = \mathbf{W}^{(\ell+1)} \sum_{w \in \mathcal{N}(v) \cup \{v\}} \frac{1}{c_{w,v}} \cdot \mathbf{x}_w^{(\ell)}$$

# GNN: Node Classification GCN



## Define GCN

Dropout is only applied in the training step, but not for predictions

We have 2 Message Passing Layers and one Linear output layer

We use the softmax function for the classification problem

The output of the model are 7 probabilities, one for each class

```
class GCN(torch.nn.Module):
    def __init__(self, hidden_channels):
        super(GCN, self).__init__()
        torch.manual_seed(42)

        # Initialize the layers
        self.conv1 = GCNConv(dataset.num_features, hidden_channels)
        self.conv2 = GCNConv(hidden_channels, hidden_channels)
        self.out = Linear(hidden_channels, dataset.num_classes)

    def forward(self, x, edge_index):
        # First Message Passing Layer (Transformation)
        x = self.conv1(x, edge_index)
        x = x.relu()
        x = F.dropout(x, p=0.5, training=self.training)

        # Second Message Passing Layer
        x = self.conv2(x, edge_index)
        x = x.relu()
        x = F.dropout(x, p=0.5, training=self.training)

        # Output layer
        x = F.softmax(self.out(x), dim=1)
        return x
```

we will use one of the most simple GNN operators, the **GCN layer** ([Kipf et al. \(2017\)](#)), which is defined as

$$\mathbf{x}_v^{(\ell+1)} = \mathbf{W}^{(\ell+1)} \sum_{w \in \mathcal{N}(v) \cup \{v\}} \frac{1}{c_{w,v}} \cdot \mathbf{x}_w^{(\ell)}$$

# GNN: Node Classification GCN

```
from IPython.display import Javascript # Restrict height of output cell.
display(Javascript('''google.colab.output.setIframeHeight(0, true, {maxHeight: 300})'''))

model = GCN(hidden_channels=16)
optimizer = torch.optim.Adam(model.parameters(), lr=0.01, weight_decay=5e-4)
criterion = torch.nn.CrossEntropyLoss()

def train():
    model.train()
    optimizer.zero_grad() # Clear gradients.
    out = model(data.x, data.edge_index) # Perform a single forward pass.
    loss = criterion(out[data.train_mask], data.y[data.train_mask]) # Compute the loss solely based on the training set.
    loss.backward() # Derive gradients.
    optimizer.step() # Update parameters based on gradients.
    return loss

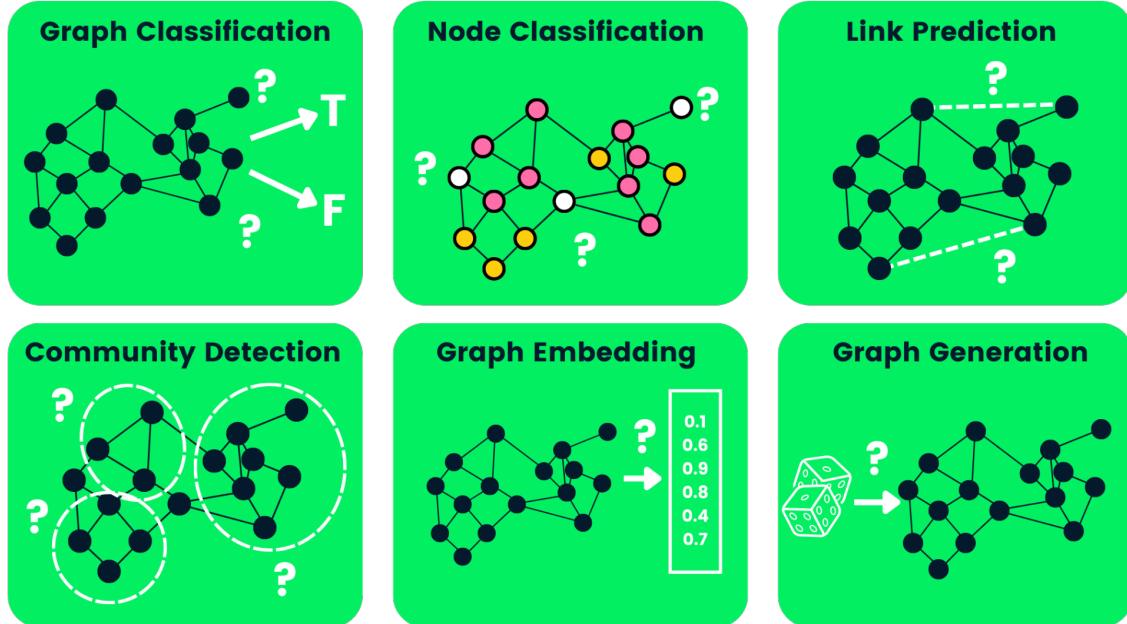
def test():
    model.eval()
    out = model(data.x, data.edge_index)
    pred = out.argmax(dim=1) # Use the class with highest probability.
    test_correct = pred[data.test_mask] == data.y[data.test_mask] # Check against ground-truth labels.
    test_acc = int(test_correct.sum()) / int(data.test_mask.sum()) # Derive ratio of correct predictions.
    return test_acc

for epoch in range(1, 101):
    loss = train()
    print(f'Epoch: {epoch:03d}, Loss: {loss:.4f}')
```

```
▶ test_acc = test()
print(f'Test Accuracy: {test_acc:.4f}')

Test Accuracy: 0.8150
```

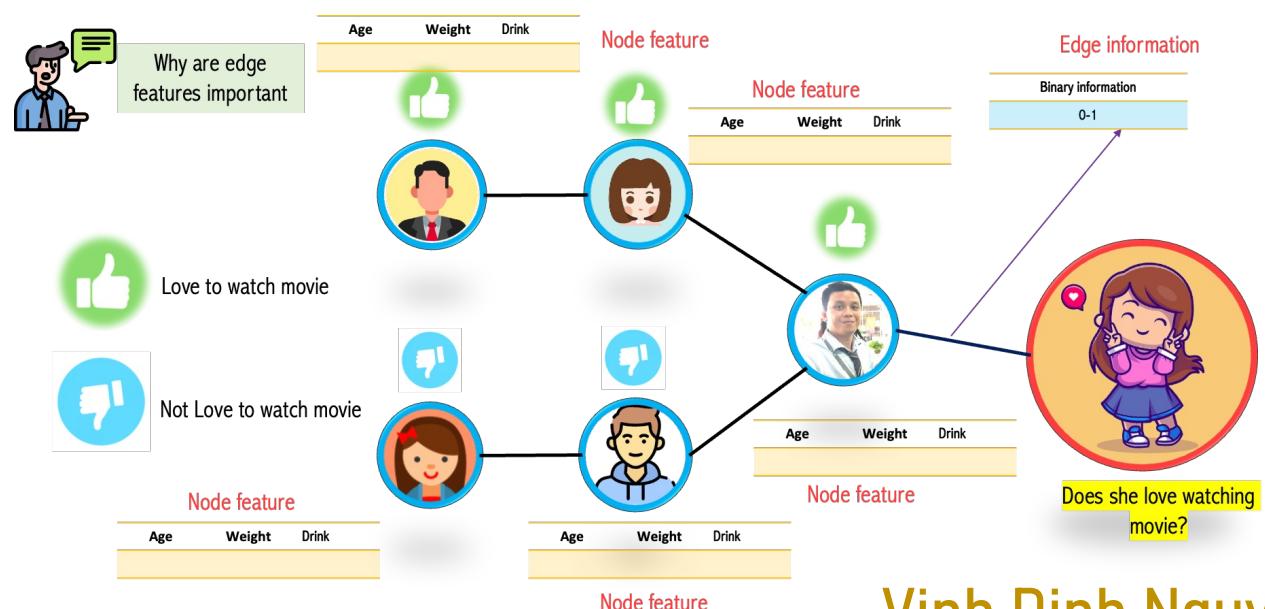
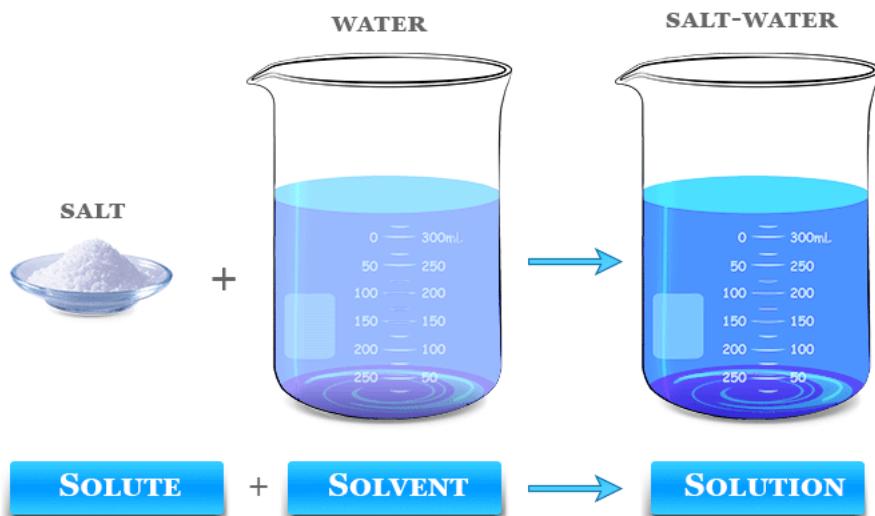
# Summary



- 
- A vertical stack of four colored triangles (yellow, green, teal, blue) is positioned to the left of a list of four learning objectives, each enclosed in a rounded rectangle.
- What is Graph Data Around Us
  - Understand Graph Neural Network
  - Understand Graph Convolutional Neural Network
  - Node Classification with Cora Citation Dataset

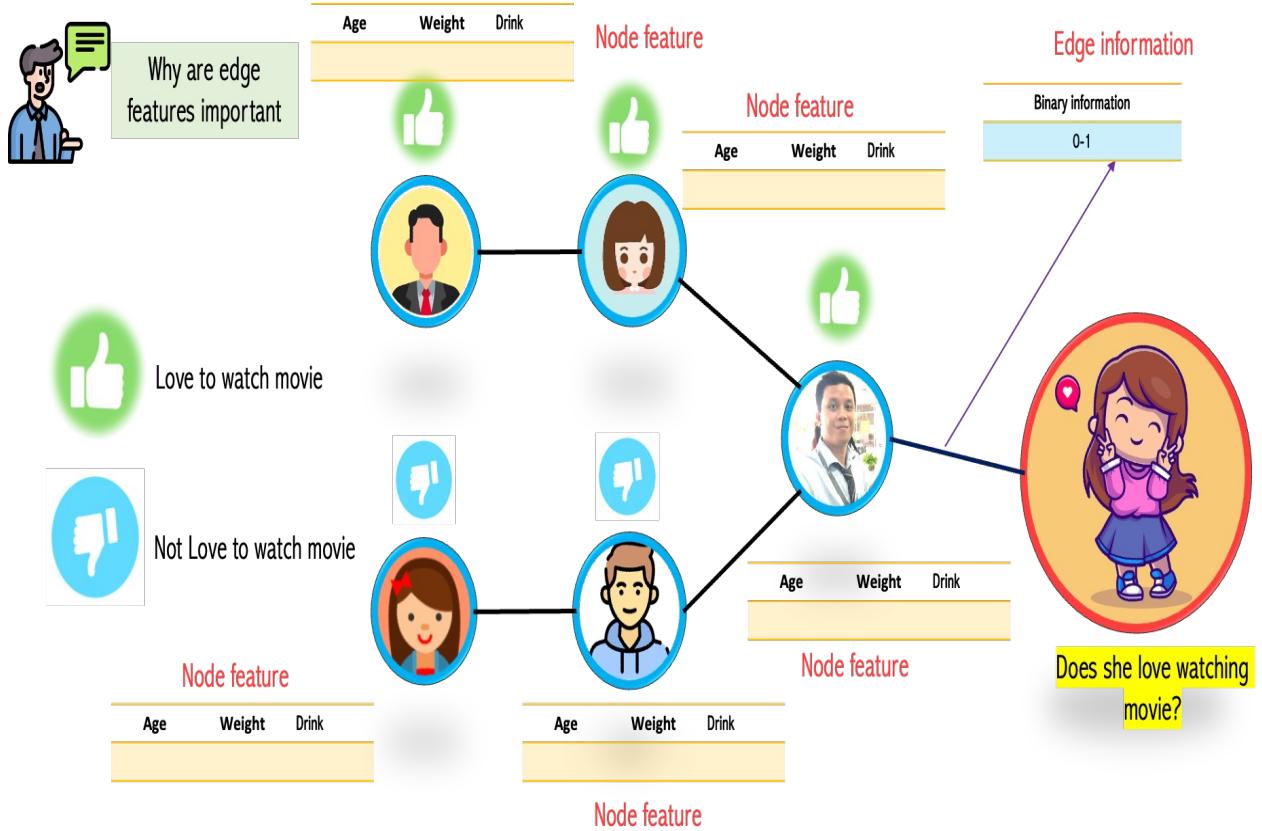


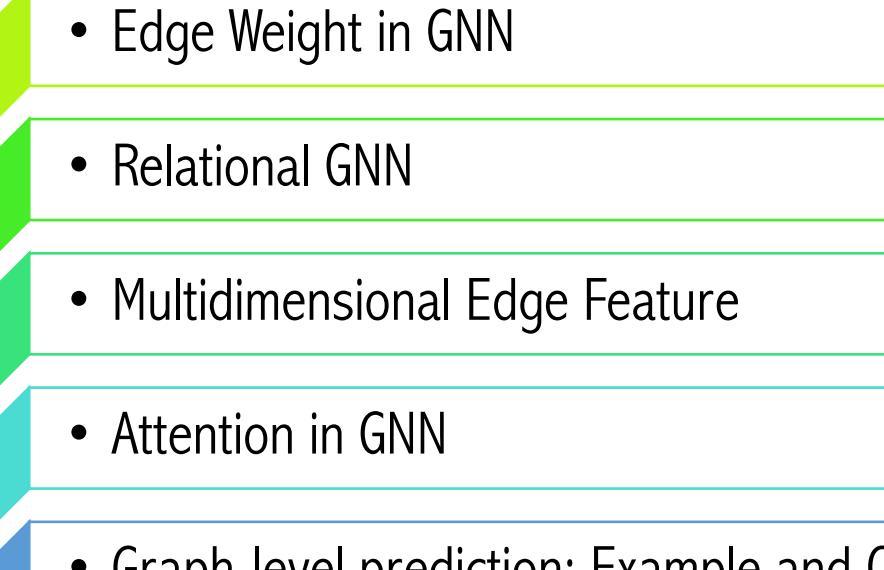
# Advanced Graph Neural Network (GCN, Graph Relational, Attention & Level-Prediction)



Vinh Dinh Nguyen  
PhD in Computer Science

# Objective



- 
  - How to integrate edge feature to GNN
  - Edge Weight in GNN
  - Relational GNN
  - Multidimensional Edge Feature
  - Attention in GNN
  - Graph-level prediction: Example and Code

# Outline

- Edge Feature in GNN
- Edge Weight in GNN
- Relational GNN
- Multidimension Edge Feature
- Attention in GNN
- Example: Graph-Level Prediction
- Summary

# Outline

- Edge Feature in GNN
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- Summary

# Edge Feature in GNN: Last But Not Least



Why are edge features important



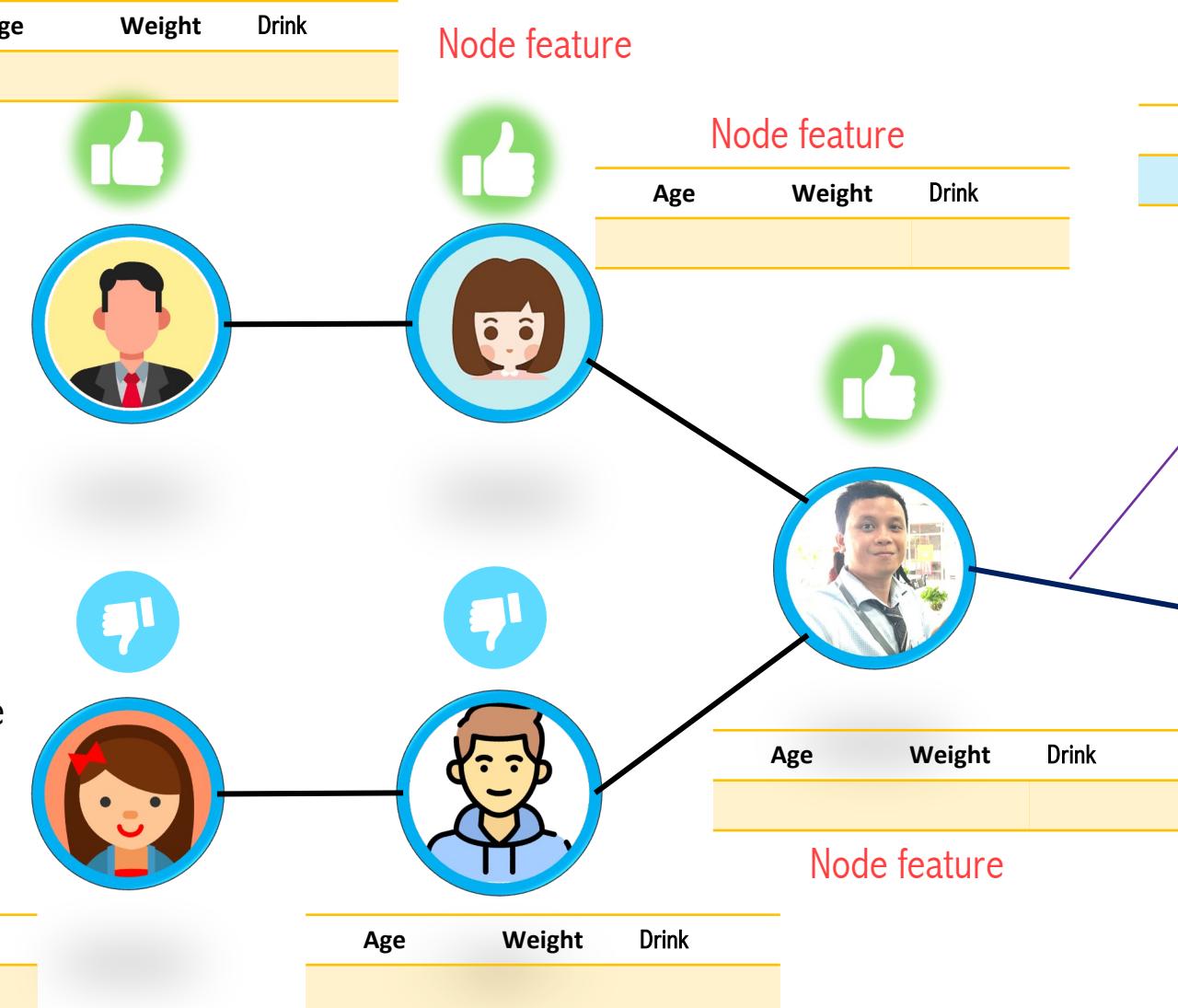
Love to watch movie



Not Love to watch movie

Node feature

Age	Weight	Drink



Edge information

Binary information
0-1

Does she love watching movie?

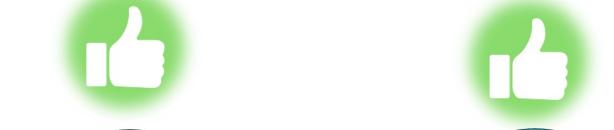
# Edge Feature in GNN: Last But Not Least



Why are edge features important

Age	Weight	Drink

Node feature



Node feature

Age	Weight	Drink

Edge information

Friend	Friend Since	Live together
Yes	9	No

How do edge features utilize in GNN?



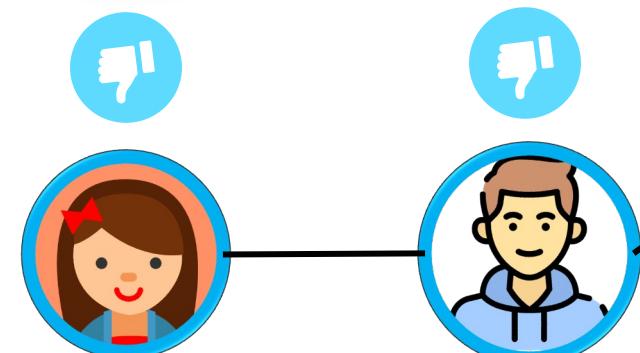
Love to watch movie



Not Love to watch movie

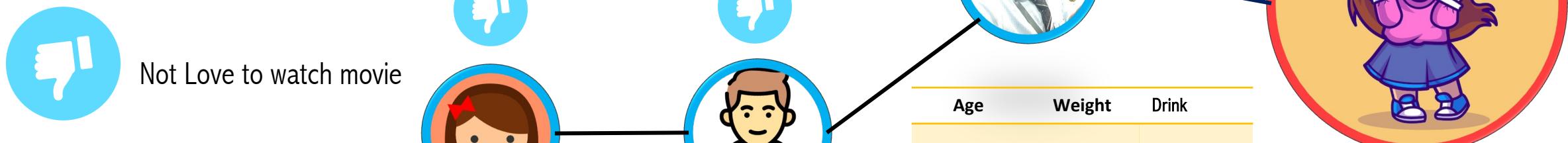
Node feature

Age	Weight	Drink

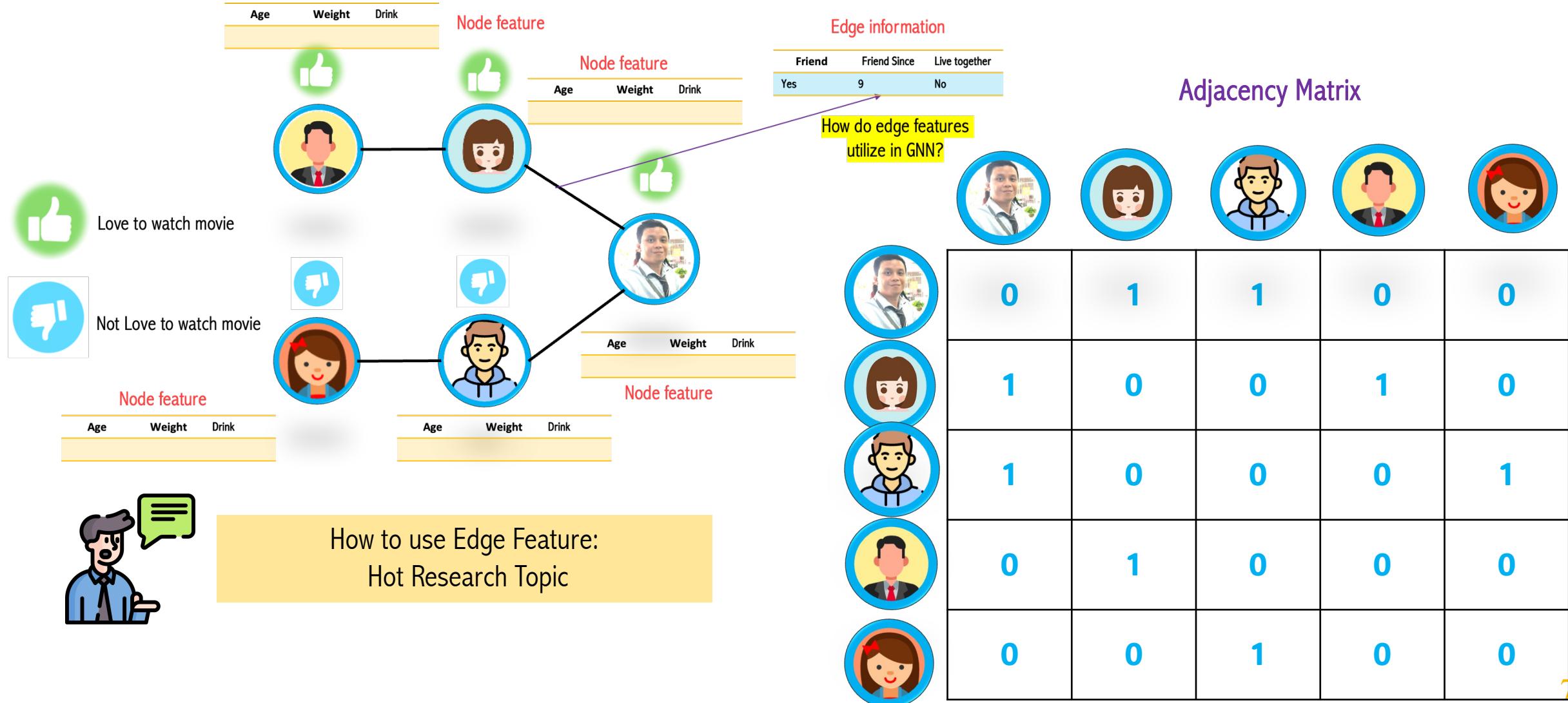


Age	Weight	Drink

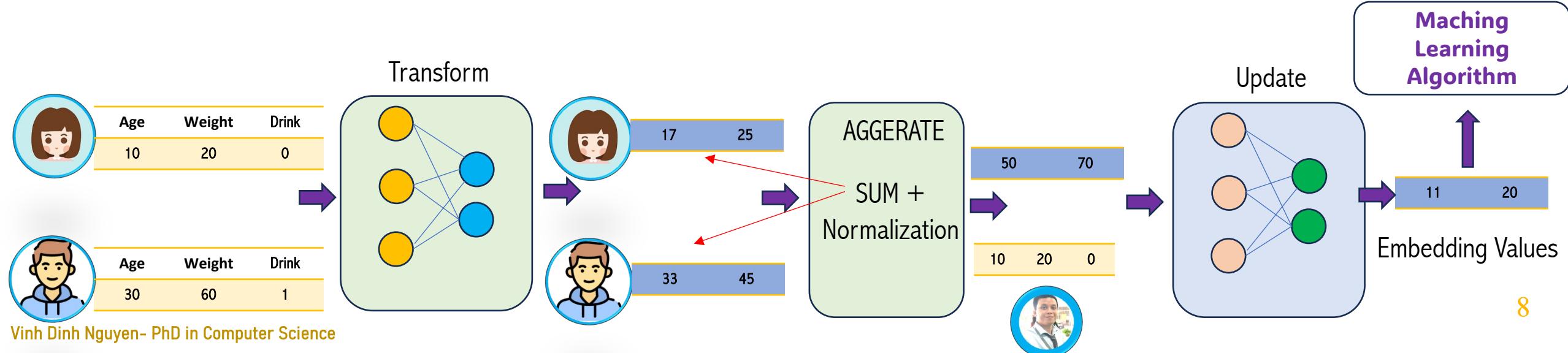
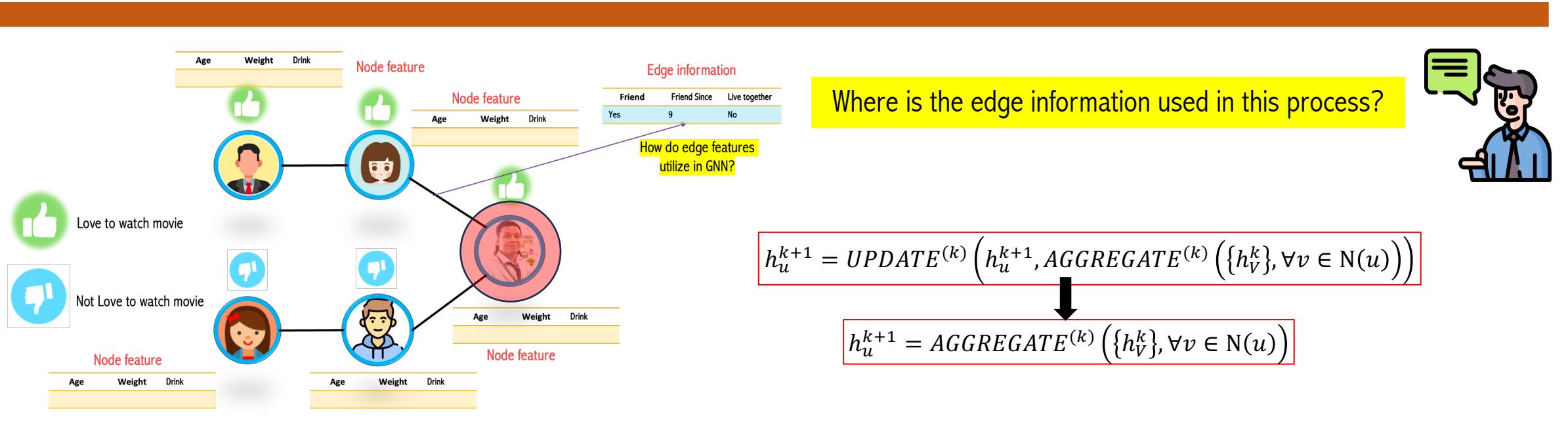
Node feature



# Edge Feature in GNN



# Node Embedding for Vinh Nguyen



# Node Embedding for Vinh Nguyen



Adjacency Matrix

0	1	1	0	0
1	0	0	1	0
1	0	0	0	1
0	1	0	0	0
0	0	1	0	0

Where is the edge information used in this process?

Binary approach: use directly when we select the node

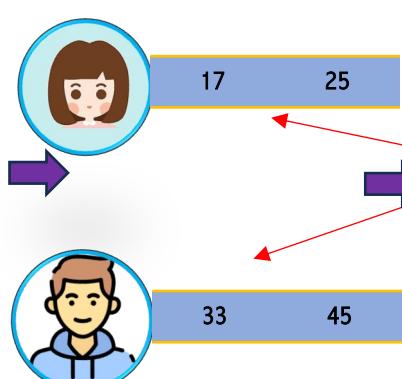
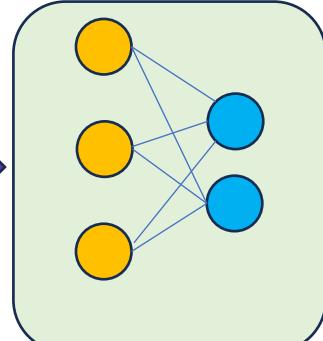
$$h_{\text{Vinh}}^{k+1} = \text{UPDATE}^{(k)}(h_{\text{Vinh}}^k, \text{AGGREGATE}_{j \in N(\text{Vinh})} \text{TRANSFORM}(h_j^k))$$

$$h_{\text{Vinh}}^{k+1} = \text{AGGREGATE}_{j \in N(\text{Vinh})} \text{TRANSFORM}(h_j^k)$$

Maching Learning Algorithm

Transform

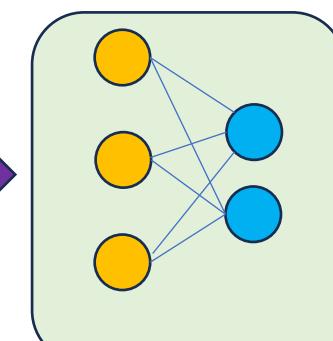
	Age	Weight	Drink
10	20	0	
30	60	1	



AGGERATE  
SUM + Normalization

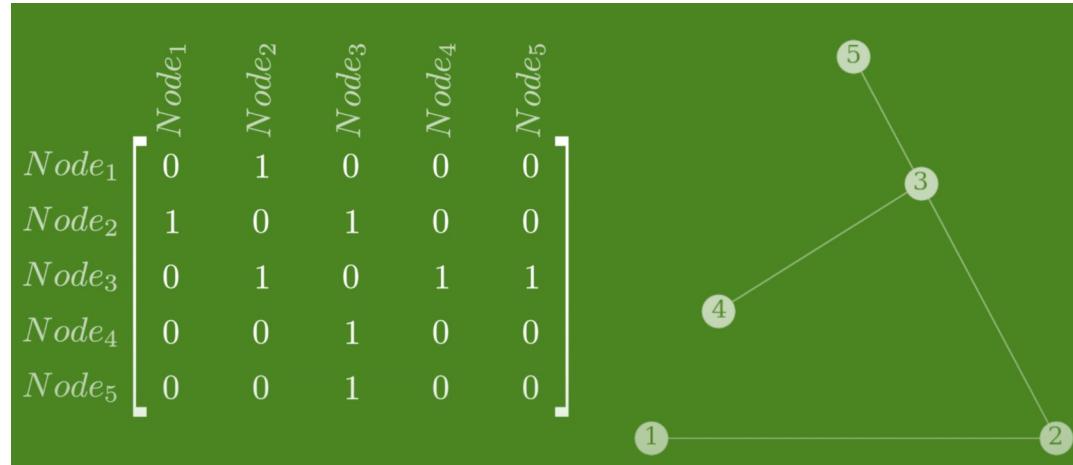
$$\begin{matrix} 50 & 70 \\ + & \\ 10 & 20 & 0 \end{matrix}$$

Update



Embedding Values

# Example



The '0' of the adjacency matrix cancel the contribution of all connected nodes, resulting a sum of just the connected nodes.

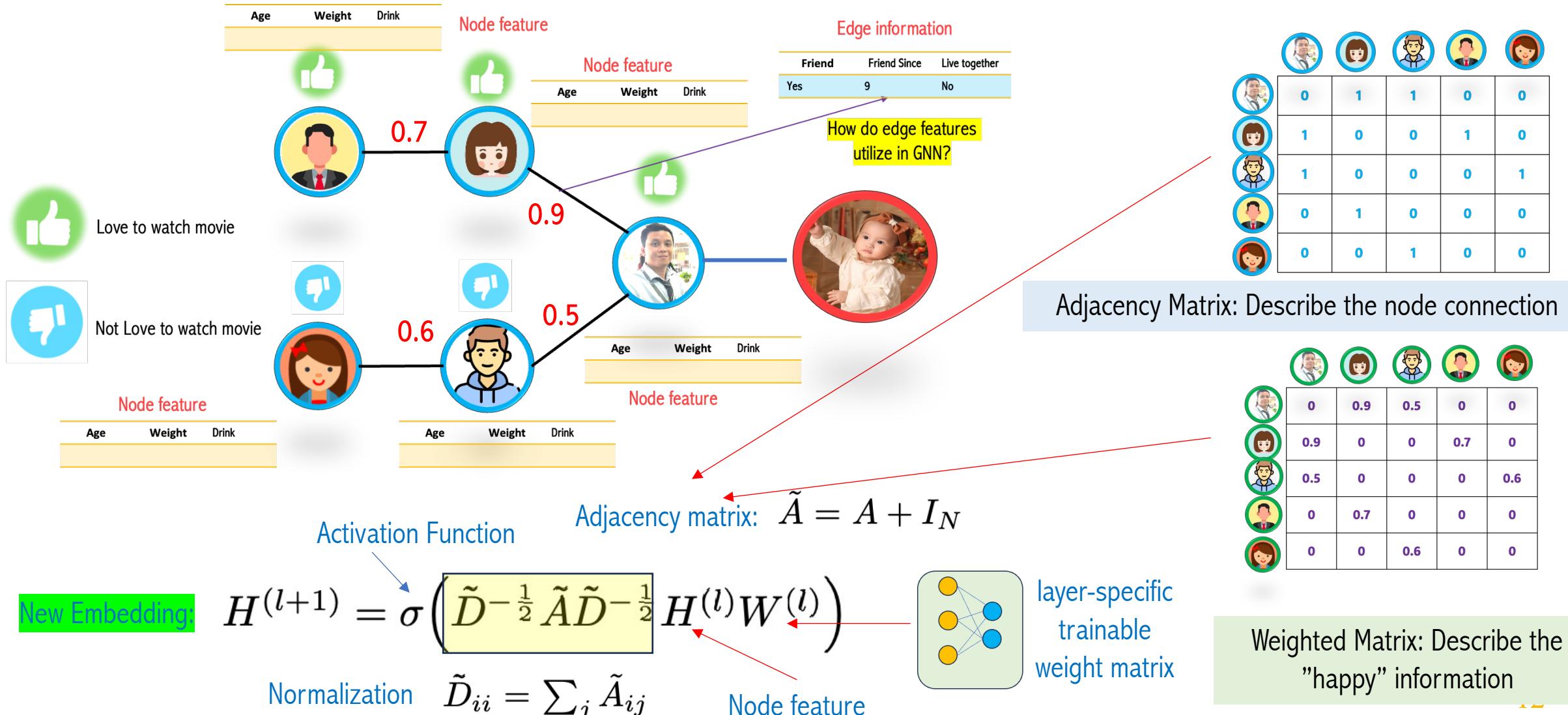


$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}
 \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}
 = 
 \begin{bmatrix}
 (0)(1) + (1)(2) + (0)(3) + (0)(4) + (0)(5) \\
 (1)(1) + (0)(2) + (1)(3) + (0)(4) + (0)(5) \\
 (0)(1) + (1)(2) + (0)(3) + (1)(4) + (1)(5) \\
 (0)(1) + (0)(2) + (1)(3) + (0)(4) + (0)(5) \\
 (0)(1) + (0)(2) + (1)(3) + (0)(4) + (0)(5)
 \end{bmatrix}
 = 
 \begin{bmatrix} 2 \\ 4 \\ 11 \\ 3 \\ 3 \end{bmatrix}$$

# Outline

- Edge Feature in GNN
- Edge Weight in GNN
- Relational GNN
- Multidimension Edge Feature
- Attention in GNN
- Example: Graph-Level Prediction
- Summary

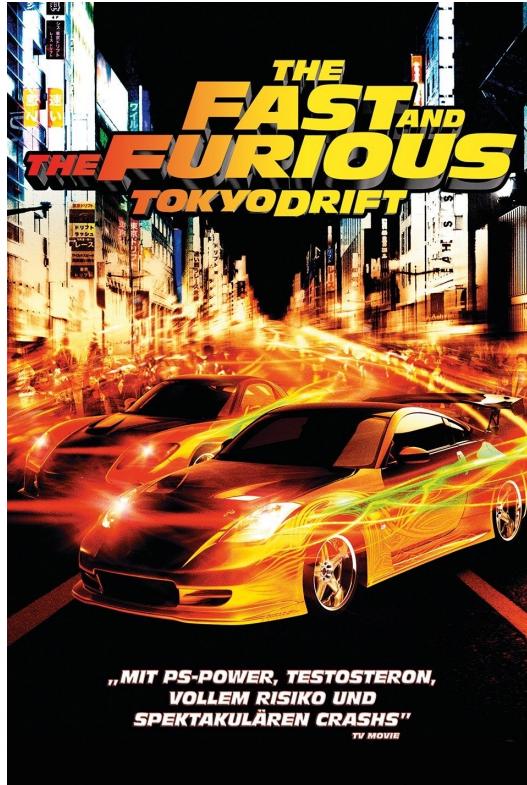
# Edge Weight: Common Approach



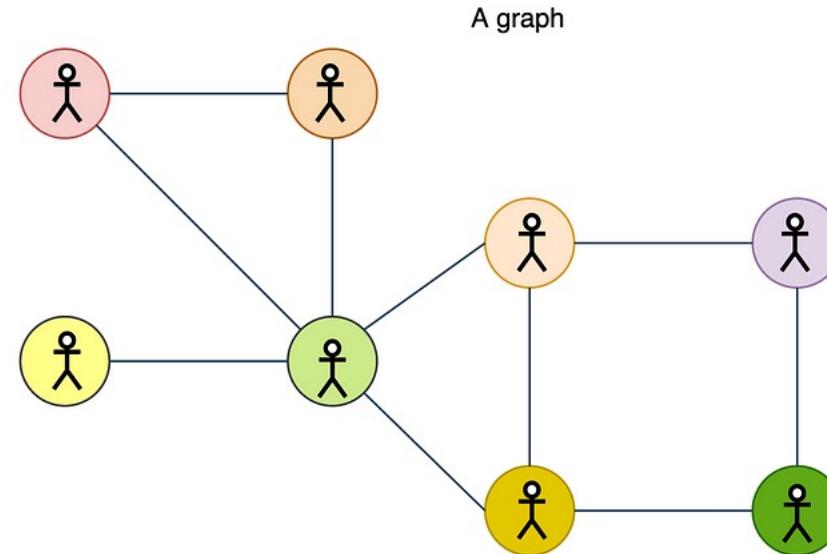
# GNN: Review

*You know, who you choose to be around you, let's you know who you are.*

*The Fast and the Furious: Tokyo Drift.*

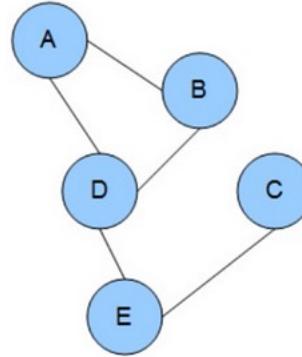


# GNN: Review

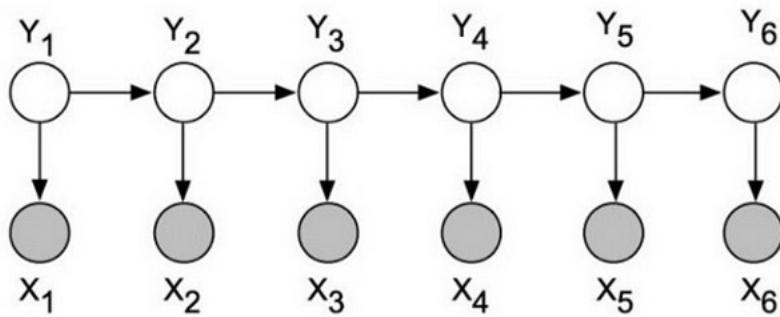


In social networks, friend connections can be realized by a social graph.

# GNN: Review

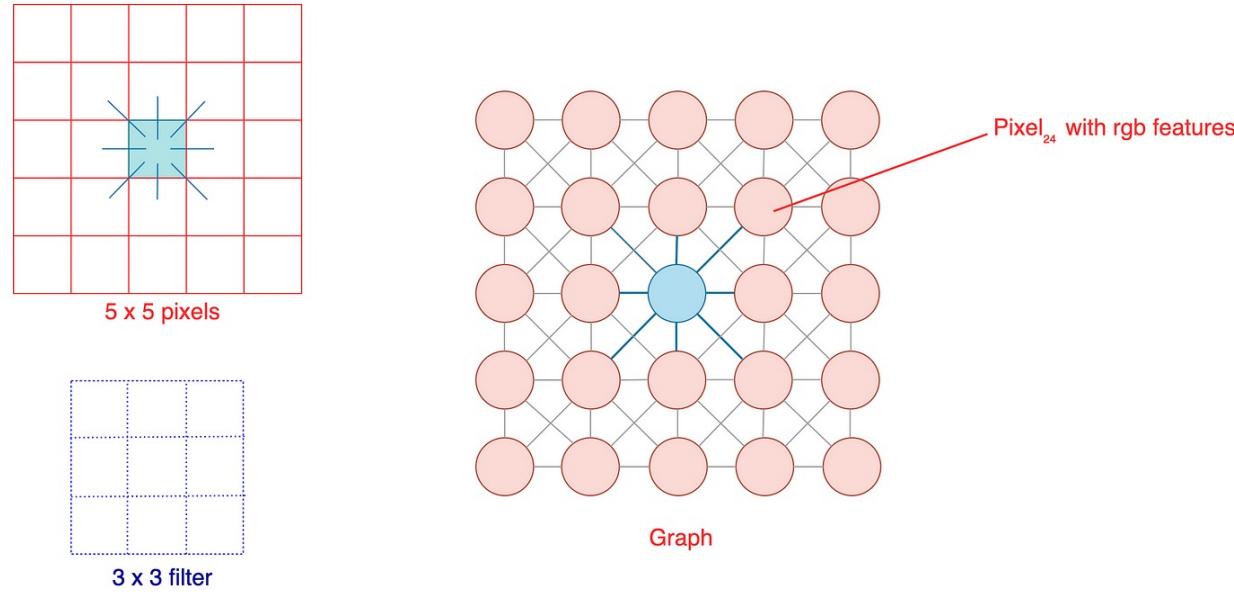


HMM



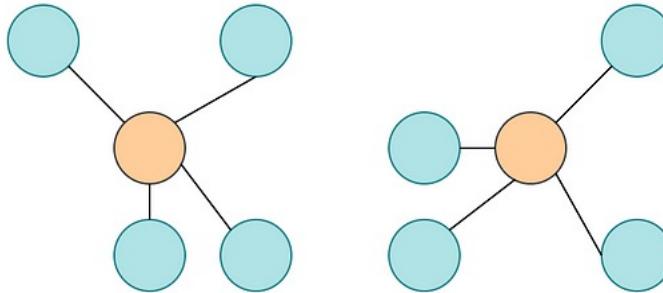
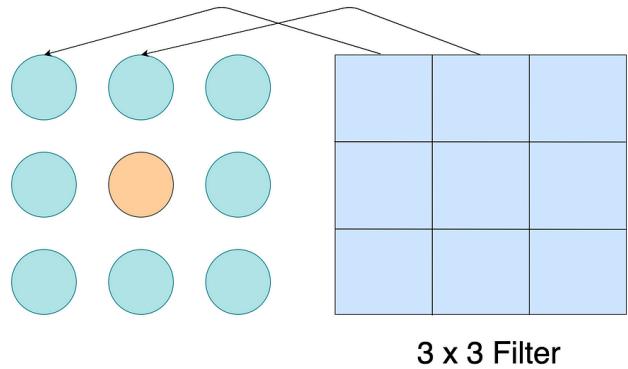
In speech recognition, the phoneme  $Y_i$  and the acoustic model  $x_i$  form an HMM (a graph for speech recognition).

# GNN: Review



Even on CNN, an input image can be modeled as a graph. For example, the graph for a  $5 \times 5$  image. Each node represents a pixel and for the case of a  $3 \times 3$  filter, every node is connected to its eight immediate neighbors.

# GNN: Review

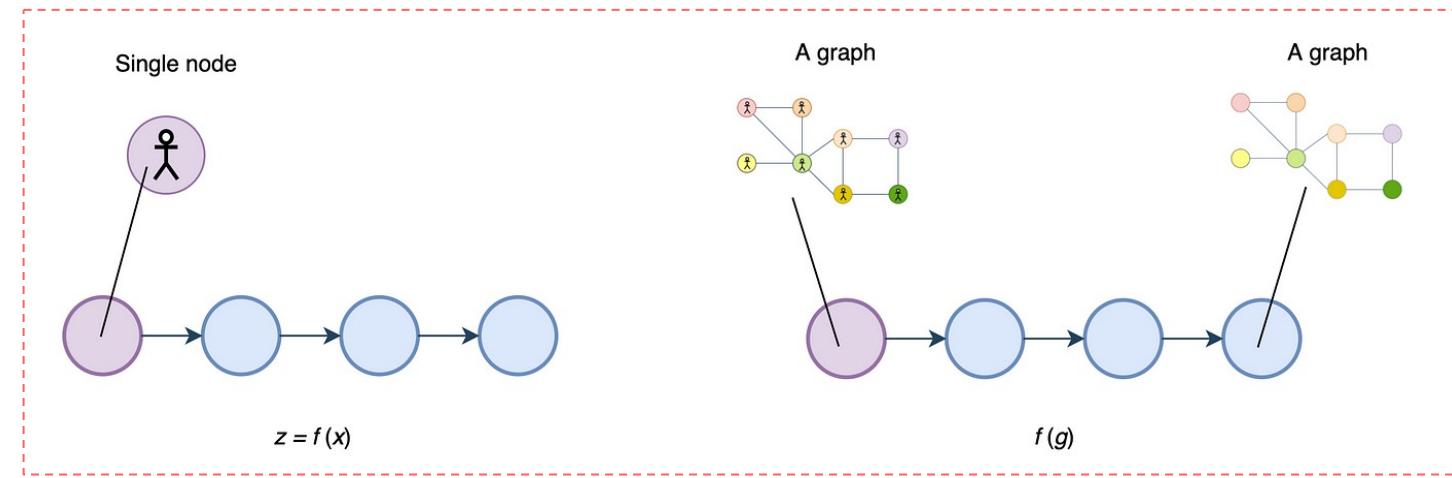
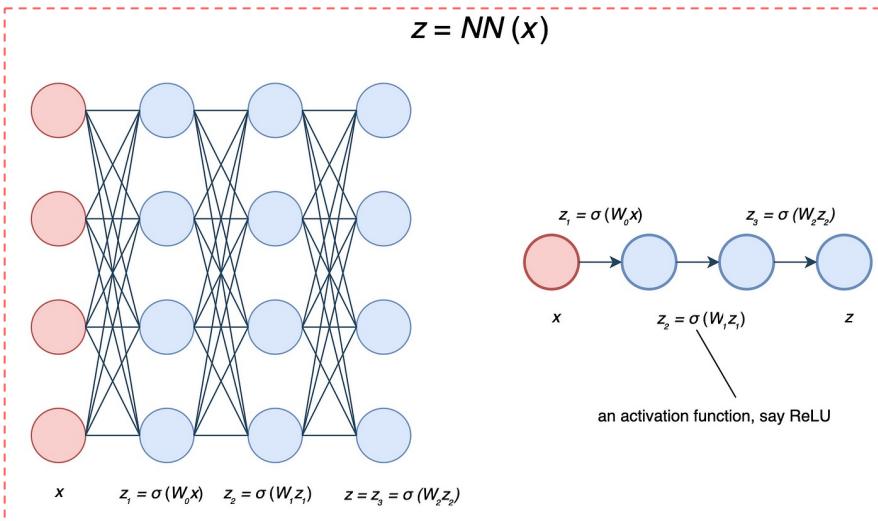


In particular, when the relationships between neighboring nodes are irregular and high dimensional, we need to define them explicitly in order to solve them efficiently. In CNN, we work in a Euclidean space. How weights are associated with the input features (pixels) is well defined.



But this is not the case for a graph. For example, the graphs above are the same even though it looks different spatially.

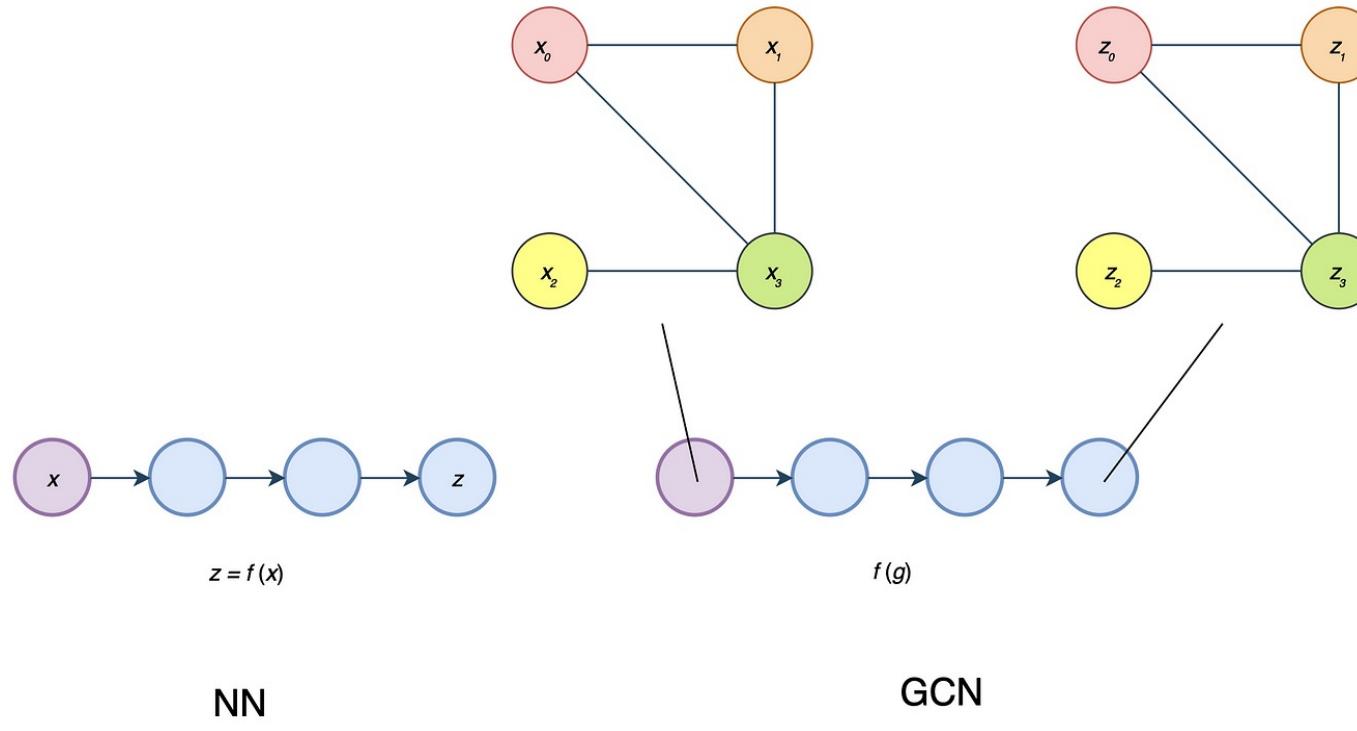
# GNN: Review



In general, neural networks (NNs) takes an input  $x$  to predict  $z$ .

This leads us to the challenge of how a NN can process a graph directly.

# GNN: Review



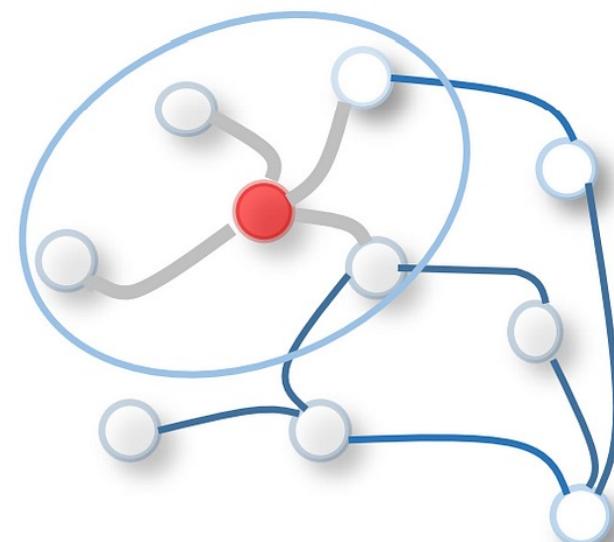
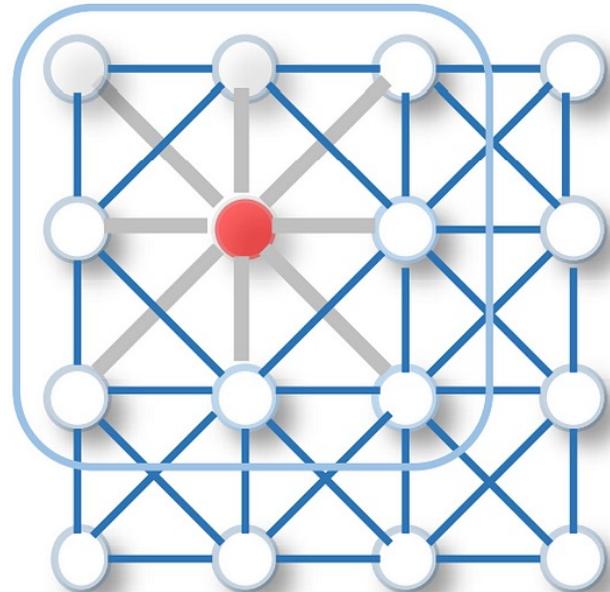
In GCN (Graph Convolutional Network), the input to the NN will be a graph. Also, instead of inferring a single  $z$ , it infers the value  $z_i$  for each node  $i$  in the graph. And to make predictions for  $Z_i$ , GCN utilizes both  $X_i$  and its neighboring nodes in the calculation.

# GNN: Review

## Graph Convolutional Networks (GCN)



The general idea of GCN is to apply convolution over a graph. Instead of having a 2-D array as input, GCN takes a graph as an input.



# GNN: Review

## Graph Convolutional Networks (GCN)



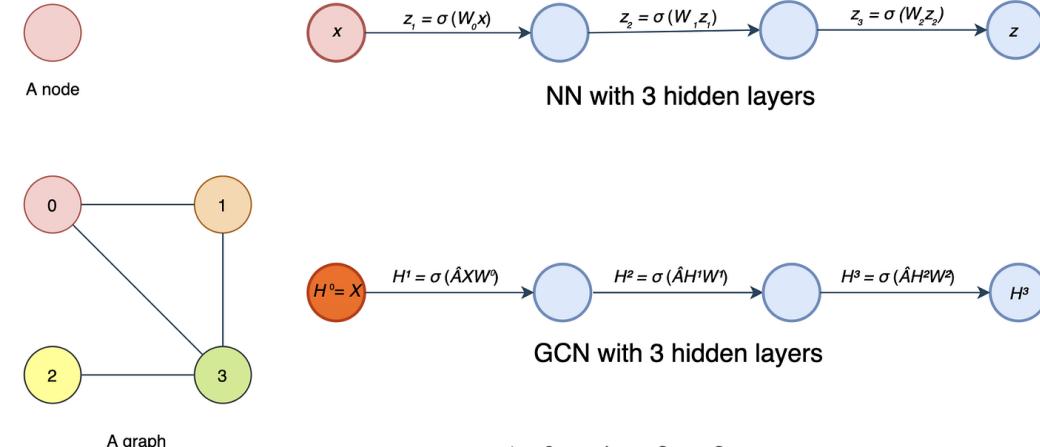
That comes to the output of the hidden layer to be  $\sigma(\hat{A}H^iW^i)$ . If we ignore  $W$  for a second, for each node in a hidden layer,  $\hat{A}H^i$  sums up features on each node with its neighbors.



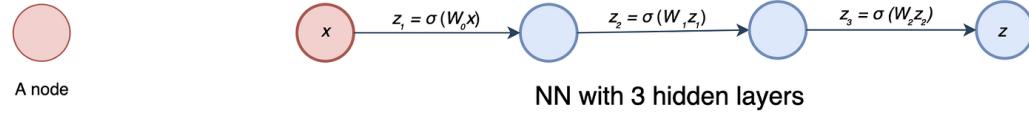
Diminishing or exploding problem in a NN



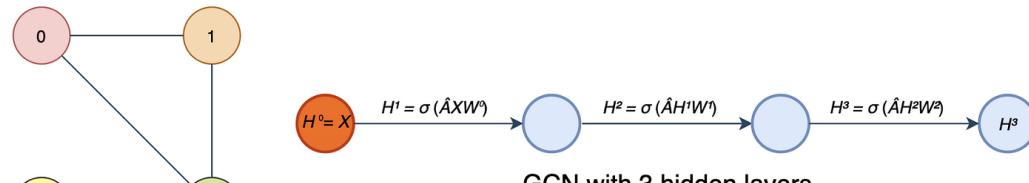
In specific, GCN wants  $\hat{A}$  to be normalized to maintain the scale of the output feature vectors



A graph



NN with 3 hidden layers



GCN with 3 hidden layers

$\sigma$  is an activation function, like ReLU

	0	1	2	3	
0	1	1	0	1	Node 0 and 3 are connected
1	1	1	0	1	
2	0	0	1	1	
3	1	1	1	1	

$\hat{A}$

Mathematically,  $\hat{A}$  equals  $A + I$

All diagonal elements are 1  
(All nodes are self-connected)

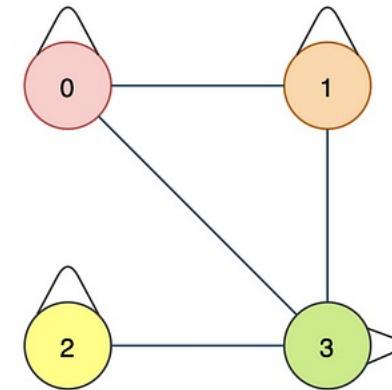
One possibility is to multiple  $\hat{A}$  with  $D^{-1}$  where  $D$  is the diagonal node degree matrix of  $\hat{A}$  in measuring the degree of each node

# GNN: Review

	0	1	2	3	
0	1	1	0	1	
1	1	1	0	1	
2	0	0	1	1	
3	1	1	1	1	

	0	1	2	3	
0	4	0	0	0	
1	0	4	0	0	
2	0	0	3	0	
3	0	0	0	5	

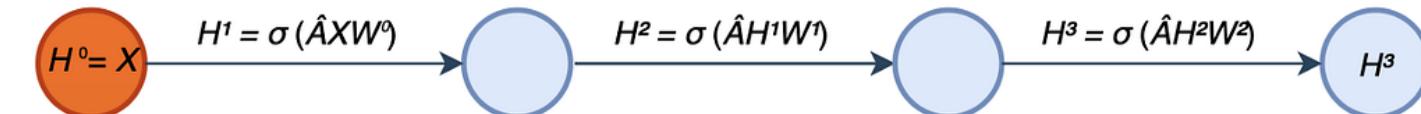


For an undirected graph, the degree of a node is counted as the number of times an edge terminates at that node. So a self-loop will count twice. In our example, node 0 has 2 edges connecting to its neighbors plus a self-loop. Its degree equals 4 (i.e. 2 + 2). For node 3, its degree equals 5 (3 + 2).

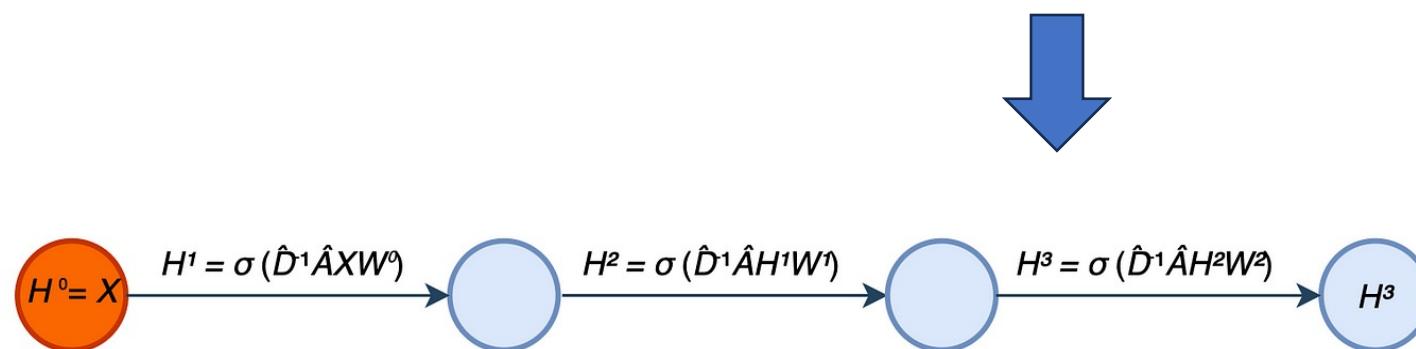
	0	1	2	3	
0	1/4	0	0	0	
1	0	1/4	0	0	
2	0	0	1/3	0	
3	0	0	0	1/5	

# GNN: Review

## Graph Convolutional Networks (GCN)



GCN with 3 hidden layers



GCN

$\xrightarrow{\sigma(\hat{D}^{-1}\hat{A}H^1W^1)}$   
a layer-wise propagation rule

The diagram summarizes the model discussed so far. In this example, it has 3 hidden layers and for each hidden layer, it computes its output as  $\sigma(\hat{D}^{-1}\hat{A}H^lW^l)$ . The equation used to compute a hidden layer output from the last layer output is called the propagation rule.

$$H^{(l+1)} = f(H^{(l)}, A)$$

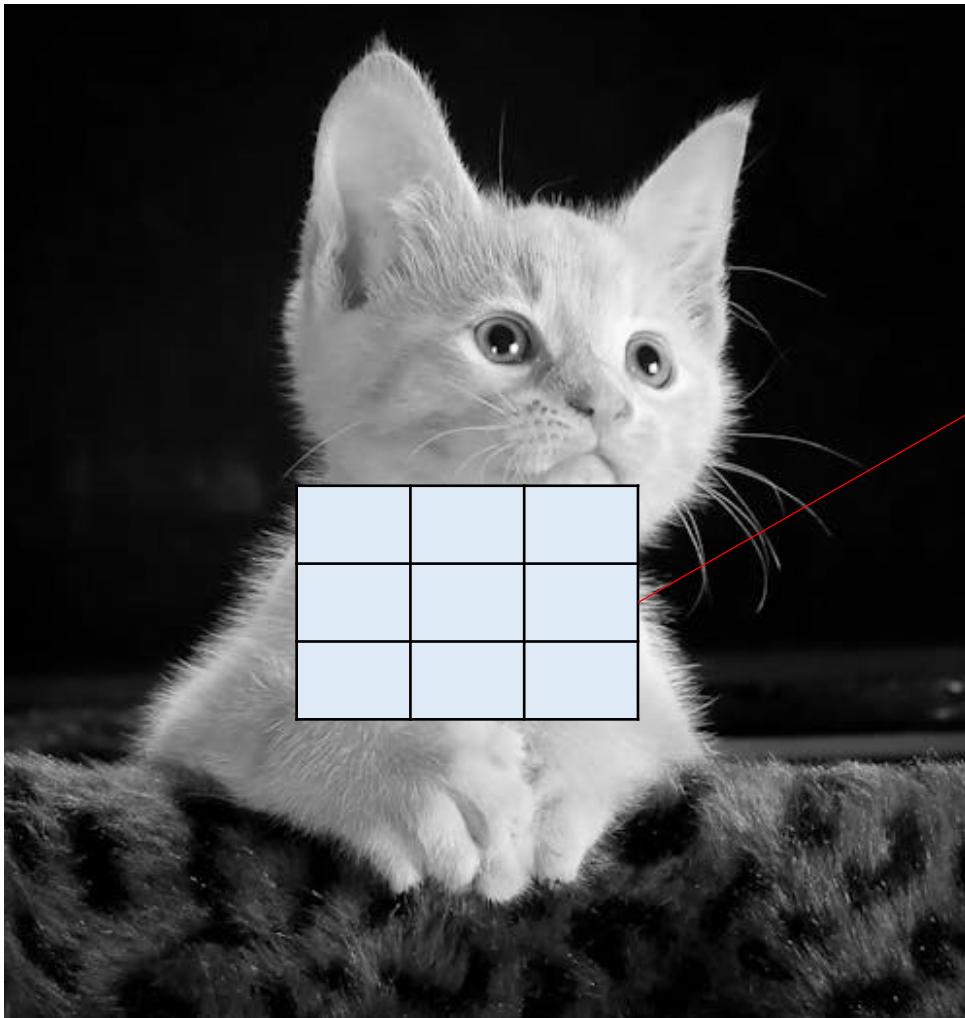
$$H^{(l+1)} = \sigma\left(\tilde{D}^{-\frac{1}{2}}\tilde{A}\tilde{D}^{-\frac{1}{2}}H^{(l)}W^{(l)}\right)$$

where

$$\tilde{A} = A + I_N$$

$$\tilde{D}_{ii} = \sum_j \tilde{A}_{ij}$$

# From CNN to GNN



*Convolutional Operation*

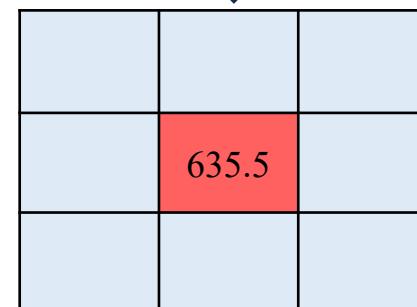
*ROI*

48	109	57
17	52	126
13	13	64

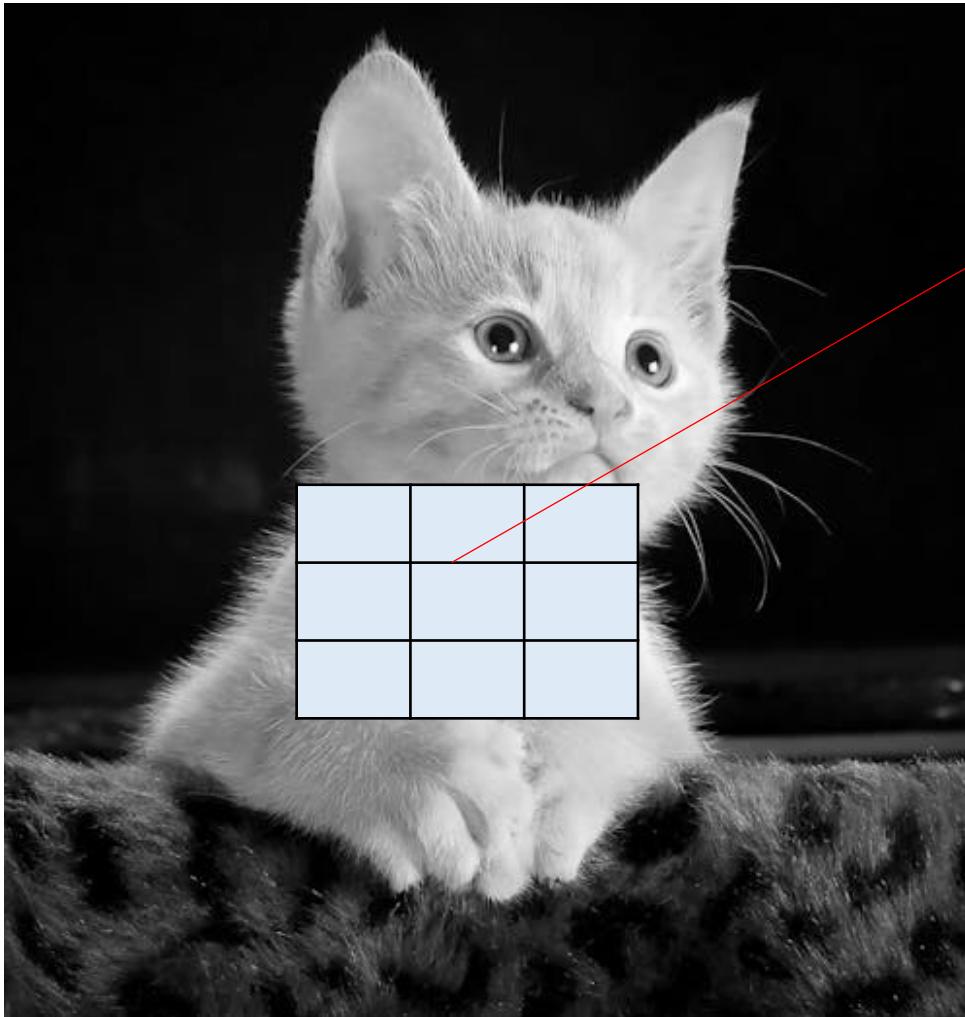
*Kernel*

$$\begin{bmatrix} 0.12 & 0.58 & 1.17 \\ -0.44 & 3.11 & -0.8 \\ 5.11 & -0.31 & 4.17 \end{bmatrix}$$

$$0.12 \times 48 + 0.58 \times 109 + 1.17 \times 57 - 0.44 \times 17 + 3.11 \times 52 - 0.8 \times 126 + 5.11 \times 13 - 0.31 \times 13 + 4.17 \times 64 = 635.5$$



# From CNN to GNN

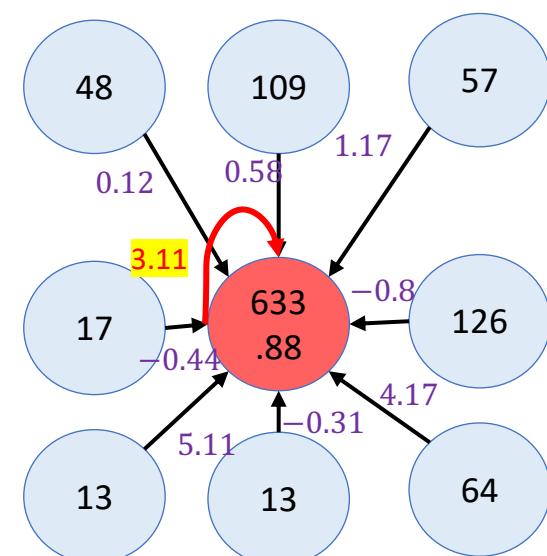


*ROI*

48	109	57
17	52	126
13	13	64

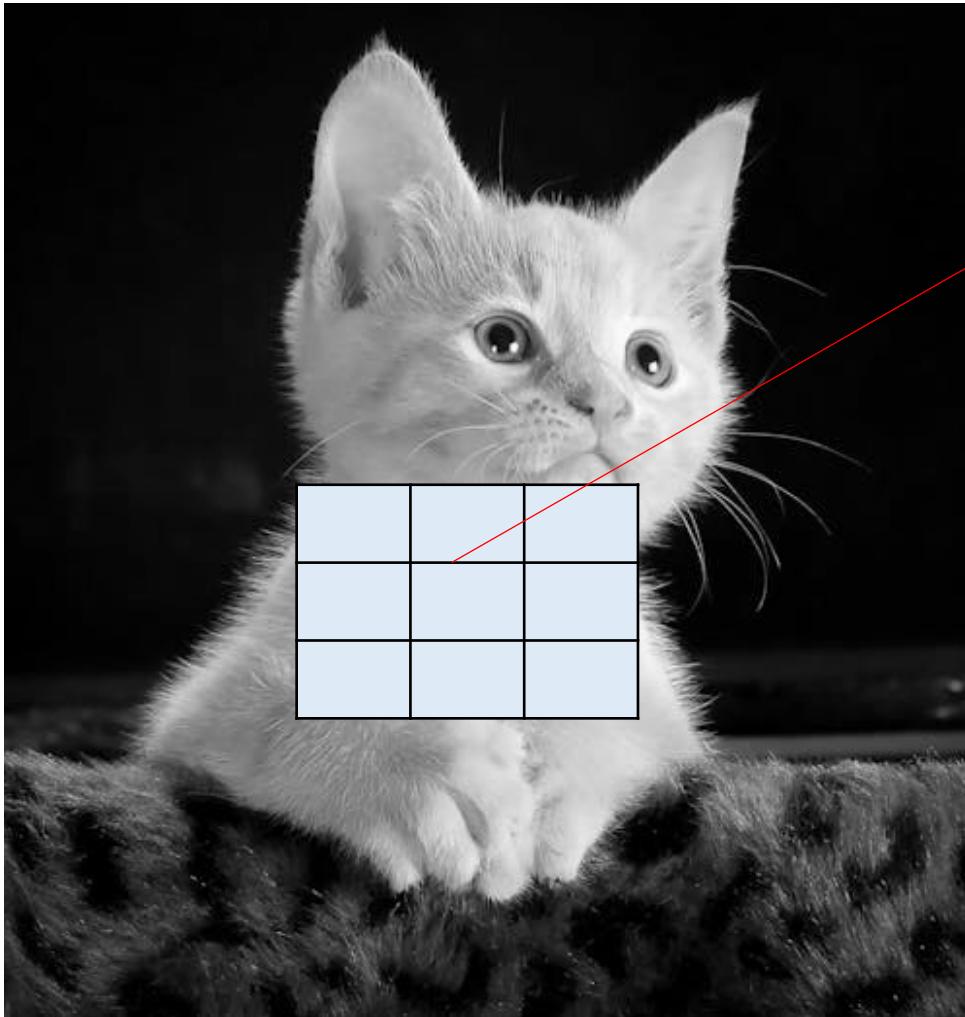
*Kernel*

$$\begin{bmatrix} 0.12 & 0.58 & 1.17 \\ -0.44 & \textcolor{blue}{3.11} & -0.8 \\ 5.11 & -0.31 & 4.17 \end{bmatrix}$$



$$0.12 \times 48 + 0.58 \times 109 + 1.17 \times 57 - 0.44 \times 17 - 0.8 \times 126 + 5.11 \times 13 - 0.31 \times 13 + 4.17 \times 64 = 633.88$$

# From CNN to GNN

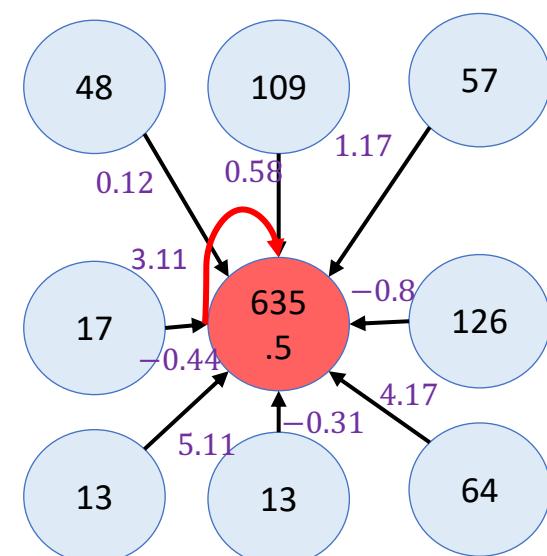


*ROI*

48	109	57
17	52	126
13	13	64

*Kernel*

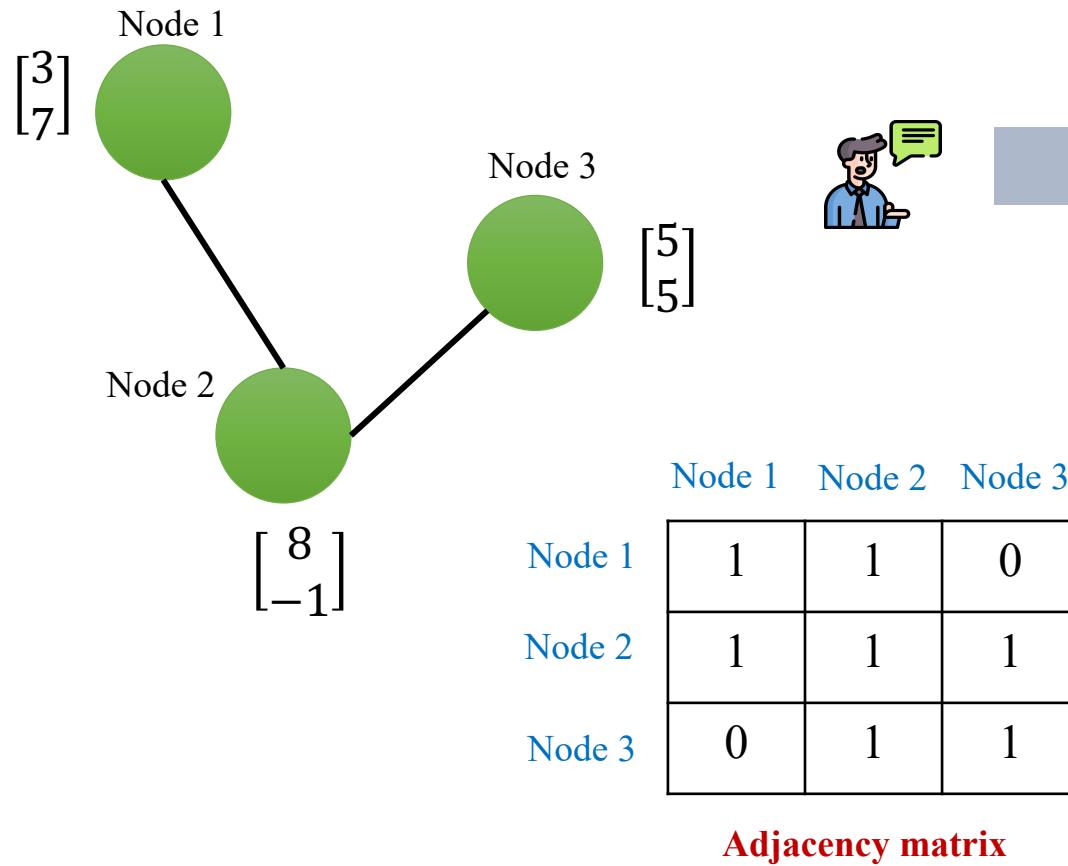
$$\begin{bmatrix} 0.12 & 0.58 & 1.17 \\ -0.44 & \textcolor{red}{3.11} & -0.8 \\ 5.11 & -0.31 & 4.17 \end{bmatrix}$$



$$0.12 \times 48 + 0.58 \times 109 + 1.17 \times 57 - 0.44 \times 17 - 0.8 \times 126 + 5.11 \times 13 - 0.31 \times 13 + 4.17 \times 64 = 633.88$$

$$0.12 \times 48 + 0.58 \times 109 + 1.17 \times 57 - 0.44 \times 17 + \textcolor{red}{3.11} \times \textcolor{red}{52} - 0.8 \times 126 + 5.11 \times 13 - 0.31 \times 13 + 4.17 \times 64 = 635.5$$

# From CNN to GNN



What if information in one channel is more important than other channel?



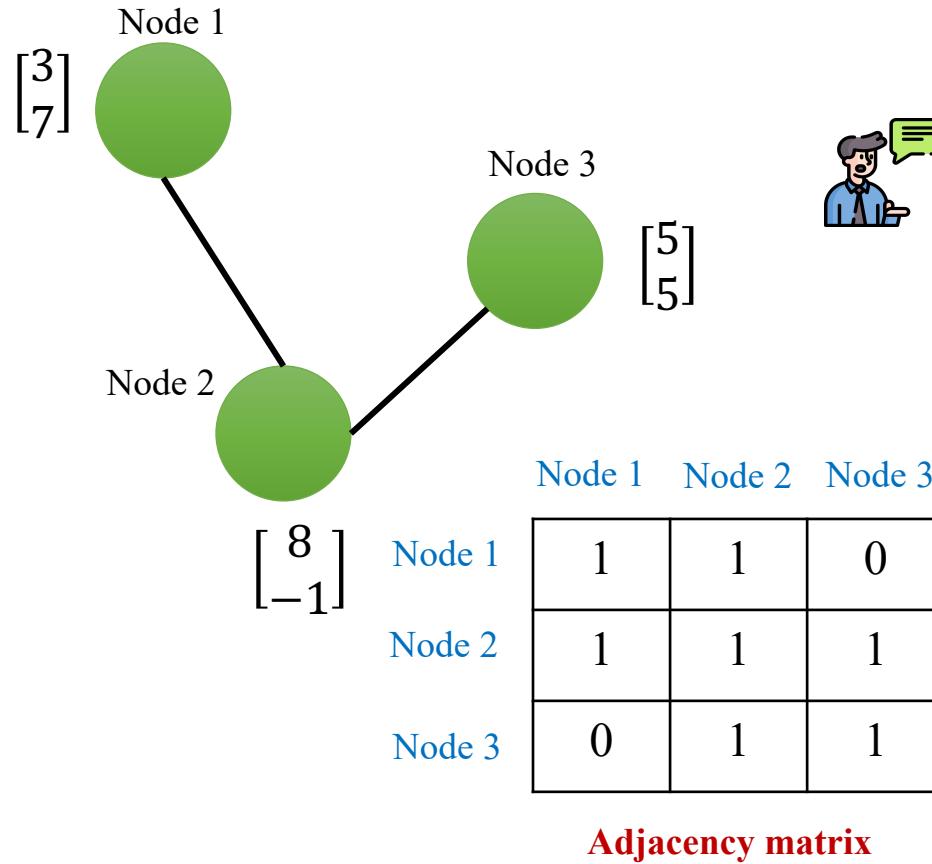
Node 1  
Node 2  
Node 3

N1 + N2
N1 + N2 + N3
N2 + N3

Node 1  
Node 2  
Node 3

3+8	7+-1
3+8+5	7-1+5
8+5	-1+5

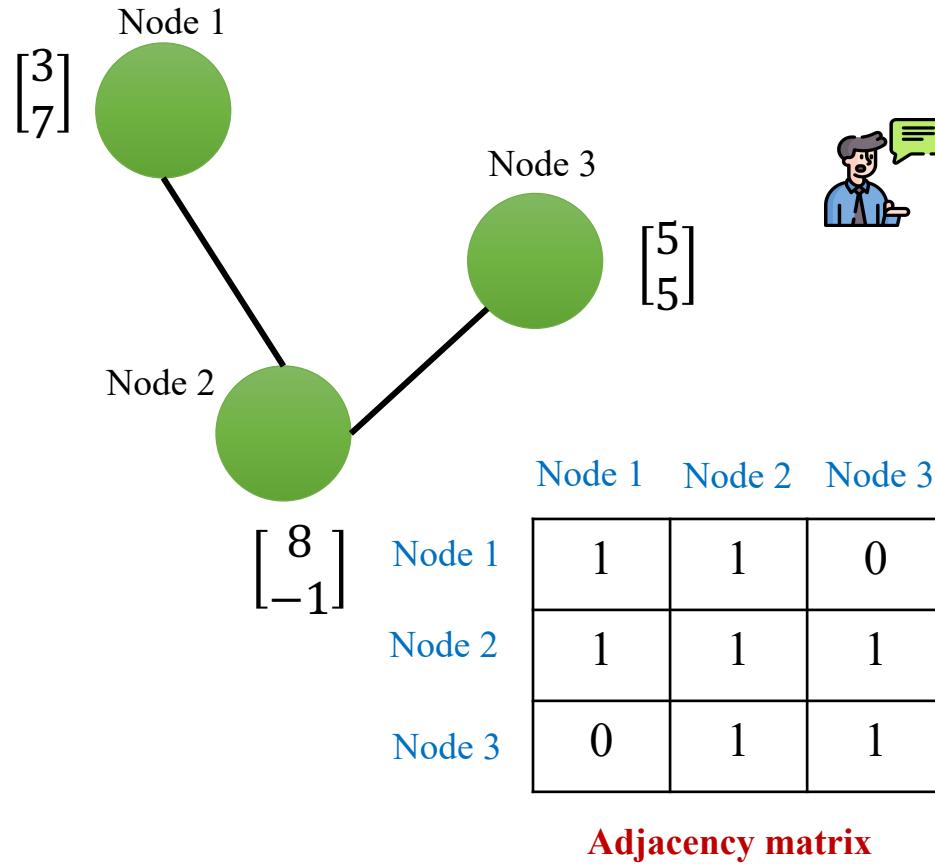
# From CNN to GNN



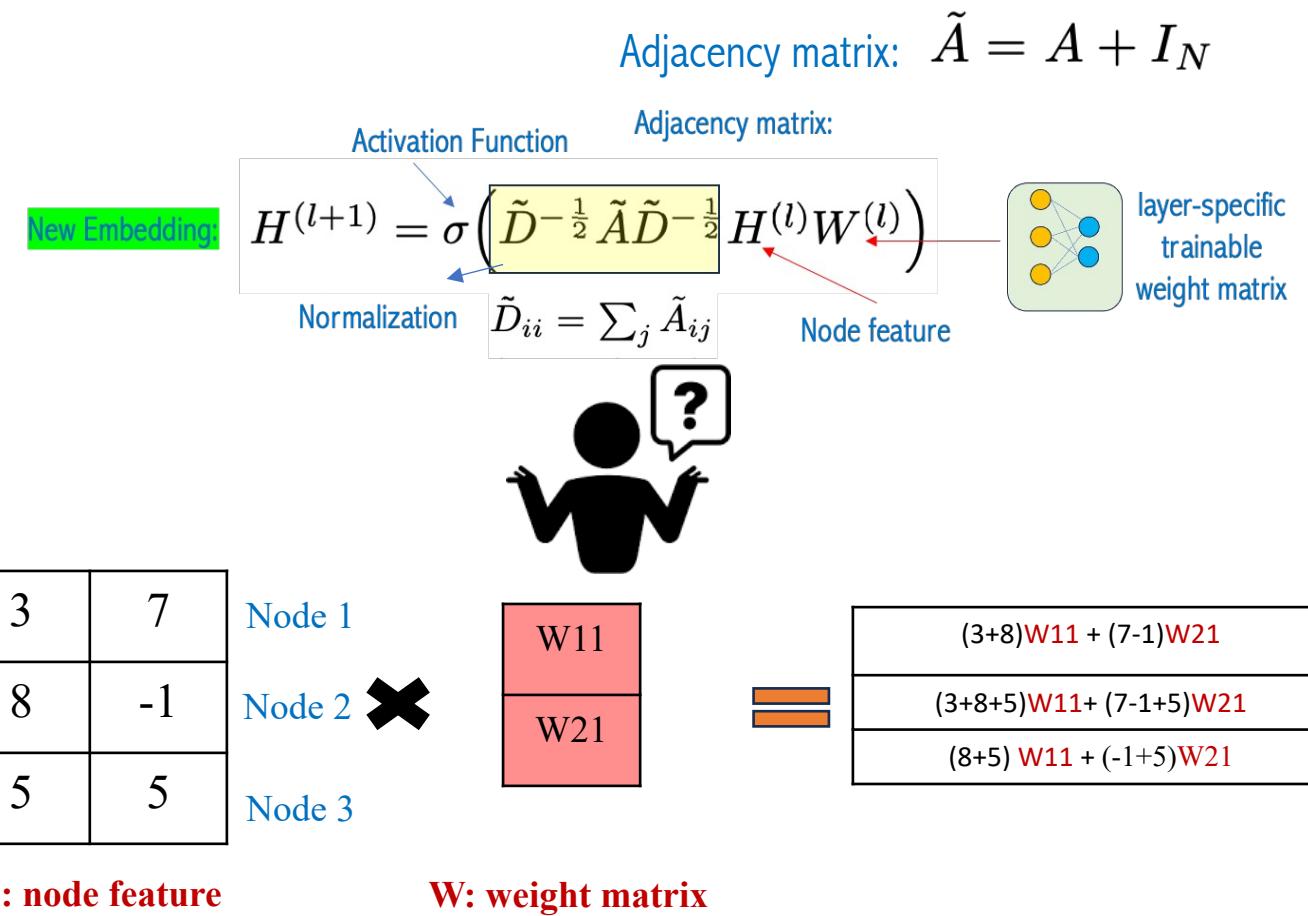
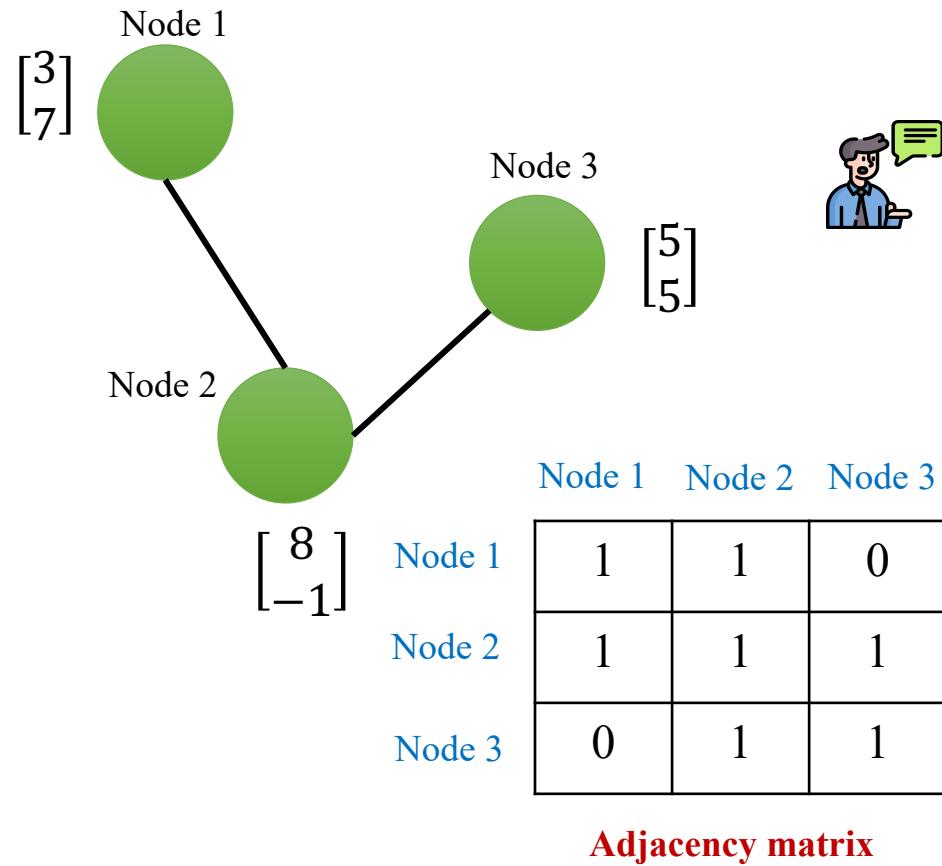
What if information in one channel is more important than other channel?

$$\begin{array}{c}
 \text{Adjacency matrix} \quad \times \quad \text{H: node feature} \quad \times \quad \text{W: weight matrix} \\
 \hline
 \begin{matrix} & \text{Node 1} & \text{Node 2} & \text{Node 3} \\ \text{Node 1} & 1 & 1 & 0 \\ \text{Node 2} & 1 & 1 & 1 \\ \text{Node 3} & 0 & 1 & 1 \end{matrix} \quad \times \quad \begin{matrix} & \text{Node 1} & \text{Node 2} & \text{Node 3} \\ \text{Node 1} & 3 & 7 & \\ \text{Node 2} & 8 & -1 & \\ \text{Node 3} & 5 & 5 & \end{matrix} \quad \times \quad \begin{matrix} & \text{Node 1} & \text{Node 2} \\ \text{Node 1} & W_{11} & W_{12} \\ \text{Node 2} & W_{21} & W_{22} \\ \text{Node 3} & & \end{matrix} = \begin{matrix} & \text{Node 1} & \text{Node 2} \\ \text{Node 1} & (3+8)W_{11} + (7-1)W_{21} & (3+8)W_{12} + (7-1)W_{22} \\ \text{Node 2} & (3+8+5)W_{11} + (7-1+5)W_{21} & (3+8+5)W_{12} + (7-1+5)W_{22} \\ \text{Node 3} & (8+5)W_{11} + (-1+5)W_{21} & (8+5)W_{12} + (-1+5)W_{22} \end{matrix}
 \end{array}$$

# From CNN to GNN



# From CNN to GNN



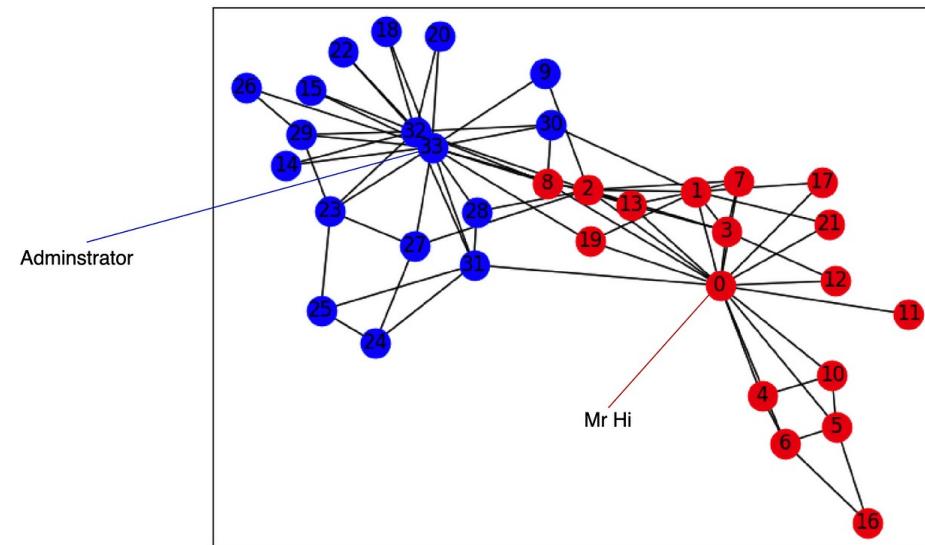
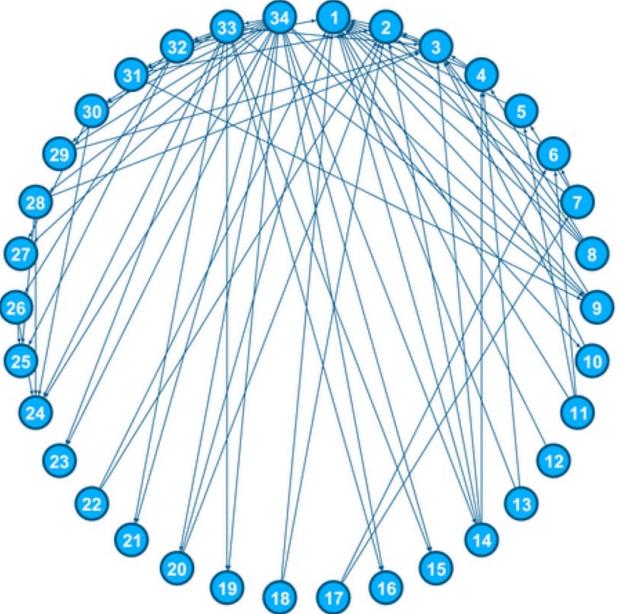
What if information in one node is more important than other node?



# Attention GNN

# Zachary's karate club

There is a karate club that has two major stakeholders: the instructor (Mr. Hi) and the administrator. Unfortunately, the dispute between them causes it to split into 2 clubs. The original members will need to choose a side and pick which one to join. Their decisions will be based on how well they are connected with Mr. Hi or the administrator. This also includes how well they are connected to members that are associated with them. The diagram below is the social graph in representation connections between members.



# Zachary's karate club



```
A = nx.to_numpy_array(g)
A

array([[0.,  4.,  5.,  ...,  2.,  0.,  0.],
       [4.,  0.,  6.,  ...,  0.,  0.,  0.],
       [5.,  6.,  0.,  ...,  0.,  2.,  0.],
       ...,
       [2.,  0.,  0.,  ...,  0.,  4.,  4.],
       [0.,  0.,  2.,  ...,  4.,  0.,  5.],
       [0.,  0.,  0.,  ...,  4.,  5.,  0.]])
[39] A_mod = A + np.eye(g.number_of_nodes()) # add self-connections

D_mod = np.zeros_like(A_mod)
np.fill_diagonal(D_mod, np.asarray(A_mod.sum(axis=1)).flatten())

D_mod_invroot = np.linalg.inv(sqrtm(D_mod))

A_hat = D_mod_invroot @ A_mod @ D_mod_invroot
```

$$\tilde{A} = A + I$$

$$\hat{A} = \tilde{D}^{-\frac{1}{2}} \tilde{A} \tilde{D}^{-\frac{1}{2}}$$

# Zachary's karate club

```
[40] X = np.eye(g.number_of_nodes())
```



```
print(X)
```

```
[[1. 0. 0. ... 0. 0. 0.]
 [0. 1. 0. ... 0. 0. 0.]
 [0. 0. 1. ... 0. 0. 0.]
 ...
 [0. 0. 0. ... 1. 0. 0.]
 [0. 0. 0. ... 0. 1. 0.]
 [0. 0. 0. ... 0. 0. 1.]]
```



Input Feature X

# Zachary's karate club

```

class GCNLayer():
    def __init__(self, n_inputs, n_outputs, activation=None, name=''):
        self.n_inputs = n_inputs
        self.n_outputs = n_outputs
        self.W = glorot_init(self.n_outputs, self.n_inputs)
        self.activation = activation
        self.name = name

    def __repr__(self):
        return f"GCN: W{'_'+self.name if self.name else ''} ({self.n_inputs}, {self.n_outputs})"

    def forward(self, A, X, W=None):
        """
        Assumes A is (bs, bs) adjacency matrix and X is (bs, D),
        where bs = "batch size" and D = input feature length
        """
        self._A = A
        self._X = (A @ X).T # for calculating gradients. (D, bs)

        if W is None:
            W = self.W

        H = W @ self._X # (h, D)*(D, bs) -> (h, bs)
        if self.activation is not None:
            H = self.activation(H)
        self._H = H # (h, bs)
        return self._H.T # (bs, h)

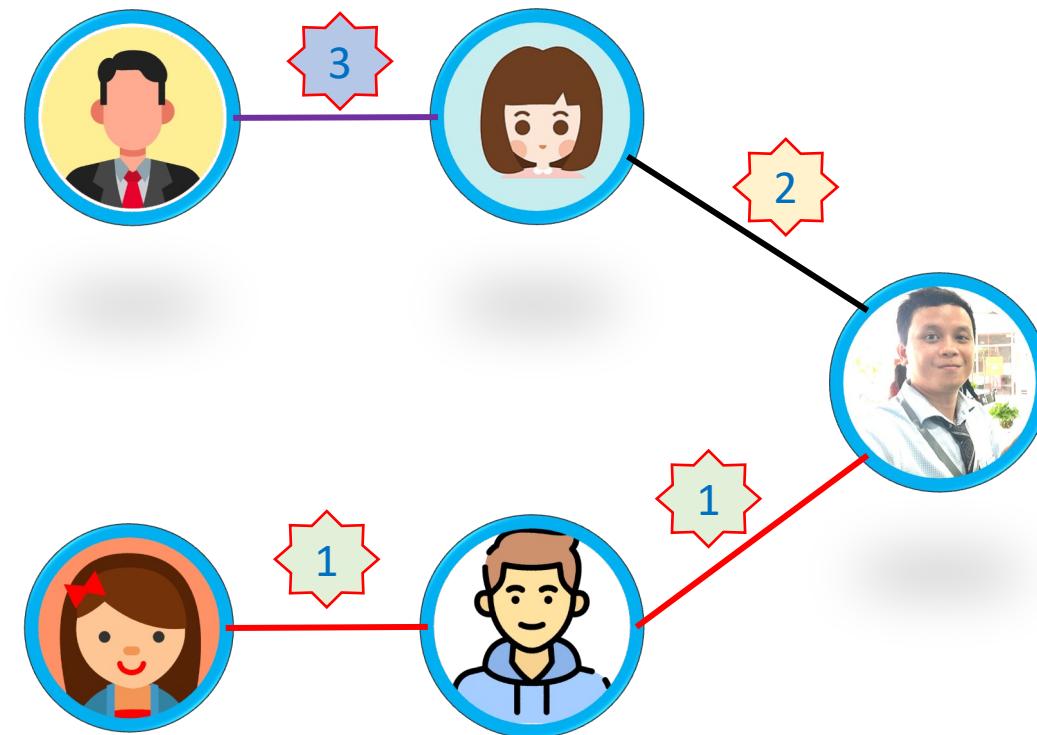
```

$$H^{l+1} = \sigma(\underbrace{W \hat{A} H^l}_{\text{Message Passing}})$$

# Different Types of Edge Connections



How to include  
various types of  
Edges in GNN



Friend



Family



Colleagues

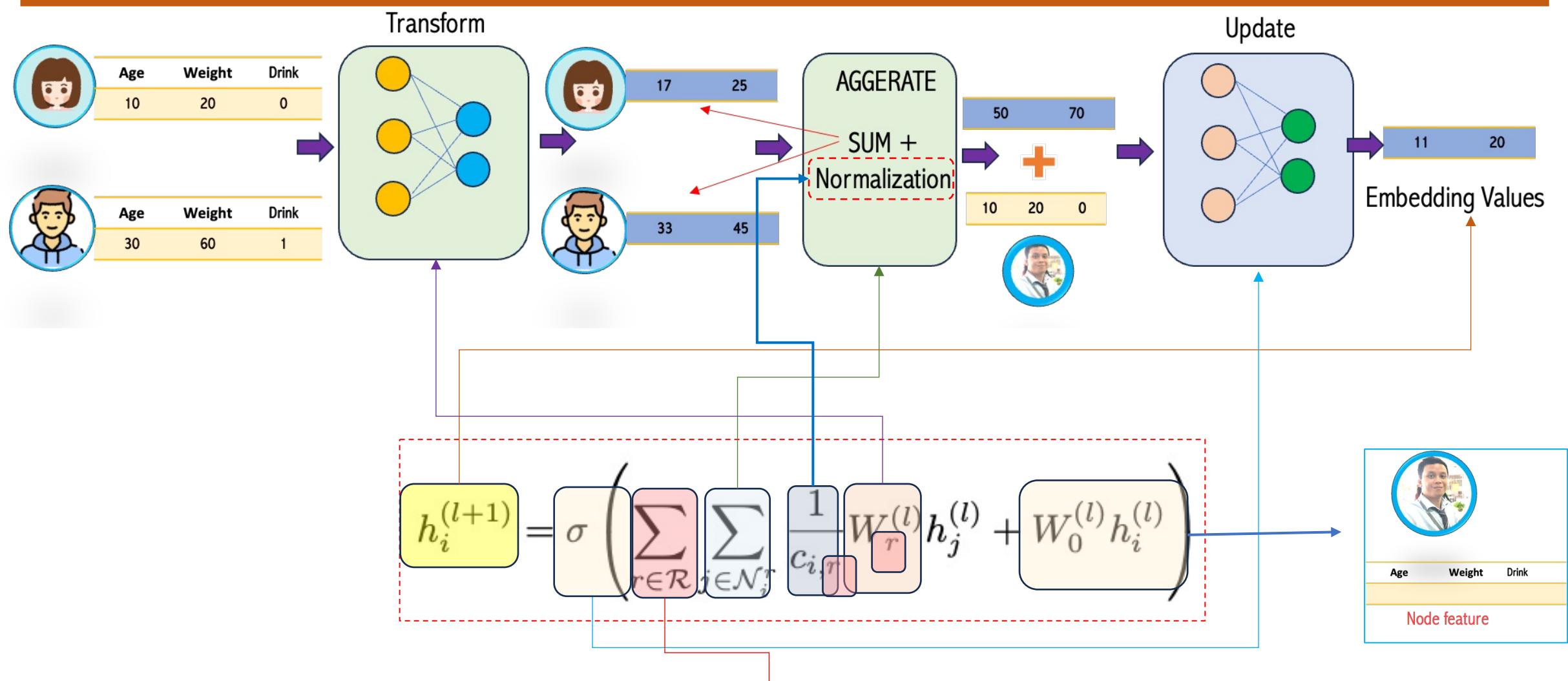
Relational Graph  
Convolutional  
Neural Network



# Outline

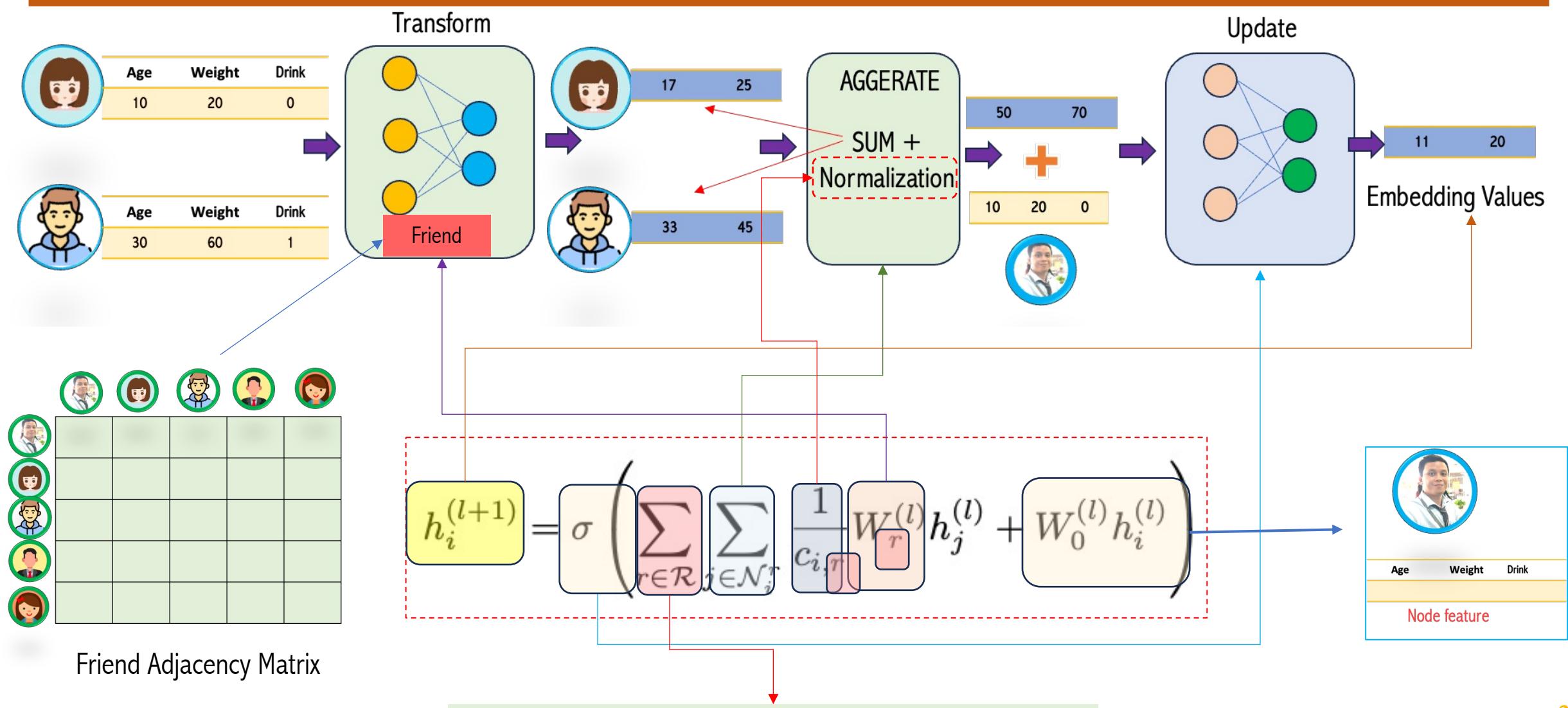
- Edge Feature in GNN
- Edge Weight in GNN
- Relational GNN
- Multidimension Edge Feature
- Attention in GNN
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- Summary

# Relational GNN



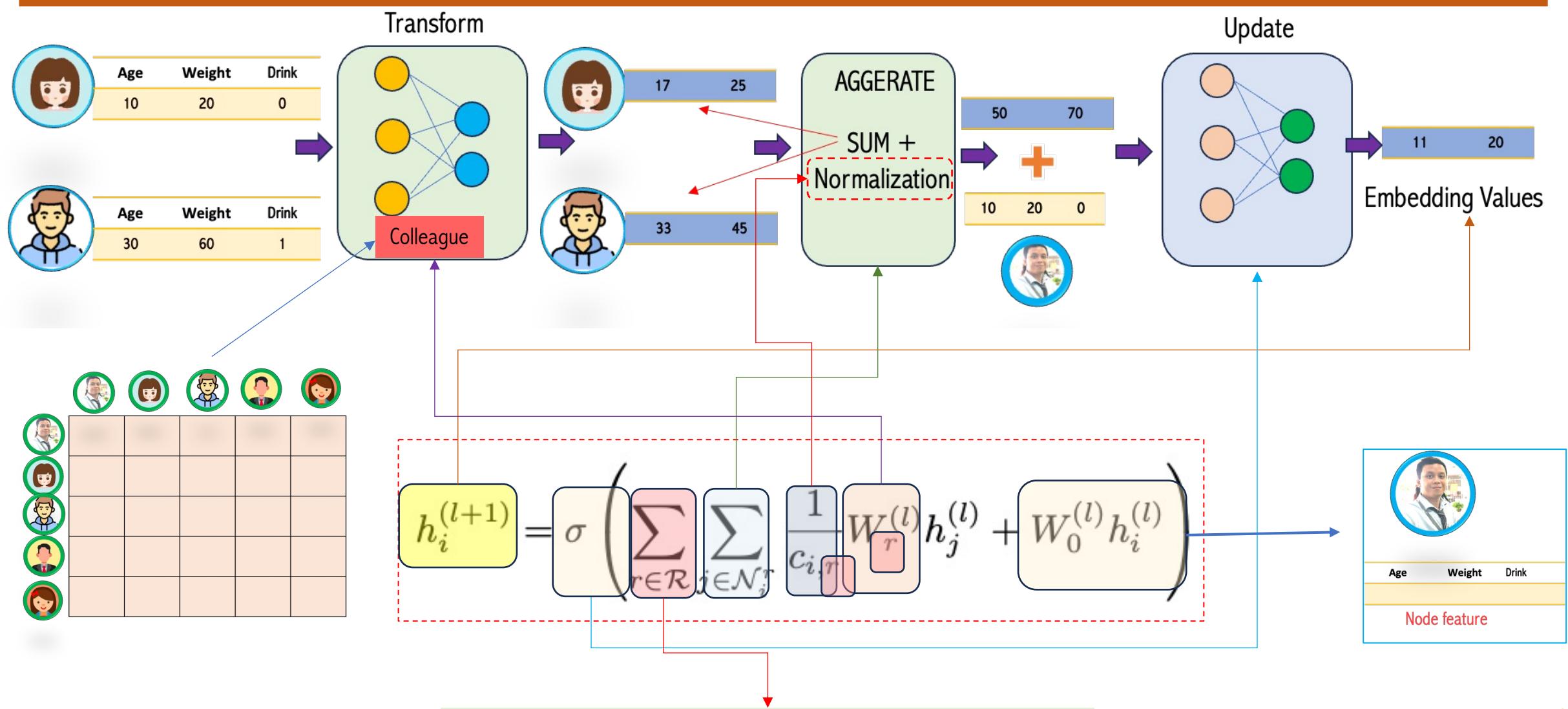
Various types of relationship: Edge conditions GNN

# Relational GNN



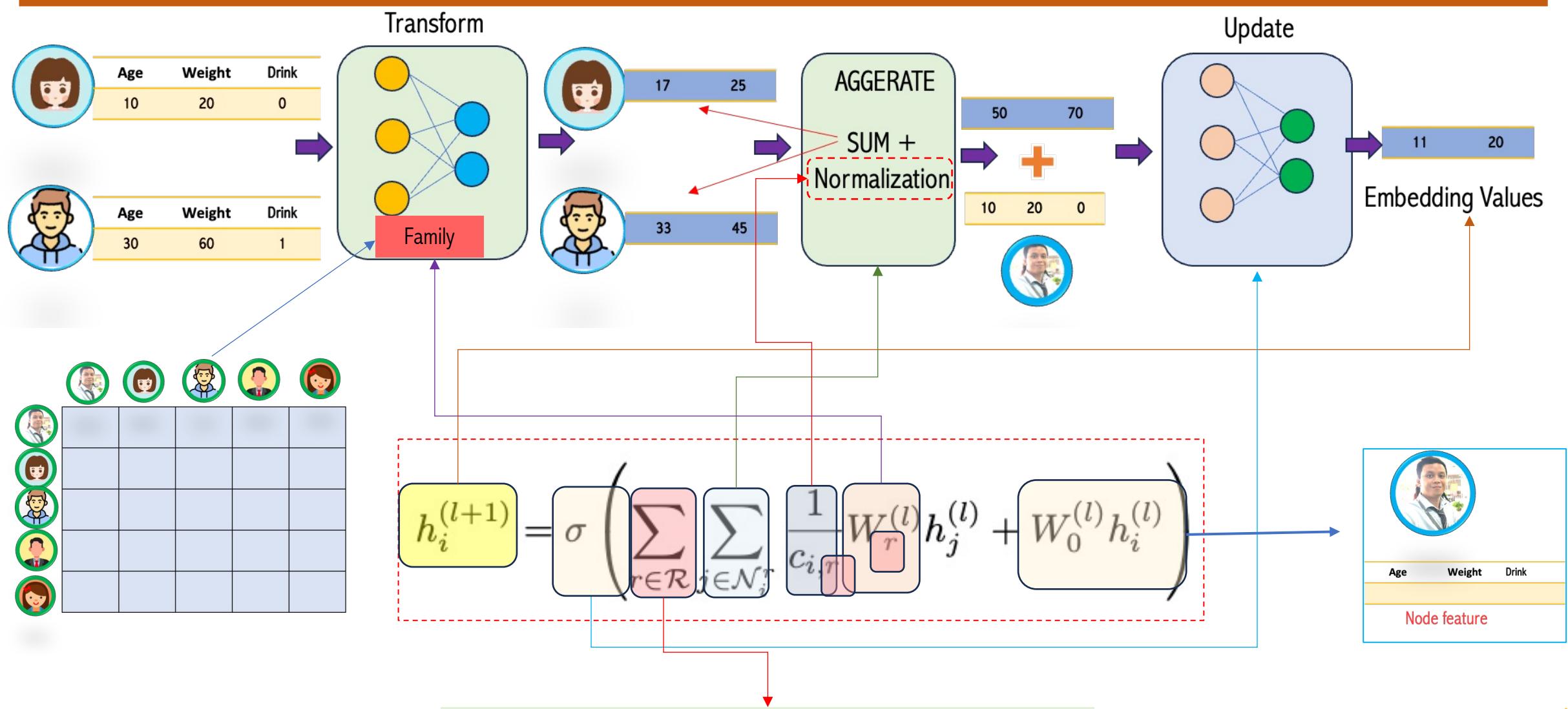
Various types of relationship: Edge conditions GNN

# Relational GNN

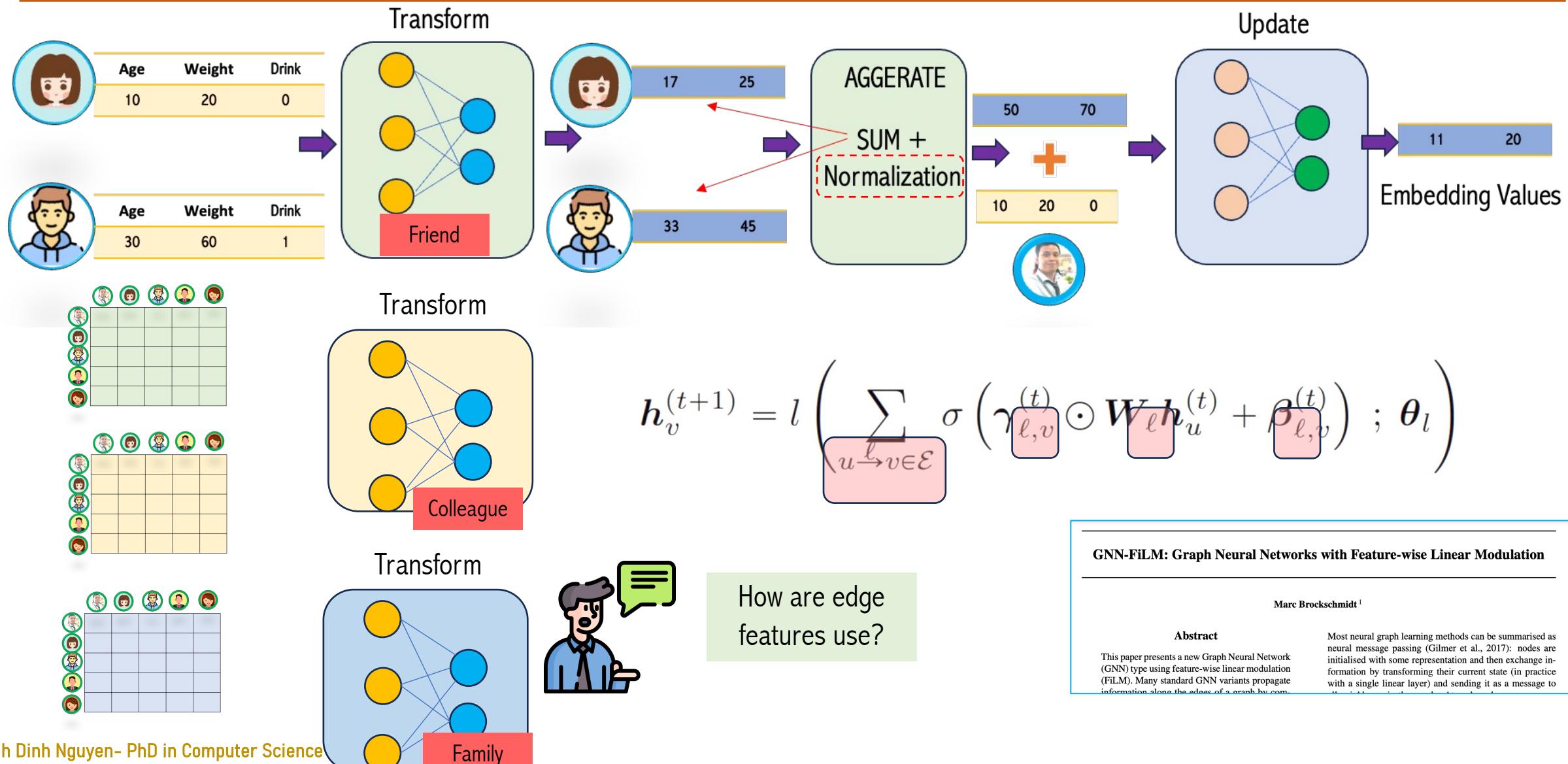


Various types of relationship: Edge conditions GNN

# Relational GNN



# GNN: Feature-wise Linear Modulation



# GNN Edge Feature: Variants

GGNN: $A' = GRU(A)$	$A$	$\mathbf{W}_1 \cdot B + \mathbf{W}_2 \cdot C + \mathbf{W}_1 \cdot D$
R-GCN: $A' = \sigma(\mathbf{W}_G \cdot A + \mathbf{W}_1 \cdot B + \mathbf{W}_2 \cdot C + \mathbf{W}_1 \cdot D)$	$\mathbf{W}_G \cdot A + \mathbf{W}_1 \cdot B + \mathbf{W}_2 \cdot C + \mathbf{W}_1 \cdot D$	
R-GAT: $A' = \sigma((\mathbf{a}_{A'})_{A \xrightarrow{A} A} \cdot \mathbf{W}_G \cdot A + (\mathbf{a}_{A'})_{B \xrightarrow{1} A} \cdot \mathbf{W}_1 \cdot B + (\mathbf{a}_{A'})_{C \xrightarrow{2} A} \cdot \mathbf{W}_2 \cdot C + (\mathbf{a}_{A'})_{D \xrightarrow{1} A} \cdot \mathbf{W}_1 \cdot D)$	$(\mathbf{a}_{A'})_{A \xrightarrow{A} A} \cdot \mathbf{W}_G \cdot A + (\mathbf{a}_{A'})_{B \xrightarrow{1} A} \cdot \mathbf{W}_1 \cdot B + (\mathbf{a}_{A'})_{C \xrightarrow{2} A} \cdot \mathbf{W}_2 \cdot C + (\mathbf{a}_{A'})_{D \xrightarrow{1} A} \cdot \mathbf{W}_1 \cdot D$	
R-GIN: $A' = \sigma(MLP_G(A) + MLP_1(B) + MLP_2(C) + MLP_1(D))$	$MLP_G(A) + MLP_1(B) + MLP_2(C) + MLP_1(D)$	
GNN-MLP: $A' = \sigma(MLP_G(A \  A) + MLP_1(B \  A) + MLP_2(C \  A) + MLP_1(D \  A))$	$MLP_G(A \  A) + MLP_1(B \  A) + MLP_2(C \  A) + MLP_1(D \  A)$	
RGDCN: $A' = \sigma(\mathbf{W}_{G,A} \cdot A + \mathbf{W}_{1,A} \cdot B + \mathbf{W}_{2,A} \cdot C + \mathbf{W}_{1,D} \cdot D)$	$\mathbf{W}_{G,A} \cdot A + \mathbf{W}_{1,A} \cdot B + \mathbf{W}_{2,A} \cdot C + \mathbf{W}_{1,D} \cdot D$	
GNN-FiLM: $A' = \sigma(\beta_{G,A} + \gamma_{G,A} \odot \mathbf{W}_G \cdot A + \beta_{1,A} + \gamma_{1,A} \odot \mathbf{W}_1 \cdot B + \beta_{2,A} + \gamma_{2,A} \odot \mathbf{W}_2 \cdot C + \beta_{1,D} + \gamma_{1,D} \odot \mathbf{W}_1 \cdot D)$	$\beta_{G,A} + \gamma_{G,A} \odot \mathbf{W}_G \cdot A + \beta_{1,A} + \gamma_{1,A} \odot \mathbf{W}_1 \cdot B + \beta_{2,A} + \gamma_{2,A} \odot \mathbf{W}_2 \cdot C + \beta_{1,D} + \gamma_{1,D} \odot \mathbf{W}_1 \cdot D$	

## GNN-FiLM: Graph Neural Networks with Feature-wise Linear Modulation

Marc Brockschmidt<sup>1</sup>

### Abstract

This paper presents a new Graph Neural Network (GNN) type using feature-wise linear modulation (FiLM). Many standard GNN variants propagate information along the edges of a graph by com-

Most neural graph learning methods can be summarised as neural message passing (Gilmer et al., 2017): nodes are initialised with some representation and then exchange information by transforming their current state (in practice with a single linear layer) and sending it as a message to

# Outline

- Edge Feature in GNN
- Edge Weight in GNN
- Relational GNN
- Multidimension Edge Feature
- Attention in GNN
- Example: Graph-Level Prediction
- Summary

# Multidimensional Edge Feature



How to integrate multi-dimensional edge features to the transformation of the neighborhood states?



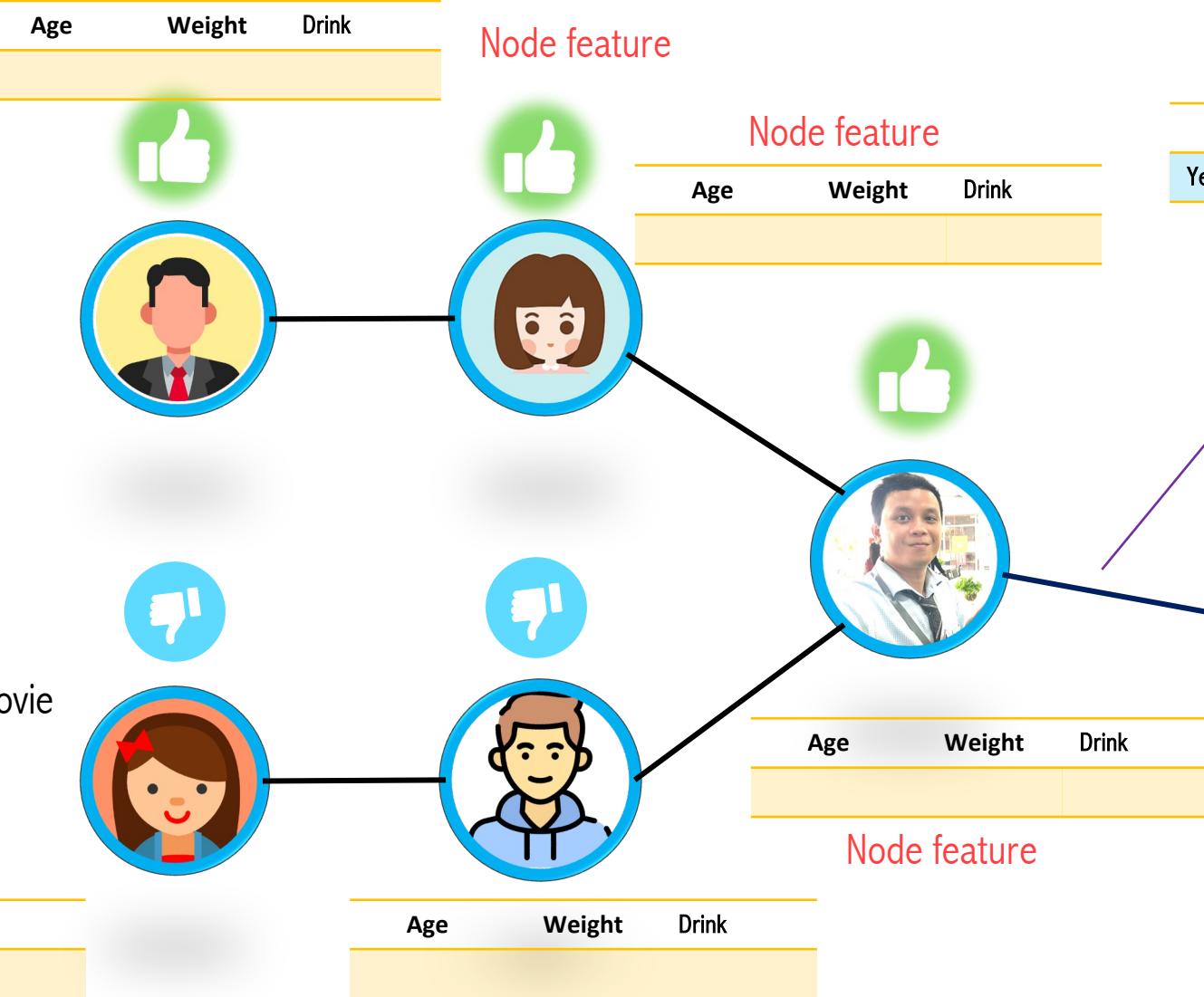
Love to watch movie



Not Love to watch movie

Node feature

Age	Weight	Drink



Edge information

Friend	Friend Since	Live together
Yes	9	No

How do edge features utilize in GNN?

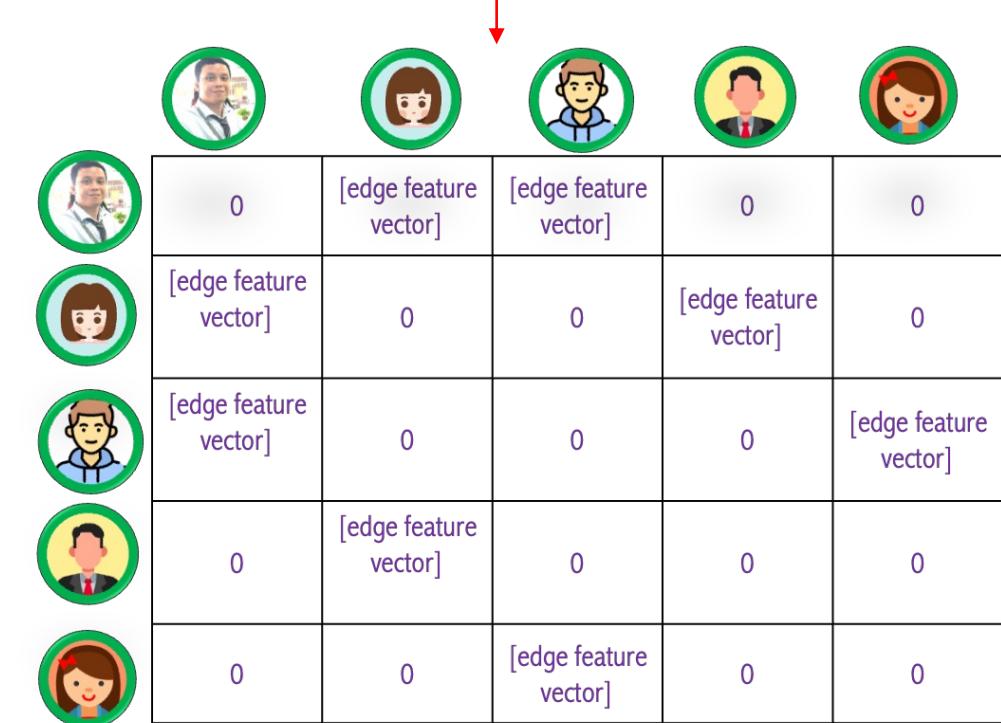


# Multidimensional Edge Features

$$h_v = \gamma \left( x_v, \bigoplus_{w \in N(v)} \phi(x_v, x_w, e_{vw}) \right)$$

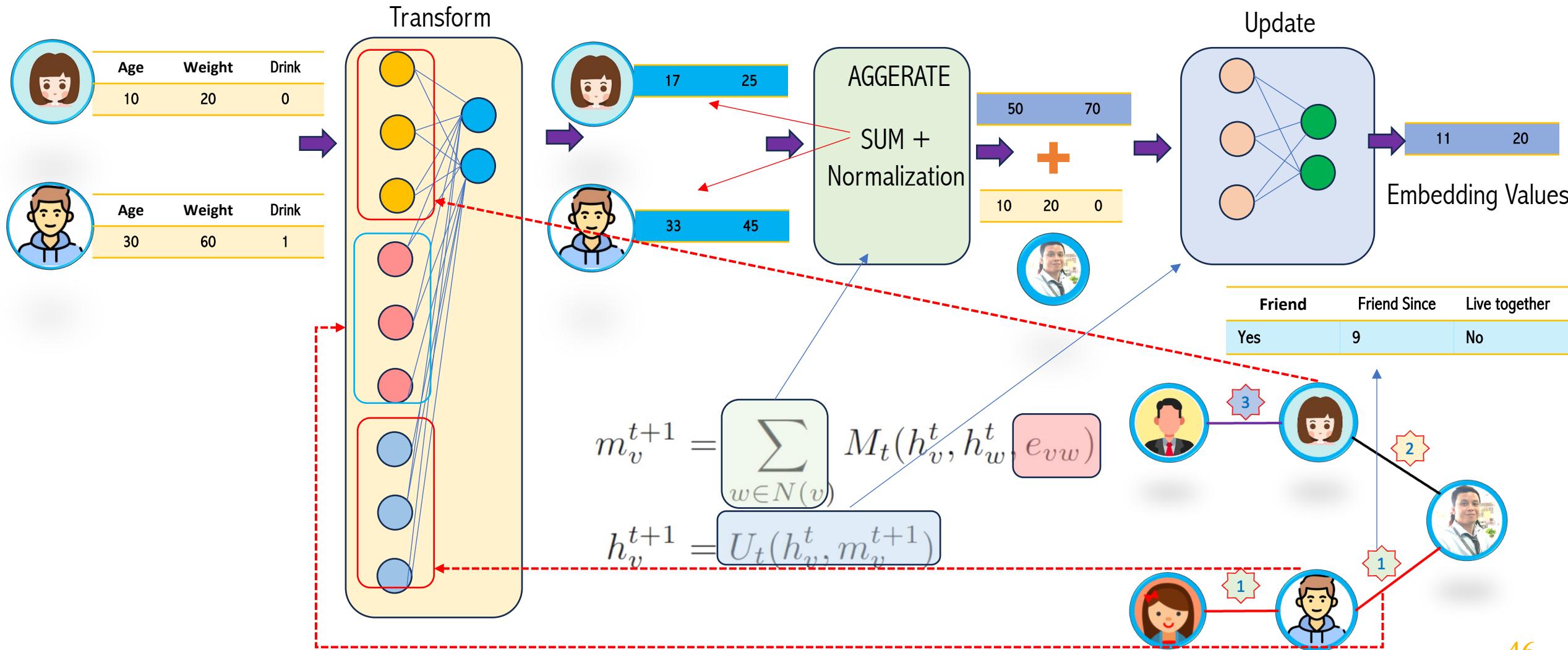
↑ UPDATE      ↑ AGGREGATE      ↑ TRANSFORM

Message Passing in Neural Network

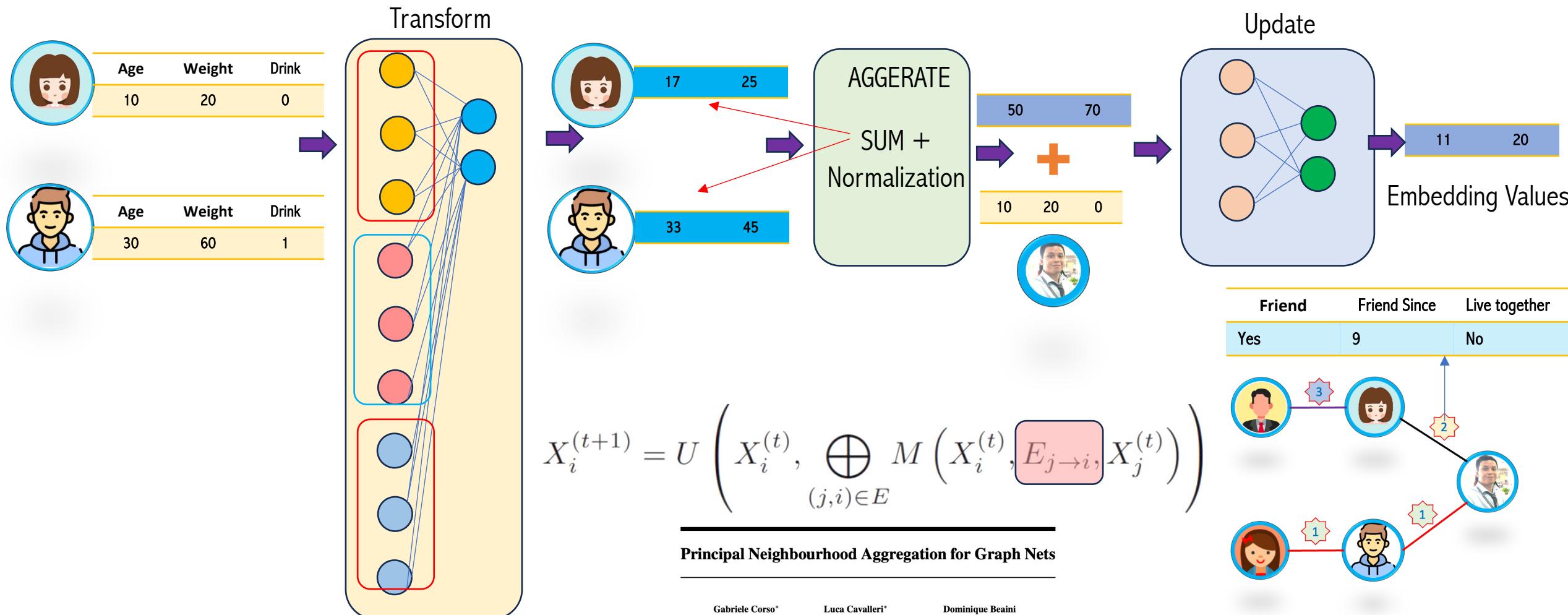


What is the size of this adjacency matrix?

# Multidimensional Edge Features: MLP



# Multidimensional Edge Features: PNAConv



Gabriele Corso\*  
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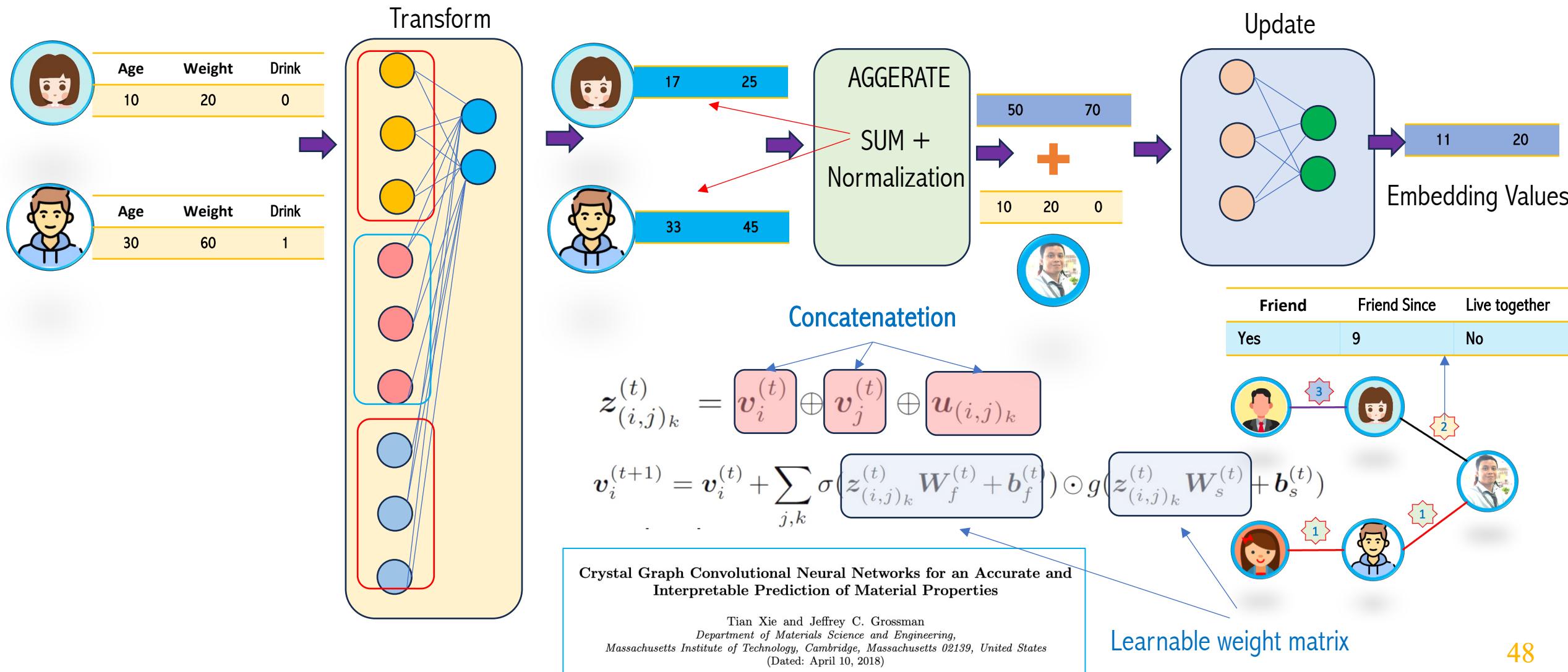
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# Multidimensional Edge Features: Crystal GCN



## NENN: Incorporate Node and Edge Features in Graph Neural Networks

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Dongsheng Li

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LDS1201@163.COM

Hierachical dual-level attention mechanism

Learn how important specific nodes and edges are for the new embedding

$$\mathbf{x}_{\mathcal{N}_i} = \sigma(W_n \cdot \text{MEAN}(\{\alpha_{ij}^n \mathbf{x}_j, \forall j \in \mathcal{N}_i\}))$$

$$\mathbf{x}_{\mathcal{E}_i} = \sigma(W_e \cdot \text{MEAN}(\{\alpha_{ik}^e \mathbf{e}_k, \forall k \in \mathcal{E}_i\}))$$

$$\mathbf{x}_i^{(l+1)} = \text{CONCAT}(\mathbf{x}_{\mathcal{N}_i}^{(l)}, \mathbf{x}_{\mathcal{E}_i}^{(l)})$$

Node-level Attention

$$\mathbf{e}_{\mathcal{E}_i} = \sigma(W_e \cdot \text{MEAN}(\{\beta_{ik}^e \mathbf{e}_k, \forall k \in \mathcal{E}_i\}))$$

$$\mathbf{e}_{\mathcal{N}_i} = \sigma(W_n \cdot \text{MEAN}(\{\beta_{ij}^n \mathbf{x}_j, \forall j \in \mathcal{N}_i\}))$$

$$\mathbf{e}_i^{(l+1)} = \text{CONCAT}(\mathbf{e}_{\mathcal{N}_i}^{(l)}, \mathbf{e}_{\mathcal{E}_i}^{(l)})$$

Edge-level Attention

Edge feature are used in both layers to generate new embeeding.

# Edge Feature Embedding: NENN

$$\mathbf{x}_{\mathcal{N}_i} = \sigma(W_n \cdot \text{MEAN}(\{\alpha_{ij}^n \mathbf{x}_j, \forall j \in \mathcal{N}_i\}))$$

$$\mathbf{x}_{\mathcal{E}_i} = \sigma(W_e \cdot \text{MEAN}(\{\alpha_{ij}^e \mathbf{e}_k, \forall k \in \mathcal{E}_i\}))$$

$$\mathbf{x}_i^{(l+1)} = \text{CONCAT}(\mathbf{x}_{\mathcal{N}_i}^{(l)}, \mathbf{x}_{\mathcal{E}_i}^{(l)})$$

Node-level Attention

$$\mathbf{e}_{\mathcal{E}_i} = \sigma(W_e \cdot \text{MEAN}(\{\beta_{ij}^e \mathbf{e}_k, \forall k \in \mathcal{E}_i\}))$$

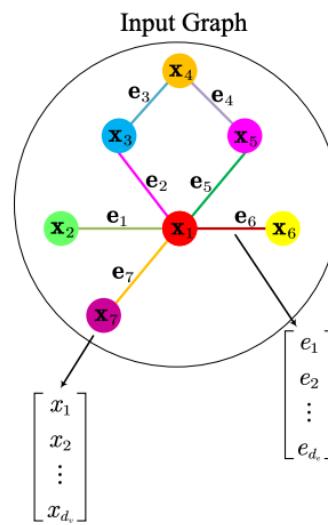
$$\mathbf{e}_{\mathcal{N}_i} = \sigma(W_n \cdot \text{MEAN}(\{\beta_{ij}^n \mathbf{x}_j, \forall j \in \mathcal{N}_i\}))$$

$$\mathbf{e}_i^{(l+1)} = \text{CONCAT}(\mathbf{e}_{\mathcal{N}_i}^{(l)}, \mathbf{e}_{\mathcal{E}_i}^{(l)})$$

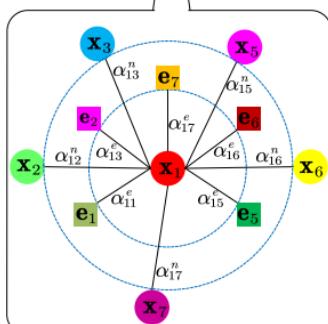
Edge-level Attention



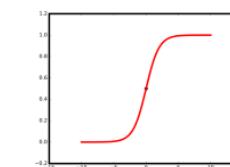
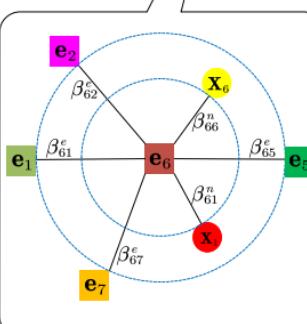
This approach iteratively updates node and edge embeddings in order to merge both information together



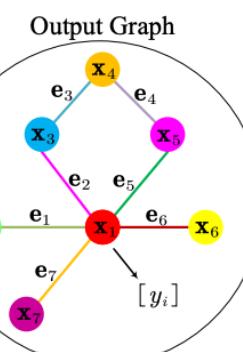
Node-level Attention Layer



Edge-level Attention Layer



more layers



# Edge Feature: How to Use



`edge_weight`  
`edge_type`  
`edge_attr`

→ GNN Layer can use `weight values` on the adjacency matrix  
→ GNN Layer can use different `edge types / relations`  
→ GNN Layer can use `edge features`

## torch\_geometric.utils



Search docs

### INSTALL PYG

Installation

### GET STARTED

Introduction by Example

Colab Notebooks and Video Tutorials

### TUTORIALS

Design of Graph Neural Networks

Working with Graph Datasets

Use-Cases & Applications

Distributed Training

FAQ

Contributing

Code of Conduct

Code of Ethics

Code of Safety

sort

`scatter`  
Reduces all values from the `src` tensor at the indices specified in the `index` tensor along a given dimension `dim`.

`group_argsort`  
Returns the indices that sort the tensor `src` along a given dimension in ascending order by value.

`group_cat`  
Concatenates the given sequence of tensors `tensors` in the given dimension `dim`.

`segment`  
Reduces all values in the first dimension of the `src` tensor within the ranges specified in the `ptr`.

`index_sort`  
Sorts the elements of the `inputs` tensor in ascending order.

`cumsum`  
Returns the cumulative sum of elements of `x`.

`degree`  
Computes the (unweighted) degree of a given one-dimensional index tensor.

`softmax`  
Computes a sparsely evaluated softmax.

`lexsort`  
Performs an indirect stable sort using a sequence of keys.

`sort_edge_index`  
Row-wise sorts `edge_index`.

torch\_geometric.sampler  
torch\_geometric.datasets  
torch\_geometric.transforms  
torch\_geometric.utils  
torch\_geometric.explain  
torch\_geometric.metrics  
torch\_geometric.distributed  
torch\_geometric.contrib  
torch\_geometric.graphgym  
torch\_geometric.profile

### CHEATSHEETS

GNN Cheatsheet  
Graph Neural Network Operators  
Heterogeneous Graph Neural Network Operators  
Hypergraph Neural Network Operators  
Point Cloud Neural Network Operators

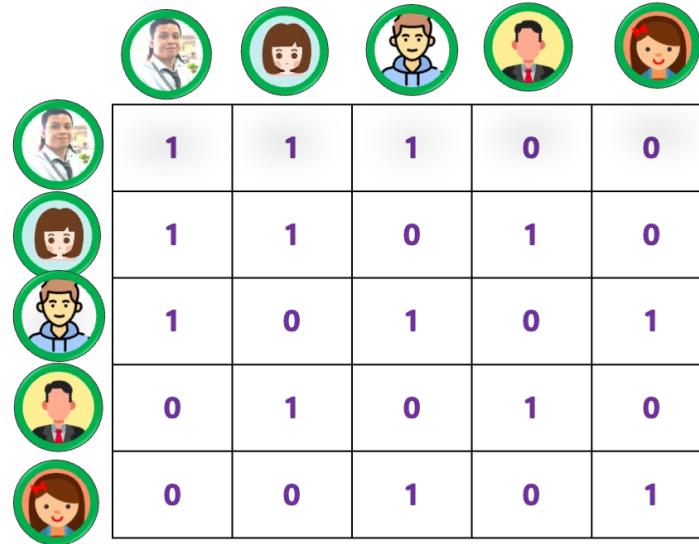
## Heterogeneous Graph Neural Network Operators

Name	SparseTensor	edge_weight	edge_attr	bipartite	static	lazy
RGCNConv <a href="#">(Paper)</a>	✓					
FastRGCNConv	✓					
CuGraphRGCNConv <a href="#">(Paper)</a>						✓
RGATConv <a href="#">(Paper)</a>	✓			✓		
FiLMConv <a href="#">(Paper)</a>	✓			✓	✓	✓
HGTConv <a href="#">(Paper)</a>	✓					✓
HEATConv <a href="#">(Paper)</a>	✓			✓		✓
HeteroConv						✓
HANConv <a href="#">(Paper)</a>	✓					✓

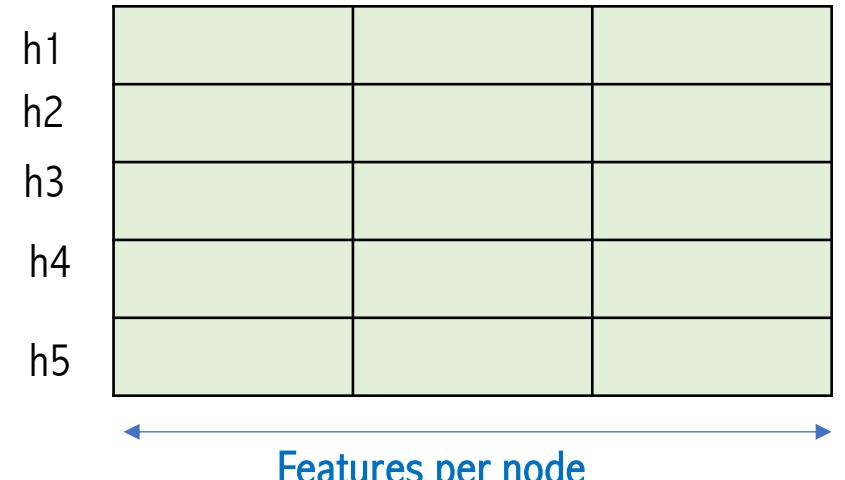
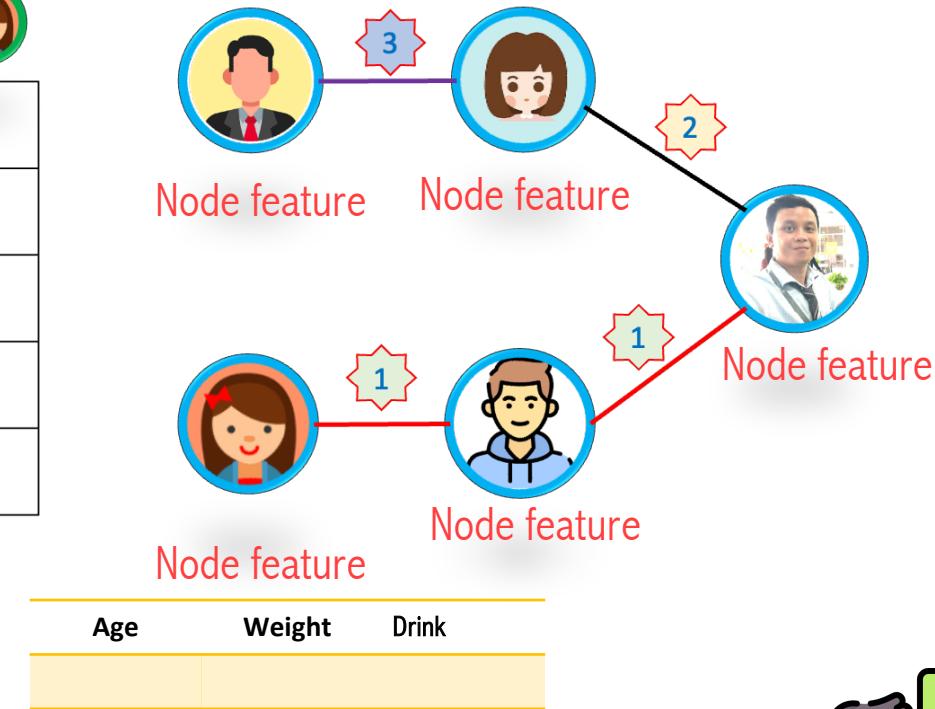
# Outline

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# Attention in Graph Neural Network



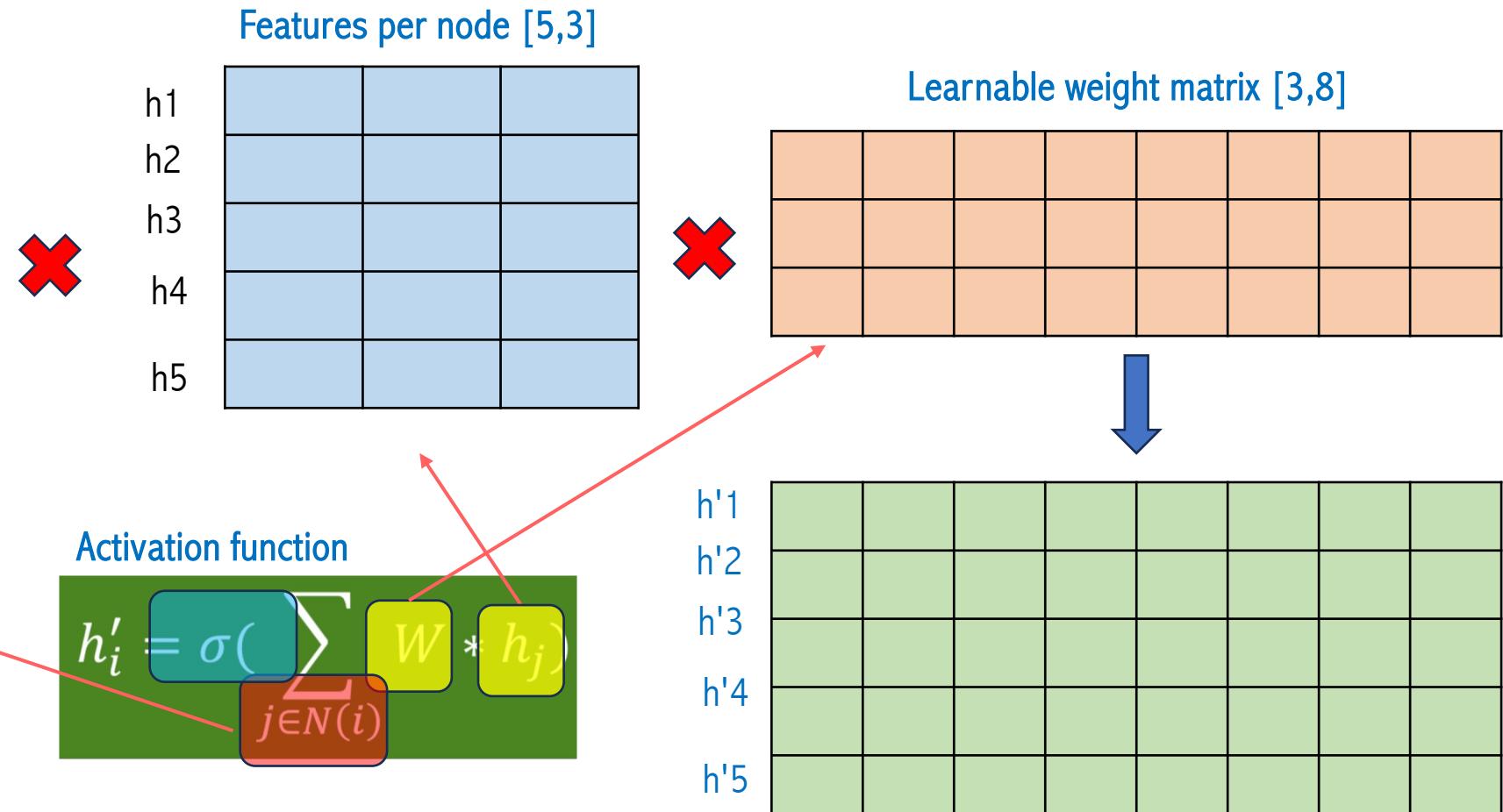
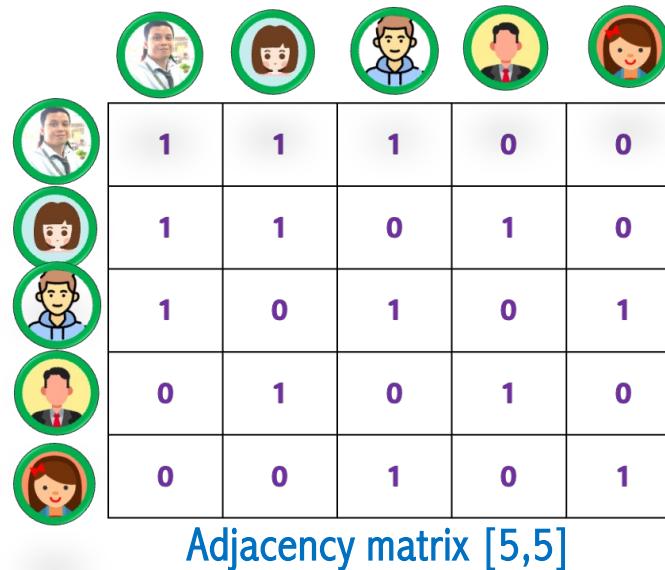
Adjacency Matrix



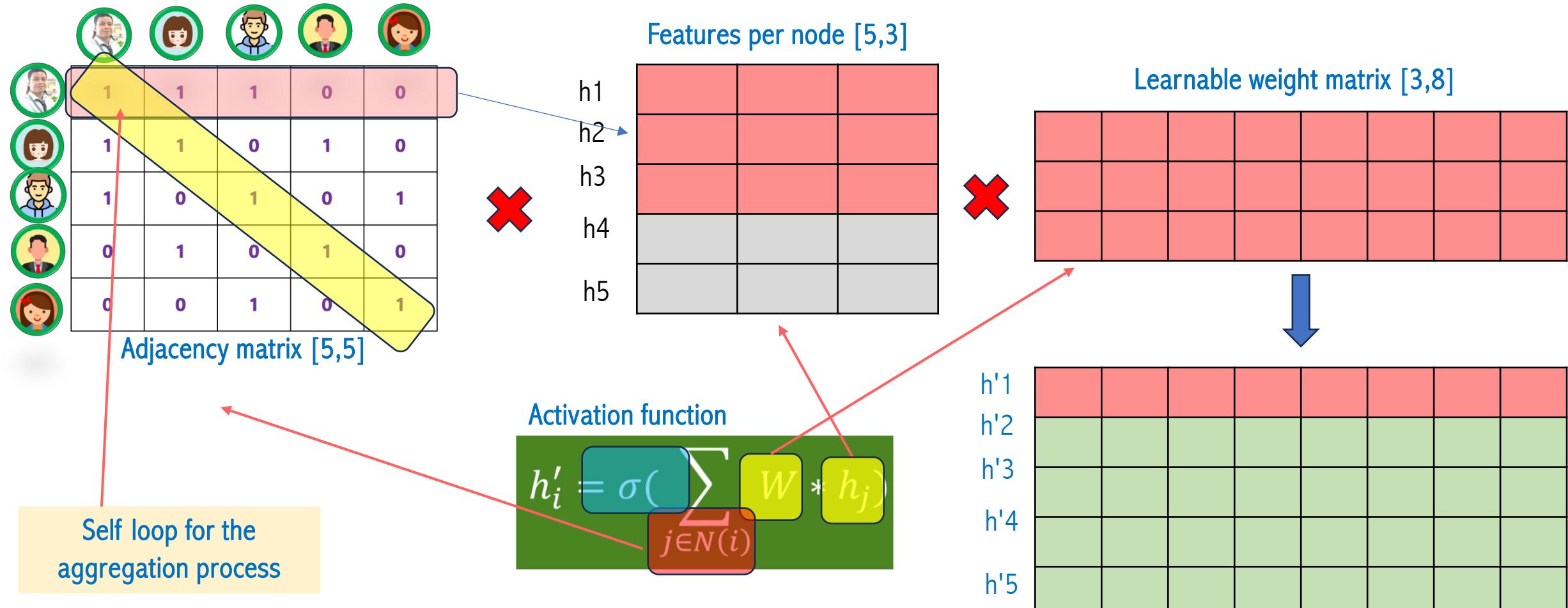
How to use this feature matrix in GNN



# GNN: Look at the 1st Layer



# GNN: Look at the 1st Layer

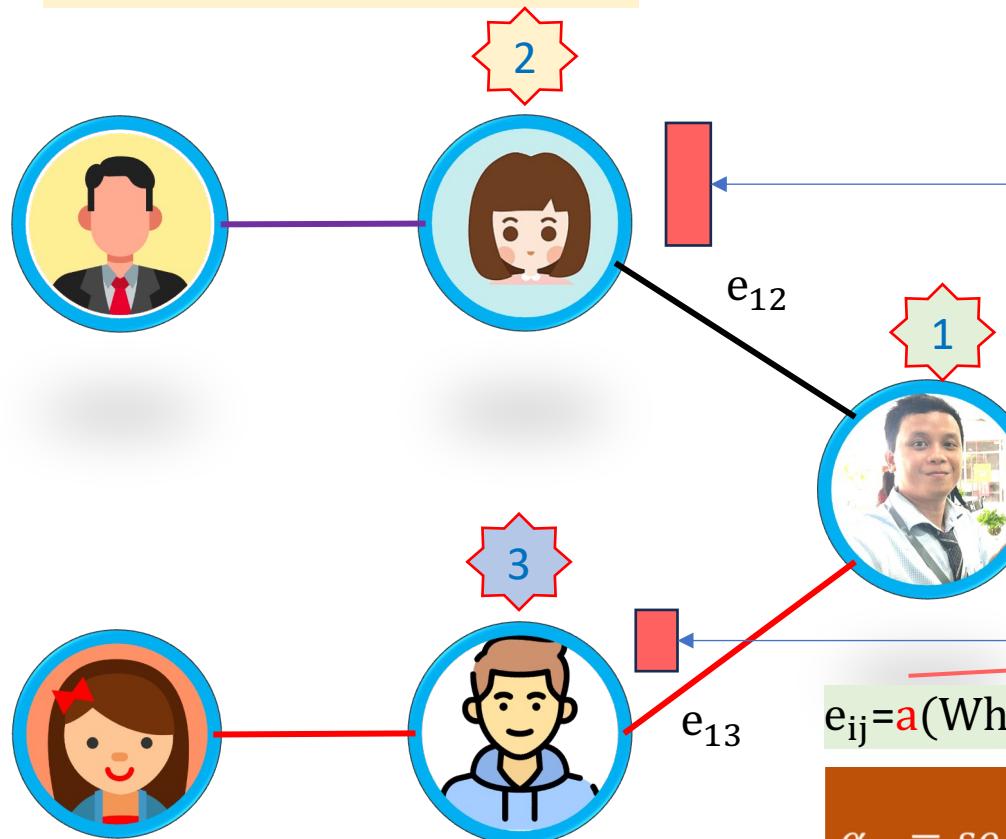


Embedding Features per node[1,8]: information of their own node feature and neighbor node features 55

# GNN: Attention Mechanism



Learn how important node J's features are for node I: attention coefficient



For the word 'play', the word 'children' is more important than the word 'the'



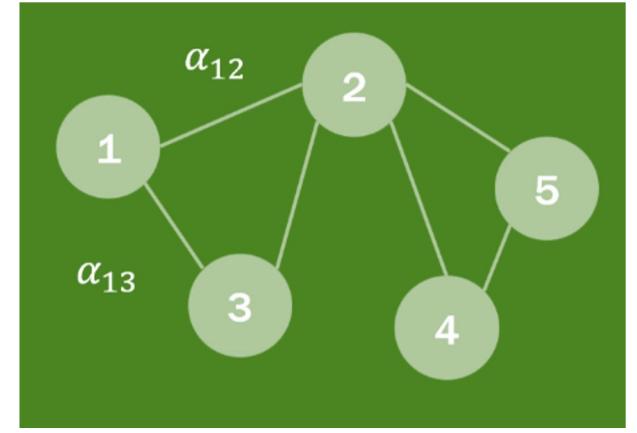
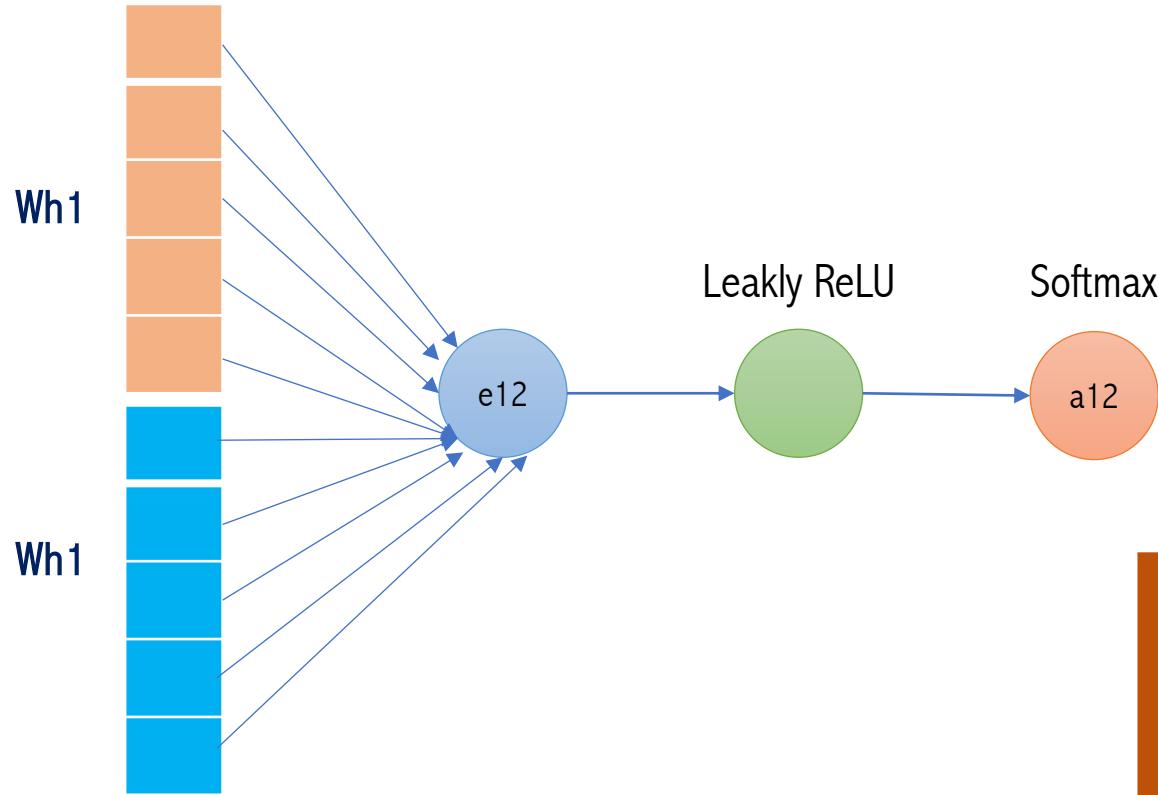
The model should pay more attention to node 2

Attention mechanism

Only considering neighbor nodes

$$\alpha_{ij} = \text{softmax}_j(e_{ij}) = \frac{\exp(e_{ij})}{\sum_{k \in N(i)} \exp(e_{ik})} = \frac{\exp(a(Wh_i, Wh_j))}{\sum_{k \in N(i)} \exp(a(Wh_i, Wh_j))}$$

# GNN: Self-Attention Mechanism



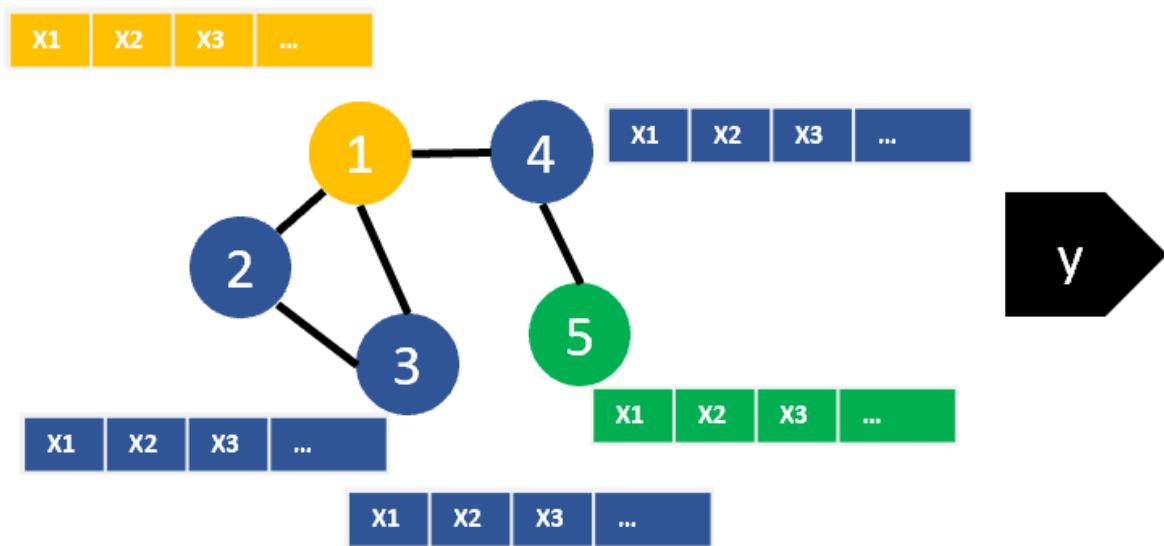
$$\alpha_{ij} = \frac{\exp(\text{LeakyReLU}(\vec{w}_a^T [Wh_i || Wh_j])))}{\sum_{k \in N(i)} \exp(\text{LeakyReLU}(\vec{w}_a^T [Wh_i || Wh_j])))}$$

QUIZ TIME

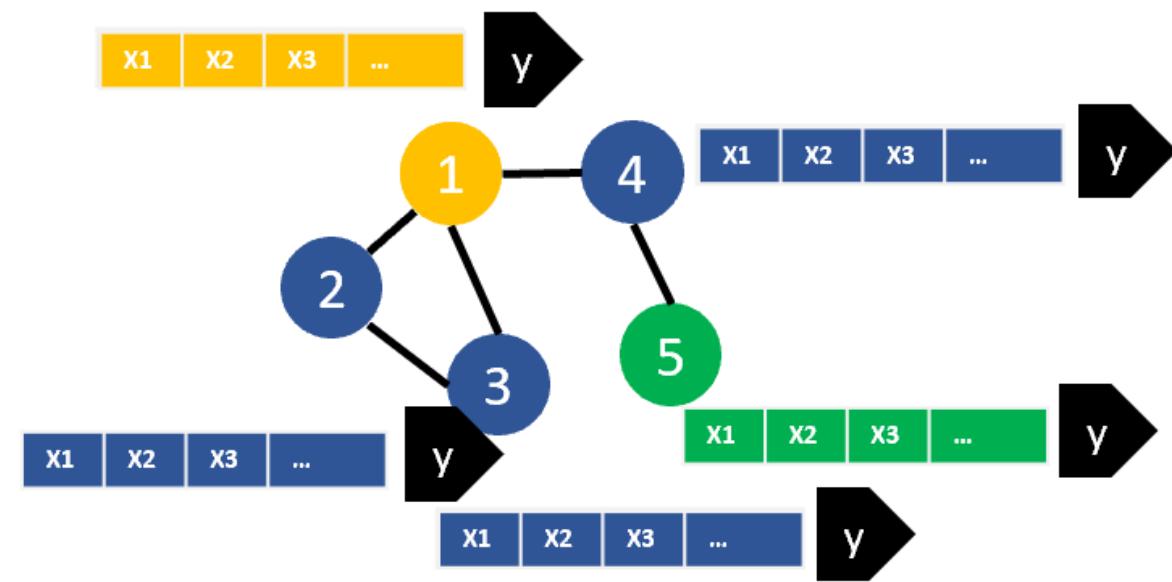
# Outline

- Edge Feature in GNN
- Edge Weight in GNN
- Relational GNN
- Multidimension Edge Feature
- Attention in GNN
- Example: Graph-Level Prediction
- Summary

# GNN: Graph Prediction



Graph-level prediction



Node-level prediction

# GNN: Graph Prediction

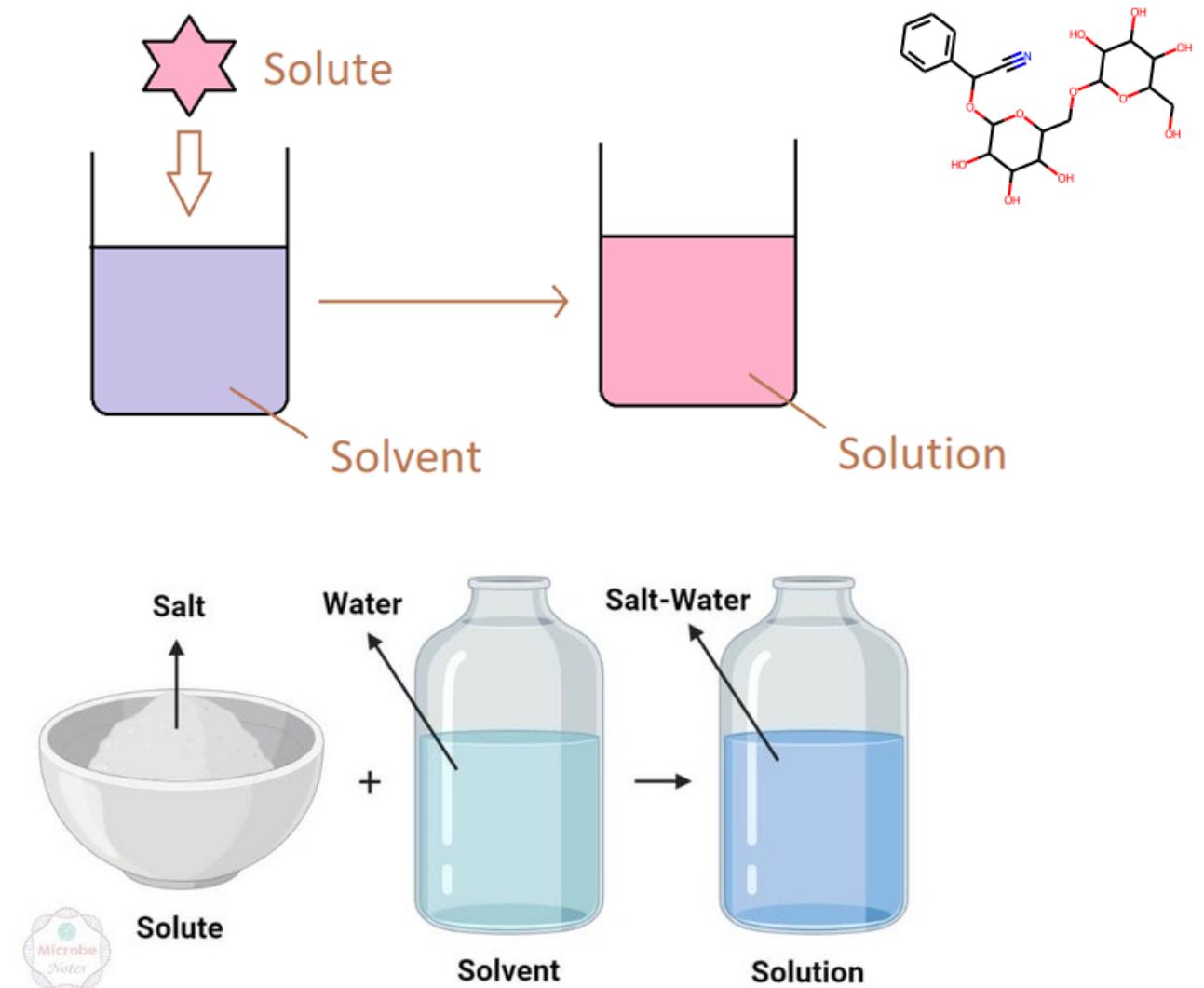
Installing Pytorch Geometric and RDKit

Background info on the Dataset

Looking into the Dataset

Visualizing molecules

Implementing the Graph Neural Network



# GNN: Graph Prediction

Installing Pytorch  
Geometric and RDKit

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Neural Network

```
!pip install torch-scatter      -f https://pytorch-geometric.com/whl/torch-{TORCH}+{CUDA}.html
!pip install torch-sparse       -f https://pytorch-geometric.com/whl/torch-{TORCH}+{CUDA}.html
!pip install torch-cluster      -f https://pytorch-geometric.com/whl/torch-{TORCH}+{CUDA}.html
!pip install torch-spline-conv  -f https://pytorch-geometric.com/whl/torch-{TORCH}+{CUDA}.html
!pip install torch-geometric
```

```
!pip install rdkit
import rdkit
from torch_geometric.datasets import MoleculeNet
```

# GNN: Graph Prediction

Installing Pytorch  
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Dataset

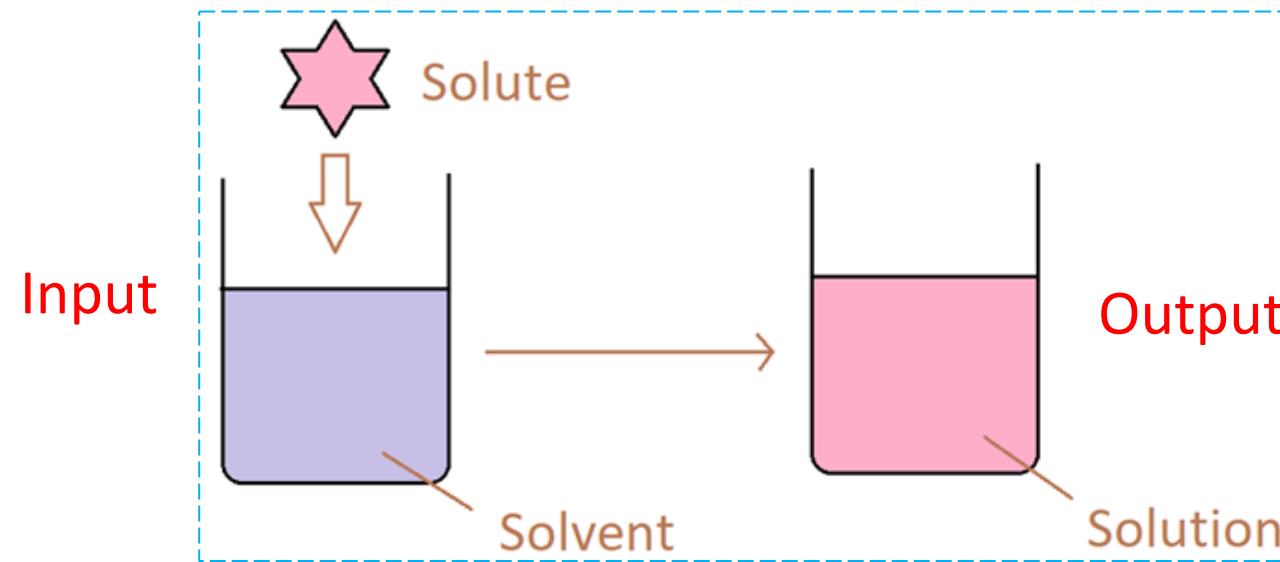
Looking into the Dataset

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How are different molecules dissolving in water?



ESOL is a small dataset consisting of water solubility data for 1128 compounds. The dataset has been used to train models that estimate solubility directly from chemical structures (as encoded in SMILES strings). Note that these structures don't include 3D coordinates, since solubility is a property of a molecule and not of its particular conformers.

# GNN: Graph Prediction

Installing Pytorch  
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```
# Investigating the dataset
print("Dataset type: ", type(data))
print("Dataset features: ", data.num_features)
print("Dataset target: ", data.num_classes)
print("Dataset length: ", data.len)
print("Dataset sample: ", data[0])
print("Sample nodes: ", data[0].num_nodes)
print("Sample edges: ", data[0].num_edges)

# edge_index = graph connections
# smiles = molecule with its atoms
# x = node features (32 nodes have each 9 features)
# y = labels (dimension)
```

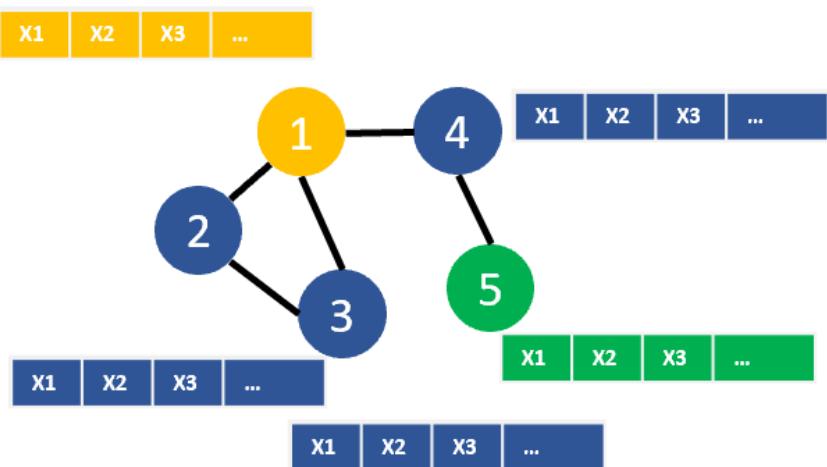
```
Dataset type: <class 'torch_geometric.datasets.molecule_net.MoleculeNet'>
Dataset features: 9
Dataset target: 734
Dataset length: <bound method InMemoryDataset.len of ESOL(1128)>
Dataset sample: Data(x=[32, 9], edge_index=[2, 68], edge_attr=[68, 3], smiles='OCC3OC(OCC2OC(OC(C#N)
```

# GNN: Graph Prediction

## Looking into the Dataset

# Implementing the Graph Neural Network

```
# Investigating the features  
# Shape: [num_nodes, num_node_features]  
data[0].x
```



```
tensor([[8, 0, 2, 5, 1, 0, 4, 0, 0],  
       [6, 0, 4, 5, 2, 0, 4, 0, 0],  
       [6, 0, 4, 5, 1, 0, 4, 0, 1],  
       [8, 0, 2, 5, 0, 0, 4, 0, 1],  
       [6, 0, 4, 5, 1, 0, 4, 0, 1],  
       [8, 0, 2, 5, 0, 0, 4, 0, 0],  
       [6, 0, 4, 5, 2, 0, 4, 0, 0],  
       [6, 0, 4, 5, 1, 0, 4, 0, 1],  
       [8, 0, 2, 5, 0, 0, 4, 0, 1],  
       [6, 0, 4, 5, 1, 0, 4, 0, 1],  
       [8, 0, 2, 5, 0, 0, 4, 0, 0],  
       [6, 0, 4, 5, 1, 0, 4, 0, 0],  
       [6, 0, 2, 5, 0, 0, 2, 0, 0],  
       [7, 0, 1, 5, 0, 0, 2, 0, 0],  
       [6, 0, 3, 5, 0, 0, 3, 1, 1],  
       [6, 0, 3, 5, 1, 0, 3, 1, 1],  
       [6, 0, 3, 5, 1, 0, 3, 1, 1],  
       [6, 0, 3, 5, 1, 0, 3, 1, 1],  
       [6, 0, 3, 5, 1, 0, 3, 1, 1],  
       [6, 0, 4, 5, 1, 0, 4, 0, 1],  
       [8, 0, 2, 5, 1, 0, 4, 0, 0],  
       [6, 0, 4, 5, 1, 0, 4, 0, 1],  
       [8, 0, 2, 5, 1, 0, 4, 0, 0],  
       [6, 0, 4, 5, 1, 0, 4, 0, 1],  
       [8, 0, 2, 5, 1, 0, 4, 0, 0],  
       [6, 0, 4, 5, 1, 0, 4, 0, 1],  
       [8, 0, 2, 5, 1, 0, 4, 0, 0],  
       [6, 0, 4, 5, 1, 0, 4, 0, 1],  
       [8, 0, 2, 5, 1, 0, 4, 0, 0]])
```

# GNN: Graph Prediction

Installing Pytorch  
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Background info on the  
Dataset

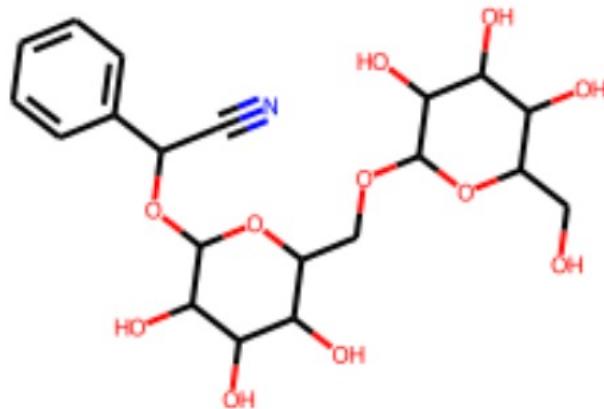
Looking into the Dataset

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```
data[0]["smiles"]  
  
'OCC3OC(OCC2OC(OC(C#N)c1ccccc1)C(OC(OC2)C(OC(OC3O
```

```
from rdkit import Chem  
from rdkit.Chem.Draw import IPythonConsole  
molecule = Chem.MolFromSmiles(data[0]["smiles"])  
molecule
```



# GNN: Graph Prediction

Installing Pytorch  
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```
import torch
from torch.nn import Linear
import torch.nn.functional as F
from torch_geometric.nn import GCNConv, TopKPooling, global_mean_pool
from torch_geometric.nn import global_mean_pool as gap, global_max_pool as gmp
embedding_size = 64

class GCN(torch.nn.Module):
    def __init__(self):
        # Init parent
        super(GCN, self).__init__()
        torch.manual_seed(42)

        # GCN layers
        self.initial_conv = GCNConv(data.num_features, embedding_size)
        self.conv1 = GCNConv(embedding_size, embedding_size)
        self.conv2 = GCNConv(embedding_size, embedding_size)
        self.conv3 = GCNConv(embedding_size, embedding_size)

        # Output layer
        self.out = Linear(embedding_size*2, 1)
```

# GNN: Graph Prediction

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```
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        self.conv2 = GCNConv(embedding_size, embedding_size)
        self.conv3 = GCNConv(embedding_size, embedding_size)

        # Output layer
        self.out = Linear(embedding_size, 1)

    def forward(self, data):
        x = self.initial_conv(data.x, data.edge_index)
        x = F.relu(x)
        x = global_mean_pool(x, data.batch)

        x = self.conv1(x, data.edge_index)
        x = F.relu(x)
        x = global_mean_pool(x, data.batch)

        x = self.conv2(x, data.edge_index)
        x = F.relu(x)
        x = global_mean_pool(x, data.batch)

        x = self.conv3(x, data.edge_index)
        x = F.relu(x)
        x = global_mean_pool(x, data.batch)

        x = self.out(x)

        return x

Number of parameters: 13249
```

# GNN: Graph Prediction

Installing Pytorch  
Geometric and RDKit

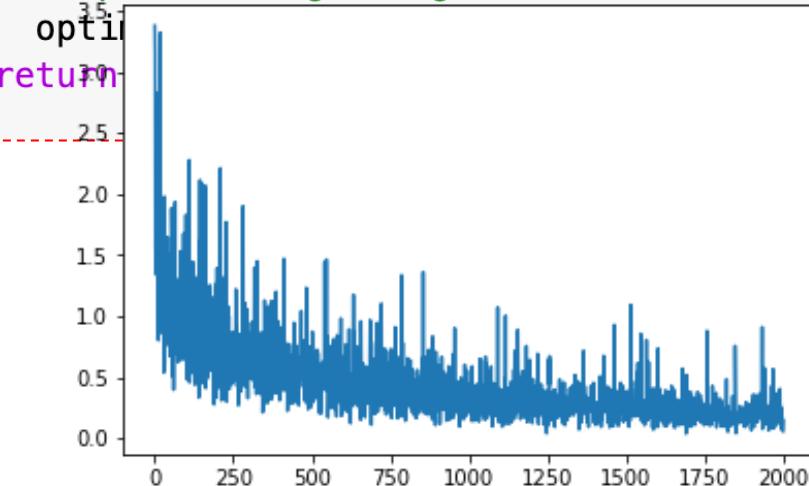
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```
def train(data):
    # Enumerate over the data
    for batch in loader:
        # Use GPU
        batch.to(device)
        # Reset gradients
        optimizer.zero_grad()
        # Passing the node features and the connection
        pred, embedding = model(batch.x.float(), batch.edge_index)
        # Calculating the loss and gradients
        loss = loss_fn(pred, batch.y)
        loss.backward()
        # Update using the gradients
        optimizer.step()
    return embedding
```



```
Starting training...
Epoch 0 | Train Loss 3.377596378326416
Epoch 100 | Train Loss 0.9617947340011597
Epoch 200 | Train Loss 1.0771363973617554
Epoch 300 | Train Loss 0.6295697093009949
Epoch 400 | Train Loss 0.37517455220222473
Epoch 500 | Train Loss 0.465716689825058
Epoch 600 | Train Loss 0.5129485726356506
Epoch 700 | Train Loss 0.21677978336811066
Epoch 800 | Train Loss 0.33871856331825256
Epoch 900 | Train Loss 0.3640660345554352
Epoch 1000 | Train Loss 0.20501013100147247
Epoch 1100 | Train Loss 0.18023353815078735
Epoch 1200 | Train Loss 0.2812242805957794
Epoch 1300 | Train Loss 0.18207958340644836
Epoch 1400 | Train Loss 0.1321338415145874
Epoch 1500 | Train Loss 0.18665631115436554
Epoch 1600 | Train Loss 0.1817774772644043
Epoch 1700 | Train Loss 0.09456530958414078
Epoch 1800 | Train Loss 0.23615044355392456
Epoch 1900 | Train Loss 0.11381624639034271
```

# GNN: Graph Prediction

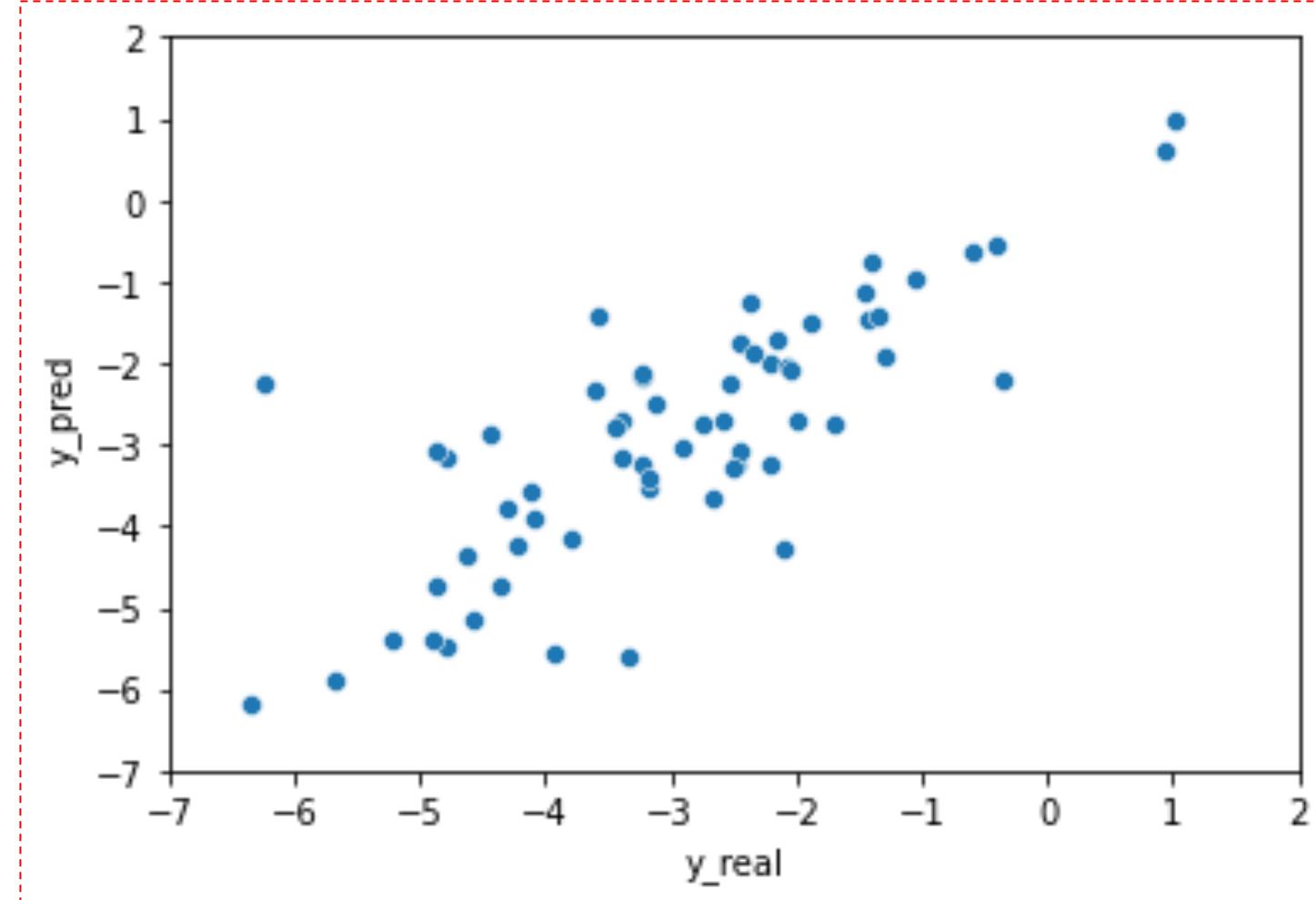
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## Vision GNN: An Image is Worth Graph of Nodes

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<sup>2</sup>Huawei Noah's Ark Lab

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### Abstract

Network architecture plays a key role in the deep learning-based computer vision system. The widely-used convolutional neural network and transformer treat the image as a grid or sequence structure, which is not flexible to capture irregular and complex objects. In this paper, we propose to represent the image as a graph structure and introduce a new *Vision GNN* (ViG) architecture to extract graph-level feature for visual tasks. We first split the image to a number of patches which are viewed as nodes, and construct a graph by connecting the nearest neighbors. Based on the graph representation of images, we build our ViG model to transform and exchange information among all the nodes. ViG consists of two basic modules: Grapher module with graph convolution for aggregating and updating graph information, and FFN module with two linear layers for node feature transformation. Both isotropic and pyramid architectures of ViG are built with different model sizes. Extensive experiments on image recognition and object detection tasks demonstrate the superiority of our ViG architecture. We hope this pioneering study of GNN on general visual tasks will provide useful inspiration and experience for future research.

The PyTorch code is available at <https://github.com/huawei-noah/Efficient-AI-Backbones> and the MindSpore code is available at <https://gitee.com/mindspore/models>.

# Vision GNN: Read and Understand

## Abstract

Network architecture plays a key role in the deep learning-based computer vision system. The widely-used convolutional neural network and transformer treat the image as a grid or sequence structure, which is not flexible to capture irregular and complex objects. In this paper, we propose to represent the image as a graph structure and introduce a new *Vision GNN* (ViG) architecture to extract graph-level feature for visual tasks. We first split the image to a number of patches which are viewed as nodes, and construct a graph by connecting the nearest neighbors. Based on the graph representation of images, we build our ViG model to transform and exchange information among all the nodes. ViG consists of two basic modules: Grapher module with graph convolution for aggregating and updating graph information, and FFN module with two linear layers for node feature transformation. Both isotropic and pyramid architectures of ViG are built with different model sizes. Extensive experiments on image recognition and object detection tasks demonstrate the superiority of our ViG architecture. We hope this pioneering study of GNN on general visual tasks will provide useful inspiration and experience for future research.

The widely-used convolutional neural network and transformer treat the image as a grid or sequence structure, which is not flexible to capture irregular and complex objects



In this paper, we propose to represent the image as a graph structure and introduce a new Vision GNN (ViG) architecture to extract graph level feature for visual tasks



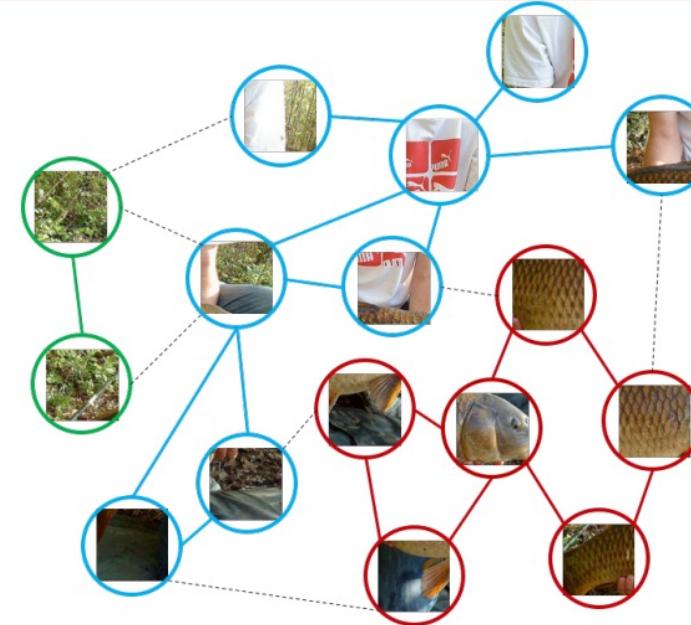
# Vision GNN: Read and Understand



(a) Grid structure.



(b) Sequence structure.

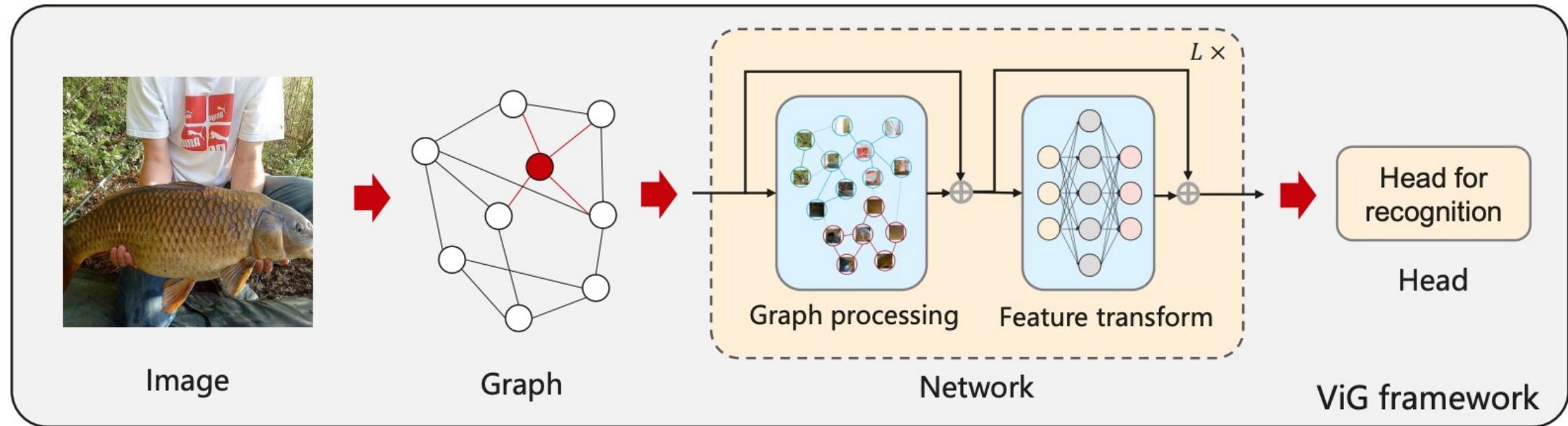


(c) Graph structure.



Illustration of the grid, sequence and graph representation of the image

# Vision GNN: Read and Understand

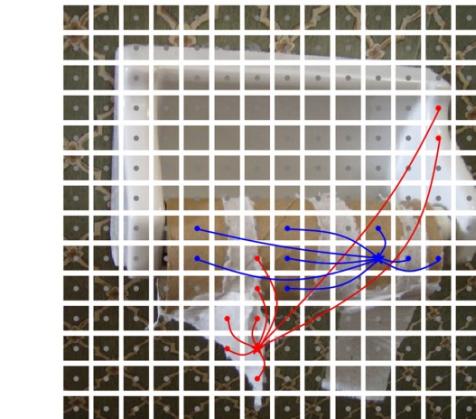
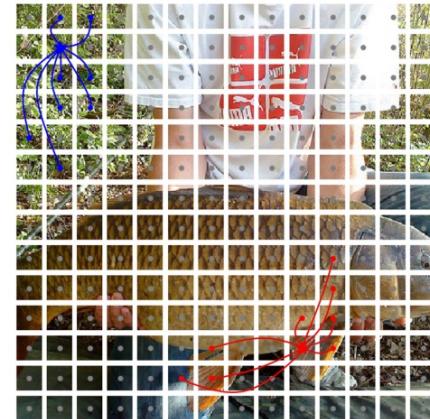


The framework of the proposed ViG model

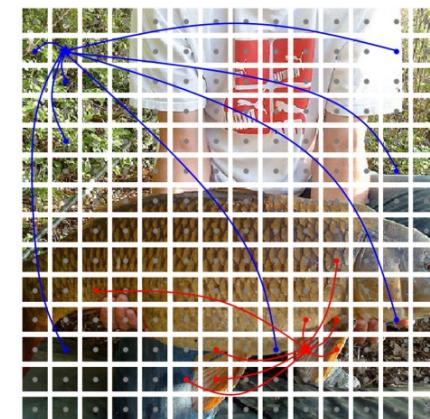
# Vision GNN: Read and Understand



(a) Input image.



(b) Graph connection in the 1st block.



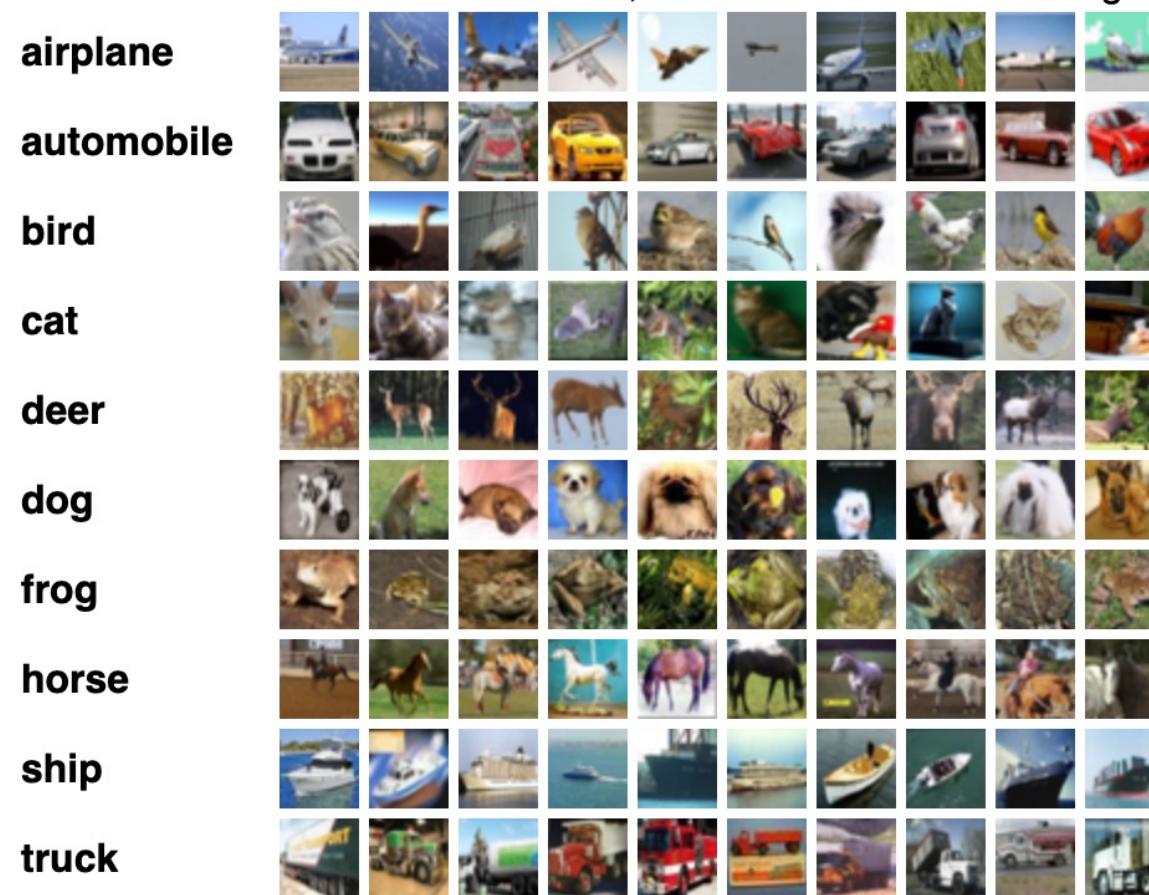
(c) Graph connection in the 12th block.

Visualization of the constructed graph structure. The pentagram is the center node, and the nodes with the same color are its neighbors in the graph



# Example: CIFAR-10 Dataset

The CIFAR-10 dataset consists of 60000 32x32 colour images in 10 classes, with 6000 images per class. There are 50000 training images and 10000 test images.



# Example: CIFAR-10 Dataset

	#params	Eval Accuracy
CNN (LeNet)	5.6M	70.34
ViT	5.6M	48.29
ViG	5.7M	<b>74.93</b>

# Example: CIFAR-10 Dataset

```
class LeNet(nn.Module):
    def __init__(self, imdim=3, num_classes=10):
        super(LeNet, self).__init__()

        self.conv1 = nn.Conv2d(imdim, 64, kernel_size=5, stride=1, padding=0)
        self.mp = nn.MaxPool2d(2)
        self.relu1 = nn.ReLU(inplace=True)
        self.conv2 = nn.Conv2d(64, 128, kernel_size=5, stride=1, padding=0)
        self.relu2 = nn.ReLU(inplace=True)
        self.fc1 = nn.Linear(128 * 5 * 5, 1024)
        self.relu3 = nn.ReLU(inplace=True)
        self.fc2 = nn.Linear(1024, 1024)
        self.relu4 = nn.ReLU(inplace=True)

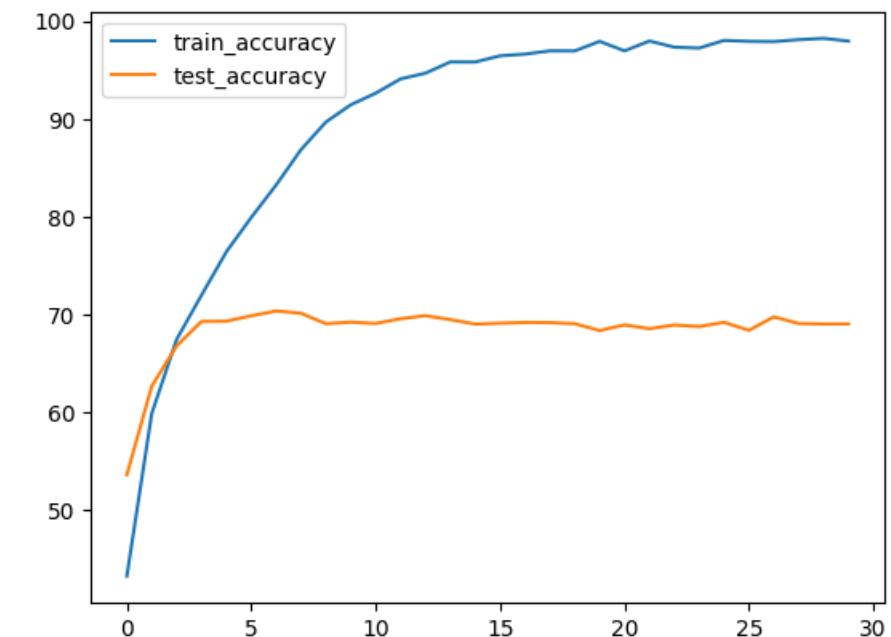
        self.fc3 = nn.Linear(1024, 1024)
        self.fc4 = nn.Linear(1024, num_classes)

    def forward(self, x):
        in_size = x.size(0)
        out1 = self.mp(self.relu1(self.conv1(x)))
        out2 = self.mp(self.relu2(self.conv2(out1)))
        out2 = out2.view(in_size, -1)
        out3 = self.relu3(self.fc1(out2))
        out = self.relu4(self.fc2(out3))

        out = self.fc3(out)

        return self.fc4(out)
```

CNN for Image Classification



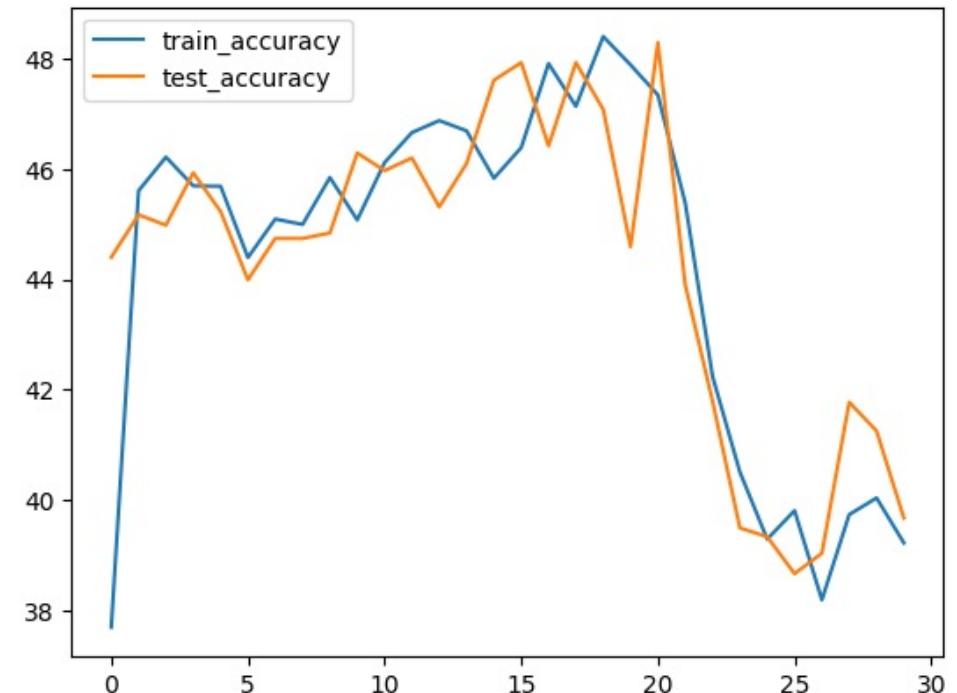
# Example: CIFAR-10 Dataset

```
class ViT(nn.Module):
    def __init__(self,
                 *,
                 image_size,
                 patch_size,
                 num_classes,
                 dim,
                 depth,
                 heads,
                 mlp_dim,
                 pool="cls",
                 channels=3,
                 dim_head=64,
                 dropout=0.0,
                 emb_dropout=0.0,
                 ):
        super().__init__()
        image_height, image_width = pair(image_size)
        patch_height, patch_width = pair(patch_size)

        assert (
            image_height % patch_height == 0 and image_width % patch_width == 0
        ), "Image dimensions must be divisible by the patch size."

        num_patches = (image_height // patch_height) * (image_width // patch_width)
        patch_dim = channels * patch_height * patch_width
        assert pool in {
            "cls",
```

Vision Transformer for Image Classification



# Example: CIFAR-10 Dataset

```
# train
for epoch in range(max_epoch):
    model.train()
    running_loss = 0.0
    running_correct = 0 # to track number of correct predictions
    total = 0 # to track total number of samples

    for i, (inputs, labels) in enumerate(trainloader, 0):
        # Move inputs and labels to the device
        inputs, labels = inputs.to(device), labels.to(device)

        # Zero the parameter gradients
        optimizer.zero_grad()

        # Forward pass
        outputs = model(inputs)

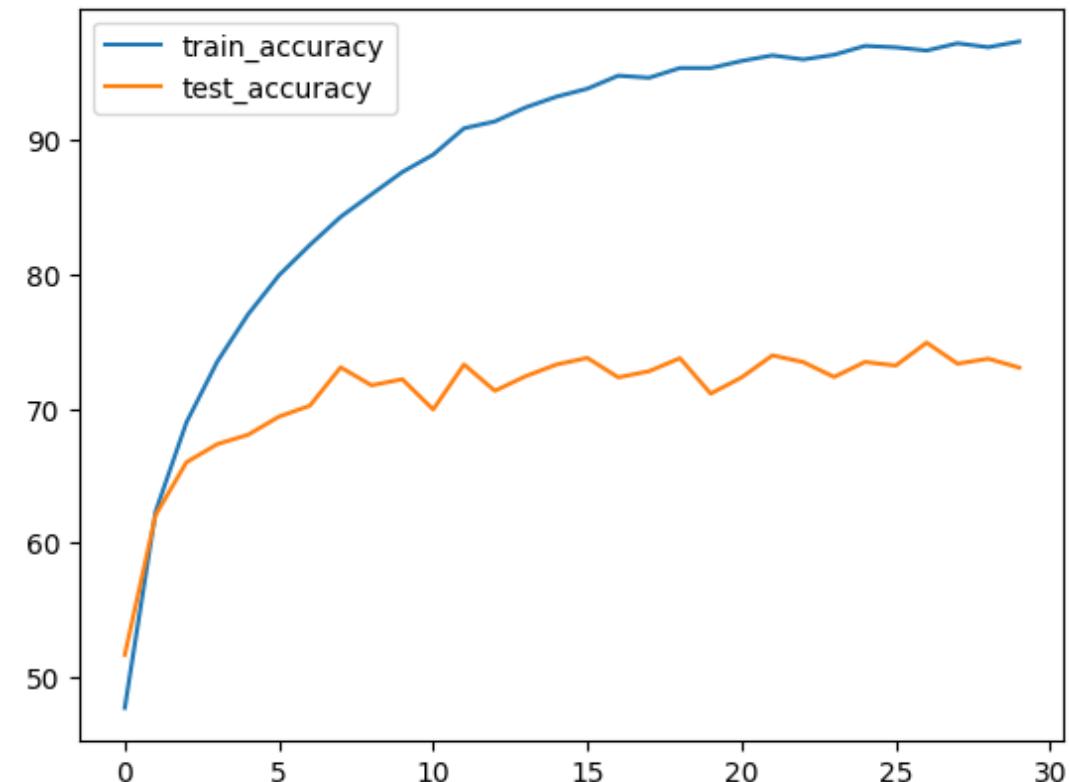
        loss = criterion(outputs, labels)

        running_loss += loss.item()

        # Backward pass and optimization
        loss.backward()
        optimizer.step()

        # Determine class predictions and track accuracy
        _, predicted = torch.max(outputs.data, 1)
        total += labels.size(0)
        running_correct += (predicted == labels).sum().item()
```

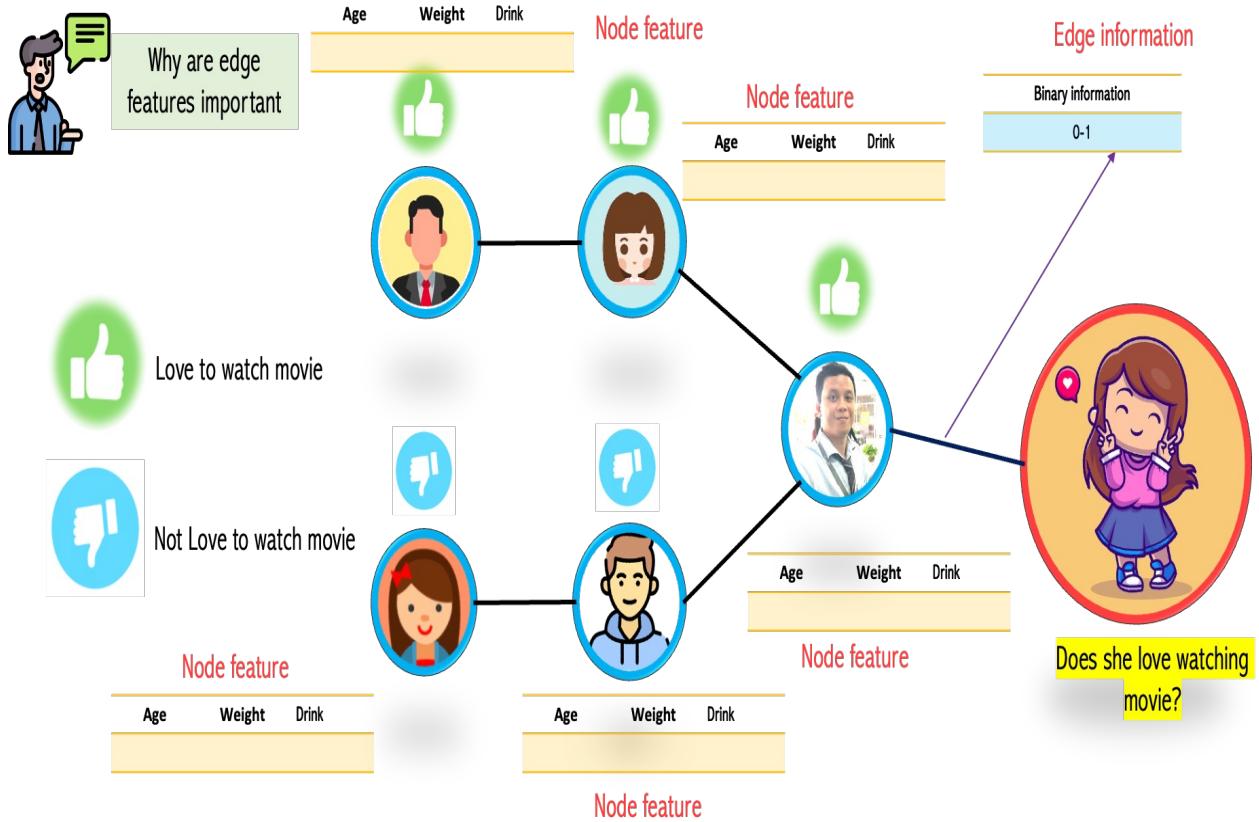
Vision GNN for Image Classification



# Outline

- Edge Feature in GNN
- Edge Weight in GNN
- Relational GNN
- Multidimension Edge Feature
- Attention in GNN
- Example: Graph-Level Prediction
- Summary

# Summary



- How to integrate edge feature to GNN
- Edge Weight in GNN
- Relational GNN
- Multidimensional Edge Feature
- Attention in GNN
- Graph-level prediction

