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Stochastic Modeling of MEMS Inertial Sensors

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Abstract: *A detailed methodology that allows the developing of stochastic discrete-time models of MEMS gyroscope and accelerometer noises is presented. The methodology is based on the frequency-domain and time-domain characteristics of the sensors noises and is illustrated for the case of Analog Devices tri-axis Inertial measurement Sensor ADIS16350. It is shown that the gyro and accelerometer noises have similar second-order models which are appropriate for usage in the development of Kalman filters for navigation and control systems.*

Keywords: *MEMS inertial sensors, Stochastic Error models, Kalman filtering.*

1. Introduction

The Micro Electro Mechanical Systems (MEMS) inertial sensors have several applications in low-cost navigation and control systems [4, 5, 9]. A common disadvantage of these sensors are the significant errors which accompany the corresponding measurements. This necessitates the development of adequate error models which may be used to achieve sufficient measurements accuracy with the aid of appropriate filtering.

The inertial sensor errors consist of deterministic and stochastic parts. The deterministic part includes constant biases, scale factors, axis nonorthogonality, axis misalignment and so on, which are removed from row measurements by the corresponding calibration techniques. The stochastic part contains random errors (noises) which cannot be removed from the measurements and should be modeled as stochastic processes.

The MEMS gyroscope noise typically consists of the following terms:

- *Bias instability*. This is a stationary stochastic process which may be considered as a low-order zero-mean Gauss-Markov process.
- *Angular random walk*. This is an angular error process which is due to white noise in angular rate.
- *Rate random walk*. This is a rate error due to white noise in angular acceleration.
- *Discretization error*. This is an error representing the quantization noise.

Similarly, the MEMS accelerometer noise may be represented as a sum of the following terms.

- *Bias instability*.
- *Velocity random walk*. This is a velocity error due to white noise in acceleration.
- *Acceleration random walk.* This is an acceleration error due to white noise in jerk.
- *Discretization error*.

Several other noise terms are described in detail in [6].

Different techniques for building models of MEMS sensor noises are presented in [4, 7, 8], to name a few. Usually, they exploit the autocorrelation function of the noise in order to obtain 1-st order Gauss-Markov or higher order Auto-Regressive models. Note that it is desirable to keep the model order as low as possible since the model is frequently used in the design of Kalman filter to determine optimal estimates based on the sensor measurements.

The aim of this paper is to present a detailed methodology that allows the development of stochastic discrete-time models of MEMS gyro and accelerometer noises. The methodology is based on the frequency-domain and time-domain characteristics of the sensors noises and is illustrated for the case of Analog Devices tri-axis Inertial measurement Sensor ADIS16350. It is shown that the gyroscope and accelerometer noises have similar second-order models which are appropriate for usage in the development of Kalman filters for navigation and control systems.

The units used in the paper conform to the units used in [2] for comparison purposes.

2. Allan variance

The Allan variance provides a means of identifying various noise terms in the original data set [1, 6].

Assume that a quantity $\theta(t)$ is measured at discrete time moments $t=kT_0$, $k=1, 2, \ldots, L$. The average value of θ between times t_k and $t_k + \tau$ where $\tau = mT_0$, is given by

$$\hat{\Omega}_k(\tau) = \frac{\theta_{k+m} - \theta_k}{\tau}$$

where $\theta_k = \theta(kT_0)$.

The Allan variance is defined as

$$\sigma^2(\tau)=\frac{1}{2}\left\langle(\hat{\Omega}_{k+m}-\hat{\Omega}_k)^2\right\rangle=\frac{1}{2\tau^2}\left\langle(\theta_{k+2m}-2\theta_{k+m}+\theta_k)^2\right\rangle \tag{1}$$

where $\langle\ \rangle$ is the ensemble average.

The Allan variance is estimated as follows:

$$\sigma^2(\tau)=\frac{1}{2\tau^2(L-2m)}\sum_{k=1}^{L-2m}(\theta_{k+2m}-2\theta_{k+m}+\theta_k)^2\ . \tag{2}$$

The Allan variance is related to the PSD of the measured quantity and is connected with the parameters of the noise terms described in Section 1. This connection is exploited in the next sections to determine the gyroscope and accelerometer noises models.

3. Stochastic models of gyroscopic sensors

The stochastic discrete-time model of the gyro sensors is derived on the base of the gyro noise measured at rest. For this aim we use L = 1 000 000 samples of one of the gyro outputs of ADIS16350

Inertial Measurement Sensor [2] measured with frequency f_s = 100 Hz during the period of 10 000 s. After removing a small constant bias of –0.1027 deg/s. we found that the noise mean square value is (to six digits)

$$\sigma_{\text{gyro\_noise}} = 0.581614 \text{ deg/s.}$$

The centered output gyro noise is shown in Fig. 1.

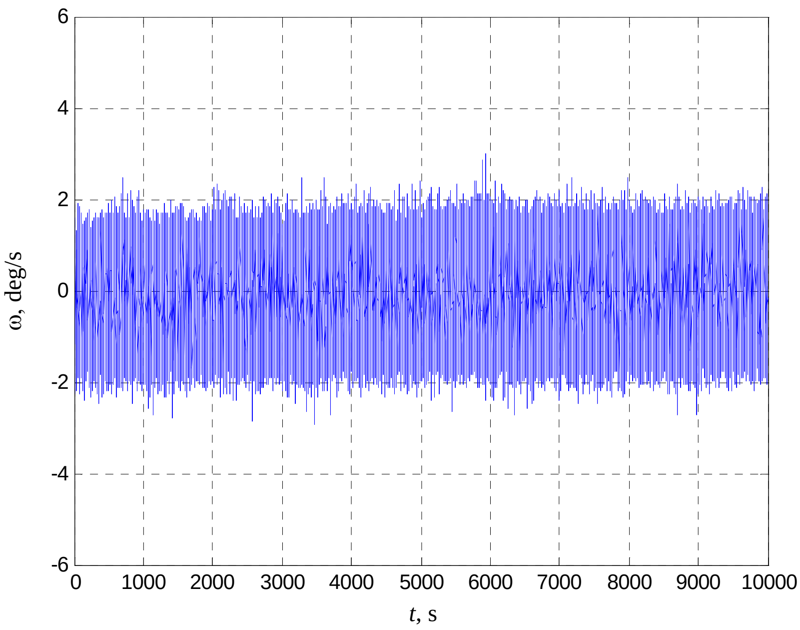

Fig. 1. Output gyro noise

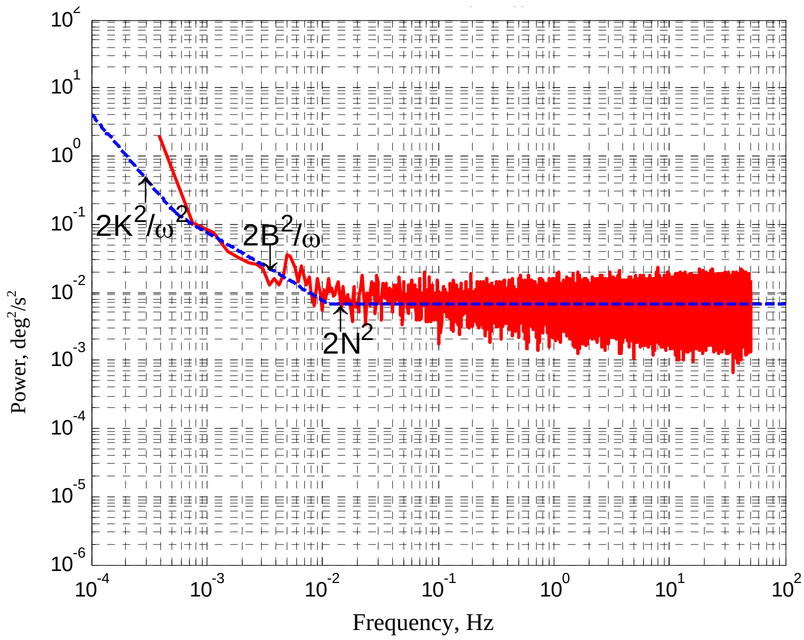

Fig. 2. Power spectral density of gyro noise

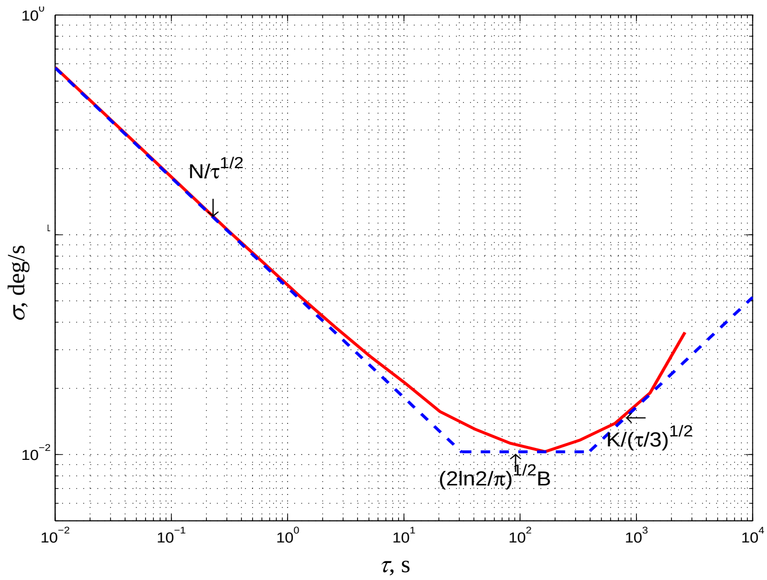

Fig. 3. Allan variance of gyro noise

To determine the different terms of the gyro noise we study its frequency domain and time domain characteristics. In the frequency domain we make use of the one sided power spectral density (PSD) of the gyro noise, determined by the function *pwelch* from MATLAB and shown in Fig. 2. Taking into account the connection between the slopes of the PSD and the slopes of the spectral densities of different kinds of gyro noises [6], we conclude that there are three noise components, namely bias instability, angular random walk and rate random walk with mean square values B, N and K, respectively. Since the gyro has 14-bit resolution its discretization error is negligible.

To determine the parameters B, N and K precisely, we use the Allan variance of the gyro noise, computed with overlapping estimates and shown in Fig. 3. The connection between parameters B, N and K and the Allan variance, taken from [6], is shown in the same figure. As a result one obtains the values $B = 1.55\times10^{-2}$ deg/s, $N = 5.75\times10^{-2}$ deg/s$^{1/2}$, and $K = 9\times10^{-4}$ deg/s$^{1/2}$.

The bias instability is usually described by the first order lag

(3) $$T\dot{x}(t) + x(t) = v(t)\,,$$

where $x(t)$ is a Gauss-Markov process and $v(t)$ is an input white noise. According to Fig. 3 we take $T = 120$ s.

Since we are interested to determine a discrete-time model of the noise, we discretize equation (3) to obtain

(4) $$x(k+1) = a_d x(k) + b_d \eta(k)$$

where $a_d = e^{(-\frac{1}{T})\Delta T} = 0.999917$, $b_d = \int_0^{\Delta T} e^{(-\frac{1}{T})\tau} d\tau = 8.33299\times10^{-5}$, $\Delta T = \frac{1}{f_s} = 0.01$,

and $\eta(k)$ is a discrete-time white noise with unknown variance σ_η^2.

It is well known that for the time-invariant discrete-time system

$$x(k+1) = A_d x(k) + B_d \eta(k)$$

the state covariance matrix P is related to the input covariance matrix Q by the matrix equation [3]

(5) $$P = A_d P A_d^T + B_d Q B_d^T\,.$$

That is why the variances of the discrete-time stochastic processes $x(k)$ and $\eta(k)$ in respect to the 1-st order system (4) are related by

$$(1 - a_d^2)\sigma_x^2 = b_d^2 \sigma_\eta^2\,.$$

Since $\sigma_x = B$, we have that

(6) $$\sigma_\eta = \sqrt{1 - \frac{a_d^2 B}{b_d}}\,.$$

As a result one finds σ_η=2.40125.

Consider now how to determine the model of rate random walk, which will be denoted by rrw. The effect of this noise on the gyro output may be represented by the response of an integrator to a white noise. The corresponding discrete-time relationship is given by

(7) $$\mathrm{rrw}(z) = K\mathrm{dfilt}(z)\omega(z),$$

where

(8) $$\mathrm{dfilt}(z) = \frac{\Delta T}{z-1}$$

is the integrator discrete-time transfer function and ω is a white noise with unknown variance σ_ω^2.

The value of σ_ω is found by approximating the low-frequency part of the gyro noise PSD by the PSD of rate random walk (7), as shown in Fig. 4. As a result we find $\sigma_\omega = 5$. Finally, the angular random walk is modeled as a white noise arw whose variance σ^2_{arw} is determined according to the expression

(9) $$\sigma_{\text{arw}} = \sqrt{\sigma^2_{\text{gyro\_noise}} - \sigma_x^2 - \sigma_{\text{yyw}}^2}.$$

This gives $\sigma_{\text{arw}} = 0.581188$.

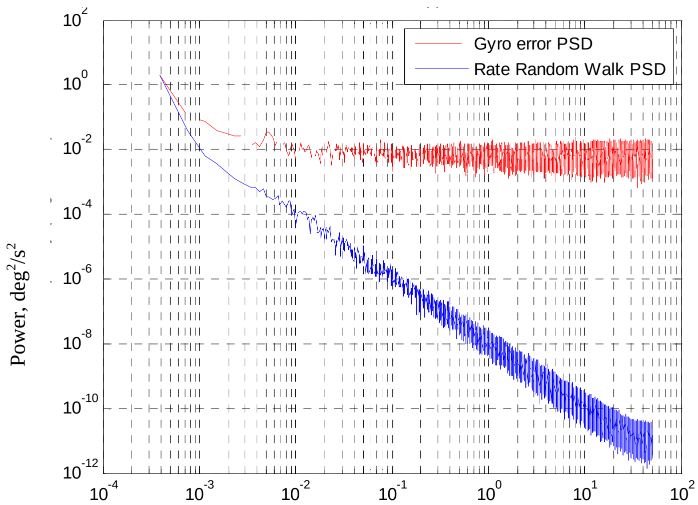

Fig. 4. Rate random walk approximation

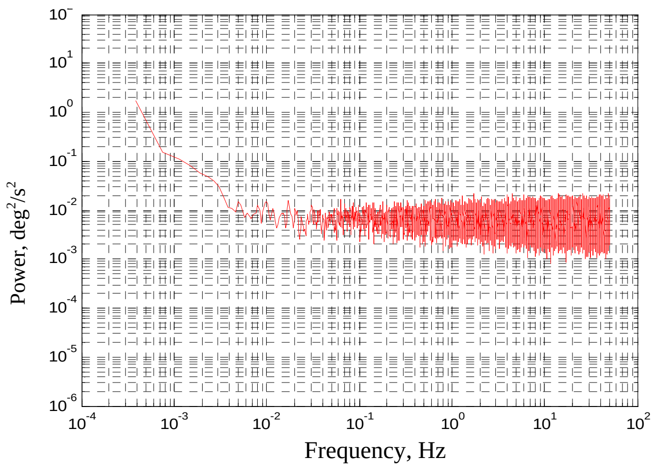

Fig. 5. Power spectral density of the gyro model noise

In this way the gyro noise is represented as

(10) $$\text{gyro\_noise} = x + \text{arw} + \text{rrw},$$

where x, rrw and arw are described by (4), (7), and (9), respectively.

As it is seen from Fig. 5, the PSD of the model noise, generated according to (10), coincides well with the PSD of the measured noise, shown in Fig. 2.

In Fig. 6 we show the measured gyro noise x_{means} along with the noise x_{model} generated by the gyro noise model. The closeness of both signals is estimated by the quantity

$$\text{fit} = 1 - \frac{\sqrt{\sum_{k=1}^{L}(x_{\text{means}}(k) - x_{\text{model}}(k))^2}}{\sqrt{\sum_{k=1}^{L}(x_{\text{means}}(k))^2}} 100\%. \tag{11}$$

In the given case one obtains fit = 96% which shows that the signals are quite close.

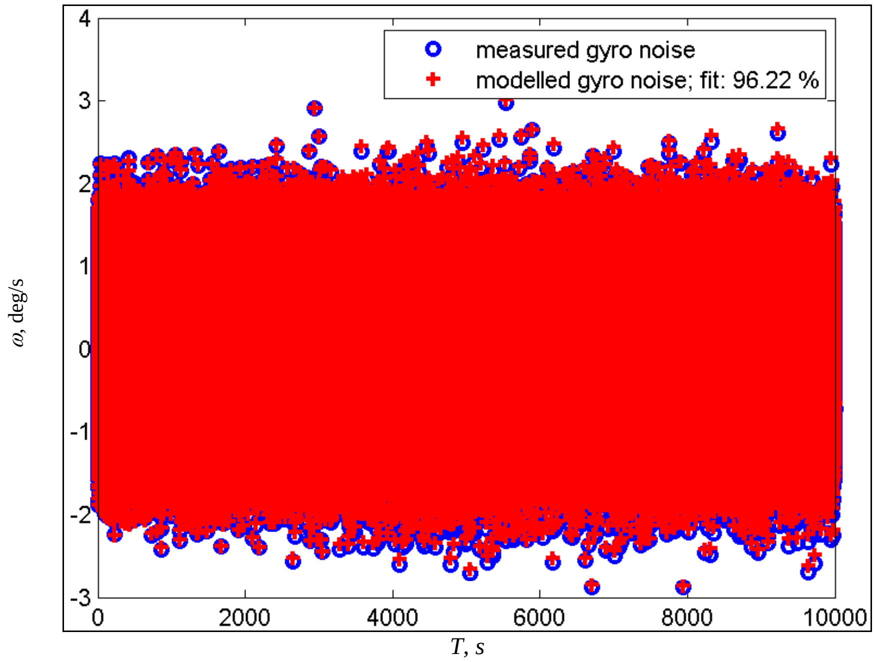

Fig. 6. Measured gyro noise and the gyro model noise

4. Stochastic models of accelerometers

In developing a model of the accelerometer noise we implement the same methodology as in the case of gyroscopes. This is due to the fact that the PSD and Allan variance of the accelerometer noise have properties similar to the corresponding gyro characteristics.

In the given case we use again 1 000 000 samples of the accelerometer noise measured at rest. The mean square value of the noise is found to be $\sigma_{\text{accellerometer\_noise}} = 5.62452\times10^{-3}$, g. (Note that the acceleration values are measured in the earth gravitation unit (g)).

The PSD of the accelerometer noise is shown in Fig. 7 and the Allan variance is shown in Fig. 8. From Allan variance we find that the accelerometer noise has the following parameters:

- Bias instability B = 0.325 mg;
- Velocity random walk. N = 0.294 (m/s/ $\sqrt{\text{h}}$);
- Acceleration random walk K = 0.206 (m/s^2/ $\sqrt{\text{h}}$);
- Time-constant T = 80 s.

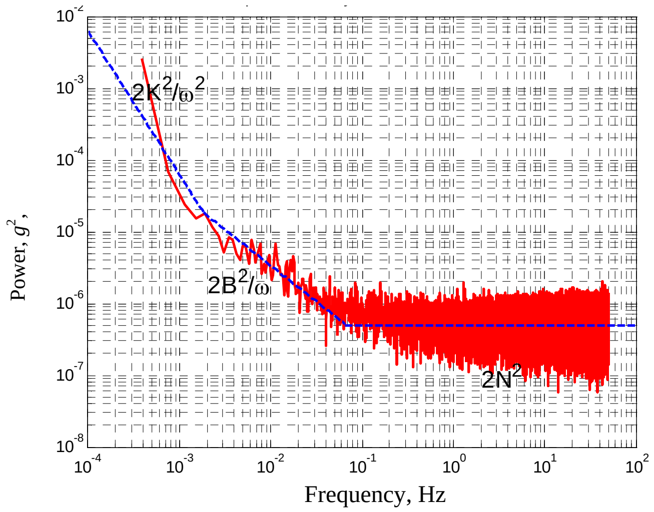

Fig. 7. Power spectral density of accelerometer noise

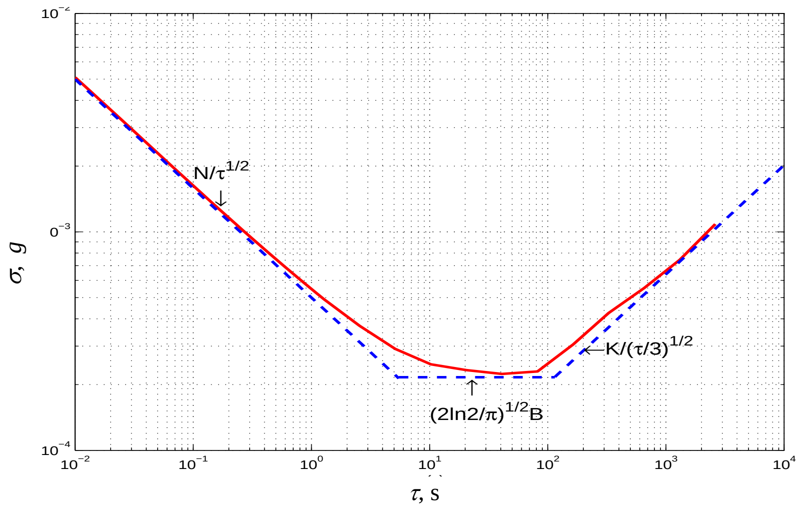

Fig. 8. Allan variance of accelerometer noise

As in the case of gyro noise, the bias instability is described by equation of the type (4), with coefficients a_d and b_d computed for the new value of the time constant T. The acceleration random walk is described by the equation (7) with the same integrator transfer function and white noise mean square value $\sigma_\omega = 5$. The velocity random walk is taken as a white noise with variance σ^2_{vrw} determined by an equation of the type (9). This gives $\sigma_{\text{vrw}} = 5.57178\times10^{-3}$ g. The final model of the acceleration noise is obtained by relationship of the type (10).

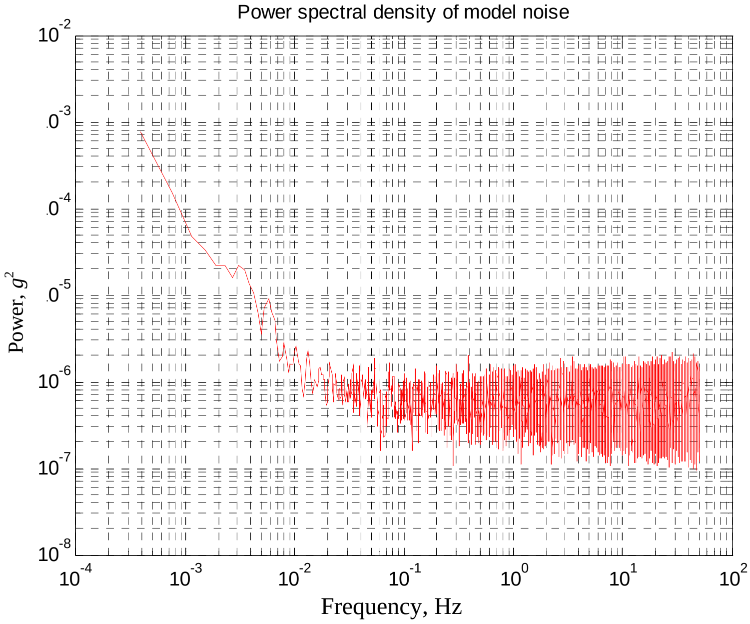

Fig. 9. Power spectral density of the accelerometer model noise

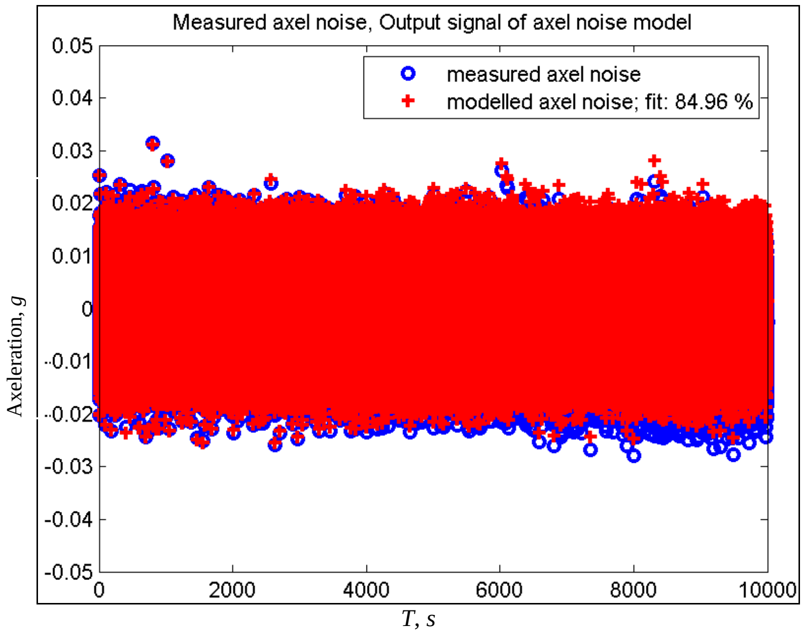

Fig. 10. Measured accelerometer noise and the accelerometer model noise

The power spectral density of the acceleration noise generated according to the model described is shown in Fig. 9. This PSD is close to the PSD computed from the measurements.

In Fig. 10 we show the measured accelerometer noise along with the noise generated by the accelerometer noise model. Using the expression (11) one obtains fit = 85% which shows that the signals are sufficiently close.

Finally, it should be noted that when the gyro and accelerometer noise models are used in the design of a Kalman filter instead of deg/s one should express the model parameters in rad/s and instead of g, the corresponding parameters should be expressed in m/s^2.

5. Conclusions

It is shown that the gyroscope and accelerometer noises pertaining to Analog Devices MEMS Inertial Measurement Sensor ADIS16350 have similar second-order models which may be determined with sufficient accuracy by using the frequency-domain and time-domain characteristics of the noises. The error models are implemented as programs in MATLAB® and Simulink® and are appropriate for usage in the development of Kalman filters for navigation and control systems.

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References

1. Allan, D. W. Statistics of Atomic Frequency Standards. – In: Proc. of the IEEE, **54**, 1966, 221-230.
2. Analog Devices. ADIS16350/ADIS16355: High Precision Tri-Axial Inertial Sensor. **http: www.analog.com.**
3. Anderson, B. D. O., J. B. Moore. Optimal Filtering. Englewood Cliffs, NJ, Prentice-Hall, 1979.
4. Farrel, J. A. Aided Navigation: GPS with High Rate Sensors. New York, McGraw Hill, 2008.
5. Grewal, M. S., L. R. Weill, A. P. Andrews. Global Positioning Systems, Inertial Navigation and Integration. 2nd ed. Wiley-Interscience, 2007.
6. IEEEStd 952-1997, IEEE Standard Specification Format Guide and Test Procedure for Single-Axis Interferometric Fiber Optic Gyros. IEEE Aerospace and Electronic Systems Society, 1998.
7. Nassar, S. Improving the Inertial Navigation System (INS) Error Model for INS and INS/DGPS Applications. PhD Dissertation, Department of Geomatics Engineering, University of Calgary, Alberta, Canada, 2003.
8. Park, M., Y. Gao. Error and Performance Analysis of MEMS-Based Inertial Sensors with a Low-Cost GPS Receiver. – Sensors, **8**, 2008, 2240-2261.
9. Titterton, D., J. Weston. Strapdown Inertial Navigation Technology. 2nd ed. The Institutionof Electrical Engineers, UK, 2004.