[2] van Graas, F., et al. (1995)
FAA/Ohio University/UPS autoland flight test results. Presented at the *Institute of Navigation National Technical Meeting*, Jan. 1995.

[3] Bartone, C. (1996)
Advanced pseudolite for dual-use precision approach applications.
In *Proceedings of the Institute of Navigation GPS-96*, Sept. 17–20, 1996.

[4] Bartone, C., and van Graas, F. (1997)
Airport pseudolite for precision approach applications.
In *Proceedings from the Institute of Navigation GPS-97*, Sept. 16–19, 1997, 1843–1849.

[5] GPS Joint Program Office (1992)
GPS-ICD-200, IPN-200B-PR-001, 01 July 1992.

[6] van Graas, F., et al. (1997)
Ohio University/FAA flight test demonstration results of the local area augmentation system (LAAS).
In *Proceedings from the Institute of Navigation GPS-97*, Sept. 16–19, 1997, 1626–1627.

[7] Winer, B., et al. (1996)
GPS Receiver Laboratory RFI Tests.
In *Proceedings from the Institute of Navigation GPS-96*, Jan. 1996, 674–675.

[8] van Graas, F. (1996)
GNSS augmentation for high precision navigation services.
Advisory Group for Aerospace Research & Development, Lecture Series 207, June 1996, 8-1 to 8-3.

[9] Liu, F., et al. (1997)
LAAS signal-in-space integrity monitoring description and verification plan.
In *Proceedings from the Institute of Navigation GPS-97*, Sept. 16–19, 1997, 487–489.

[10] Radio Technical Commission for Aeronautics (1998)
RTCA SC-159, Proposed wideband pseudolite interface control document, Sept. 1998.

Equivalent ARMA Model Representation for RLG Random Errors

It is shown that a mixture of random noises can be represented by a single equivalent ARMA (autoregressive moving average) model that is simple to implement. We applied the scheme to model RLG (ring laser gyroscope) random errors. An identification result from real test data confirms the validity of this approach.

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I. INTRODUCTION

Sensors inherently contain errors in their output. The errors have to be modeled and identified before the sensors are engaged in a system so that they are properly compensated or filtered. This procedure is especially important in developing alignment algorithms for inertial navigation systems to initialize sensor errors [1–3]. We propose an efficient way to represent random errors in which a number of different forms of noises are mixed as commonly seen in inertial sensors such as gyroscopes and accelerometers. It is shown that a mixture of several noises such as white noise, random walk, quantization noise, Markov process, and other forms of general autoregressive moving average (ARMA) processes can be represented by a single equivalent ARMA model. Once the ARMA model is obtained, we can employ widely known ARMA model identification methods [4] to represent the random errors.

This scheme is applied to a ring laser gyroscope (RLG) to model its random errors. RLG is widely used for medium- to high-grade inertial navigation systems because of its good stability and small size. Various methods of modeling RLG random errors have been studied in the past [5–8]. In [5], RLG random errors are assumed to be modeled by a non-stationary ARMA model, and its parameters are estimated using data differencing. As a method of determining model order and identifying parameters, a method called DDS (data dependent system) is introduced in [6]. In [7], an identification method using state space and Kalman filtering is proposed to identify RLG random errors modeled by a mixture of stationary and non-stationary autoregression (AR) models. In [5–7], it is assumed that RLG random errors are already modeled by an ARMA model, but an *efficient* way of modeling RLG random errors in an ARMA form is needed. On other forms of errors, Ng and Pines applied the Allan variance method for modeling various RLG errors including deterministic as well as random noises [8]. The Allan variance method is known to be reliable in identifying a model from a signal of mixed noises.

For verification purposes, our proposed method is applied to real test data of an RLG. The results are then compared with the Allan variance method employed by Ng and Pines in [8]. The proposed method, which is easy to implement, give results as good as those of the Allan variance method.

II. EQUIVALENT ARMA MODEL

In this section, an equivalent ARMA model for a mixture of ARMA processes representing different forms of noises is derived. It is sufficient to show that a mixture of two ARMA processes can be simplified by a single equivalent ARMA model.

Consider two ARMA processes x_t of order (m,n) and y_t of order (p,q). The expression (m,n) means x_t consists of an mth-order AR part and an nth-order moving average (MA) part. Then we get

$$\begin{aligned} & x_t + a_1 x_{t-1} + a_2 x_{t-2} + \cdots + a_m x_{t-m} \\ &\qquad = u_t + b_1 u_{t-1} + b_2 u_{t-2} + \cdots + b_n u_{t-n} \\ & y_t + c_1 y_{t-1} + c_2 y_{t-2} + \cdots + c_p y_{t-p} \\ &\qquad = v_t + d_1 v_{t-1} + d_2 v_{t-2} + \cdots + d_q v_{t-q} \end{aligned} \tag{1}$$

where u_t and v_t are white Gaussian processes, respectively, and are independent of one another.

If we rewrite (1) using backward shift operator B, which, for example, means $Bx_t = x_{t-1}$, then we get

$$x_t = \frac{(1 + b_1 B + b_2 B^2 + \cdots + b_n B^n)}{(1 + a_1 B + a_2 B^2 + \cdots + a_m B^m)} u_t,$$

$$y_t = \frac{(1 + d_1 B + d_2 B^2 + \cdots + d_q B^q)}{(1 + c_1 B + c_2 B^2 + \cdots + c_p B^p)} v_t.$$

Introduce a new variable that corresponds to the mixture of x_t and y_t as $z_t = x_t + y_t$, then

$$\begin{aligned} & (1 + a_1 B + a_2 B^2 + \cdots + a_m B^m) \\ &\qquad \times (1 + c_1 B + c_2 B^2 + \cdots + c_p B^p) z_t \\ &\quad = (1 + f_1 B + f_2 B^2 + \cdots + f_{pn} B^{pn}) u_t \\ &\qquad + (1 + g_1 B + g_2 B^2 + \cdots + g_{qm} B^{qm}) v_t \end{aligned} \tag{2}$$

where $(1 + f_1 B + f_2 B^2 + \cdots + f_{pn} B^{pn})$ is an abbreviation of $(1 + b_1 B + b_2 B^2 + \cdots + b_n B^n)(1 + c_1 B + c_2 B^2 + \cdots + c_p B^p)$ and similarly $(1 + g_1 B + g_2 B^2 + \cdots + g_{qm} B^{qm})$ is an abbreviation of $(1 + d_1 B + d_2 B^2 + \cdots + d_q B^q)(1 + a_1 B + a_2 B^2 + \cdots + a_m B^m)$. At this point, it should be noted that the left-hand side of (2) corresponds to the AR part of the equivalent ARMA model.

Now let's derive the MA part of the equivalent ARMA model. The final result we want to show is that the right-hand side of (2) is represented by an equivalent MA model which is driven by white Gaussian processes. Define

$$\begin{aligned} s_t &= (1 + f_1 B + \cdots + f_{pn} B^{pn}) u_t \\ &\quad + (1 + g_1 B + \cdots + g_{qm} B^{qm}) v_t. \end{aligned}$$

Clearly $E[s_t] = 0$ where E denotes the expectation. And we can verify that $E[s_t s_{t-k}]$ depends only on k, that is, s_t is WSS (wide sense stationary). This verification can be shown by calculating $E[s_t s_{t-k}]$ directly and using the property that u_t and v_t are WSS. Moreover, since s_t is a linear combination of u_t and v_t, that are by assumption jointly Gaussian, respectively, and independent of one another. It is also jointly Gaussian [10–12]. Now we are ready to prove our theorem.

EQUIVALENT THEOREM. *Define r_k as $r_k \triangleq E[s_t s_{t-k}]$ since s_t is WSS. If e_is exist satisfying*

$$\begin{aligned} r_0 &= e_0^2 + e_1^2 + \cdots + e_l^2 \\ r_1 &= e_1 e_0 + e_2 e_1 + \cdots + e_l e_{l-1} \\ r_2 &= e_2 e_0 + e_3 e_1 + \cdots + e_l e_{l-2} \\ &\vdots \\ r_{l-1} &= e_{l-1} e_0 + e_l e_1 \\ r_l &= e_l e_0 \end{aligned} \tag{3}$$

where $l \triangleq \max(pn, qm)$ and $e_0 + e_1 B + \cdots + e_l B^l = 0$ has stable roots, then

$$w_t \triangleq \frac{1}{e_0 + e_1 B + \cdots + e_l B^l} s_t$$

is a white Gaussian process with zero mean and unit variance.

PROOF First we want to show that w_t is white. Introduce the following equation,

$$\frac{1}{e_0 + e_1 B + \cdots + e_l B^l} = h_0 + h_1 B + \cdots$$

which implies

$$\begin{aligned} 1 &= (e_0 + e_1 B + \cdots + e_l B^l)(h_0 + h_1 B + h_2 B^2 + \cdots) \\ &= e_0 h_0 + (e_0 h_1 + e_1 h_0) B + (e_0 h_2 + e_1 h_1 + e_2 h_0) B^2 + \cdots \end{aligned}$$

Therefore the following relations hold

$$\begin{aligned} e_0 h_0 &= 1 \\ e_0 h_1 + e_1 h_0 &= 0 \\ e_0 h_2 + e_1 h_1 + e_2 h_0 &= 0 \\ &\vdots \end{aligned} \tag{4}$$

Now evaluate $E[w_t w_{t-k}]$. Assume that $k > 0$ at first. Using $w_t = (h_0 + h_1 B + \cdots) s_t$ and since $r_k = 0$ $(k > l)$ by the definition of s_t, we obtain

$$\begin{aligned} E[w_t w_{t-k}] &= E[\{(h_0 + h_1 B + \cdots) s_t\} \\ &\qquad \times \{(h_0 B^k + h_1 B^{k+1} + \cdots) s_t\}] \\ &= h_0 (h_0 r_k + h_1 r_{k+1} + \cdots + h_{l-k} r_l) \\ &\quad + h_1 (h_0 r_{k-1} + h_1 r_k + \cdots + h_{l-k+1} r_l) \\ &\quad + h_2 (h_0 r_{k-2} + h_1 r_{k-1} + \cdots + h_{l-k+2} r_l) \\ &\quad \vdots \end{aligned} \tag{5}$$

Substituting (3) to (5) and using the relations in (4), we can show that $E[w_t w_{t-k}] = 0$ for $k \neq 0$. Now, let's consider when $k = 0$. Substituting $w_t =$

$(h_0 + h_1 B + \cdots)s_t$ to $E[w_t w_t]$ and using r_k, we get

$$
\begin{aligned}
E[w_t w_t] &= E\left[\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} h_i h_j s_{t-i} s_{t-j}\right] \\
&= (h_0^2 + h_1^2 + \cdots)r_0 \\
&\quad + (h_0 h_1 + h_1 h_0 + h_1 h_2 + h_2 h_1 + \cdots)r_1 \\
&\quad + \cdots + (h_0 h_l + h_1 h_{l+1} + \cdots + h_l h_0 + h_{l+1} h_1 + \cdots)r_l \\
&\le r_0 \sum_{i=0}^{\infty} h_i^2 + 2\bar{r} \sum_{|i-j|\neq 0}^{\infty} |h_i h_j| \qquad (6)
\end{aligned}
$$

where $\bar{r} \triangleq \max_i\{|r_i|\}$, $(i = 1,\ldots,l)$. According to the Schwartz inequality, it can be seen that the right-hand side of (6) is bounded by $2r_{\max}\sum_{i=0}^{\infty} h_i^2$ where $r_{\max} \triangleq \max\{r_0, \bar{r}\}$. Since it is assumed that $e_0 + e_1 B + \cdots + e_l B^l = 0$ has stable roots, $\sum_{i=0}^{\infty} h_i^2$ converges [9]. Therefore $E[w_t w_t]$ exists and is constant over time t because s_t in (6) is WSS. This shows that w_t is white.

Secondly, it can be seen that w_t is jointly Gaussian since w_t is a linear combination of s_t that is jointly Gaussian. We want to calculate the mean and variance of w_t. First, $E[s_t] = 0$ implies $E[w_t] = 0$. For the variance, define $E[w_t w_t]$ as σ_w^2 and use $E[w_t w_{t-k}] = 0$ when $k \neq 0$. Then, we obtain

$$r_0 = E[s_t s_t] = (e_0^2 + e_1^2 + \cdots + e_l^2)\sigma_w^2.$$

This implies $\sigma_w^2 = 1$ according to (3), and completes the proof.

From the Equivalent Theorem, we get

$$s_t = (e_0 + e_1 B + \cdots + e_l B^l) w_t$$

where w_t is a white Gaussian process. This implies that the right-hand side of (2) can be represented by an equivalent MA model.

Until now we have shown that the mixture of any two ARMA processes can be represented by a single equivalent ARMA model. We can easily extend this result to the mixture of many ARMA processes.

III. IDENTIFICATION OF RLG RANDOM ERRORS

In this section, as an application of the result obtained in Section II, an identification procedure for a medium-grade RLG[1] is presented. We assume that the RLG contains a white noise, a quantization noise, a random walk, and a first-order Markov process as its random errors [5, 8, 13]. All possible combinations of the four noises are considered as candidates for equivalent ARMA models to represent the random errors. The candidate models are examined using rate table tested data to identify the best model, which is then compared with the one obtained by the Allan variance method in [8].

A. Noises in RLG

For RLG, white noise, quantization noise, random walks, and first-order Markov processes are considered to be important forms of its random errors. A spontaneous laser emission and a dithering to compensate rock-in phenomenon cause a white noise. A discrete nature of RLG outputs results in a quantization noise. A random walk and an exponentially correlated first-order Markov process are other common forms of noise found in gyroscopes (mechanical as well as laser). The four noises are represented by y_{wh^t}, y_{qt^t}, y_{rw^t}, and y_{Mk1^t}, respectively, and they are mathematically expressed by

$$
\begin{aligned}
y_{\mathrm{wh}^t} &= w_{\mathrm{wh}^t} \\
y_{\mathrm{qt}^t} &= w_{\mathrm{qt}^t} - w_{\mathrm{qt}^{t-1}} \\
y_{\mathrm{rw}^t} &= y_{\mathrm{rw}^{t-1}} + w_{\mathrm{rw}^t} \\
y_{\mathrm{Mk1}^t} &= \phi y_{\mathrm{Mk1}^{t-1}} + w_{\mathrm{Mk1}^t}
\end{aligned}
$$

where w_{wh^t}, w_{qt^t}, w_{rw^t}, and w_{Mk1^t} are mutually independent white Gaussian with mean zero and variances σ_{wh}, σ_{qt}, σ_{rw}, and σ_{Mk1}, respectively. ϕ is a coefficient of the first-order Markov process that is related to a correlation time.

B. Equivalent ARMA Model

According to the procedure explained in Section II and the Equivalent Theorem, we want to represent a mixture of the four noises by a corresponding equivalent ARMA model. For example, let's consider a mixture of white noise and quantization noise to derive an equivalent ARMA model. The mixture of white noise and quantization noise is represented by z_t, and it can be written as

$$
\begin{aligned}
z_t &= y_{\mathrm{wh}^t} + y_{\mathrm{qt}^t} \\
&= w_{\mathrm{wh}^t} + (1 - B) w_{\mathrm{qt}^t}. \qquad (7)
\end{aligned}
$$

Applying the Equivalent Theorem, the right-hand side of (7) can be written as

$$(e_0 + e_1 B) w_t \qquad (8)$$

where e_0 and e_1 satisfy the conditions described in the Equivalent Theorem, that is, first, they are the roots of (3) and, secondly, $e_0 + e_1 B = 0$ has stable roots. e_0 and e_1 are solved as

$$
\begin{aligned}
e_0 &= \sqrt{\frac{2\sigma_{\mathrm{qt}}^4}{(\sigma_{\mathrm{wh}}^2 + 2\sigma_{\mathrm{qt}}^2) - \sqrt{\sigma_{\mathrm{wh}}^4 + 4\sigma_{\mathrm{wh}}^2\sigma_{\mathrm{qt}}^2}}} \\
e_1 &= \frac{-(\sigma_{\mathrm{wh}}^2 + 2\sigma_{\mathrm{qt}}^2) + \sqrt{\sigma_{\mathrm{wh}}^4 + 4\sigma_{\mathrm{wh}}^2\sigma_{\mathrm{qt}}^2}}{2\sigma_{\mathrm{qt}}^2}.
\end{aligned} \qquad (9)
$$

[1]HG1700 (Honeywell IMU, gyroscope bias error: 1 deg/h).

TABLE I
Equivalent ARMA Models

Noises	ARMA Model Order	Noises	ARMA Model Order
wh + qt	(0,1)	wh + qt + Mk1	(1,2)
Wh + rw	(0,1)[2]	rw + Mk1	(1,1)
Wh + Mk1	(1,1)	wh + rw + Mk1	(1,2)
Rw + qt	(0,2)	rw + qt + Mk1	(1,3)
Qt + Mk1	(1,2)	wh + rw + qt + Mk1	(1,3)
wh + rw + qt	(0,2)		

TABLE II
Parameter Estimation Results Using RLG Real Data

Data Set	Proposed Method (e_0, e_1)	Allan Variance Method (e_0, e_1)
1	(133.7, −98.1)	(134.8, −99.4)
2	(116.3, −87.0)	(114.1, −85.8)

Hence, the mixture of white noise and quantization noise can be represented by an ARMA(0,1) with corresponding parameters given by (9). Equivalent ARMA models of all possible mixtures of the four noises of RLG are summarized in Table I. In the table, abbreviations are used for white noise (wh), quantization noise (qt), random walk (rw), and 1st-order Markov process (Mk1).

C. Identification Procedure

To identify the best representation for the Honeywell RLG, input and output data have been collected from a static rate table test. No rate input is applied to the gyroscope. Output data are compensated for the Earth rate input and a constant bias. The order and parameter values of ARMA models are going to be determined from the data set.

Equivalent ARMA models in Table I are examined one by one using the test data. The PEM (prediction error method) in [4] is employed to find the best fit model. The criterion of best fit is the whiteness test of prediction errors described in [14]. For the purpose of comparison, the Allan variance method described in [8] has also been applied to the test data.

The identification procedure using the Allan variance method is composed of the following sequences. The first step is to divide the data set into a number of fixed size clusters. Secondly, compute the average of each cluster. The third step is to compute a mean-squared, moving average (Allan variance) of the clusters. The fourth step is to vary the size of the clusters and compute the corresponding Allan variances. The fifth step is to plot the Allan variances versus the cluster size. Finally, judging from the shape of the plot, one can identify the types of noises contained in the data.

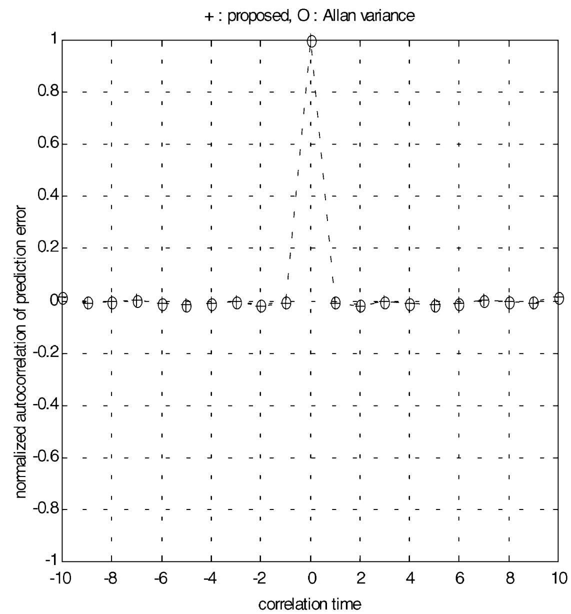

Fig. 1. Normalized autocorrelation function of prediction error.

The identification results show that both the proposed and the Allan variance methods yield the first-order MA as the best fit, that is, white noise plus quantization noise in Table I fits the test data best. The identified parameters from both methods are given in Table II where e_0 and e_1 are the parameters defined in (8). Thus, the two methods produce similar results.

To verify the results, normalized autocorrelations of prediction errors calculated between test output and identified model output are plotted in Fig. 1. From the figure, we can confirm that both methods produce models that make their prediction errors satisfy the whiteness property. Consequently, we can say that both the proposed and the Allan variance methods work well for modeling RLG random errors. But it should be noted that the proposed method does not require the data clustering procedure engaged in the Allan variance method. Employing the proposed method to model RLG random errors reduces computational efforts significantly without sacrificing modeling accuracy.

IV. CONCLUSION

In this paper, we propose an efficient way of modeling random errors inherent to inertial sensors.

[2]Note that the order of an ARMA model containing a random walk can be reduced by introducing a new variable. For example, consider a mixture of a white noise and a random walk:

$$z_t = z_{t-1} + e_0 w_t + e_1 w_{t-1}.$$

Define $y_t = z_t - z_{t-1}$, and then the above equation can be rewritten as

$$y_t = e_0 w_t + e_1 w_{t-1}.$$

Now the mixture of a white noise and a random walk is expressed by ARMA(0,1) instead of ARMA(1,1). Another benefit of this operation is that a non-stationary process can be changed to a stationary process [5].

It was shown that a mixture of ARMA models driven by white Gaussian processes can be represented by an equivalent ARMA model. In deriving the equivalent ARMA model, the AR part was obtained by using a back shift operator. The MA part was derived by introducing a new white Gaussian process driving the model. A procedure to find the MA part is presented in a form of theorem. Thus the equivalency between a mixture of ARMA models driven by white Gaussian processes and a newly derived single ARMA model is established mathematically.

The proposed method is then applied to identifying an ARMA model for RLG random errors. The identified model is checked by test data collected from a Honeywell RLG. The data are assumed to consist of white noise, quantization noise, random walks, and 1st-order Markov processes. The identification results give an equivalent ARMA model of order $(0,1)$ which implies that the white noise and quantization noise is mixed in real data. The model is compared with another ARMA model obtained by employing the Allan variance method. The comparison confirms that the proposed method fittingly and efficiently generates an equivalent ARMA model for RLG random errors. The method may be used for other applications of modeling sensor errors that contain a mixture of random noises.

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REFERENCES

[1] Siouris, G. M. (1993)
Aerospace Avionics Systems—A Modern Synthesis.
New York: Academic Press, 1993.

[2] Lee, J. G., Park, C. G., and Park, H. W. (1996)
Multiposition alignment of strapdown inertial navigation system.
IEEE Transactions on Aerospace and Electronic Systems, **32**, 4 (July 1996).

[3] Lee, J. G., Park, C. G., and Park, H. W. (1996)
Strapdown INS error model for multiposition alignment.
IEEE Transactions on Aerospace and Electronic Systems, **32**, 4 (July 1996).

[4] Ljung, L. (1987)
System Identification Theory for the User.
New York: Simon & Schuster, 1987.

[5] Oravetz, A. S., and Sandberg, H. J. (1970)
Stationary and nonstationary characteristics of gyro drift rate.
AIAA Journal (Oct. 1970).

[6] Pandit, S. M., and Zhang, W. (1986)
Modeling gyro drift rate by data dependent systems.
IEEE Transactions on Aerospace and Electronic Systems, **22**, 4 (July 1986).

[7] Jiang, H., Yang, W. Q., and Yang, Y. T. (1996)
State space modeling of random drift rate in high-precision gyro.
IEEE Transactions on Aerospace and Electronic Systems, **32**, 3 (July 1996).

[8] Ng, L. C., and Pines, D. J. (1997)
Characterization of ring laser gyro performance using the Allan variance method.
Journal of Guidance, Control, and Dynamics, **20**, 1 (Jan. 1997).

[9] Box, G. E. P., Jenkins, G. M., and Reinsel, G. C. (1994)
Time Series Analysis Forecasting and Control (3rd ed.).
Englewood Cliffs, NJ: Prentice-Hall, 1994.

[10] Rohatgi, V. K. (1976)
An Introduction to Probability Theory and Mathematical Statistics.
New York: Wiley, 1976.

[11] Papoulis, A. (1984)
Probability, Random Variables, and Stochastic Process (2nd ed.).
New York: McGraw-Hill, 1984.

[12] Davenport, W. B., Jr. (1970)
Probability and Random Process—An Introduction for Applied Scientists and Engineers.
Englewood Cliffs, NJ: Prentice-Hall, 1970.

[13] Tehrani, M. M. (1983)
Ring laser gyro data analysis with cluster sampling technique.
Proceedings of the SPIE, **412** (1983).

[14] Landau, I. D. (1990)
System Identification and Control Design.
Englewood Cliffs, NJ: Prentice-Hall, 1990.

Loran-C Cycle Identification in Hard-Limiting Receivers

A new technique is presented for use in hard-limiting receivers. It is based on the widely used analogue "half-cycle peak ratio" (HCPR) technique, from which it differs by being entirely digital; no additional analogue signal processing or other hardware is required. Drawing on recent advances, the new method makes decisions based on a large number of sample points in each Loran-C pulse, thereby increasing the resilience of the identification process in the presence of noise. Performance of the algorithm in the face of continuous-wave interference (CWI) is equal to or better than that of other cycle-identification techniques.

I. INTRODUCTION AND DEFINITIONS

Loran-C is a very widely used hyperbolic radio navigation system which can provide an accurate

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