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Online estimation method of Allan variance coefficients for MEMS IMU

Z.Y. Miao,[a] F. Shen,[a,1] D.J. Xu,[a] C.M. Tian[b] and K.P. He[a]

[a] *Department of Automation, Harbin Engineering University, Harbin, Heilongjiang Province, China*
[b] *Department of Information and Communication Engineering, Harbin Engineering, Harbin, Heilongjiang Province, China*

E-mail: shenfeng@hrbeu.edu.cn

ABSTRACT: As a noise analysis of MEMS IMU, the traditional Allan variance methods have large computational burden because of requiring to store a large amount of data. Moreover, the procedure of drawing slope lines for estimation is also painful. In order to overcome these drawbacks, a online method is proposed to estimate the Allan variance parameters, which directly model sensors random errors including quantization noise, angular random walk, bias instability, rate random walk and rate ramp into a nonlinear state space model and then implemented by sage-husa adaptive Kalman filter algorithm. The comparison of results of real ADIS16405 IMU static gyro noise analyzed by Allan variance method and the proposed approach shows that the results from the proposed method are well within the error limits of Allan variance method. Moreover, the technique proposed here estimates the Allan variance coefficients in real time, effectively avoids storage of history data and manual analysis for an Allan variance graph

KEYWORDS: Data Handling; On-board data handling; Instrumental noise

[1] Corresponding author.

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Contents

1 Introduction

Nowadays, with the rapid development of micro-electro- mechanical system (MEMS), MEMS inertial sensors have been widely used in low-cost navigation and positioning system [1]–[2]. Although the MEMS inertial sensors have advantages such as fabricate cheap, lightweight and low-power, they can not compete with the established sensors in high-accuracy application areas because of the high random noise and time varying biases. In order to improve the performance of MEMS inertial sensors, the random errors inherently in these sensors output have to be modeled and identified so that they can properly be compensated after using in a navigation and positioning system [3].

Generally speaking, As a type of time-domain analysis technique, the Allan variance is simple but useful to determine the characteristics of the underlying random processes which gives rise to the data noise. Put simply, the Allan variance is a method of representing the root mean square (RMS) random-drift errors as a function of averaging times [4]. It has been widely used and accepted as a preferred method for identifying stochastic processes, such as quantization noise, white noise, correlated noise, sinusoidal noise, random walk and flicker noise in inertial sensors [5]–[7]. However, as a common and standard method to determine random noise sources of MEMS IMU, there are several disadvantages in it. One disadvantage is that this technology is offline in nature. Another disadvantage is that it requires a large amount of static data to be stored. Besides, in order to effectively estimate the noise coefficients, it requires manual selection of data section which easy introduces human error.

Although, some modified Allan variance methods such as sliding average Allan variance [8], fully and not fully overlapping Allan variance [9] have been developed to improve the analysis performance in recently years, those methods are still offline analysis and need to store a large amount of history data. Additionally, some online estimation methods presented in [10]–[11], which estimate online Allan variance coefficients and decrease the requirement of data store, but the state-space model is difficult to be established.

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Focusing on the disadvantages, a new online estimation method is proposed to estimate Allan variance coefficients in this paper. In the proposed method, the recursive expressions of Allan variance is modeled into an accurate nonlinear state space model and then implemented by adaptive extended Kalman filter algorithm in this filed. Because the state equation is directly modeled by random errors and zero-mean Gauss white noises, the measurement equation is a new recursive expressions of Allan variance and adaptive extended Kalman filter algorithm is implemented to estimate the Allan variance coefficients, the method proposed here avoids the limitation and drawback of the existing Allan variance methods.

This paper is organized as follows. Firstly, a description of stochastic error sources in inertial sensors is given in section II. Then, the proposed method will be detailed description in section III. Thirdly, simulations based on MEMS ADIS16405 IMU gyro data are used to verify the validity of the online estimation method. Eventually, the conclusion and future work are given.

2 Stochastic error sources in inertial sensors: a review

In this paper, the inertial sensors stochastic model are related to five basic noise terms as shown in figure 1. The five basic noise terms are quantization noise, angle random walk, bias instability, rate random walk, and rate ramp. The definition of above noise sources are defined in [12], and their detailed derivations can be found in [13].

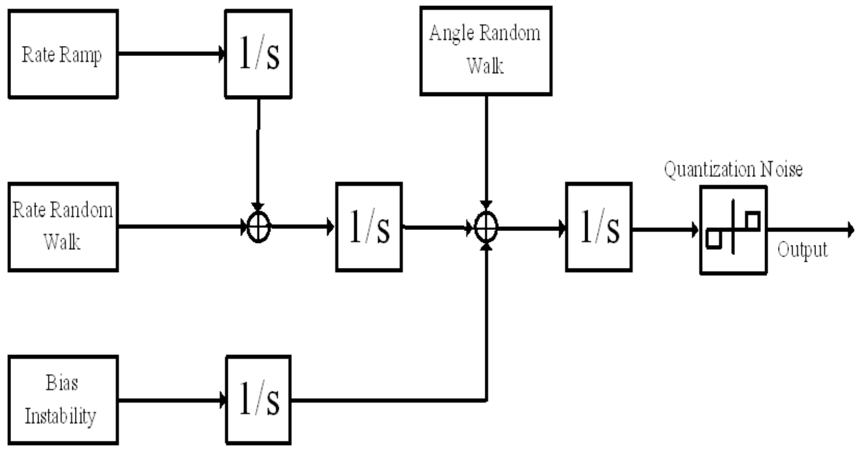

Figure 1. Inertial Sensor Stochastic Model.

Quantization Noise: the quantization noise is one of the errors introduced into an analog signal by encoding it in digital form. It represents the minimum resolution level of the sensor. The Allan variance of quantization noise is given as follow:

$$\delta^2(\tau) = \frac{3Q^2}{\tau^2} \tag{2.1}$$

where Q is the quantization noise coefficient and τ is the sample interval.

Angle random walk: angle random walk is a high frequency noise and characterized by a white-noise rational spectrum on the gyro rate output. The Allan variance for angle random walk are given by:

$$\delta^2(\tau) = \frac{N^2}{\tau} \tag{2.2}$$

where N is the angle random-walk coefficient.

Bias Instability: the origin of this noise is the electronics or other components that are susceptible to random flickering. It is also known as flicker noise and approximated by the the first order Gauss-Markov process. The Allan variance of bias instability is:

$$\delta^2(\tau) = \left(\frac{B}{0.6648}\right)^2 \tag{2.3}$$

where B is the flicker noise parameter.

Rate Random Walk: rate random walk is a random process of uncertain origin, possibly a limiting case of an exponentially correlated noise with a very long correlation time. It is associated with the rate PSD. The the Allan variance of it can be formulated as:

$$\delta^2_{rrw}(\tau) = \frac{K^2\tau}{3} \tag{2.4}$$

where K is the rate random-walk coefficient.

Rate Ramp: the error terms considered so far are of random character. It could also be due to a very small acceleration of the platform in the same direction and persisting over a long period of time.Allan variance of ramp noise is:

$$\delta^2_{rr} = \frac{R^2\tau^2}{2} \tag{2.5}$$

where R is the ramp noise parameter.

From the standard Allan variance plot, the stochastic errors above can be straightforwardly identified. The Allan variance coefficients and corresponding curve slope are shown in table 1.

Table 1. Summary of characteristic noise coefficients and curve slope.

Noise Types	Noise Coefficient	Curve Slope
Quantization Noise	Q	-1
Angle Random Walk	N	$-1/2$
Bias Instability	B	0
Rate Random Walk	K	$1/2$
Drift Rate Ramp	R	1

3 Principle of the online estimation method

3.1 Nonlinear state space model

In order to estimate the Allan variance coefficients directly, the $\delta^2_{\text{total}}(\tau)$ should be dynamic expression at time k. However, the traditional $\delta^2_{\text{total}}(\tau)$ is not used directly because it is a ensemble average of the stored data. When the computation of $\delta^2_{\text{total}}(\tau)$ can be carried out as soon as a new

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sample arrives, the recursive algorithm is the best choice. Now the detailed derivation of recursive formulation of Allan variance is given as follow:

Assume that τ_0 is sample time, N is total number of data points, according to [10], the two steps of Allan variance computation are:

$$\overline{w}_k(M) = \frac{1}{M}\sum_{i=1}^{M} w_{ki}; \qquad k = 1, 2 \cdots K \tag{3.1}$$

$$\delta_A^2(\tau_m) = \frac{1}{2}\left\langle \left(\overline{w}_{k+1}(M) - \overline{w}_k(M)\right)^2 \right\rangle \cong \frac{1}{2(K-1)}\sum_{k=1}^{K-1}\left(\overline{w}_{k+1}(M) - \overline{w}_k(M)\right)^2 \tag{3.2}$$

where M is data cluster length, $K = N/M$ is the number of clusters, $\tau_m = M\tau_0$ is correlation time.

The eq. (3.1) is the average of each cluster and each of them can be rewritten by recursive form as follow:

$$\begin{aligned}
\overline{w}_k(m) &= \frac{1}{m}\sum_{i=1}^{m} w_{ki} \\
&= \frac{w_{km}}{m} + \left(\frac{m-1}{m}\right)\left(\frac{1}{m-1}\right)\sum_{i=1}^{m-1} w_{ki} \\
&= \left(\frac{m-1}{m}\right)\overline{w}_k(m-1) + \frac{1}{m} w_{km}
\end{aligned} \tag{3.3}$$

where $1 \le m \le M$ and $\overline{w}_k(0) = 0$.

Eq. (3.2) is the Allan variance the traditional expression of Allan variance. Using the similar theory it can be written as follow:

$$\begin{aligned}
\delta_k^2(m) &= \frac{1}{2(k-1)}\sum_{i=1}^{k-1}\left(\overline{w}_{i+1}(m) - \overline{w}_i(m)\right)^2 \\
&= \frac{1}{2(k-1)}\left(\overline{w}_k(m) - \overline{w}_{k-1}(m)\right)^2 + \left(\frac{k-2}{k-1}\right)\left(\frac{1}{2(k-2)}\right)\sum_{i=1}^{k-2}\left(\overline{w}_{i+1}(m) - \overline{w}_i(m)\right)^2 \\
&= \left(\frac{k-2}{k-1}\right)\delta_{k-1}^2(m) + \frac{1}{2(k-1)}\left(\overline{w}_k(m) - \overline{w}_{k-1}(m)\right)^2
\end{aligned} \tag{3.4}$$

where $k \ge 2$ and $\delta_1^2(m) = 0$.

When $M = 1$, the Allan variance of each data point can be calculated by above equation. Therefore, the eq. (3.4) is the recursion equation of Allan variance.

Assume that Allan variance coefficients are composed of the true values and zero-mean Gauss white noises as follows:

$$x_{k+1} = [Q \quad N \quad B \quad K \quad R]_{k+1}^T = x_k + \varsigma_k \tag{3.5}$$

where x_k is state vector at time k, ς_k is a zero-mean Gauss white noise vector. Undoubtedly, eq. (3.5) can be used as state equation.

Allan variance can be expressed in another way as follows:

$$\delta_k^2(\tau_m) = \frac{3Q^2}{\tau_m^2} + \frac{N^2}{\tau_m} + \frac{2\ln 2B^2}{\pi} + \frac{K^2\tau_m}{3} + \frac{R^2\tau_m^2}{2} \tag{3.6}$$

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According to eq. (3.4) and eq. (3.6), the Allan variance with $m = M = 1$ can be shown as follow:

$$\left(\frac{k-2}{k-1}\right)\delta_{k-1}^2 + \frac{1}{2(k-1)}\left(\overline{w}_k - \overline{w}_{k-1}\right)^2 = \frac{3Q^2}{\tau_0^2} + \frac{N^2}{\tau_0} + \frac{2\ln 2B^2}{\pi} + \frac{K^2\tau_0}{3} + \frac{R^2\tau_0^2}{2} \tag{3.7}$$

Eq. (3.7) is a dynamic equation of Allan variance and can be used as measurement equation. Then, the nonlinear state space model of Allan variance has been completed.

3.2 Sage-husa adaptive Kalman filter

The sage-husa adaptive Kalman filter (SHAKF) method is considered as the most promising method for general online applications. Moreover, it can also estimate noise statistical characteristic at the same time of estimating states. In the system with unknown noises, the unknown noise statistics are modified simultaneously in the process of estimating system states result in a accurate states estimation [14]. It has been successfully used in navigation field [15]–[16]. This technology is a recursive algorithm instead of storage large amounts of history data. Therefore the computational burden is greatly reduced.

For the nonlinear state space models (3.5) and (3.6) with unknown statistic noise, the explicit procedure of SHAKF is given as follows:

$$x_{k|k-1} = x_{k-1} + q_{k-1} \tag{3.8}$$

$$P_{k|k-1} = P_{k-1} + Q_{k-1} \tag{3.9}$$

$$K_k = P_{k|k-1}H_k^T\left(H_k P_{k|k-1} H_k^T + R_{k-1}\right)^{-1} \tag{3.10}$$

$$\varepsilon_k = \left(\frac{k-2}{k-1}\right)\delta_{k-1}^2 + \frac{1}{2(k-1)}\left(\overline{w}_k - \overline{w}_{k-1}\right)^2 - H_k x_{k|k-1} - r_k \tag{3.11}$$

$$x_k = x_{k|k-1} + K_k\varepsilon_k \tag{3.12}$$

$$P_k = \left(I - K_k H_k\right)P_{k|k-1} \tag{3.13}$$

$$D_k = H_k^T\left(H_k P_{k|k-1} H_k^T + R_k\right)^{-1} \tag{3.14}$$

$$\gamma_k = (1-b)/\left(1-b^{k+1}\right) \tag{3.15}$$

$$q_k = q_{k-1} + \gamma_k Q_{k-1}\varepsilon_k \tag{3.16}$$

$$Q_k = Q_{k-1} + \gamma_k Q_{k-1} D_k\left(\varepsilon_k\varepsilon_k^T - H_k P_{k|k-1} H_k^T - R_{k-1}\right)D_k^T Q_{k-1} \tag{3.17}$$

$$r_k = (1-\gamma)\,r_{k-1} + \gamma\left(\left(\frac{k-2}{k-1}\right)\delta_{k-1}^2 + \frac{1}{2(k-1)}\left(\overline{w}_k - \overline{w}_{k-1}\right)^2 - H_k x_k\right) \tag{3.18}$$

$$R_k = (1-\gamma)\,R_{k-1} + \gamma\left[\left(I - H_k K_k\right)\varepsilon_k\varepsilon_k^T\left(I - H_k K_k\right)^T + H_k P_k H_k^T\right] \tag{3.19}$$

where q_k, Q_k are the mean and variance of time-varying process noise,respectively. r_k, R_k are variable measuring noise mean and variance, respectively. H_k is the Jacobian matrix of measurement equation. I is unit matrix. b is a forgetting factor with $0 \le b \le 1$, and b should be closer to 1 for slow time-varying statistic noise.

The SHAKF is initiated with an initial guess for the state vector which is composed by the main stochastic noises according to the plot of the Allan standard deviation, then subsequent state estimate and covariance estimate for every new data are generated in the process of recursive operation. The block diagram of the new method can be summarized as shown in figure 2.

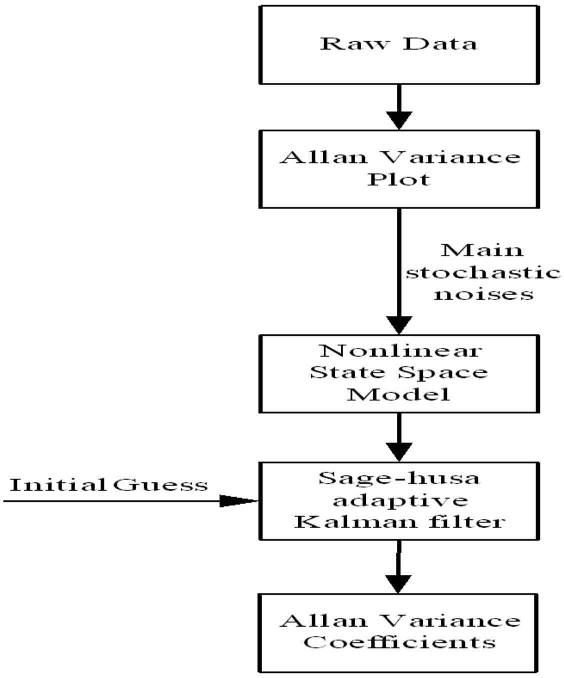

Figure 2. Flow Chart of the New Method.

4 Simulations and result

In order to compare the performance of Allan variance method with the approach proposed in the above sections, they are applied to an actual MEMS sensors ADIS16405 as shown in figure 3. The 5-h static data which was collected from the ADIS16405 on room temperature with 100Hz is shown in figure 4. By applying the Allan variance method to the whole data set, a log-log plot of the Allan standard deviation versus the cluster time is shown in figure 5.

Figure 3. ADIS 16405.

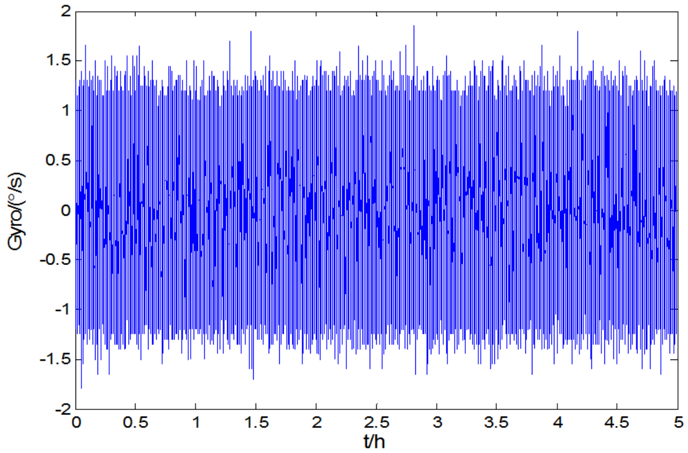

Figure 4. Raw Data of Gyro Y.

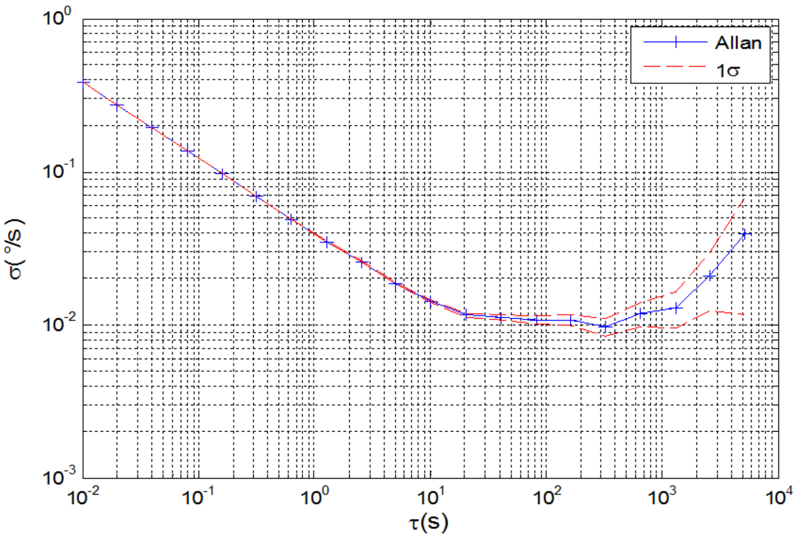

Figure 5. Allan Variance Plot of Gyro Y.

According to Allan variance plot analysis, there are three main stochastic noises to be considered for the Gyro Y in this test: namely angle random walk, bias instability and angular rate random walk with mean square values N, B and K, respectively. Therefore, the nonlinear state space model of Gyro Y can be deduced as follow:

$$\bar{x}_{k+1} = \begin{bmatrix} N_{k|k} \\ B_{k|k} \\ K_{k|k} \end{bmatrix} + \zeta_{k|k}$$

$$\left(\frac{k-2}{k-1}\right)\delta_{k-1}^2 + \frac{1}{2(k-1)}\left(\overline{w}_k - \overline{w}_{k-1}\right)^2 = \frac{N^2}{\tau_0} + \frac{2\ln 2B^2}{\pi} + \frac{K^2\tau_0}{3}$$

At this point, the nonlinear state space model of y gyro used as an example to elaborate the procedure in this paper is obtained. Similarly, the nonlinear state space model for other gyros and accelerometers consist any errors in the 5 kinds of basic error can be obtained by following the above procedure.

Initial the N, B and K as $0.48\left(^\circ/h^{1/2}\right)$, $20.8\left(^\circ/h\right)$ and $55.44\left(^\circ/h^{3/2}\right)$, respectively. In order to illustrate initial change trend clearly in the estimation process, only 20000 samples for N, B and K are shown in figure 6 to figure 8. The comparison between Allan parameters and this online estimation method is shown in table 2.

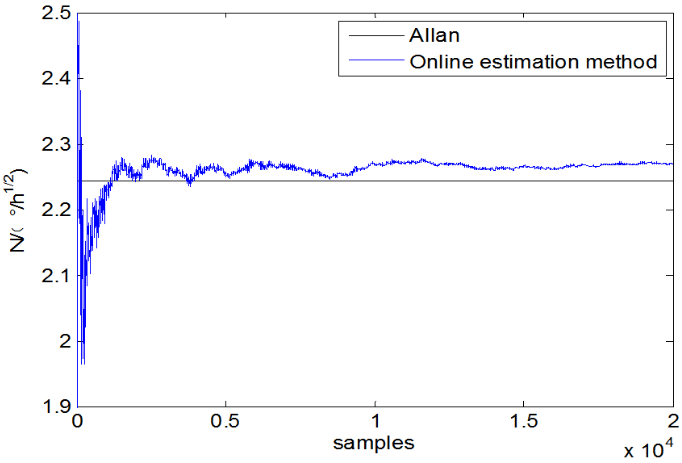

Figure 6. Estimates of N for Gyro Y.

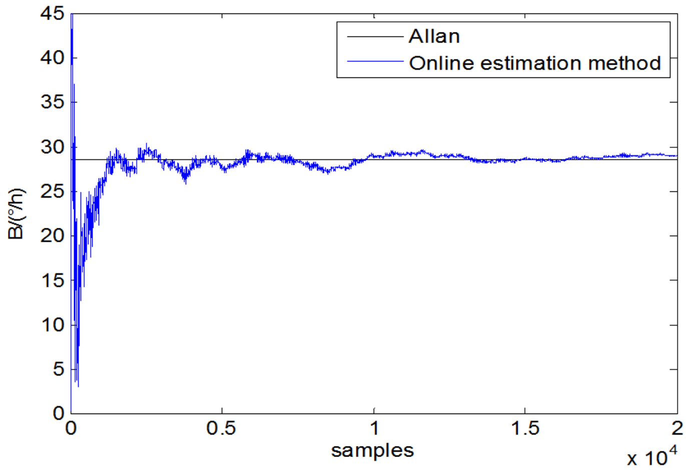

Figure 7. Estimates of B for Gyro Y.

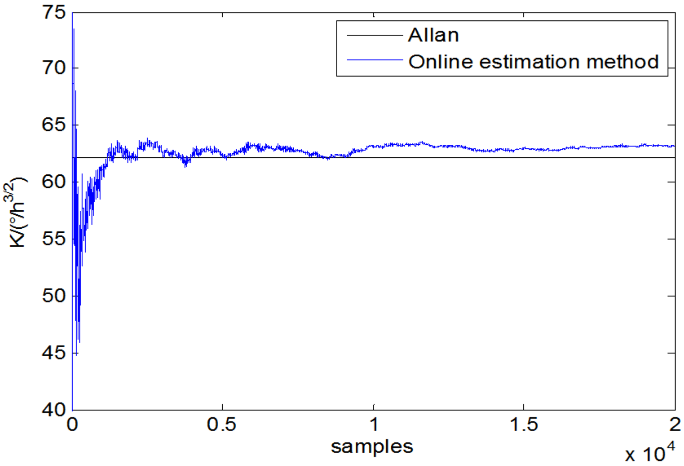

Figure 8. Estimates of K for Gyro Y.

In order to prove the flexibility of new approach in estimation Allan variance coefficients, another experiment based on Gyro X of ADIS16405 with 50Hz is shown in following. The 5-h static data which was collected on room temperature is shown in figure 9 and the Allan variance plot is shown in 10.

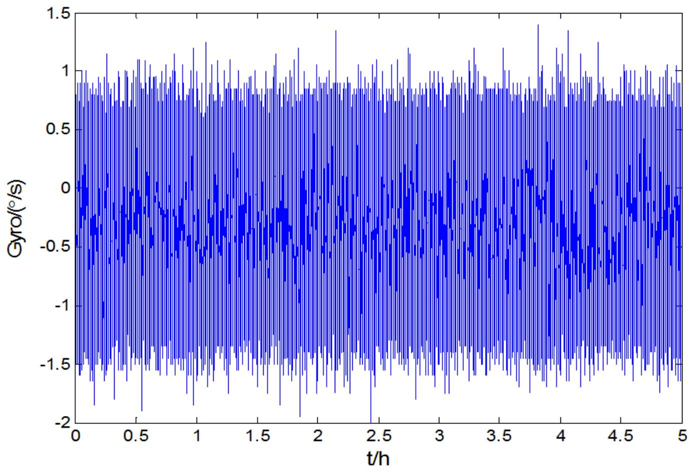

Figure 9. Raw Data of Gyro X.

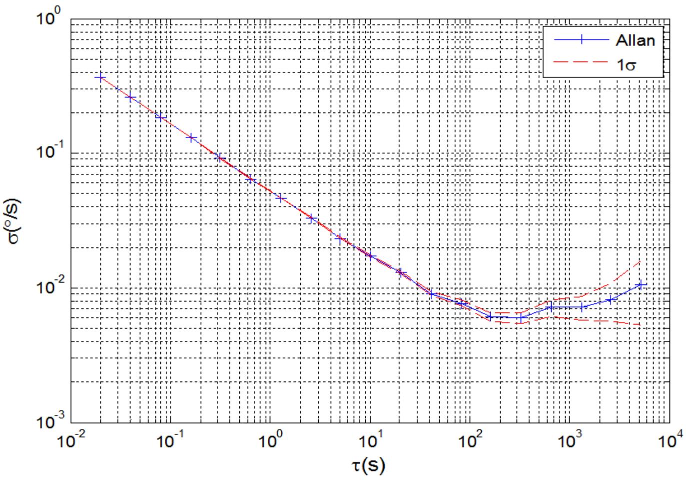

Figure 10. Allan Variance Plot of Gyro X.

According to figure 10, angle random walk and bias instability are considered as the main stochastic noises for the Gyro X in this test Therefore, the nonlinear state space model of Gyro X can be deduced as follow:

$$\bar{x}_{k+1} = \begin{bmatrix} N_{k|k} \\ B_{k|k} \end{bmatrix} + \zeta_{k|k}$$

$$\left(\frac{k-2}{k-1}\right)\delta_{k-1}^{2} + \frac{1}{2(k-1)}\left(\overline{w}_{k} - \overline{w}_{k-1}\right)^{2} = \frac{N^{2}}{\tau_{0}} + \frac{2\ln 2B^{2}}{\pi}$$

The N,B are initialled as $1.25\left({}^{\circ}/h^{1/2}\right)$ and $18.6\left({}^{\circ}/h\right)$, respectively. Only 20000 samples of N and B are shown in figure 11 and figure 12 to illustrate initial change trend in the estimation process. The comparison between Allan variance parameters and this online estimation method is also shown in table 2.

Table 2. Comparison of Estimated Parameters between Allan Variance and Online Method.

Item	100 Hz		50 Hz	
	Allan Variance	Online estimation method	Allan Variance	Online estimation method
$N\left({}^{\circ}/h^{1/2}\right)$	2.2437	2.2697	3.1178	3.1308
$B\left({}^{\circ}/h\right)$	28.3976	29.1681	25.5637	28.1728
$K\left({}^{\circ}/h^{3/2}\right)$	62.1752	63.2137		

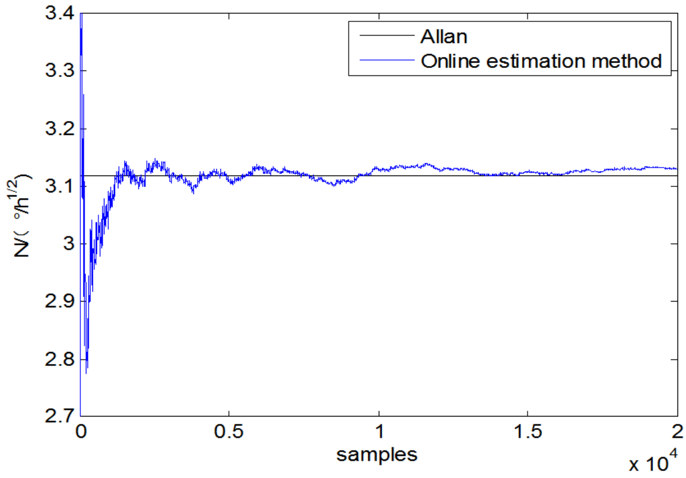

Figure 11. Estimates of N for Gyro X.

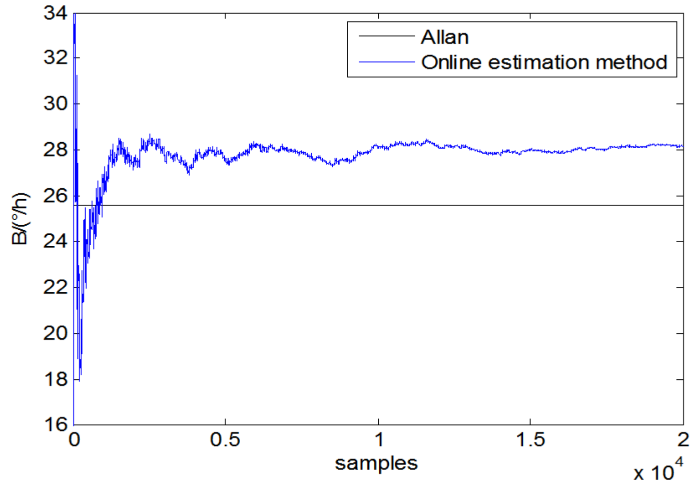

Figure 12. Estimates of B for Gyro X.

Figure 6 to figure 8 and figure 11 to figure 12 show that the parameters estimated by the proposed online estimation method are basically close to results of Allan variance method. With reference to the value estimated by Allan variance method in table 2, the value of estimating N, B and K estimated after one pass over 20000 data can also prove the validity of the proposed method. From table 2, the relative errors of the estimation of N, B and K by the proposed online estimation method relative to the Allan variance method calculated by $(|\text{Allan} - \text{Online}|/\text{Allan})\,\%$ are 1.1588%, 2.7133% and 1.6352%, respectively in the first experiment. The relative errors of the estimation for N and B in the second experiment are 0.4169%10.2063%, respectively.

According to above analysis, the online estimation method has a simply modeling process and can be used to estimate ADIS 16405 random errors in real time. Moreover, the sage-husa adaptive Kalman filter is implemented in the new method, resulting in a accurate estimation. Therefore, the new method is a valid online estimation method to estimate the Allan variance coefficients.

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5 Conclusion and future work

In this paper, a online estimation method based on nonlinear state space model and Sage-husa adaptive kalman filter is proposed. In the test of actual ADIS 16405 gyro data, the estimation results of the proposed method are within the error limits of Allan variance method. However, unlike Allan variance methods, the new method avoids to store a large amount of data and to analysis an Allan variance graph manually.

The success of the proposed technique indicate an encouraging direction in accurately estimating the Allan variance parameters for inertial sensors with recursive online analysis. Although, the proposed method performs well in static data, what about the onboard performance? Theoretically, the new method can be used for autonomous estimation of Allan variance coefficients for onboard inertial sensors. But in practice, it must be implemented carefully because the online estimation method can be affected by initial values of parameters and the stability of the SHAKF are still a matters that needed to be considered. Onboard performance analysis and improvement of the new online method are our main work in next step.

Acknowledgments

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