W5453 – Advanced Time Series Analysis and Forecasting
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Applied Project II: Value-at-Risk Evaluation of Four Financial Stocks Portfolio Using DCC-GARCH Model.

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Introduction

It is crucial in finance to accurately model volatility, which is also known as risk. However, as risk cannot be directly observed, various modeling techniques have been developed to estimate and predict risk. One such popular approach is the generalized autoregressive conditional heteroskedasticity (GARCH) model, which has led to the development of autoregressive conditional volatility models. GARCH models have been successful in capturing several key characteristics of financial returns, including time-varying volatility, persistence and clustering of volatility, and asymmetrical reactions to equal-magnitude positive and negative shocks. These stylized facts of financial returns are recognized to the effectiveness of GARCH models (Mcaleer & da Veiga, 2008)

Various GARCH models have been introduced in recent years to capture the asymmetric patterns in time series. One of the extended models is the exponential GARCH, also known as the EGARCH model, which not only considers the magnitude and sign of shocks but also explains the leverage effect. Another model applied in this paper is the asymmetry power ARCH (APARCH) model, which effectively explains the impact of both positive and negative shocks separately.

Although modelling the volatility of returns has been a primary focus, comprehending the comovements of financial returns holds significant practical value. It’s been observed by may researchers that financial volatilities move together more or less closely over time across assets and markets. Thus, to estimate the variance of a portfolio, it is necessary to use multivariate GARCH models that capture correlations among various return series. Thus, Dynamic Conditional Correlation GARCH model (DCC-GARCH) which is a multivariate volatility model is implemented in this study. It models the time-varying conditional covariance separately as well as conditional correlations between different financial assets. The DCC-GARCH model is popular in finance and economics for analyzing and forecasting the volatility and correlation of financial asset returns, particularly in portfolios with multiple assets.

This research primary objective is to construct and evaluate the value-at-risk of a portfolio of four companies (SAP, SIE, DTE, and ALV) by utilizing logarithmic returns over a 12-year period. Before moving toward the analysis of the study, the detailed descriptions of DCC model and VaR measure with its mathematical application were presented.

In the next part, to achieve the portfolio, firstly various univariate GARCH model specifications were applied to each stock, and a DCC- GARCH (1,1) model was fitted. With the out-of-sample one-step rolling forecasting, time-varying conditional correlations were obtained between all four financial stocks. Based on the forecasted results, different weights were assigned to construct the portfolio. Then, the fitted portfolio's 1% value-at-risk was obtained, and violations were observed. Lastly, the conclusion of the estimations and forecasts as well as references utilized are presented.

Detailed Description of DCC-GARCH Model

The Dynamic conditional correlations (DCC) model is one of the most used multivariate GARCH model. It computes conditional covariance matrix, which consists of the univariate GARCH models. Before moving further it's important to define univariate GARCH models used in the study i.e., GARCH, APARCH and EGARCH models.

The generalized autoregressive conditionally heteroscedastic (GARCH) model considers conditional variance to be a linear combination between squared of residual and a part of lag of conditional variance. It was proposed by Bollerslev in 1986 to overcome the weaknesses of the ARCH model (Bollerslev, 1986). The conditional variance is defined as (Teräsvirta, 2009) :

$$h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}.$$

The conditional variance has the property that the unconditional autocorrelation function of ε_t^2; if it exists, can decay slowly, although still exponentially. The most popular GARCH model in application is GARCH(1,1), that is $p = q = 1$ in the conditional variance equation. A sufficient condition for the conditional variance to be positive with the probability one is $\alpha_0 > 0, \alpha_j \geq 0, j = 1, \dots, q$; $\beta_j \geq 0, j = 1, \dots, p$ (Teräsvirta, 2009).

The EGARCH is the exponential GARCH, which was developed by Nelson in 1991. It allows asymmetric effects between positive and negative shocks on the conditional variance of future observations. One of the additional advantage of this model is that there are no restrictions on the parameters. The conditional variance of the EGARCH model is an asymmetric function of lagged disturbances. The model EGARCH (p, q) is defined as follows (Olowe, 2009) :

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - \sqrt{\frac{2}{\pi}} \right| + \sum_{j=1}^{q} \beta_j \log(\sigma_{t-j}^2) + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}}$$

Where $\omega, \alpha_i, \beta_i$ and Y_k are the constant parameters. The asymmetric effect of past shocks is captured by the γ coefficient, which is usually negative, that is, cetteris paribus positive shocks generate less volatility than negative shocks .The leverage effect can be tested if $\gamma < 0$. If $\gamma \neq 0$, the information impact is asymmetric (Olowe, 2009).

The APARCH model is the asymmetry power ARCH model proposed by Ding, Granger, and Engle in 1993. It also allows asymmetry effects of shocks on the conditional volatility. This model can well express the Fat tails, Excess kurtosis, and Leverage Effects. the power parameter of the standard deviation can be estimated rather than imposed, and the optional γ parameters are added to capture asymmetry of up to order r . The APARCH (p, q) model is stated as below (Olowe, 2009):

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta$$

Where $\delta > 0, |\gamma_i| \leq 1$ for $i = 1, \dots, r$. If $\gamma \neq 0$, the information impact is asymmetric.

The DCC model introduced by Engle and Sheppard in 2001 is an extension of Bollerslev' s (1990) Constant Conditional Correlation (CCC) estimator. The DCC directly parameterizes the conditional correlations and is estimated in two stages – in first stage univariate GARCH model is assessed, while in second stage, residuals are transformed by their standard deviation and used to estimate parameters for dynamic correlation. The correlation process involves a fixed number of parameters, which is independent of the number of series being correlated. As a result, the DCC method has clear computational advantages over other multivariate GARCH models and can be used to estimate large correlation matrices with ease. Moreover, the DCC approach is more realistic as the dependence between returns is likely to vary over time (Lee et al., 2006).

The model is states as follows (Engle & Sheppard, 2001):

$$\mathrm{H_t} = \mathrm{D_t R_t D_t}$$

Where $\mathrm{H_t}$ is the covariance matrix and $\mathrm{R_t}$ is a $\mathrm{n} \times \mathrm{n}$ matrix of the conditional correlation of the returns. The diagonal matrix $\mathrm{D_t}$ consists of the univariate GARCH models is expressed as :

$$D_t = \begin{bmatrix} \sqrt{h_{1t}} & 0 & \cdots & 0 \\ 0 & \sqrt{h_{2t}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sqrt{h_{nt}} \end{bmatrix}$$

Additionally, $\mathrm{H_t}$ must be a positive definitive, which is inherently achieved as long as $\mathrm{R_t}$ is symmetric. In defining the matrix, two conditions must be satisfied. Firstly, $\mathrm{H_t}$ must be positive definite since it is a covariance matrix. Secondly, the elements within $\mathrm{R_t}$ must be bounded by values less than one. This requirements are met through a decomposition (Rombouts & Verbeek, 2009) :

$$\mathrm{R_t} = (\mathrm{diag}\mathrm{Q_t})^{-\frac{1}{2}} \mathrm{Q_t} (\mathrm{diag}\mathrm{Q_t})^{-\frac{1}{2}}$$

$$\mathrm{Q_t} = (1 - \alpha - \beta)\overline{\mathrm{Q}} + \alpha u_t u'_{t-1} + \beta \mathrm{Q_{t-1}}$$

Where the $u_{it} = \epsilon_{it}/\sqrt{\mathrm{h_{iit}}}$, $\overline{\mathrm{Q}}$ is the $\mathrm{N} \times \mathrm{N}$ unconditional variance matrix of u_t. Furthermore, the parameters α (≥ 0) and β (≥ 0) are scalars and must be larger than zero but the sum has to be less than one i.e., $\alpha + \beta < 1$. One of the condition of univariate GARCH of being stationary is also applied in the DCC model. The estimate of $\overline{\mathrm{Q}}$ is as follows (Hartman & Sedlak, 2013):

$$\overline{\mathrm{Q}} = \frac{1}{\mathrm{T}} \sum_{t=1}^{\mathrm{T}} \epsilon_t \epsilon'_t$$

From the joint return distribution we can calculate the quantiles of the marginal distributions $\mathrm{r_{it}}|\mathrm{I_{t-1}}, \mathrm{i} = 1, \ldots, \mathrm{N}$. These marginal densities are given as follows (Rombouts & Verbeek, 2009):

$$f_{r_{it}|I_{t-1}}(r_i) = \int_{R^{N-1}} f_{r_t|I_{t-1}}(r_i, \bar{r}_{-i})\, \mathrm{d}\bar{r}_{-i},$$

Where $\bar{r}_{-i}$ indicates everything in r except r_i. The main interest, however, lies in the distribution of a linear combination of the vector of returns, $w'_t r_t$ or a portfolio, which depends upon the salient dependencies between the different returns.

The DCC-GARCH model is a popular tool for modeling time-varying conditional correlation between financial assets, but it has several limitations. Firstly, the model assumes that the conditional correlation between variables is constant over time, which may not reflect the true dynamics of financial markets, resulting in misspecification of the model and biased estimation of the correlation structure. Secondly, the model may not capture all the relevant information in the data, leading to potential misspecification. Additionally, the model's performance is sensitive to the choice of the underlying distribution and estimation method used, and it can be computationally intensive and unsuitable for large datasets. Lastly, the model may not perform well in forecasting during periods of high volatility or extreme events, which are precisely the periods where accurate forecasting is most critical (Caporin & McAleer, 2013).

The Value-at-Risk (VaR) is a measure of the market risk of the portfolio and measures loss that it could generate over a given time horizon with a given degree of confidence. In general, the VaR is a function of the confidence level α , the density $g(.)$, the portfolio weights w_t, the functional form of the mean vector μ_t and of the covariance matrix H_t, where the w_t , μ_t and H_t are time dependent. The VaR of a portfolio with weights w_t at level α is as follows (Rombouts & Verbeek, 2009):

$$P(w'_t r_t < \mathrm{VaR}_\alpha) = \alpha$$

In the case where $g(.)$ is the multivariate normal density the definition of the VaR reduces to the following formula :

$$\mathrm{VaR}_\alpha = w'_t \mu_t + (w'_t H_t w_t)^{1/2} z_\alpha$$

where z_α is the αth quantile of the univariate standard normal distribution.

To evaluate the goodness of the VaR estimates, backtesting is applied. The backtesting is a process of determining how well a strategy performs using historical data. It calculates the percentage of times the observed portfolio returns, p_t, falls below the VaR estimates and evaluates that number to the confidence level used. If the observed values of p_t, is outside the confidence interval than violation is occurred.

The next part of the paper will be regarding implementation of DCC model and forecasting VaR of a portfolio using the R programming.

Application of DCC-GARCH Model to Four German Companies Stocks.

This study analyzes the daily closing prices of the four highest market capitalization companies in the German stock market: SAP (SAP.DE), Siemens (SIE.DE), Deutsche Telekom (DTE.DE), and Allianz (ALV.DE), from January 1, 2010, to December 31, 2022. The data was sourced from Yahoo Finance and included a total of 3,302 observations for each of the four stocks.

For preforming the advanced time series analysis and forecasting, R programming is used and following libraries were installed: “rugarch”, “rmgarch”, “purrr”, “zoo”, “magrittr”, “parallel” , and “R.utils”. Since the data for the four stocks was collected separately, the first step involved merging all the datafiles. The common factor used to merge the time series was the "Date" variable, which resulted in the creation of a dataset for the daily closing prices.

Financial time series models show improved performance when applied to log-returns rather than normal returns. As log- return exhibit more symmetric behavior, are less skewed, and allow for better comparison of asset performance as they account for differences in scale. Thus, closing prices are converted to log-returns to normalize for such differences.

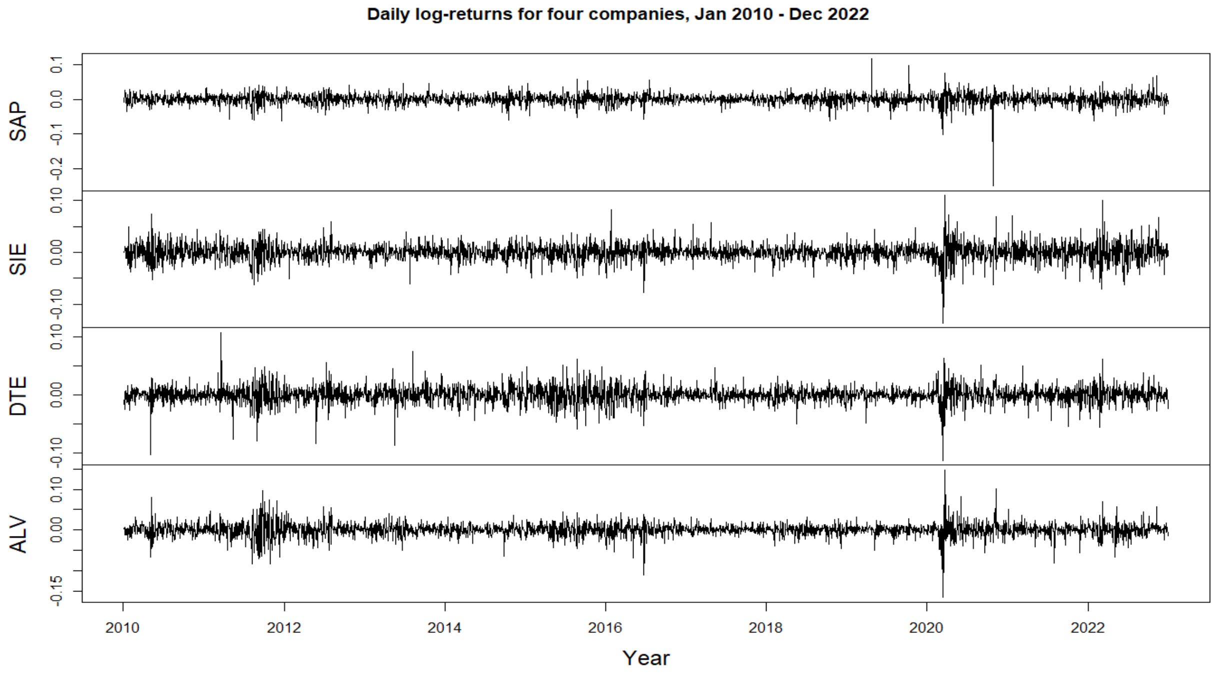

Figure 1: Daily log returns of four companies from January 2010 to December 2022

According to figure 1 prior to 2020, the stocks haven’t exhibited extremely high volatility. However, the COVID-19 pandemic caused a sudden increase in volatility, which is represented by the large spikes in the graph in year 2020. After the pandemic's impact on the market, SAP has demonstrated relatively low volatility compared to the other three companies. The other three companies continued to experience some fluctuations, potentially due to their industries being more sensitive to pandemic-related factors. However, SAP being a software company may have been less affected by these factors, leading to its comparatively more stable performance.

Looking at the correlation matrix, its visible that the correlation coefficients between SAP and the other stocks are all positive, with the strongest correlation being with SIE (0.5818). There is a positive correlation between DTE and ALV (0.5819). Similarly, there is a positive correlation between SIE and the other stocks, with the strongest correlation being with ALV (0.6774). The SIE and ALV has the highest correlation among all other series in the matrix.

Correlation Matrix				
	SAP	**SIE**	**DTE**	**ALV**
SAP	1.0000	0.5818	0.4485	0.5353
SIE	0.5818	1.0000	0.5306	0.6774
DTE	0.4485	0.5306	1.0000	0.5819
ALV	0.5353	0.6774	0.5819	1.0000

Table 1: Correlation Matrix for stocks : SAP, SIE, DTE and ALV

The Dynamic Conditional Correlations (DCC) GARCH Model

We are interested in fitting the DCC-GARCH model to forecast Value-at-Risk for the weighted portfolio of our time series. For that the "rmgrach" package is installed in R. Primarily various univariate GARCH models were fitted using the ugarchspec() function. This function allows the user to specify the characteristics of the GARCH model, such as the variance model, the mean model, and the distribution model.

The emphasis is to fit a combination of GARCH, APARCH and EGARCH model. All four stock, the models have the same mean model (ARMA(0,0)) and conditional normal distribution. However, the variance model is different for each series. For instance, the first model of SAP stocks uses an asymmetric power ARCH model (APARCH(1,1)), while the second model for SIE stocks uses a standard GARCH model (GARCH(1,1)). The third model for DTE stocks uses an exponential GARCH model (EGARCH(1,1)), and the fourth model for ALV stocks uses again the (GARCH(1,1)) model.

After specifying the models, the ugarchfit() function is used to fit the models to the log returns of each stock. The goal of the study is to construct a DCC-GARCH model based on the previously fitted univariate GARCH models. To accomplish this, a list of GARCH specifications for each individual series was generated using the multispec() function from the "rugarch" package and stored as the variable "mspec". The following R code employs the dccspec() function to create the specification for DCC-GARCH model:

dccspec <- dccspec(uspec = mspec, dccOrder = c(1, 1),model = "DCC", distribution = "mvnorm")

The DCC process is fitted with an order of (1,1), and the multivariate normal distribution is used for the distribution. To speed up the computation, a cluster of parallel processes is created using the makePSOCKcluster() function from the "parallel" package. Although our dataset is not very large, parallel processing has been implemented for convenience.

Further to fit the DCC-GARCH(1,1) model using the “dccspec”, the multifit() and dccfit() functions from the "rugarch" package were employed. The following code illustrates the usage of these functions to generate the fit:

multfit <- multifit(mspec, rt, out.sample = 250, cluster = cl)

dccfit <- dccfit(dccspec, data = rt, out.sample = 250,fit = multfit,cluster = cl)

The argument "out.sample" indicates that 250 observations are reserved for out-of-sample forecasting. This shows that for the study, the model will be estimated using all available data except the final 250 observations, which will serve as a benchmark for evaluating the model's forecasting performance. The AIC value of model is -24.208 and BIC value is -24.155, which states that the model might be a good fit to the data. However, it's important to note that the goodness of fit shouldn't be solely determined based on the AIC or BIC values. Additional measures such as MSE, RMSE, Adjusted R-squared or others could be utilized to assess the model's performance.

One-step rolling forecasts is used for the DCC model for estimating time-varying conditional correlations between all four financial stocks. In a rolling forecast, the model is trained on a fixed window of past data and then used to make forecasts on a rolling basis, updating the window of past data by one observation at a time. This approach allows for an evaluation of the model's performance on out-of-sample data and provides a way to monitor the model's ability to adapt to changing market conditions.

Thus, using the dccforecast() function with rolling window of 250 days minus 1, one step rolling forecasted values for model is generated. This means that 249 days of training data and 1 day of testing data.

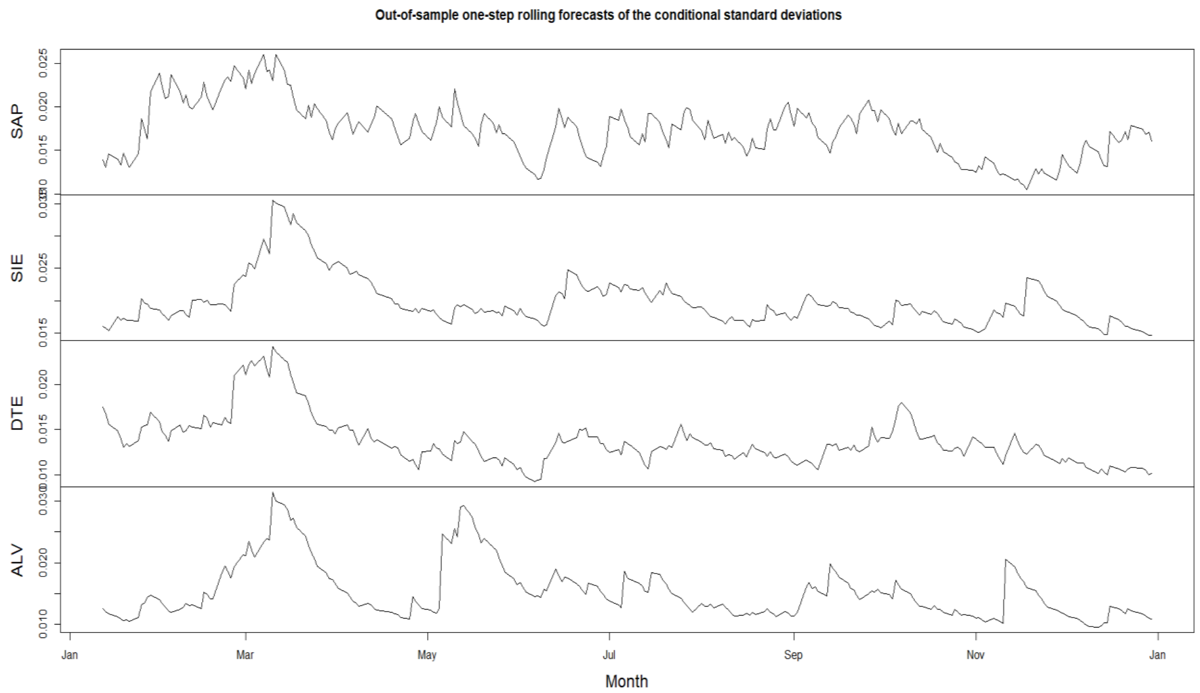

Figure 2: Out-of-sample one-step rolling forecasts of the conditional standard deviations of each time series.

From figure 2, it can be observed that the conditional standard deviations of each series exhibit a fluctuating trend overall, indicating that the volatility of each series is changing over time and is not stable. However, starting from November, all series except for SAP show a decreasing trend in their conditional standard deviations, which suggests that their volatility is either decreasing or becoming more stable. On the other hand, the increasing trend in SAP's conditional standard deviation after November indicates that the volatility of SAP's stock may be increasing.

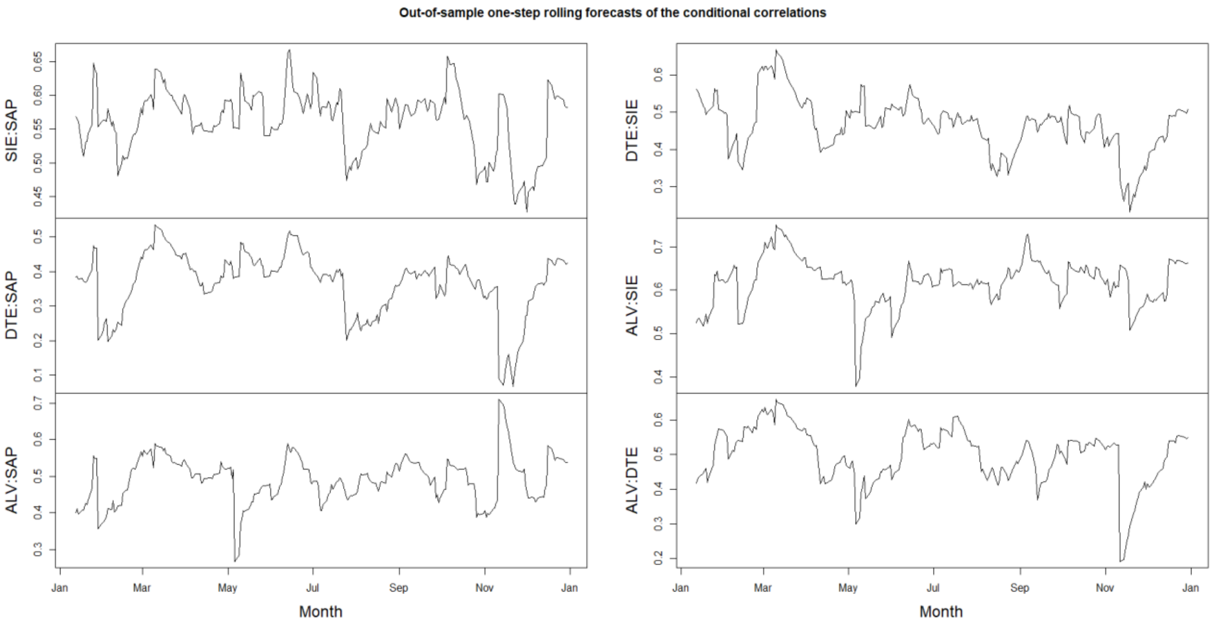

Figure 3: Out-of-sample one-step rolling forecasts of the conditional correlations.

After examining figure 3 which is representing conditional correlation, it is evident that the correlations among all-time series are volatile and subject to fluctuations over time. This inconsistency may indicate that the relationships among them are more complicated, or that external factors have influenced it. Among them, only the correlation between ALV and SAP shows an increase in November, while the others experience a significant decrease. Additionally, the conditional correlation between ALV and SIE presents some consistency after the decline in May compared to the others, and a strong relationship is noticeable.

Forecasted Pairwise Correlation Coefficients Matrix						
Date	**SIE:SAP**	**DTE:SAP**	**ALV:SAP**	**DTE:SIE**	**ALV:SIE**	**ALV:DTE**
2022-01-11	0.5684	0.3834	0.3998	0.5620	0.5244	0.4169
2022-01-12	0.5660	0.3872	0.4107	0.5585	0.5320	0.4264

Table 2 : Forecasted Pairwise Correlation Coefficients Matrix between Stock Price Returns for Four Companies on January 11th and 12th, 2022.

Based on the above table, it is evident that the time series of SAP and SIE have the highest correlation coefficients in comparison to all other pairs. Additionally, the correlation coefficients of SIE with ALV and DTE are also relatively high on both dates, indicating a strong positive linear relationship between the returns of these stocks. Therefore, it can be concluded that SAP and SIE are the best performing series that move together, while SIE also exhibit a strong correlation with DTE and ALV.

Portfolio Forecasting

The DCC-GARCH model was utilized to establish a portfolio and its risk is evaluated using the VaR measure. The allocation of weights for each stock in the portfolio is critical to maintain an appropriate balance. Analysis of the graphs and tables showed that SAP stocks exhibit higher volatility as compared to SIE, DTE, and ALV. Additionally, SIE was observed to have a higher correlation with all other series.

Therefore, the portfolio was constructed with the following weight allocation: SAP-25%, SIE-30%, DTE-25%, and ALV-20%. The highest weight was given to SIE as it moves together with all other series, ensuring an appropriate balance and diversification of the portfolio. Using the apply() function code as below portfolio returns are calculated for last 250 observations:

Portfolio <- apply(tail(rt, 250), MARGIN = 1, FUN = function(x){weighted.mean(x, w = weights)}) %>% zoo(order.by = dates_test)

The 1% quantile of the portfolio returns distribution, which represents the Value-at-Risk (VaR) of the portfolio, is calculated using the qdist() function. The following code is utilized for this purpose:

q01 <- qdist(distribution = dm, p = 0.01, mu = marg[,1], sigma = marg[,2], lambda = -0.5, skew = marg[,3], shape = marg[,4]) %>% zoo(order.by = dates_test)

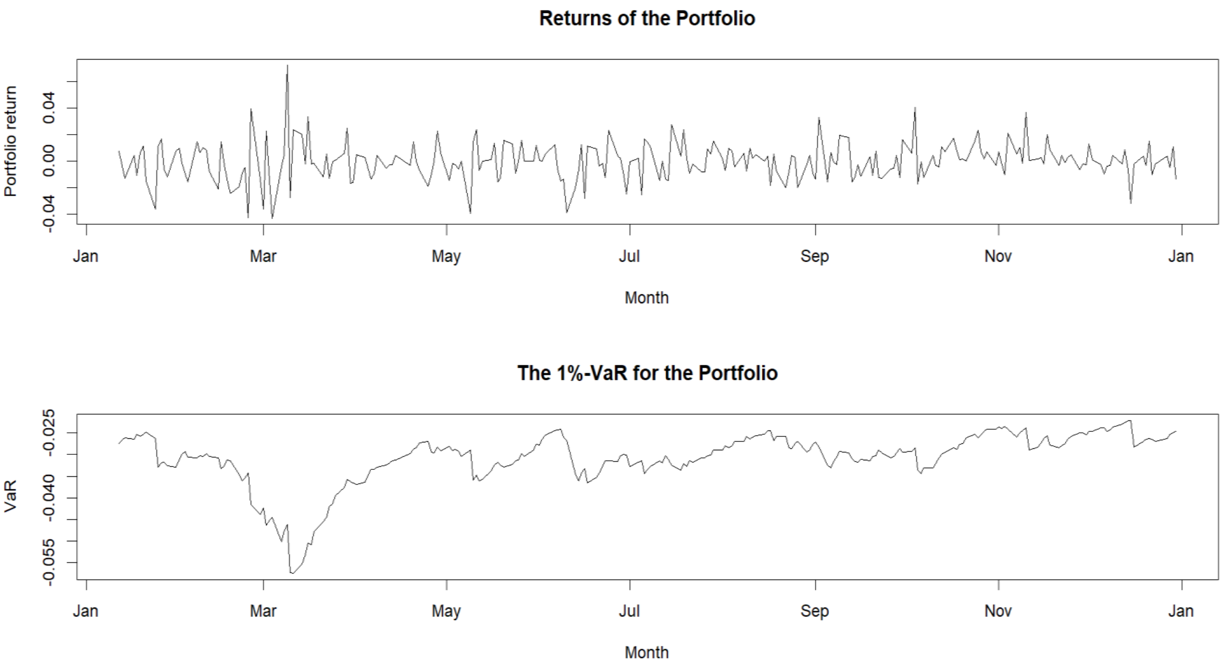

Figure 4: Portfolio Returns and its values at 1 % VaR.

The figure 4 displaying the 1% VaR of the portfolio indicates a sharp decline in the month of March, followed by a subsequent increase. This increasing trend over time implies that the portfolio is becoming progressively riskier, with the potential for greater losses. While the returns of the portfolio demonstrate consistent fluctuations, they do not show a significant increase in returns over the same period. Therefore, the portfolio is becoming increasingly risky without providing appropriate higher returns.

A "violations" vector is created to demonstrate the cases where the portfolio's returns fall below the 1% VaR measure, which indicates a 99% level of confidence that the portfolio's returns will not exceed a particular loss threshold during a specified period.

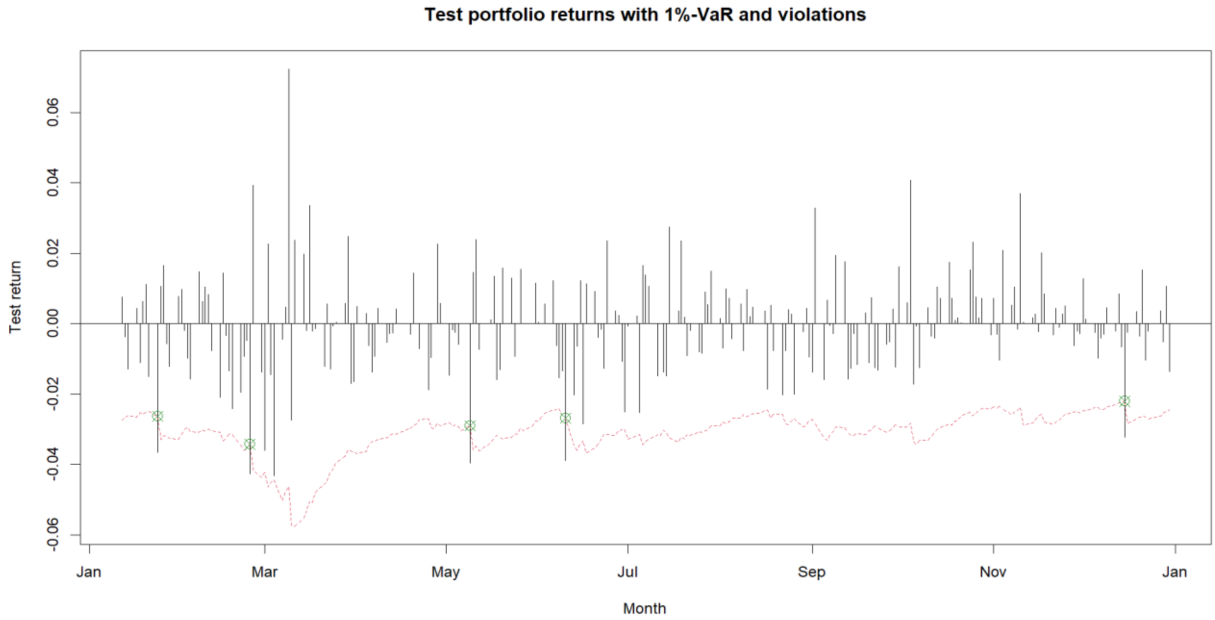

Figure 5 : Portfolio returns with 1% VaR and violations.

According to Figure 5, the portfolio experienced five instances in which its returns fell below the 1% VaR measure, indicating a higher level of volatility than anticipated and an increased probability of significant losses. This violation of the 1% VaR measure suggests that the actual returns exceeded what was predicted by the model, which may be attributed to unforeseen market conditions, changes in underlying assets, or other unaccounted factors. The presence of five such violations indicates that the portfolio may carry a higher level of risk than initially estimated and may require modifications to effectively ease weakness.

Conclusion

In this study, we utilize the DCC-GARCH model, which is a multivariate volatility model that combines the generalized autoregressive conditional heteroscedasticity (GARCH) and dynamic conditional correlation (DCC) models. The model is employed to estimate the time-varying correlations and conditional variances of the log daily closing prices of four highest market capitalization companies in the German stock market, namely SAP, Siemens, Deutsche Telekom, and Allianz, from January 1, 2010, to December 31, 2022.

The goal of our study is to forecast the value-at-risk for the weighted portfolio of our multivariate time series using the DCC-GARCH model. To achieve this, we install various R packages, including "rugarch," "rmgarch," and "zoo". Primarily a combination of GARCH, APARCH, and EGARCH models to each return series of the four stocks were fitted to create a specification for the DCC-GARCH model. The DCC process was fitted with an order of (1,1), and the multivariate normal distribution.

To fit a portfolio and calculate its VaR, rolling forecasting of volatility is essential. Thus, we applied one-step rolling forecasting to the out-of-sample data of the fitted DCC-GARCH model. From the conditional correlations, we observed that SIE had a positive correlation with all series and the highest correlation with SAP. Therefore, a weighted portfolio with the highest weight of 30% allotted to SIE was fitted. After obtaining the portfolio returns, we calculated a 1% VaR measure.

The 1% VaR of the portfolio shows a slight increasing trend, while the returns of the portfolio demonstrate consistent fluctuations and do not show a significant increase in returns over time. This implies that the portfolio is becoming increasingly risky without providing appropriate higher returns. There were five instances in which the returns fell below the 1% VaR measure, indicating that the portfolio may carry a higher level of risk than initially estimated and may require modifications to effectively ease weakness.

One of the major limitations of this study is that the focus was not on fitting the best GARCH model, and the p and q value order was (1,1) for each model specification. Additionally, the study did not compare other weighted portfolios or other models such as the Constant Conditional Correlation (CCC) model, Copula-Based GARCH (CoGARCH) model, or other multivariate volatility models. This study could be extended by including more companies in the portfolio, applying other multivariate volatility models to overcome the limitations of the DCC model, comparing VaR measures of different models and portfolios, and exploring different forecasting techniques and comparing their performance to the DCC-GARCH model. There are enormous opportunities for extending this study.

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