Implementation and Validation of the Magnetized Poisson-Vlasov-Fokker-Planck Framework

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Motivation

Accurate and time effective simulation of plasma flow is of great importance in many areas of engineering and energy technology. The main difficulty arise from the presence of a wide range of length and time scales, the need to include Coulomb interaction between particles and their electromagnetic interactions. The Boltzmann transport equation provides the starting point for a kinetic description of such systems. Efforts have been made in the years to simplify the collision operator in order to reduce its high computational cost.

Poisson-Vlasov-Fokker-Planck Equation

Assuming the external magnetic field is stationary, we obtain the following coupled equations describing the space and time evolution of the mass density function $\mathcal{F}_s$ of species s (e.g electrons, ions):

$$\frac{\partial \mathcal{F}_s}{\partial t} + U_i \frac{\partial \mathcal{F}_s}{\partial x_i} + \frac{\partial}{\partial U_i}((a_{s,i} - b_{s,ik} U_k)\mathcal{F}_s) = \frac{\partial^2}{\partial U_i \partial U_j}\left(\frac{C_{s,ik}C_{s,jk}}{2}\mathcal{F}_s\right)$$

and the electric potential Poisson equation:

$$\nabla^2 \varphi(x,t) = -\frac{\rho(x,t)}{\varepsilon_0} \quad ; \quad \mathbf{E} = -\nabla_x \varphi(x,t)$$

Such Fokker-Planck type equations are solved using a discrete particle approximation of the mass density function. The computational particles evolution equation are given by the Langevin evolution equation

$$d\mathbf{x}_p = \mathbf{U}_p dt$$

$$d\mathbf{U}_p = (\mathbf{a}_p - \mathbf{b}_p \mathbf{U}_p)dt + \mathbf{c}_p d\mathbf{W}_p$$

These stochastic differential equations are solved using the Accurate Time Integration scheme proposed by *Jenny and Gorji (2019)*:

$$\hat{\mathbf{x}}_p^{n+1} = \hat{\mathbf{x}}_p^n + \mathbf{f} + \mathbf{B}^{1/2}\boldsymbol{\xi}_{p,x}^{n+1}$$

$$\hat{\mathbf{U}}_p^{n+1} = \mathbf{d} + \mathbf{A}^{1/2}\boldsymbol{\xi}_{p,u}^{n+1}$$

Momentum-Energy Correction

The imperfect random sampling of the random vector $\boldsymbol{\xi}$ results in a flow field which does not conserves momentum and energy anymore. Researchers at IFD have proposed a methodology to compute a correction field, which can be evaluated at each particle position and returns the corrected velocity field which has the desired conservation properties. Using the global momentum conservation error and test functions, we get a LSE for the correction coefficients r_i which corrects the mean particle velocity in a cell:

$$\sum_{j=1}^{N} r_j \underbrace{\left[\sum_{p=1}^{N_p} \phi_j(\mathbf{x}_p)\phi_i(\mathbf{x}_p)\right]}_{A_{ij}} = \underbrace{\sum_{p=1}^{N_p} R_p \phi_i(\mathbf{x}_p)}_{b_i} \; ; \; \forall i = 1, \ldots, N$$

Using the energy conservation criteria and applying a similar procedure we get a scalar coefficient κ_i which corrects the particles velocity fluctuations:

$$\kappa_i = \left[1 - \sum_{p=1:\ x_p \in \Omega_i}^{N_p} \frac{\hat{R}(\mathbf{x}_p)}{\hat{K}_{p,i}}\right]^{1/2}$$

Exemplary Results

Shown is a 2-dimensional plasma flow over an obstacle. The dots represent charged particles colored with horizontal velocity.

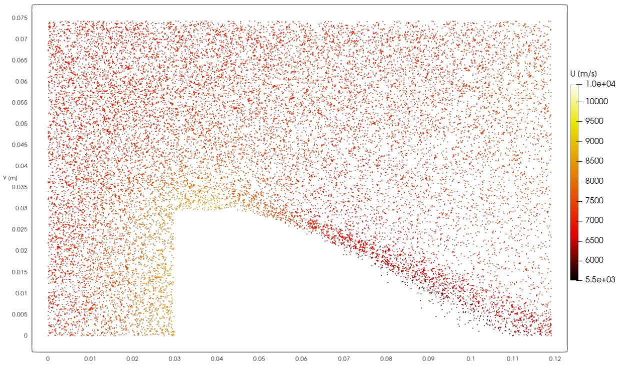

Figure 1: 2d plasma flow over obstacle

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