Tristan Nguyen
303 774 2957

Problem 1:

Count $|S|$: In bijection with the # of 3 element subsets of an 8 element set where the 3 elements are not distinguishable: $\binom{8}{3}$.

Count $|T|$: Count # ~~of~~ of bit strings with 1 one, 0 ones and subtract it from the # of 7-bit strings: $2^7 - \binom{7}{1} - \binom{7}{0}$.

The number of injective functions from S to T is in bijection with the number of $|S|$ permutations of a $|T|$ element set: Choose $|S|$ elements from a $|T|$ element set, and map them each to a different element of S: Answer: $\dfrac{(2^7-\binom{7}{1}-\binom{7}{0})!}{(2^7-\binom{7}{1}-\binom{7}{0}-\binom{8}{3})!}$

Problem 2:

Let us count the number of 12-variable strings of length n in two different ways.

Method 1: For each position $1,\dots,n$, we choose 1 of 12 variables: 12^n.

Method 2: Remove one variable from all 12-variable strings. This gives us 11-variable strings of length $1,\dots,n$. Wlog, assume the variable removed is x.

Case 0: 0 x's were removed. The number of such strings is 11^n.

Case 1: 1 x was removed. The number is $\underbrace{\binom{n}{1}}_{\text{\# of ways to remove x from a position } 1,\dots,n} \overbrace{11^{n-1}}^{\text{\# of bit strings with x removed.}}$

⋮

Case n: n x's were removed. The number is $\binom{n}{n} 11^{n-n}$.

Adding up all disjoint cases gives us

$$\sum_{k=0}^{n}\binom{n}{k}11^{n-k} = \sum_{k=0}^{n}\binom{n}{n-k}11^{n-k} \underset{0\le n-k\le n}{=} \sum_{p=0}^{n}\binom{n}{p}11^{p}$$

Since we counted the same thing twice, $12^n = \sum_{k=0}^{n}\binom{n}{k}11^k$.