Proof. We will proceed by induction

Basis Step: Let $P(n)$ be the statement that $(1+x)^n \geq 1+nx$. Let $x \in \mathbb{Z}_+$ be arbitrary. We must show that $P(1)$ is true:

$$(1+x)^1 \geq 1+(1)x$$
$$1+x \geq 1+x$$
$$1+x = 1+x$$

So $(1+x)^n \geq 1+nx$ when $n=1$.

Inductive step: Let ~~k~~ $k, x \in \mathbb{Z}_+$ be arbitrary. Suppose $(1+x)^k \geq 1+kx$ is true. We must show that $(1+x)^{k+1} \geq 1+(k+1)x$. Observe the algebraic manipulations on $(1+x)^k \geq 1+kx$:

$$(1+x)^k \geq 1+kx$$
$$(1+x)^{k+1} \geq (1+kx)(1+x)$$
$$(1+x)^{k+1} \geq 1+x+kx+kx^2$$
$$(1+x)^{k+1} \geq 1+(k+1)x+kx^2$$
$$\geq 1+(k+1)x$$

$*$ $kx^2 \geq 0$

$1+(k+1)x+kx^2 \geq 1+(k+1)x$

We have $(1+x)^{k+1} \geq 1+(k+1)x$ as desired.

$\square$

Proof. We will proceed by induction.

Basis step: We need to prove that $\cancel{\sum_{i=1}^{n-1} r_i s_i}$ $\sum_{i=1}^{n-1} r_i s_i = \frac{n(n-1)}{2}$

for $n=1$. Observe that we cannot split it since we already have n piles of 1 stone. So $\sum_{i=1}^{n-1} r_i s_i = 0 = \frac{1(1-1)}{2} = \frac{n(n-1)}{2}$

Inductive step: Suppose that $k \geq 1$ is arbitrary and that $\sum_{i=1}^{1-1} r_i s_i = \frac{1(1-1)}{2}$, $\sum_{i=1}^{2-1} r_i s_i = \frac{2(2-1)}{2}$, ..., $\sum_{i=1}^{k-1} r_i s_i = \frac{k(k-1)}{2}$.

We must show that $\sum_{i=1}^{k+1-1} r_i s_i = \frac{(k+1)(k)}{2}$. Observe that when we first split a pile of $k+1$ stones, we get two piles ~~less than~~ with less than $k+1$ stones. Call these piles P_1 and P_2.
~~Since P_1 and P_2 have from $1, 2, \ldots, k$ stones, we have a~~
$|P_1| = p_1$ and $|P_2| = p_2$ ($|P|$ denotes number of stones in P) where $p_1 + p_2 = k+1$. We can apply our inductive hypothesis to P_1 and P_2:

$$\cancel{p_1 p_2 + \frac{p_1(p_1-1)}{2} + \frac{p_2(p_2-1)}{2} = \sum_{i=1}^{k+1-1} r_i s_i}$$

$$\cancel{\frac{p_1^2 - p_1 + p_2^2 - p_2}{2}} =$$

$$\sum_{i=1}^{k+1-1} r_i s_i = \sum_{i=1}^{p_1-1} r_i s_i + \sum_{i=1}^{p_2-1} r_i s_i + p_1 p_2$$

$$= \frac{p_1(p_1-1)}{2} + \frac{p_2(p_2-1)}{2} + \frac{2p_1p_2}{2}$$

$$= \frac{p_1^2 - p_1 + p_2^2 - p_2 + 2p_1p_2}{2}$$

$$= \frac{(p_1+p_2)(p_1+p_2-1)}{2}$$

$$= \frac{(k+1)(k)}{2}$$

as desired. $\square$