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Problem 1:

Proof. Suppose for contradiction that we have a simple graph $G=(V,E)$, $|V|=n$, where no two vertices have the same degree. Let $V=\{v_1, v_2, \ldots, v_n\}$. Observe that ~~as~~ $deg(v_i) \geq 0$ but each vertex can be ~~some~~ adjacent to at most $n-1$ other vertices, so $0 \leq deg(v_i) \leq n-1$ for $i=1,\ldots,n$. Since $deg(v_1) \neq deg(v_2) \neq \ldots \neq deg(v_n)$, ~~we must have there must be a vertex~~ and there are $(n-1)-0+1$ numbers between 0 and $n-1$ inclusive, there must be a vertex v_k with degree 0 and a vertex v_z with degree $n-1$ in G. Contradiction. Since v_z has degree $n-1$, it must be adjacent to v_k. But $deg(v_k)=0$ which is impossible. $\square$

Problem 2.

Proof. Suppose that G is connected ($G=(V,E)$).

($\rightarrow$) Suppose G is not complete. We wish to show that we can remove $v_1, \dots, v_m$ such that G becomes disconnected. Observe that G can only be connected but not complete when $n>2$. ~~Let $G = G(V - \{v_1, \dots, v_m\}, E')$ where E' is the subgraph of E but with edges incident to $v_1, \dots, v_m$ removed~~ Since G is ~~disconnected~~ not complete, there exists $v_{m+1}, v_{m+2} \in V$ such that $\{v_{m+1}, v_{m+2}\} \notin E$. Now remove all vertices and their incident edges except for v_{m+1}, v_{m+2}. We have a disconnected graph.

($\leftarrow$) Suppose that we can remove $v_1, \dots, v_m$ to make G disconnected. Let G' be the subgraph of G with $v_1, \dots, v_m$ and their incident edges removed. Since G' is disconnected, we can find $v_{m+1}, v_{m+2} \in G'$ such that no path exists between them. Thus, $\{v_{m+1}, v_{m+2}\} \notin G'$. But $\{v_{m+1}, v_{m+2}\}$ is not incident with $v_1, \dots, v_m$, so it cannot be one of the edges we removed to get G'. Therefore, $\{v_{m+1}, v_{m+2}\} \notin G$ and G is not complete as desired. $\square$