# Math 124 - Programming for Mathematical Applications

UC Berkeley, Spring 2023

# Homework 1

Due Wednesday Jan 25

### Problem 1: Markdown / LaTeX

A page of mathematical text is provided as a PNG image below. Write this entire text segment as a Markdown / LaTeX cell.

The following online resources can be helpful:

- [Markdown Cheat-sheet (https://github.com/adam-p/markdown-here/wiki/Markdown-Here-Cheatsheet)](https://github.com/adam-p/markdown-here/wiki/Markdown-Here-Cheatsheet)
- [Wiki-books LaTeX/Mathematics (https://en.wikibooks.org/wiki/LaTeX/Mathematics)](https://en.wikibooks.org/wiki/LaTeX/Mathematics)
- [LaTeX Math Cheat-sheet (http://tug.ctan.org/info/undergradmath/undergradmath.pdf)](http://tug.ctan.org/info/undergradmath/undergradmath.pdf)

# This is the top-level header

*This text is emphasized*, **this text is bold**, and ***this text is both***.

## A sub header

Some inline math $\hat{u}(x) = a\varphi(x)$ and some display-style math:

$$\int_{-1}^{1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

where $x_i, w_i, i = 1, \ldots, n$ are specified points and weights.

Example of cases:

$$\varphi(x) = \begin{cases} 2x, & x \leq \frac{1}{2}, \\ 2 - 2x, & x > \frac{1}{2}. \end{cases}$$

### A sub-sub header

Some bullet points with math:

- $A$ is symmetric, that is, $a_{ij} = a_{ji}$
- $A$ is tridiagonal, that is, $a_{ij} = 0$ whenever $|i - j| > 1$

and some enumerated points:

1. First
2. Second
3. Third

Inline Julia code with syntax highlighting:

```julia
function myfunc(x)
    return 2x + 1
end
```

Multiple equations with aligned first equal signs:

$$\begin{aligned} a_{11} &= 4 \cdot 4 \cdot \frac{1}{4} + (-4)(-4)\frac{1}{4} = 8 \\ a_{21} &= a_{12} = a_{23} = a_{32} = -4 \end{aligned}$$

and some vectors and matrices

$$\begin{pmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Finally a horizontal line:

---

# This is a top-level header

*This text is italicized*, **this text is bold**, and ***this text is both***.

## A sub-header

Some inline math $\hat{u}(x) = a\phi(x)$ and some display-style math:

$$\int_{-1}^{1} f(x)\,dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

where $x_i, w_i, i = 1, \ldots, n$ are specified points and weights.

Example of cases:

$$\phi(x) = \begin{cases} 2x, & x \le \frac{1}{2}, \\ 2 - 2x, & x > \frac{1}{2}. \end{cases}$$

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Inline Julia code with syntax highlighting:

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function myfunc(x)
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Multiple equations with aligned equal signs:

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and some vectors and matrices:

$$\begin{pmatrix} 8 & -4 & 0 \\ -4 & 8 & 4 \\ 0 & -4 & 8 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Finally, a horizontal line:

Type *Markdown* and LaTeX: $\alpha^2$

$\alpha^2$

## Problem 2

Fill in all the question marks in the comments (first try without running any code).

In [76]:
```
X = 8;
# X = 8, Y = ?
Y = X;
# X = 8, Y = 8
X = Y;
# X = 8, Y = 8
X *= 2;
# X = 16, Y = 8
Y /= 2;
# X = 16, Y = 4
```

## Problem 3

(from **Insight**, P1.1.6)

An ellipse with semiaxes $a$ and $b$ is specified by

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

If $r = a = b$, then this defines a circle whose perimeter is given by $P = 2\pi r$. Unfortunately, if $a \neq b$, then there is no simple formula for the perimeter and we must resort to approximation. Numerous possibilities have been worked out:

$$P_1 = \pi(a+b) \qquad P_5 = \pi(a+b)\left(1 + \frac{3h}{10 + \sqrt{4-3h}}\right)$$

$$P_2 = \pi\sqrt{2(a^2+b^2)} \qquad P_6 = \pi(a+b)\frac{64-3h^2}{64-16h}$$

$$P_3 = \pi\sqrt{2(a^2+b^2) - \frac{(a-b)^2}{2}} \qquad P_7 = \pi(a+b)\frac{256-48h-21h^2}{256-112h+3h^2}$$

$$P_4 = \pi(a+b)\left(1+\frac{h}{8}\right)^2 \qquad P_8 = \pi(a+b)\left(\frac{3-\sqrt{1-h}}{2}\right)$$

Here,

$$h = \left(\frac{a-b}{a+b}\right)^2$$

can be regarded as a departure from "circlehood."

## Problem 3 (a)

Write a function `printallP(a,b)` which computes each of the 8 approximations and prints each value using `println`.

In [1]:
```
function printallP(a,b)
    h = ((a - b)/(a + b))^2
    println(pi*(a + b))
    println(pi*(sqrt(2(a^2 + b^2))))
    println(pi*(sqrt(2(a^2 + b^2) - ((a - b)^2)/2)))
    println(pi*(a + b)*(1 + h/8)^2)
    println(pi*(a + b)*(1 + 3h / (10 + sqrt(4 - 3h))))
    println(pi*(a + b)*(64 - 3h^2)/(64 - 16h))
    println(pi*(a + b)*(256 - 48h - 21h^2)/(256 - 112h + 3h^2))
    println(pi*(a + b)*(3 - sqrt(1 - h))/2)
end
```

Out[1]:
```
printallP (generic function with 1 method)
```

## Problem 3 (b)

Run `printallP` for the parameters $(a, b) = (1, 1)$, $(1, 0.5)$, and $(1, 0.1)$.

In [6]:
```
printallP(1,1)
```

```
6.283185307179586
6.283185307179586
6.283185307179586
6.283185307179586
6.283185307179586
6.283185307179586
6.283185307179586
6.283185307179586
```

In [7]:
```
printallP(1,0.5)
```

```
4.71238898038469
4.967294132898051
4.841519436353346
4.844197699936344
4.844224108065043
4.844223672097832
4.844224098583075
4.847142001497851
```