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Section 102

Writing Workshop

4a) Suppose x, y are rational with $x<y$. Observe the following computations

$$\begin{array}{cc} x<y & x<y \\ +x \quad +x & +y \quad +y \\ 2^{-1}(2x)<(y+x)2^{-1} & 2^{-1}(x+y)<(2y)2^{-1} \\ x<\frac{y+x}{2} & \frac{x+y}{2}<y \end{array}$$

Since $x<\frac{x+y}{2}<y$ and $\frac{x+y}{2}$ is rational ($x+y$ and 2 are non zero integers), we have found a $z \in \mathbb{Q}$ where $x<z<y$.

b) Suppose x, y are in $\mathbb{Q}$ with $x<y$. By the previous part, we can find a z_1 such that

$$x<z_1<y$$

and a z_2

$$x<z_2<z_1<y$$

so

$$x<z_i<\ldots<z_2<z_1<y$$

It is clear that the set of z is infinite and a subset of S. So S is infinite.

Alt proof: Suppose $x, y \in \mathbb{Q}$ and $x<y$. Suppose S is finite for contradiction. By the well ordering principle, it must have a least element or be the empty set. Since we can find a z_1 such that

$$x<z_1<y$$

S is not empty. Consider all the z_i in our set S:

$$x<z_i<z_{i-1}<\ldots<z_1<y$$

Let z_i be that least element. But x and z_i are rational and we can find a z_{i+1}:

$$x<z_{i+1}<z_i$$

So z_i is not a least element. Contradiction.

Reflection:

4a) This proof was very similar to the one I wrote on the exam but now I specified that z is rational and provided an explicit statement why.

Ob- Neglected to show one of the implications:

$\forall x, y\, Q(x<y \leftrightarrow \exists x(z \in Q \wedge x<z<y))$

which was $z \in Q$.

4b) I constructed S using a repeated algorithm with

step 1: Finding z_1 such that $x<z_1<y$

step 2: Finding z_2, z_3 such that $x<z_2<z_1<z_3<y$

⋮

Although the proof worked, I liked the simpler 1st proof I wrote in this rewrite because it proved something weaker: that a subset of S was infinite. I believe I can use this idea in later proofs where instead of considering the entirety of a set, I can consider a smaller portion of it.

The alternate proof by contradiction was written because I did not think to use contradiction during the exam. I think it would be a good habit to consider types of proof methods before jumping into a proof.