4 a) $\{\ldots, -5, -4, -2, -1, 1, 2, 4, 5, \ldots\}$

$$1, -1, 2, -2, 4, -4, 5, -5, 7, -7, \ldots$$
$$\underbrace{1 \quad 2 \quad 3 \quad 4}_{n=1} \quad \underbrace{5 \quad 6 \quad 7 \quad 8}_{n=2} \quad 9 \quad 10$$

$4(n-1)+a \qquad 4(n-1)+a$

then!.

$4(n-1)+1 \to 3(n-1)+1$ $\boxed{\text{It is countable}}$

$4(n-1)+2 \to -3(n-1)-1$

$4(n-1)+3 \to 3(n-1)+2$

$4(n\cancel{-1}) \to -3(n-1)-2$

For $n \in \mathbb{N}$, take the number $a \in \mathbb{Z}_+$ in one of the forms $4(n-1)+1$, $4(n-1)+2$, $4(n-1)+3$, $4n$ and assign it to the corresponding $3(n-1)+1$, $-3(n-1)-1$, $3(n-1)+2$, or $-3(n-1)-2$.

b) Order the #s $5^{a_n}7^{b_n}$ where $a_n \in \mathbb{Z}_{\geq 0}$ $b_n \in \mathbb{Z}_+$ in increasing order:

$$5^{a_1}7^{b_1}, 5^{a_2}7^{b_2}, \ldots\ldots, 5^{a_n}7^{b_n}$$

where $5^{a_1}7^{b_1} < 5^{a_2}7^{b_2} < 5^{a_3}7^{b_3} < \ldots < 5^{a_n}7^{b_n}$

Let $S_i = \{s : s \text{ is a multiple of } 5 \text{ and } 5^{a_i}7^{b_i} < s < 5^{a_{i+1}}7^{b_{i+1}}\}$

$S_{j_i} = \{-s : s \in S_i\}$

~~The sequence~~ The sequence:

$\cancel{\emptyset}$, $0, S_1, S_{j_1}, S_2, S_{j_2}, \ldots, S_i, S_{j_i}$

is an enumeration if we count each S_i and S_{j_i} ~~in increasing order~~ from closest element to 0 to farthest.

$\boxed{\text{Countable}}$

c) $\{r \in \mathbb{R} : r_n r_{n-1} r_{n-2} \ldots r_0 \cdot r_{n+1} r_{n+2} \ldots, r_n \in \{1\}, n \in \mathbb{Z}_{\geq 0}\}$

111.	11.	$\cancel{1.0}$	1.1	1.11	1.111
D_1	11.		11.1	11.11	11.111
D_2	111.		111.1	111.11	111.111
D_3	1111.		.	.	.
	.		.	.	.
	.		.	.	.

$D_n = \{d : d \in \mathbb{R} \text{ and has } n \text{ digits equ[al] to } 1\}$.

$D'_n = \{-d : d \in D_n\}$

The sequence:

$D_1, D'_1, D_2, D'_2, \ldots, D_n, D'_n$

is an enumeration. $\boxed{\text{Countable}}$

d) $\{r : r_n r_{n-1} r_{n-2} \cdots r_0 . r_{n+1} r_{n+2} \cdots = r$ and $r_n = 1$ or $9\}$

$f(1) = r_{11} r_{12} r_{13} \cdots$ Pick ~~a~~

$f(2) = r_{21} r_{22} r_{23} \cdots$ $r = .r_{aa} r_{bb} r_{cc} \cdots$

$f(3) = r_{31} r_{32} r_{33} \cdots$ such that $a \neq 1$ $b \neq 2$ ......

$\vdots$ $\boxed{\text{Uncountable}}$

(10) A-B

a) finite

$A = \mathbb{R}$ $B = \mathbb{R}_+ \cup \mathbb{R}_-$

$A - B = \{0\}$

b) countably ~~infinite~~ infinite

$A = \mathbb{R}$ $B = \mathbb{R} - \mathbb{Z}$

$A - B = \mathbb{Z}$

c) Uncountable

$A = \mathbb{R}$ $B = \mathbb{R}_-$

$A - B = \mathbb{R}_{\geq 0}$

~~(12) Suppose $|B| \leq |A|$ and $A \subseteq B$. For a function $f : A \to B$,~~

~~(12) Suppose $A \subseteq B$. Let $a_n \in A$ be arbitrary. If $|A|$~~

~~and We have~~ $a_1, a_2, \ldots, a_n \in B$ and it follows that $n \leq |B|$.

(12) Suppose $A \subseteq B$. Let $a_n \in A$ and $|A| = n$. We have

$a_1, a_2, \ldots, a_n \in B$

so $n \leq |B|$. But $n = |A|$ so $|A| \leq |B|$

(15) Proof. Suppose A, B are sets, A uncountable, and $A \subseteq B$. It follows that $|A| \neq |\mathbb{Z}_+|$ and $|A| \leq |B|$. Since $|A| \neq |\mathbb{Z}_+|$, $|A| > |\mathbb{Z}_+|$ or $|A| < |\mathbb{Z}_+|$. But $|A|$ cannot be less than $|\mathbb{Z}_+|$ since it would be finite and therefore countable. So:

$$|\mathbb{Z}_+| < |A| \leq |B| \rightarrow |\mathbb{Z}_+| < |B|$$

B is uncountable as desired.

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(30) $ax^2+bx+c=0$

Proof. Let a,b,c be integers. Since $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, define:

$P_z=\{p : p=\frac{-b+\sqrt{b^2-4ac}}{2a}, p\in\mathbb{R}, \text{and } abc=z\}$

$N_z=\{n : n=\frac{-b-\sqrt{b^2-4ac}}{2a}, n\in\mathbb{R}, \text{and } abc=z\}$

Let sequence t be

$t=P_0, N_0, P_1, N_1, P_{-1}, N_{-1}, P_2, N_2, P_{-2}, N_{-2}, \ldots$

Let an element of a set from t be "new" if it does not appear in any of the previous sets. Let t' be the sequence of only "new" elements.

P_z and N_z are finite since $-z\le a,b,c\le z$ so there can only be a limited combination of a,b,c such that $abc=z$.

(4b) Suppose $f: S \to P(S)$. ~~and If is onto~~ Let $T = \{s \in S \mid s \notin f(s)\}$
Suppose $f(x)$ is onto. T is an ~~s~~ element of $P(S)$ since
$T \subseteq S$. Consider

$$f(s) = T$$

Which says that $f(s) \subseteq T$ so for $s \in f(s)$, $s \in T$. Contradiction.
$s \notin T$ since $s \in f(s)$. So there is no $s \in S$ such that
$f(s) = T$ for $T \in P(S)$.

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HW4.1 (12) $a \in \mathbb{Z}_+ \rightarrow 4 \nmid a^2+2$

$4k \neq a^2+2$

~~$2 \equiv a^2 \pmod 4$~~

~~$2 = a^2 \bmod 4 = a \bmod 4$~~

$0 \equiv a^2+2 \pmod 4$

$-4n-2 \equiv a^2 \pmod 4$

$4n+2 \equiv 2 \pmod 4$

a^2+4n+2

	a^2+2	a	a^2	
4	3	1	1	☆
8	6	2	4	$a^2 \bmod 4 = 0$
12	11	3	9	$a^2 \bmod 4 = 1$
16	18	4	16	$a^2 \bmod 4 = 0$
28	27	5	25	$a^2 \bmod 4 = 1$
38	38	6	36	$a^2 \bmod 4 = 0$
		7	49	$a^2 \bmod 4 = 1$

$2 \bmod 4 = 2$

$(n+2)^2 = n^2+4n+4$

$(n+2)^2 \bmod 4 = n^2+4n+4 \bmod 4$

$= n^2 \bmod 4$

$2 \bmod 4 = 2$

so $n^2 \bmod 4 + 2 \bmod 4 = (0 \text{ or } 1+2) \bmod 4 \neq 0$

so $(n^2+2) \bmod 4 \rightarrow 4k+r = n^2+2$ where $r = 2$ or 3

Proof. Suppose a is a positive integer. We see that

$(a+2)^2 \bmod 4 = a^2+4a+4 \bmod 4 = (a^2 \bmod 4 + 4a+4 \bmod 4) \bmod 4$

$= (a^2 \bmod 4 + 0) \bmod 4 = a^2 \bmod 4$

Let $a=1$

$a^2 \bmod 4 = 1 \bmod 4 = 1$

Therefore, $1^2, 3^2, 5^2, \ldots, (2n-1)^2 \bmod 4$ is 1.

Let $a=2$

$a^2 \bmod 4 = 4 \bmod 4 = 0$

Therefore, $2^2, 4^2, 6^2, \ldots, (2n)^2 \bmod 4$ is 0.

Since $2 \bmod 4 = 2$,

$(a^2 \bmod 4 + 2 \bmod 4) \bmod 4 = (a^2 \bmod 4 + 2) \bmod 4$

Case 1: a is odd $\rightarrow 3 \bmod 4 \rightarrow 4k+3 = a^2+2$. We have a remainder so $4 \nmid a^2+2$

Case 2: a is even $\rightarrow 2 \bmod 4 \rightarrow 4k+2 = a^2+2$. We have a remainder so $4 \nmid a^2+2$

as desired.

(18) a) $a \equiv 11 \pmod{19} \rightarrow 13a \equiv 143 \pmod{19} \equiv \boxed{0 = c}$

$19 \times 7 = 133$ (written: $\overset{6}{19} \times 7 = 143$)

c) $a-b \equiv 8 \pmod{19}$ $\boxed{c = 8}$

e) $2a^2+3b^2 \pmod{19} \equiv 2(121)+3(9) \equiv 269 \equiv 3$

$\boxed{c = 3}$

$269 \div 19$: $269 - 19 = 79$, $79 - 76 = 3$

a mod m = b mod m
$a = q_1 m + r_1$
$b = q_2 m + r_2$

(21) Proof. Suppose $a \bmod m = b \bmod m$. Using the division algorithm,

$a = q_1 m + r_1$ and $b = q_2 m + r_2$

for $q_1, q_2 \in \mathbb{Z}$ and $0 \le r_1, r_2 < m$.

From those equations, $a \bmod m = r_1$ and $b \bmod m = r_2$ so $r_1 = r_2$. Observe that

$a - b = q_1 m + r_1 - (q_2 m + r_2) = q_1 m - q_2 m = (q_1 - q_2) m \rightarrow m \mid a - b$

Therefore, $a \equiv b \pmod{m}$ as desired.

(22) Proof. Suppose $a \equiv b \pmod{m}$. Then $m \mid a - b$ so $mk = a - b$ for $k \in \mathbb{Z}$. Since $a = mk + b$:

$a \bmod m = mk + b \bmod m = b \bmod m$

as desired.

(26) a) $-17 \bmod 2 = (-20 \bmod 2 + 3 \bmod 2) \bmod 2 = 3 \bmod 2 = 1$

b) $144 \bmod 7 = (140 \bmod 7 + 4 \bmod 7) \bmod 7 = 4 \bmod 7 = 4$

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(42) Proof. Let $a, b, c, m \in \mathbb{Z}$, $m \geq 2$, $c > 0$ and $a \equiv b \pmod{m}$. Then $m \mid a-b$ so there is a $k \in \mathbb{Z}$ such that $mk = a-b$. Multiply both sides by c to get $mkc = ac-bc$ so $(mc)k = ac-bc$ and $mc \mid ac-bc$. Thus, $ac \equiv bc \pmod{mc}$

(43) a) Let $a=4, b=1, c=2, m=2$

$ac = 8$ $bc = 2$

Since $8-2=6$ and $2 \mid 6$, $ac \equiv bc \pmod{m}$

but $a-b = 4-1 = 3$ and $2 \nmid 3$ so $a \equiv b \pmod{m}$ is false.

b) ~~Let a=6, b=3, b=2, a=8, c=2,~~

~~m=2~~ $m = 3$

$1 \equiv 3 \pmod{2}$ $4 \equiv 1 \pmod{3}$

$4^2 \equiv (1 \bmod 3)^2 = 1$

$5m \not\equiv 2n \pmod{3}$ $5 \equiv 2 \pmod{3}$ 2

$5^2 \equiv (2 \bmod 3)^2 = 4 = 1$ 1

1 2 $5^3 \equiv 2$ 2

even odd

$6 \equiv 9 \pmod{3}$

$5^6 \not\equiv 2^9 \pmod{3}$

$(125 \bmod 3)^2 \bmod 3$ $(8 \bmod 3)^3 \bmod 3$

$2^2 \bmod 3 = 1$ $2^3 \bmod 3 = 2$

Let $m=3$ $a = 5$ $b=2$ $c=6$ $d=9$.

Indeed, $5 \equiv 2 \pmod{3}$ and ~~6≡9≡6~~ $6 \equiv 9 \pmod{3}$. But $5^6 \not\equiv 2^9 \pmod{3}$ as desired.

(46) Proof. Let n be an odd positive integer. Observe that

$(n+2)^2 \bmod 8 = n^2+4n+4 \bmod 8 = ((n^2) \bmod 8 + (4n+4) \bmod 8) \bmod 8$

Since n is odd, $n = 2k-1$ for $k \in \mathbb{N}$. So

$4n+4 = 4(2k-1)+4 = 8k$.

Since $k \in \mathbb{N}$, $8 \mid 4n+4$. So

$(n+2)^2 \bmod 8 = n^2 \bmod 8$.

Since

$1^2 \equiv 1 \pmod{8}$

We have shown $n^2 \equiv 1 \pmod{8}$ for odd positive integers n.

4.2

(2) a) $321 = 2\cdot 160 + 2^0$
$= 2^2\cdot 80 + 2^0$
$= 2^3\cdot 40 + 2^0$
$= 2^4\cdot 20 + 2^0$
$= 2^5\cdot 10 + 2^0 = 2^6\cdot 5 + 2^0 = 2^7(2) + 2^6 + 2^0 = 2^8 + 2^6 + 2^0$

$(101000001)_2$

b) $1023 = 2\cdot 511 + 2^0 = 2^2\cdot 255 + 2^1 + 2^0$
$= 2^3\cdot 127 + 2^2 + 2^1 + 2^0 = 2^4\cdot 63 + 2^3 + 2^2 + 2^1 + 2^0$
$= 2^5\cdot 31 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 2^6\cdot 15 + 2^5 + \ldots$
$= 2^7\cdot 7 + \ldots = 2^8\cdot 3 = 2^9\cdot 1$

$(1111111111)_2$

c) $100632 = 2\cdot 50316 = 2^2\cdot 25158 = 2^3\cdot 12579 = 2^4\cdot 6289 + 2^3$
$= 2^5\cdot 3144 + 2^4 + 2^3 = 2^6\cdot 1572 + 2^4 + 2^3 = 2^7\cdot 786 + 2^4 + 2^3$
$= 2^8\cdot 393 + 2^4 + 2^3 = 2^9\cdot 196 + 2^8 + 2^4 + 2^3 = 2^{10}\cdot 98 + 2^8 + 2^4 + 2^3$
$= 2^{11}\cdot 49 + \ldots = 2^{12}\cdot 24 + 2^{11} + 2^8 + \ldots = 2^{13}\cdot 12 = 2^{14}\cdot 6 = 2^{15}\cdot 3$
$= 2^{16} + 2^{15} + 2^{11} + 2^8 + 2^4 + 2^3$

$(11000100100011000)_2$

(26) $11^{644} \bmod 645$

1 $\quad 11 \bmod 645 = 11$
2 $\quad 11^2 \bmod 645 = 121$
4 $\quad 11^{2^2} \bmod 645 = 451$
$2^3 = 8 \quad 11^{2^3} \bmod 645 = 226$
16 $\quad 11^{2^4} \bmod 645 = 121$
32 $\quad 11^{2^5} \bmod 645 = 451$
$2^6 = 64 \quad 11^{2^6} \bmod 645 = 226$
$128 \to \quad = 121$

$121 \times 121 = 14641$; $645\overline{)14641}$ → 451
$451 \times 451 = 203401$; $645\overline{)203401}$ → 226
$226 \times 226 = 51076$; $645\overline{)51076}$ → 121

$644 = 2\cdot 322 = 2^2\cdot 161 = 2^3\cdot 80 + 2^2 = 2^4\cdot 40 + 2^2 = 2^5\cdot 20 + 2^2$
$= 2^6\cdot 10 + 2^2 = 2^7\cdot 5 + 2^2 = 2^8\cdot 2 + 2^7 + 2^2$
$= 2^9 + 2^7 + 2^2$

$2^9 \to 226$
$2^7 \to 121$
$2^2 \to 451$

$11^{644} \bmod 645 = 226\cdot 121\cdot 451 \bmod 645$
$= 226\cdot 391 \bmod 645$
$= \boxed{1}$

$451 \times 121 = 54571$; $645\overline{)54571}$ → 391
$391 \times 226 = 88366$; $645\overline{)88366}$ → 1

(32) Proof. ($\rightarrow$) Suppose $n > 0$ is divisible by 11. Then there is some $k \in \mathbb{Z}_+$ such that $n = 11k$. We have

$$k = k_0 \cdot 10^0 + k_1 \cdot 10^1 + k_2 \cdot 10^2 + \ldots + k_a \cdot 10^a$$

where $k_a \in \{0, 1, \ldots, 9\}$ and $a \in \mathbb{Z}_{\geq 0}$. So

$$n = 10k + k = k_0 \cdot 10^0 + k_1 \cdot 10^1 + k_2 \cdot 10^2 + \ldots + k_a \cdot 10^a$$
$$+ k_0 \cdot 10^1 + k_1 \cdot 10^2 + \ldots + k_{a-1} \cdot 10^a + k_a \cdot 10^{a+1}$$

Observe that the sum of the k_a in odd positions is equal to the sum of the k_a in even positions:

$$k_0 + (k_1 + k_2) + \ldots + k_a = (k_0 + k_1) + \ldots + k_a$$

Clearly,

$$k_0 + k_1 + k_2 + \ldots + k_a \equiv k_0 + k_1 + \ldots + k_a \pmod{11}$$

But since $0 \leq k_a \leq 9$, it is possible that there is a ~~$k_b + k_{b+1}$~~ $k_b + k_{b+1} \geq 10$.
To show ~~this~~ that the sum of k_a in even positions minus the sum in odd positions, ~~note~~ is divisible by 11 for base 10, note that

$$-10 \equiv 1 \pmod{11}$$

So if $k_b + k_{b+1} \geq 10$, we need to add 1 to the next digit and subtract 10 from $k_b + k_{b+1}$:

$$k_b + k_{b+1} - 10 < 10 \qquad k_{b+1} + k_{b+2} + 1$$

Since $-10 \equiv 1 \pmod{11}$, we can perform this algorithm on

$$k_0 + (k_1 + k_2) + (k_3 + k_4) + \ldots + k_a \equiv (k_0 + k_1) + (k_2 + k_3) + \ldots + k_a \pmod{11}$$

to obtain, for $n = \sum_{i=0}^{a+1} n_i \cdot 10^i$

$$n_0 + n_2 + n_4 + \ldots + n_{2\lfloor \frac{a+1}{2} \rfloor} \equiv n_1 + n_3 + \ldots + n_{2\lceil \frac{a+1}{2} \rceil - 1} \pmod{11}$$

as desired.

($\leftarrow$) Suppose the difference of the sum of an integer's even and odd positions is divisible by 11. Let that integer be n. So for

$$n = \sum_{i=0}^{a} n_i \cdot 10^i$$

we have

$$(n_0 - n_1 + n_2 - \ldots \mp (-1)^a n_a) \bmod 11 = 0$$
$$(n_0 \bmod 11 + (-n_1) \bmod 11 + n_2 \bmod 11 + \ldots + (-1)^a n_a \bmod 11) \bmod 11 = 0$$

Proposition 1: If $n \in \mathbb{N}$, $10^{2n} \bmod 11 = 1$

Since $100 \bmod 11 = 1$, it follows that $100^n \bmod 11 = 1^n \bmod 11 = 1$

& $100^n = 10^{2n}$ as desired.

Proposition 2: Any positive integer with even number of digits, each digit being 1 is divisible by 11.

We can write that integer in the form of

$$11 \cdot 10^0 + 11 \cdot 10^2 + \ldots + 11 \cdot 10^{2a} \qquad a \in \mathbb{Z}_{\geq 0}$$

$$11(10^0 + 10^2 + \ldots + 10^{2a})$$

as desired.

Proposition 3: Any positive integer in the form 100...001 with an even number of digits is divisible by 11.

We have $10\ldots01 = \underbrace{999\ldots9}_{\text{even number of digits}} + 11$

$= 9(11\ldots11) + 11$ &From proposition 2

$= 11(9(10^0 + 10^2 + \ldots) + 1)$

as desired.

So from propositions 1 and 3:

$$n_0 \bmod 11 + (-n_1) \bmod 11 + n_2 \bmod 11 + \ldots + (-1)^a n_a \bmod 11 = 0$$

$$+ 11 n_1 \bmod 11 \qquad + 1001 n_3 \bmod 11 \ldots = 0$$

$$n_0 \bmod 11 + 10 n_1 \bmod 11 + n_2 \bmod 11 + 1000 n_3 \bmod 11 + \ldots = 0$$

$$\times 100 \bmod 11 \qquad \times 10000 \bmod 11 \ldots$$

$$n_0 \bmod 11 + 10 n_1 \bmod 11 + 100 n_2 \bmod 11 + 1000 n_3 \bmod 11 + \ldots + 10^a n_a \bmod 11 = 0$$

$$\left(\sum_{i=0}^{a} n_i 10^i\right) \bmod 11 = 0$$

$$n \bmod 11 = 0$$

as desired.

(33) Proof: ($\rightarrow$) Suppose $n>0$ is divisible by 3. Then there is a $k \in \mathbb{Z}_+$ such that $n=3k$. We have for ~~$k=\sum k_i 2^i$~~, $a \in \mathbb{Z}_{\geq 0}$, $k=\sum_{i=0}^{a} k_i 2^i$, $0 \leq k_i \leq 1$, $k_i \in \mathbb{Z}_+$

$$n=(2^0+2^1)k = k_0 2^0 + k_1 2^1 + k_2 2^2 + \ldots + k_a 2^a$$
$$+ k_0 2^1 + k_1 2^2 + \ldots + k_{a-1} 2^a + k_a 2^{a+1}$$

Observe that the sum of k_a in even positions is equal to the sum of k_a in odd positions so

$$(k_0+k_1) + \ldots + k_a \equiv k_0 + (k_1+k_2) + \ldots + k_a \pmod 3 \quad (1)$$

But since $k=0,1$, it is possible that $k_b + k_{b+1} = 2$. This means that we must subtract 2 from $k_b + k_{b+1}$ and add 1 to $k_{b+1} + k_{b+2}$:

$$k_b + k_{b+1} - 2 \qquad k_{b+1} + k_{b+2} + 1$$

But notice that $-2 \equiv 1 \pmod 3$. (2)

Observe that we can perform this algorithm on statement (1) to obtain for $n=\sum_{i=0}^{a+1} n_i 2^i$

$$n_0 + n_2 + \ldots + n_{2\lfloor\frac{a+1}{2}\rfloor} \equiv n_1 + n_3 + \ldots + n_{2\lceil\frac{a+1}{2}\rceil - 1} \pmod 3$$

as desired.

($\leftarrow$) Suppose $n>0$, $n=\sum_{i=0}^{a} n_i 2^i$, $a \in \mathbb{Z}_{\geq 0}$, $n_a = 0,1$. Suppose

$$\left(\sum_{i=0}^{a} (-1)^i n_i 2^i\right) \bmod 3 = 0$$

$$\left(\sum_{i=0}^{a} (-1)^i n_i 2^i\right) \bmod 3) \bmod 3 = 0$$

Observe the following:

$$4 \bmod 3 = 1 \rightarrow 4^n \bmod 3 = 1 \rightarrow 2^{2n} \bmod 3 = 1$$

~~$(2 \bmod 3 + 2^{2n} \bmod 3) \bmod 3 = 0 \rightarrow$~~

$$((2^{2n} \bmod 3)(2 \bmod 3)) \bmod 3 = 2$$
$$2^{2n+1} \bmod 3 = 2$$
$$(2^{2n+1} \bmod 3 + 1 \bmod 3) \bmod 3 = 0$$

The important equalities to note are:

$$2^{2n} \bmod 3 = 1 \quad (1)$$
$$2^{2n+1} + 1 \bmod 3 = 0 \quad (2)$$

So from (1) and (2):

$$n_0 \bmod 3 + (-n_1) \bmod 3 + n_2 \bmod 3 + (-n_3) \bmod 3 + \ldots + (-1)^q n_q \bmod 3 = 0$$

$$\times 2^{2} \bmod 3 \qquad \times 2^{4} \bmod 3 \ldots$$

$$2^0 n_0 \bmod 3 + (-n_1) \bmod 3 + 2^2 n_2 \bmod 3 + (-n_3) \bmod 3 + \ldots = 0$$

$$+ (n_1(2^{1}+1)) \bmod 3 \qquad + (n_3(2^3+1)) \bmod 3 + \ldots$$

$$\left(\sum_{i=0}^{q} 2^i n_i\right) \bmod 3 = 0$$

$$n \bmod 3 = 0$$

as desired.