

Math 124 - Programming for Mathematical Applications ¶

UC Berkeley, Spring 2023

Project 1 - The Trapped Knight

Due Friday, February 10

Description

In this project, you will write a computer code to generate a particular sequence of numbers described in the following YouTube video: [The Trapped Knight](https://www.youtube.com/watch?v=RGQe8waGJ4w) (<https://www.youtube.com/watch?v=RGQe8waGJ4w>).

Begin by watching the video and make sure you understand exactly how the sequence is generated. Then continue to implement the code in the 3 parts described below.

Part 1 - Initialize the board

We will store the chess board in a 2d-array of integers. The size of the board is $(2n + 1)$ -by- $(2n + 1)$, for a given integer n . This means the board extends from the center square by n steps in all directions.

The first step is to initialize the board by filling it with the integers described in the video. Finish the implementation of the function definition in the cell below such that it returns this "spiral pattern" for any given input parameter n .

An example is given below: for the following input

```
board = initialize_board(3)
```

the correct output is

```
7×7 Matrix{Int64}:
 37  36  35  34  33  32  31
 38  17  16  15  14  13  30
 39  18   5   4   3  12  29
 40  19   6   1   2  11  28
 41  20   7   8   9  10  27
 42  21  22  23  24  25  26
 43  44  45  46  47  48  49
```

Test your function for various values of n to make sure it is correct before you continue.

Hints:

- Note that since Julia uses 1-based indexing, the center square of the array `board` is given by element `board[n+1,n+1]`.
- After the center 1 has been placed, there are exactly n "circles" of numbers of increasing radius. This is naturally implemented using a for-loop.
- In each "circle", there are 4 segments going up, left, down, and right. These are also naturally implemented using a sequence of 4 for-loops.

```

In [1]: # Initializes the board on a [-n:n]x[-n:n] domain with spiral numbers
#
# Example: initialize_board(2) returns
# 17 16 15 14 13
# 18  5  4  3 12
# 19  6  1  2 11
# 20  7  8  9 10
# 21 22 23 24 25
#
# Inputs:
# n      = integer size of board to allocate
# Outputs:
# board = 2n+1 x 2n+1 integer array filled with spiral numbers
function initialize_board(n)
    C = zeros{Int64, 2n+1, 2n+1}
    count = 1
    C[n+1, n+1] = 1
    for i = 1:n
        add = 1
        j = (2i-1)^2 + 1
        while C[n+1-i, n+1+i] == 0           #Run until hits top right
            C[n+1+i-add, n+1+i] = j
            j += 1
            add += 1
        end
        add = 1
        while C[n+1-i, n+1-i] == 0           #Run until hits top left
            C[n+1-i, n+1+i-add] = j
            j += 1
            add += 1
        end
        add = 1
        while C[n+1+i, n+1-i] == 0           #Run until bottom left
            C[n+1-i+add, n+1-i] = j
            j += 1
            add += 1
        end
        add = 1
        while C[n+1+i, n+1+i] == 0           #Run until bottom right
            C[n+1+i, n+1-i+add] = j
            j += 1
            add += 1
        end
    end
    C
end

```

Out[1]: initialize_board (generic function with 1 method)

```
In [2]: initialize_board(5)
```

```
Out[2]: 11×11 Matrix{Int64}:
```

101	100	99	98	97	96	95	94	93	92	91
102	65	64	63	62	61	60	59	58	57	90
103	66	37	36	35	34	33	32	31	56	89
104	67	38	17	16	15	14	13	30	55	88
105	68	39	18	5	4	3	12	29	54	87
106	69	40	19	6	1	2	11	28	53	86
107	70	41	20	7	8	9	10	27	52	85
108	71	42	21	22	23	24	25	26	51	84
109	72	43	44	45	46	47	48	49	50	83
110	73	74	75	76	77	78	79	80	81	82
111	112	113	114	115	116	117	118	119	120	121

Part 2 - Simulate the walk

Next we will write the function to simulate the walk and produce the sequence. This function will take an initialized board as input, and produce a list of numbers as well as the corresponding x- and y-coordinates.

For example, the following input:

```
board = initialize_board(2)
display(board)
seq, xs, ys = simulate_walk(board);
println("Sequence = ", seq)
println("x-coordinates = ", xs)
println("y-coordinates = ", ys)
```

should produce the following correct output:

```
5×5 Matrix{Int64}:
 17  16  15  14  13
 18   5   4   3  12
 19   6   1   2  11
 20   7   8   9  10
 21  22  23  24  25
Sequence = [1, 10, 3, 6, 9, 4, 7, 2, 5, 8, 11, 14]
x-coordinates = [0, 2, 1, -1, 1, 0, -1, 1, -1, 0, 2, 1]
y-coordinates = [0, 1, -1, 0, 1, -1, 1, 0, -1, 1, 0, -2]
```

Again test your code, first using small values of n as shown above, which makes it easier to look at the results and find errors.

Hints:

- It is convenient to create another 2d-array of booleans, indicating if a square has been visited or not.
- Make sure you never allow the knight to jump outside the board. That is, the only valid positions are n steps from the center square in either direction

```
In [3]: # Simulates the trapped knight walk on a pre-initialized board and ret
#
# Inputs:
# board    = 2n+1 x 2n+1 integer array filled with spiral numbers
# Outputs:
# sequence = integer array containing the sequence of spiral numbers
# x_path    = integer array containing the x coordinates of each step
# y_path    = integer array containing the y coordinates of each step
```

```

# y_path = integer array containing the y coordinates of each step
function simulate_walk(board)
    #Initial coordinates
    i = (size(board,1)-1)÷2 + 1
    j = (size(board,2)-1)÷2 + 1
    #Sequence and coordinate vectors
    s = Int64[]
    x, y = Int64[0], Int64[0]
    #2-d array of Booleans to monitor visited squares
    B = ones(size(board,1),size(board,2))
    B[i,j] = 0

    while true
        v, x_1, y_1 = [], [], []
        jump = [-1, -2, 1, 2]
        #Calculate all jump coordinates and save the number and coordi
        for a_1 in jump
            for a_2 in jump
                if abs(a_1) != abs(a_2) && 1 ≤ i + a_1 ≤ size(board,1)
                    if B[i + a_1,j + a_2] == 1
                        push!(v, board[i + a_1,j + a_2])
                        push!(x_1, i + a_1)
                        push!(y_1, j + a_2)
                    end
                end
            end
        end
        #Stops the code if the knight becomes trapped and cannot move
        if v == []
            break
        else
            end
            save = v[1]
            index = 1
            #Calculates the minimum value in the vector of all possible ju
            for i = 1:length(v) - 1
                if v[i+1] < save
                    save = v[i+1]
                    index = i + 1
                else
                    end
            end
            #Adds the minimum value to the sequence vector, saves its coord
            push!(s, save)
            push!(x, x_1[index] - ((size(board,1)-1)÷2 + 1))
            push!(y, y_1[index] - ((size(board,1)-1)÷2 + 1))
            i = x_1[index]
            j = y_1[index]
            B[i,j] = 0
        end
    end
    S. X. V

```

```
end
```

Out[3]: simulate_walk (generic function with 1 method)

Part 3 - Generate the full sequence and plot the path

Finally, use your code to generate the full sequence ($n = 100$ is sufficient), output the *last* number, and plot the path by straight lines between all the visited x,y-coordinates.

```
In [4]: board = initialize_board(100)
display(board)
seq, xs, ys = simulate_walk(board);
println("Sequence = ", seq)
println("x-coordinates = ", xs)
println("y-coordinates = ", ys)
```

201×201 Matrix{Int64}:

40001	40000	39999	39998	39997	...	39805	39804	39803	39802	39801
40002	39205	39204	39203	39202		39010	39009	39008	39007	39006
40003	39206	38417	38416	38415		38223	38222	38221	39006	39005
40004	39207	38418	37637	37636		37444	37443	38220	39005	39004
40005	39208	38419	37638	36865		36673	37442	38219	39004	39003
40006	39209	38420	37639	36866	...	36672	37441	38218	39003	39002
40007	39210	38421	37640	36867		36671	37440	38217	39002	39001
40008	39211	38422	37641	36868		36670	37439	38216	39001	39000
40009	39212	38423	37642	36869		36669	37438	38215	39000	38999
40010	39213	38424	37643	36870		36668	37437	38214	38999	38998
40011	39214	38425	37644	36871	...	36667	37436	38213	38998	38997
40012	39215	38426	37645	36872		36666	37435	38212	38997	38996
40013	39216	38427	37646	36873		36665	37434	38211	38996	38995
⋮										
40190	39393	38604	37823	37050		36488	37257	38034	38819	38818
40191	39394	38605	37824	37051	...	36487	37256	38033	38818	38817

```

40192 39395 38606 37825 37052 36486 37255 38032 38817 39
610
40193 39396 38607 37826 37053 36485 37254 38031 38816 39
609
40194 39397 38608 37827 37054 36484 37253 38030 38815 39
608

40195 39398 38609 37828 37055 36483 37252 38029 38814 39
607
40196 39399 38610 37829 37056 ... 36482 37251 38028 38813 39
606
40197 39400 38611 37830 37057 37249 37250 38027 38812 39
605
40198 39401 38612 37831 37832 38024 38025 38026 38811 39
604
40199 39402 38613 38614 38615 38807 38808 38809 38810 39
603
40200 39403 39404 39405 39406 39598 39599 39600 39601 39
602
40201 40202 40203 40204 40205 ... 40397 40398 40399 40400 40
401

```

```

Sequence = [10, 3, 6, 9, 4, 7, 2, 5, 8, 11, 14, 29, 32, 15, 12, 27, 2
4, 45, 20, 23, 44, 41, 18, 35, 38, 19, 16, 33, 30, 53, 26, 47, 22, 43
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, 65, 98, 139, 60, 93, 90, 129, 176, 125, 82, 119, 164, 217, 160, 111
, 154, 205, 264, 331, 200, 101, 142, 97, 138, 187, 92, 89, 128, 175,
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```


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```

In [5]: **using** PyPlot


```
In [23]: plot(ys, -1.*xs)
println("Last number = ", seq[end])
```

Last number = 2084

