Stat134Hw2

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Exercise 1: We break a stick at a uniformly chosen random location. Find the probability that the shorter piece is less than 1/4th of the original.

Proof. If we break the stick into two pieces of length d and $l - d$, then $\frac{d}{l-d} < \frac{1}{4}$ when $d < \frac{l}{5}$. If instead, $l - d$ was the shorter one, we have $\frac{l-d}{d} < \frac{1}{4}$ when $d > \frac{4l}{5}$. So the area in which we break over the total area is the probability. It is:

$$\frac{\frac{l}{5} + \frac{l}{5}}{l} = \frac{2}{5}$$

□

Exercise 2: We throw a dart at a square shaped board of side length 20 inches. Assume that the dart hits the board at a uniformly chosen random point.

(a) Describe a probability space for this experiment.

Answer. This is

$$\Omega = \{(x, y) : 0 \leqslant x, y \leqslant 20\}$$

(b) Find the probability that the dart is within 2 inches of the center of the board.

Answer. The area in the center of the board is 4π. The total area of the board is 400. So the probability is $\pi/100$.

Exercise 3: The statement

SOME DOGS ARE BROWN

has 16 letters. Choose one of the 16 letters uniformly at random. Let X denote the length of the word containing the chosen letter. Determine the possible values and probability mass function of X.

Answer.

Exercise 4: Let X be geometric random variable with parameter $1/2$ as in Lecture 1, so that

$$P(X = k) = 2^{-k}, k = 1, 2, 3, \ldots$$

(a) find the conditional probability of the even "X is divisible by 2" given the event "X is divisible by 3".

(b) Find the conditional probability of the even "X is divisible by 3 " given the even "X is divisible by 2".

Exercise 5: Three married couples (6 guests altogether) attend a dinner party. They sit at a round table randomly in such a way that each outcome is equally likely. What is the probability that somebody sits next to his or her spouse?