1 Numerical Derivatives

Preliminary: Taylor Expansion

The Taylor expansion is a method of taking numerical derivatives that takes the form:

$$f(x) = \sum_{n=0}^{\infty} f^n(x_0) * (x - x_0)^n / n!$$

This is an infinite summation of terms that approximates an unknown point in a function using a known point. Using this expansion, we can derive two schemes for numerical differentiation; the Central Differencing scheme (3) and a Third Order Scheme.

Central Differencing

Starting with the Taylor Expansion to the 1^{st} order, we can derive an equation for solving the first derivative of a function in two ways. First using a point that is beyond the point we're trying to find ($f(x+h)$) and by using a point behind the point we're trying to find ($f(x-h)$). These two methods are called Forward Differencing (1) and Backward Differencing (2). They are derived by:

$$\begin{aligned} &Forward \\ f(x+h) &= f(x) + f'(x) * h \\ f'(x) &= \frac{f(x+h) - f(x)}{h} \end{aligned} \tag{1}$$

$$\begin{aligned} &Backward \\ f(x-h) &= f(x) - f'(x) * h \\ f'(x) &= \frac{f(x) - f(x+h)}{h} \end{aligned} \tag{2}$$

By combining these two methods, we get the Central Differencing scheme (3), which no longer relies on f(x):

$$f'(x) = (f(x+h) - f(x-h)) * 2h \tag{3}$$

Third Order Scheme

There are also methods that start with the Taylor Series to the 3^{rd} order, following similar methods as above, we can derive three-point forward, backward, and central differencing formulas:

$$Forward$$

$$f'(x) = \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h} + O(h^2) \tag{4}$$

$$Backward$$

$$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h} + O(h^4) \tag{5}$$

$$Central$$

$$f'(x) = \frac{f(x-2h) - 4f(x-h) + 3f(x)}{2h} = O(h^2) \tag{6}$$

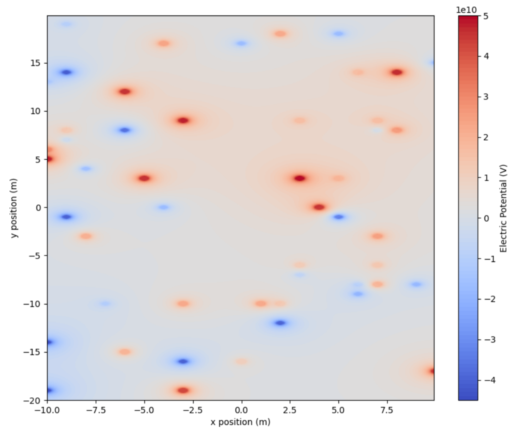

Figure 1: Electric Potential

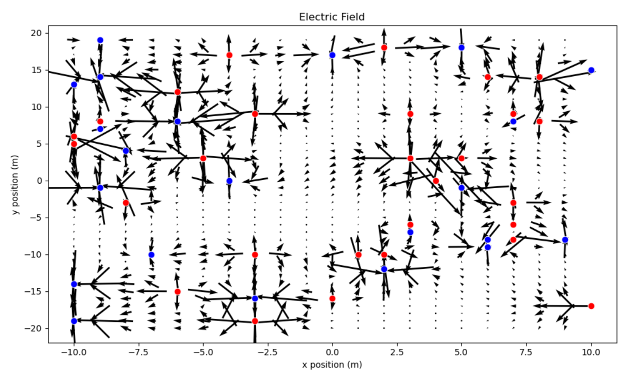

Figure 2: Electric Field

2 Electric Potential of Many Charges

2.1 Summary

This assignment was to rework our former electric potential and electric field modeling program to make it work for any number of charges, defined in a given file titled creategrid.py.

3 Harmonic Oscillator

3.1 Summary

When C^2 is 0, there is no damping, so the oscillator will oscillate forever without loss of energy 3. When the oscillator is critically damped, The system returns to equilibrium in the shortest possible time without oscillating 4. When under damped, the motion still oscillates while the amplitude decays 6. When overdamped, the system slowly returns to equilibrium without oscillating 5.

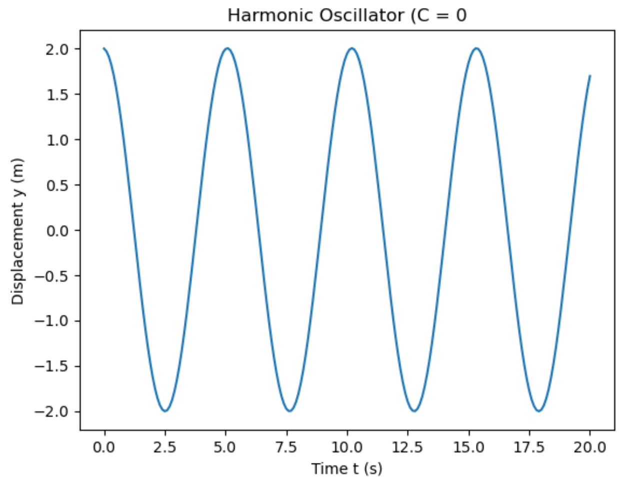

Figure 3: C=0

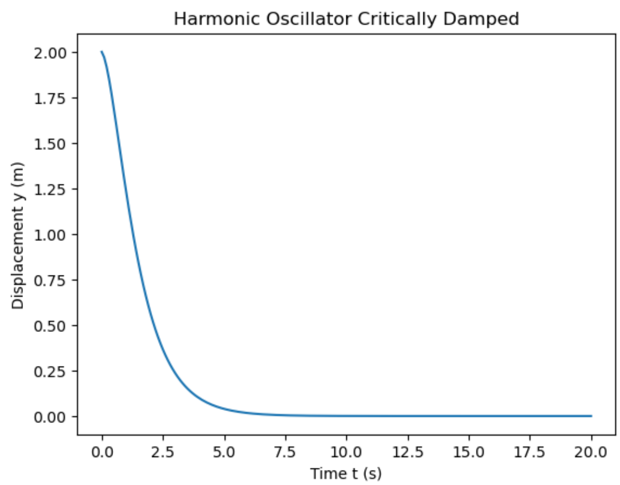

Figure 4: Critically Damped

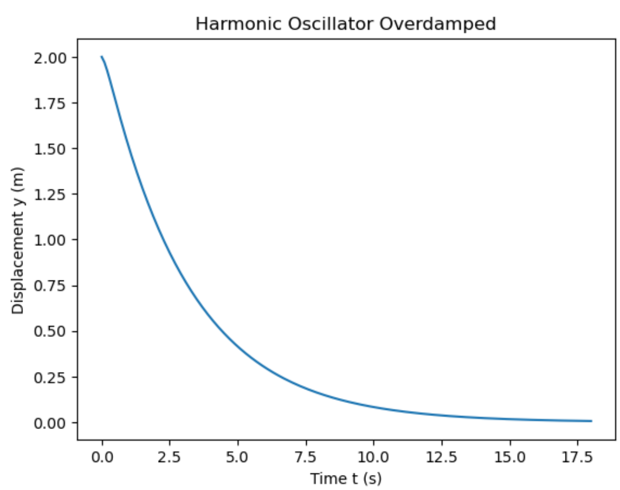

Figure 5: Overdamped

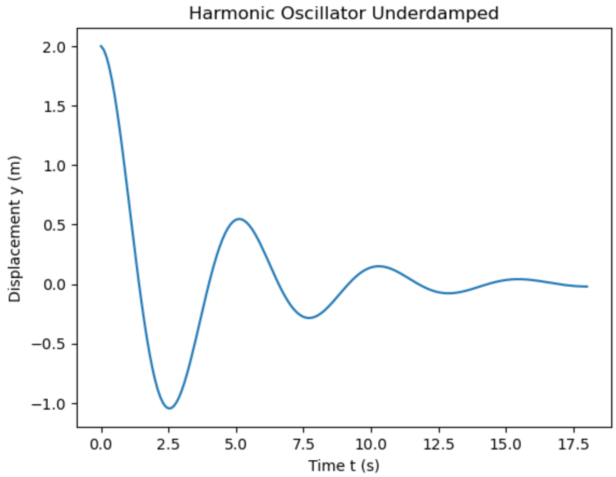

Figure 6: Under damped

4 Cycling Without Drag

4.1 Summary

For this section I used Euler's method to solve for the velocity of an ideal cyclist without drag or other degenerative forces.

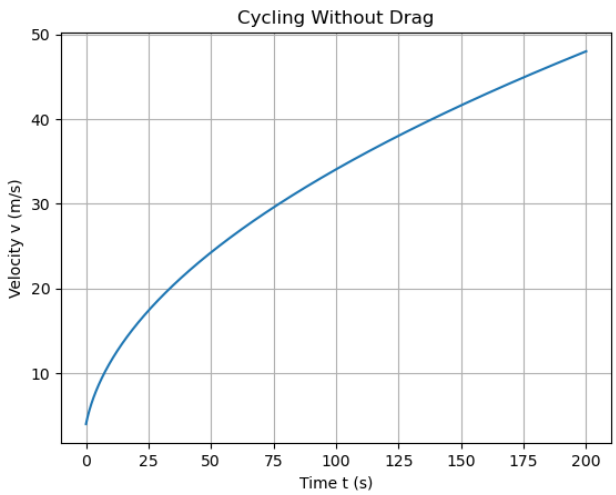

Figure 7: Velocity vs Time