CLEP
Calculus

Time—60 Minutes 45 Questions	For each question below, choose the best answer from the choices given.

1. $\lim_{x \to -5} \frac{7}{x+5}$ is

(A) -7
(B) 0
(C) $\frac{7}{10}$
(D) 7
(E) Nonexistent

2. If $f(x) = 3x^{\frac{1}{3}}$, then $f'(x) =$

(A) x
(B) $\frac{9}{4}x^{\frac{4}{3}}$
(C) $x^{-\frac{1}{3}}$
(D) $x^{-\frac{2}{3}}$
(E) $3x^{-\frac{2}{3}}$

3. Which of the following is an equation of the line tangent to the graph of $f(x) = x^3 + x^4$ at the point $x = -2$?

(A) $20x + y = -32$
(B) $20x + y = -48$
(C) $44x + y = -36$
(D) $8x - y = 4$
(E) $x - y = 2$

4. $\int\left(x+e+x^{e}+e^{x}\right)dx=$

(A) $1+ex^{e-1}+e^{x}+C$

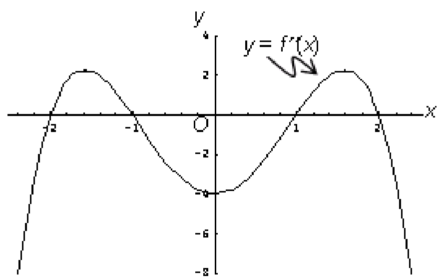
(B) $\dfrac{x^2}{2}+e+x^{e}+e^{x}+C$

(C) $\dfrac{x^2}{2}+\dfrac{e^2}{2}+\dfrac{x^{e+1}}{e+1}+\dfrac{e^{x+1}}{x+1}+C$

(D) $\dfrac{x^2}{2}+ex+\dfrac{x^{e+1}}{e+1}+e^{x}+C$

(E) $\dfrac{x^2}{2}+ex+x^{e}+e^{x}+C$

5.



The graph of f'', the second derivative of the function f, is shown above. On which of the following intervals is f concave upward?

(A) $(-\infty,-2)$
(B) $(-1, 1)$
(C) $(-\infty,-2)$ and $(0, 1)$
(D) $(-\infty,-2)$ and $(2, \infty)$
(E) $(-2, -1)$ and $(1, 2)$

6. $\int\left(x^2-\sqrt{x}\right)\sqrt{x}\ dx =$

(A) $0 + C$

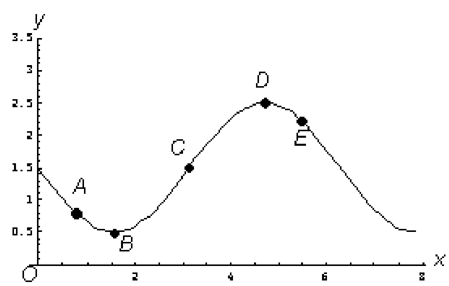
(B) $\dfrac{5x\sqrt{x}}{2}+C$

(C) $\left(\dfrac{x^3}{3}-\dfrac{2\sqrt{x^3}}{3}\right)\dfrac{2\sqrt{x^3}}{3}+C$

(D) $\dfrac{2x^3\sqrt{x}}{7}-\dfrac{x^2}{2}+C$

(E) $\dfrac{2x^5}{7}-\dfrac{x^2}{2}+C$

7.



At which of the five points on the graph in the figure above is $\dfrac{dy}{dx}$ negative and $\dfrac{d^2y}{dx^2}$ positive?

(A) A

(B) B

(C) C

(D) D

(E) E

8. The acceleration, at time t, of a particle moving along the x-axis is given by $a(t) = 4t^3 + 1$. At time $t = 0$, the velocity is 0 and the position of the particle is 12. What is the position of the particle when $t = 1$?

(A) 0.7

(B) 1.0

(C) 5.0

(D) 12.0

(E) 12.7

9. What is $\lim_{x\to\infty} \dfrac{3x^5+2x^3+5}{5x^5+3x^3+7}$?

(A) $\frac{3}{5}$
(B) $\frac{2}{3}$
(C) $\frac{5}{7}$
(D) 1
(E) The limit does not exist

10. $\int_{-3}^{-2} |x+1|\,dx =$

(A) –1.5
(B) –1.0
(C) 1.0
(D) 1.5
(E) 2.0

11. Let f and g be the functions defined by $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2$ and $g(x) = x^2 + 6x$. For which of the following positive values of a is the tangent line to f at $x = a$ parallel to the tangent line to g at $x = a$?

(A) $\frac{2}{3}$
(B) $\frac{3}{2}$
(C) 2
(D) 3
(E) 6

12. The function f is given by $f(x) = 8x^3 + 2x + 1$. What is the average value of f over the closed interval [–1, 2]?

(A) 3
(B) 9
(C) 12
(D) 20
(E) 24

13. If $f(x) = \dfrac{\tan x}{3x}$, then $F'(x) =$

(A) $\dfrac{1}{3}\sec^2 x$

(B) $\dfrac{1}{3}\csc^2 x$

(C) $\dfrac{\cos x}{3(-x\sin x + \cos x)}$

(D) $\dfrac{x\sec^2 x - \tan x}{3x^2}$

(E) $\dfrac{\tan x - x\sec^2 x}{3x^2}$

14. For selected values of x, the functions f and g and their derivatives, $f'(x)$ and $g'(x)$ are evaluated and presented in the table above. If $h(x) = g(f(x))$, what is $h'(5)$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
5	10	10	15	10
10	20	5	5	15
15	30	10	10	5

-

(A) 5
(B) 15
(C) 150
(D) 250
(E) 750

15. $\int_{-4}^{-2} \dfrac{1}{x+1}\,dx =$

(A) –4
(B) –ln 3
(C) ln 2 – ln 4
(D) ln 3 – ln 5
(E) $-\dfrac{2}{3}$

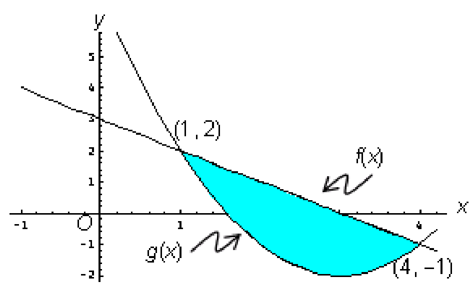
16. $\lim_{h\to 0} \dfrac{\tan\left(\frac{\pi}{3}+h\right)-\tan\left(\frac{\pi}{3}\right)}{h} =$

(A) 0
(B) $\frac{3}{4}$
(C) 1
(D) $\frac{1}{2}$
(E) 4

17. The function f is differentiable and has a relative maximum value of 5 at $x = -1$. If $h(x) = \dfrac{f(x)}{x^3}$, then $h'(-1) =$

(A) -15
(B) -5
(C) 0
(D) 15
(E) 45

18.



The shaded region in the figure above is the region bounded (enclosed) by the graphs of $f(x) = -x + 3$ and $g(x) = x^3 - 6x + 7$. Find the area of the shaded region.

(A) -41.75
(B) -38.25
(C) 8.5
(D) 38.25
(E) 41.75

19. If $x^5 + y^5 + xy = x^4$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{-x - 5x^4y^4}{5x^4y^5 + y - 4x^3}$

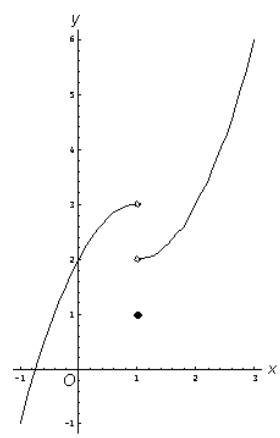
(B) $\dfrac{4x^3 - 5x^4y^5 - y}{5x^5y^4 + x}$

(C) $\dfrac{x^4}{5x^5y^4 + x}$

(D) $\dfrac{x^4}{x^5y^4 + x}$

(E) $\dfrac{5x^4y^4 + x}{y - 4x^3 + 5x^4y^5}$

20.



The graph of the function f is shown in the figure above. What is $\lim_{x \to 1} f(x)$?

(A) 1
(B) 2
(C) 2.5
(D) 3
(E) The limit does not exist

21. $\lim\limits_{x\to\infty} \dfrac{\ln x}{e^x} =$

(A) 0
(B) $\dfrac{1}{e}$
(C) 1
(D) e
(E) ∞

22. If the function f is continuous, nonpositive, and strictly increasing on the closed interval $[a, b]$, which of the following statements is NOT true?

(A) $\int_a^b f(x)dx < f(b)(b-a)$

(B) $\int_a^b f(x)dx > f(a)(b-a)$

(C) $\int_a^b f(x)dx = f(c)(b-a)$ for some number c contained in (a, b)

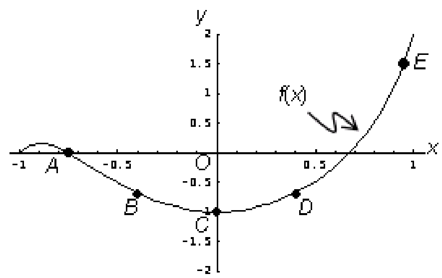
(D) $\int_a^b f(x)dx = F(b)-F(a)$, where $F'(x)=f(x)$ on the open interval (a, b)

(E) $\int_a^b f(x)dx > 0$

23. $\int x^3 e^{x^4+8}\, dx =$

(A) $\dfrac{x^4}{4}e^{x^4+8}+C$
(B) $3x^2e^{x^4+8}+C$
(C) $e^{x^4+8}+C$
(D) $e^{\frac{1}{4}\left(x^4+8\right)}+C$
(E) $\dfrac{1}{4}e^{x^4+8}+C$

24.



The function f is shown in the figure above. At which of the following points could the derivative of f be equal to the average rate of change of f over the closed interval $[-1, 1]$?

(A) A
(B) B
(C) C
(D) D
(E) E

25. For which of the following functions does $\frac{d^4y}{dx^4} = -\frac{d^2y}{dx^2}$?

I. $y = e^{-x}$
II. $y = \cos x$
III. $y = \sin x$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

26. The height, in feet, of a ball thrown vertically upward from a building 50 feet high is given by $s(t) = -16t^2 + 256t + 50$, where t is measured in seconds. What is the height of the ball, in feet, when its velocity is zero?

(A) 178
(B) 306
(C) 434
(D) 562
(E) 1074

27. If f is a function such that $\lim_{x\to -2}\dfrac{f(x)-f(-2)}{x+2}=8$, then which of the following statements must be true?

(A) $f'(-2)=-8$
(B) $f'(2)=8$
(C) $f'(-2)=8$
(D) f is continuous at $x = 2$
(E) $f(x) = -2x^2$

28. $\dfrac{d}{dx}\left(\tan\left(e^x\right)\right)=$

(A) $e^x \sec^2(e^x)$
(B) $-\dfrac{\cos e^x}{\sin e^x}$
(C) $\sec^2(e^x)$
(D) $e^x \sec^2 x + (\tan x)e^x$
(E) $e^x \tan(e^x)$

29. Which of the following statements about the function $f(x) = 3x^4 - 4x^3$ is true?

(A) f has no relative extrema and no inflection points
(B) f has one relative maximum value, one relative minimum value, and one inflection point
(C) f has one relative maximum value and two inflection points
(D) f has one relative minimum value and two inflection points
(E) f has three inflection points

30. $\int \cot^2 x \csc^2 x dx =$

(A) $-\dfrac{\cot^3 x}{3}+C$
(B) $-\dfrac{\csc^3 x}{3}+C$
(C) $\dfrac{\cot^3 x}{3}+C$
(D) $\dfrac{\csc^3 x}{3}+C$
(E) $\dfrac{\cot^2 x}{2}+C$

31. Let $s(t)$ be a differentiable function that is positive and increasing with respect to time t. For what value, $s(t)$, is the rate of increase of the volume of a cube with edge s equal to 48 times the rate of increase in s? (The volume of a cube with edge s is $V = s^3$.)

(A) 4
(B) $4\sqrt{3}$
(C) 16
(D) $16\sqrt{3}$
(E) 48

32. If $F(x) = \int_0^x \sin t\,dt$, then $F'(\frac{\pi}{6}) =$

(A) $-\frac{\sqrt{3}}{2}$

(B) $-\frac{1}{2}$

(C) $-\frac{\sqrt{3}}{2}+1$

(D) $\frac{1}{2}$

(E) $\frac{\sqrt{3}}{2}$

33. Let f be defined by $f(x) = \begin{cases} \frac{x^2-64}{x+8} \text{ for } x \neq 8 \\ 16 \text{ for } x = -8 \end{cases}$.

Which of the following statements about f are true?

I. $\lim_{x \to -8} f(x)$ exists.
II. $f(-8)$ exists.
III. f is continuous at $x = -8$.

(A) None of the above
(B) I only
(C) II only
(D) I and II only
(E) I, II, and III

34. The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. If two subintervals of equal length are used, what is the right endpoint Riemann sum approximation of $\int_0^2 f(x)dx$?

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	4	6	5	7

(A) 5.2
(B) 9.0
(C) 10.0
(D) 12.5
(E) 13.0

35. What is the average rate of change of the function f defined by $f(x) = \cos x$ on the closed interval $[0, \pi]$?

□ (A) $-\frac{2}{\pi}$
(B) $-\frac{\pi}{2}$
(C) π
(D) $\frac{\pi}{2}$
(E) $\frac{2}{\pi}$

36. Let f be continuous at $x = a$. Which of the following statements is NOT necessarily true?

(A) $\lim_{x\to a} f(x) = f\left(\lim_{x\to a} x\right)$
(B) $f(a)$ exists
(C) $\lim_{x\to a} f(x)$ exists
(D) $\lim_{h\to 0} \frac{f(a+h)}{h}$ exists
(E) $\lim_{x\to a} \left(f(x) - f(a)\right) = 0$

37. A rectangular field is to be adjacent to a river having a straight-line bank. It is to have fencing on three sides, the side on the river bank requiring no fencing. If 100 yards of fencing is available, find the length of the side of the field parallel to the river that will yield maximum area.

(A) 25 yards
(B) 30 yards
(C) 40 yards
(D) 50 yards
(E) 60 yards

38. If $f(x) = \tan^2(\cot x)$, then $f'(x) =$

(A) $2\tan(\cot x)$
(B) $2\tan(\cot x)(-\csc^2 x)$
(C) $2\tan(\cot x)(\csc^2 x)$
(D) $2\tan(\cot x)(\sec^2(\cot x))\csc^2 x$
(E) $-2\tan(\cot x)(\sec^2(\cot x))\csc^2 x$

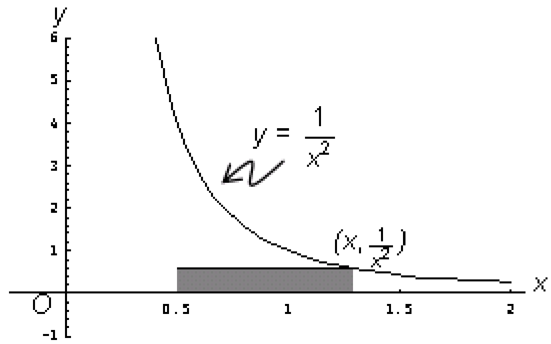
39. Let f be continuous at all points of the open interval (a, b), and let c be a point in (a, b) such that

I. f' exists at all points of (a, b), except possibly at *(C)*.
II. $f'(x) < 0$ for all values of x in some open interval having c as its right endpoint.
III. $f'(x) > 0$ for all values of x in some open interval having c as its left endpoint.

Which of the following statements must be true?

(A) $f'(c) = 0$
(B) $f'(c)$ does not exist
(C) f has a point of inflection at c
(D) f has a relative minimum value at c
(E) f has a relative maximum value at c

40.



A rectangle with one side on the x-axis and one side on the line $x = 0.5$ has its upper right vertex on the graph of $y = \frac{1}{x^2}$, as indicated in the figure above. For what value of x does the area of the rectangle attain its maximum value?

(A) 0.5
(B) 0.8
(C) 1.0
(D) 2.0
(E) ∞

41. Let $f(x) = x^3 + 2x^2 + 10x$. If h is the inverse function of f, then $h'(-9) =$

(A) -9
(B) $-\frac{1}{9}$
(C) -1
(D) $\frac{1}{9}$
(E) 9

42. If f is continuous for all x, which of the following integrals necessarily have the same value?

I. $\int_{-a}^{a} f(x)dx$

II. $2\int_{o}^{a} f(x)dx$

III. $\int_{0}^{2a} f(x)dx$

(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II, and III
(E) No two necessarily have the same value

43. Let $F(t)$ be the amount of an investment, in dollars, at time t, in years. Suppose that F is growing at a rate given by $F'(t) = (1000)e^{0.25t}$, where t is time in years. If $F(0) = 4000$, then $F(t) =$

(A) $(2000)e^{4t} + (2000)e^{4t}$
(B) $(4000)^{0.25t}$
(C) $(1000)e^{0.25t} + 4000$
(D) $(4000)e^{0.25t}$
(E) $(1000)e^{0.25t}$

44. The volume of a cube is increasing at a rate of 30 cubic feet per hour $\left(\frac{\text{ft}^3}{\text{h}}\right)$. At what rate is s, the edge of the cube, changing when the length of the edge is 5 feet?

(A) $0.17\frac{\text{ft}}{\text{h}}$

(B) $0.40\frac{\text{ft}}{\text{h}}$

(C) $2.00\frac{\text{ft}}{\text{h}}$

(D) $3.00\frac{\text{ft}}{\text{h}}$

(E) $6.00\frac{\text{ft}}{\text{h}}$

45. The Riemann sum $\sum_{i=1}^{100} \frac{1}{4}\left(\frac{i}{100}\right)^3 \frac{1}{100}$ on the closed interval [0, 1] is an approximation for which of the following definite integrals?

(A) $\int_1^{100} x^3 dx$

(B) $\int_0^2 x^3 dx$

(C) $\int_0^1 \frac{x^3}{4} dx$

(D) $\int_0^1 x^3 dx$

(E) $\int_1^{100} \left(\frac{x}{100}\right)^3 dx$

CLEP Calculus

1. **The correct answer is E.** From the definition for the limit you know that $\lim_{x\to -5} \frac{7}{x+5}$ can exist only if $\frac{7}{x+5}$ approaches a single finite value as x approaches −5 from both the left and right. As x approaches −5 from the left, the number $\frac{7}{x+5}$ is decreasing without bound; symbolically, you indicate this behavior by writing $\lim_{x\to -5^-} \frac{7}{x+5} = -\infty$. As x approaches −5 from the right, the number $\frac{7}{x+5}$ is increasing without bound; symbolically, you indicate this behavior by writing $\lim_{x\to -5^+} \frac{7}{x+5} = \infty$. Since $\frac{7}{x+5}$ does not approach a single finite value as x approaches −5 from both the left and right of −5, $\lim_{x\to -5} \frac{7}{x+5}$ is nonexistent.

2. **The correct answer is D.** If $f(x) = 3x^{\frac{1}{3}}$, then

$$\begin{aligned} f'(x) &= 3(\tfrac{1}{3})x^{\frac{1}{3}-1} \\ &= x^{-\frac{2}{3}}. \end{aligned}$$

3. **The correct answer is A.** The y-value when x is −2 is given by

$$\begin{aligned} f(2) &= (2)3 + (2)4 \\ &= 8. \end{aligned}$$

Since $f(x) = x^3 + x^4$, then $f'(x) = 3x^2 + 4x^2$. Thus, the slope of the tangent line at the point (−2, 8) is

$$\begin{aligned} f'(-2) &= 3(-2)^2 + 4(-2)^3 \\ &= -20. \end{aligned}$$

4. **The correct answer is D.**

$$\begin{aligned} \int \left(x + e + x^e + e^x\right) dx &= \int xdx + \int edx + \int x^e dx + \int e^x dx \\ &= \int xdx + e\int 1dx + \int x^e dx + \int e^x dx \\ &= \frac{x^2}{2} + ex + \frac{x^{e+1}}{e+1} + e^x + C. \end{aligned}$$

5. **The correct answer is E.** The graph of f is concave upward for values of x for which $f''(x) > 0$. Looking at the graph of f'', you can see that $f''(x)$ is positive on (−2, −1) and (1, 2). Therefore, the function f is concave upward on (−2, −1) and (1, 2).

6. **The correct answer is D.**

$$\begin{aligned}\int\left(x^2-\sqrt{x}\right)\sqrt{x}\;dx &= \int\left(x^2-x^{\frac{1}{2}}\right)x^{\frac{1}{2}}\;dx\\ &= \int\left(x^{\frac{5}{2}}-x\right)dx\\ &= \int x^{\frac{5}{2}}dx-\int x\;dx\\ &= \frac{x^{\frac{7}{2}}}{\frac{7}{2}}-\frac{x^2}{2}+C\\ &= \frac{2x^3\sqrt{x}}{7}-\frac{x^2}{2}+C.\end{aligned}$$

7. **The correct answer is A.** The slope of the tangent line at point *A* is negative, indicating that $\frac{dy}{dx}$ is negative at *A*. In a neighborhood about the point *A*, the slope of the tangent line is increasing and, thus, $\frac{dy}{dx}$ is increasing, indicating the curve is concave upward about the point *A*. Since $\frac{d^2y}{dx^2}$ is positive when the curve is concave upward, $\frac{dy}{dx}$ is negative, and $\frac{d^2y}{dx^2}$ is positive at point *A*. This condition does not occur at any of the other four points.

8. **The correct answer is E.** The $\frac{ds}{dt}$ is the velocity, *v*(*t*), of the particle; and $\frac{dv}{dt}$ is the acceleration, *a*(*t*), of the particle. Thus,

$$\begin{aligned}v(t) &= \int a(t)dt\\ &= \int(4t^3+1)dt\\ &= t^4+t+C_1.\end{aligned}$$

At $t = 0$, $v(t) = 0$.
Substituting and solving for C_1, you have

$$\begin{aligned}v(o) &= 0\\ 0^4+0+C_1 &= 0\\ C_1 &= 0.\end{aligned}$$

Now since $\frac{ds}{dt}$ is *v*(*t*),

$$\begin{aligned} s(t) &= \int v(t)dt \\ &= \int (t^4 + t)dt \\ &= \frac{t^5}{5} + \frac{t^2}{2} + C_2. \end{aligned}$$

At $t = 0$, $s(t) = 12$.
Substituting and solving for C_2, you have

$$\begin{aligned} s(0) &= 12 \\ \frac{(0)^5}{5} + \frac{(0)^2}{2} + C_2 &= 12 \\ 0 + 0 + C_2 &= 12 \\ C_2 &= 12. \end{aligned}$$

Therefore, the position function of the moving particle is $s(t) = \frac{t^5}{5} + \frac{t^2}{2} + 12.$

When $t = 1$,

$$\begin{aligned} s(1) &= \frac{(1)^5}{5} + \frac{(1)^2}{2} + 12. \\ &= 12.7. \end{aligned}$$

9. The correct answer is A. First, divide every term by the highest power of x, and then evaluate the limit. Thus,

$$\begin{aligned} \lim_{x \to \infty} \frac{3x^5 + 2x^3 + 5}{5x^5 + 3x^3 + 7} &= \lim_{x \to \infty} \frac{\frac{3x^5}{x^5} + \frac{2x^3}{x^5} + \frac{5}{x^5}}{\frac{5x^5}{x^5} + \frac{3x^3}{x^5} + \frac{7}{x^5}} \\ &= \frac{3 + \frac{2}{x^2} + \frac{5}{x^5}}{5 + \frac{3}{x^2} + \frac{7}{x^5}} \\ &= \frac{3}{5}. \end{aligned}$$

10. The correct answer is D. Note that $|x+1| = \begin{cases} x+1 \text{ if } x \geq -1 \\ -(x+1) \text{ if } x < -1 \end{cases}.$

Therefore, since the upper limit of the integral, –2, is less than –1,

$$\begin{aligned}\int_{-3}^{-2}|x+1|\,dx &= \int_{-3}^{-2}-(x+1)\ dx\\ &= -\int_{-3}^{-2}(x+1)\ dx\\ &= -\left(\frac{x^2}{2}+x\right)\Bigg|_{-3}^{-2}\\ &= -\left(\left(\frac{(-2)^2}{2}+(-2)\right)-\left(\frac{(-3)^2}{2}+(-3)\right)\right)\\ &= -\big((0)-(1.5)\big)=1.5.\end{aligned}$$

11. The correct answer is D. At $x = a$, the tangent line to f is parallel to the tangent line to g when $f'(a)=g'(a)$, that is, when $a^2 + a = 2a + 6$. Solving $a^2 + a = 2a + 6$ for a, you obtain

$$\begin{aligned}a^2+a&=2a+6\\ a^2-a-6&=0\\ (a+2)(a-3)&=0\\ a&=-2 \text{ (reject)}\\ &\text{or}\\ a&=3.\end{aligned}$$

12. The correct answer is C. The average value of a continuous function f over a closed interval $[a, b]$ is given by $\frac{1}{b-a}\int_a^b f(x)dx.$ Thus, the average value of f over the closed interval $[-1, 2]$ is

$$\begin{aligned}\frac{1}{2-(-1)}\int_{-1}^{2}\left(8x^3+2x+1\right)dx &= \frac{1}{3}\left(2x^4+x^2+x\right)\Big|_{-1}^{2}\\ &= \frac{1}{3}\left(\left(2\times2^4+2^2+2\right)-\left(2\times(-1)^4+(-1)^2+(-1)\right)\right)\\ &= \frac{1}{3}(38-2)\\ &= \frac{36}{3}\\ &= 12.\end{aligned}$$

13. The correct answer is D. Applying the rule for the derivative of the quotient of two differentiable functions, you have

$$
\begin{aligned}
f'(x) &= \frac{3x\left(\sec^2 x\right)-(\tan x)(3)}{(3x)^2} \\
&= \frac{3\left(x\sec^2 x-\tan x\right)}{9x^2} \\
&= \frac{x\sec^2 x-\tan x}{3x^2}.
\end{aligned}
$$

14. The correct answer is C. Using the chain rule for the derivative of a composite function, you have

$$
\begin{aligned}
h'(5) &= g'(f(5))\,f'(5) \\
&= g'(10)\,f'(5) \\
&= (15)(10) \\
&= 150.
\end{aligned}
$$

15. The correct answer is B.

$$
\begin{aligned}
\int_{-4}^{-2}\frac{1}{x+1}\,dx &= \ln|x+1|\Big|_{-4}^{-2} \\
&= \ln|-2+1|-\ln|-4+1| \\
&= \ln 1-\ln 3 \\
&= -\ln 3.
\end{aligned}
$$

16. The correct answer is E. Using $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ and selecting $f(x) = \tan x$ and $a=\frac{\pi}{3}$, you have

$$\begin{aligned}\lim_{h\to 0}\frac{\tan\left(\frac{\pi}{3}+h\right)-\tan\left(\frac{\pi}{3}\right)}{h} &= f'\left(\frac{\pi}{3}\right)\\ &= \sec^2\left(\frac{\pi}{3}\right)\\ &= \frac{1}{\cos^2\left(\frac{\pi}{3}\right)}\\ &= \frac{1}{\cos^2(60^\circ)}\\ &= \frac{1}{\left(\frac{1}{2}\right)^2}\\ &= 4.\end{aligned}$$

17. The correct answer is A. Since f is differentiable and has a relative maximum value of 5 at $x = -1$, you know that $f(-1) = 5$ and that $f'(-1) = 0$. Using the quotient rule for differentiation, you have

$$\begin{aligned}h'(x) &= \frac{x^3\left(f'(x)\right)-f(x)\left(3x^2\right)}{\left(x^3\right)^2}\\ &= \frac{xf'(x)-3f(x)}{x^4}.\end{aligned}$$

Thus,

$$\begin{aligned}h(-1) &= \frac{(-1)\,f'(-1)-3f(-1)}{(-1)^4}\\ & \frac{(-1)(0)-3(5)}{(-1)^4}\\ &= \frac{0-15}{1}\\ &= -15.\end{aligned}$$

18. The correct answer is B. From the figure, you can see that the graphs of f and g intersect at (1, 2) and (4, −1). Since $f(x) = -x + 3$ lies above $g(x) = x^3 - 6x + 7$ on the interval from $x = 1$ to $x = 4$, the area of the shaded region between the two graphs is given by

$$\begin{aligned}
\int_1^4 \left((-x+) - \left(x^3 - 6x + 7 \right) \right) dx &= \int_1^4 \left(-x^3 + 5x - 4 \right) dx \\
&= \left(-\frac{x^4}{4} + \frac{5x^2}{2} - 4x \right) \Bigg|_1^4 \\
&= \left(-\frac{(4)^3}{4} + \frac{5(4)^2}{2} - 4(4) \right) - \left(\frac{-(1)^4}{4} + \frac{5(1)^2}{2} - 4(1) \right) \\
&= -40 + \frac{7}{4} \\
&= -38.25.
\end{aligned}$$

19. The correct answer is B. The equation $x^5 + y^5 + xy = x^4$ is not easily solved for y in terms of x, so you should use the technique of implicit differentiation to find $\frac{dy}{dx}$. On the assumption that there is a differential function f defined implicitly by the equation $x^5 + y^5 + xy = x^4$, you have

$$\begin{aligned}
\frac{d}{dx}\left(x^5 y^5 + xy \right) &= \frac{d}{dx}\left(x^4 \right) \\
\left(x^5 \left(5y^4 \frac{dy}{dx} \right) + y^5 \left(5x^4 \right) \right) + \left(x \times \frac{dy}{dx} + y \times 1 \right) &= 4x^3 \\
5x^5 y^4 \frac{dy}{dx} + 5x^4 y^5 + x \frac{dy}{dx} + y &= 4x^3 \\
\frac{dy}{dx}\left(5x^5 y^4 + x \right) &= 4x^3 - 5x^4 y^5 - y \\
\frac{dy}{dx} &= \frac{4x^3 - 5x^4 y^5 - y}{5x^5 y^4 + x}.
\end{aligned}$$

20. The correct answer is E. From the definition for the limit you know that $\lim_{x \to 1} f(x)$ can exist only if $f(x)$ approaches a single finite value as x approaches 1 from both the left and right. As x approaches 1 from the left, the graph of f approaches the point (1, 3), so $\lim_{x \to 1^-} f(x) = 3;$ but as x approaches 1 from the right, the graph of f approaches the point (1, 2), so $\lim_{x \to 1^+} f(x) = 2.$ Because $f(x)$ approaches two different values as x approaches 1, $\lim_{x \to 1} f(x)$ does not exist.

21. The correct answer is A. Since $\lim_{x \to \infty} \ln x = \infty$ and $\lim_{x \to \infty} e^x = \infty,$ you can use L'Hôpital's rule to obtain

$$\begin{aligned}\lim_{x\to\infty}\frac{\ln x}{e^x} &= \lim_{x\to\infty}\frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}\left(e^x\right)}\\ &= \lim_{x\to\infty}\frac{\frac{1}{x}}{e^x}\\ &= \lim_{x\to\infty}\frac{1}{xe^x}\\ &= 0.\end{aligned}$$

22. The correct answer is E. Since $f(x)$ is nonpositive and strictly increasing on the closed interval $[a, b]$, then when $a < x < b$,

$$\begin{aligned}&f(x) < 0\\ &\int_a^b f(x)dx < \int_a^b 0dx = 0\\ &\int_a^b f(x)dx < 0.\end{aligned}$$

The statements given in the other answer choices are true statements based on the properties of f as given in the question stem.

23. The correct answer is E. Since $\int x^3 e^{x^4+8}\ dx$ cannot be integrated directly from commonly known formulas, use the technique of substitution to evaluate the integral. Let $u = x^4 + 8$ and, thus,

$$\begin{aligned}du &= 4x^3 dx\\ \frac{1}{4}du &= x^3 dx.\end{aligned}$$

Then, you have

$$\begin{aligned}\int x^3 e^{x^4+8}\ dx &= \frac{1}{4}\int e^u du\\ &= \frac{1}{4}e^u + C\\ &= \frac{1}{4}e^{x^4+8} + C.\end{aligned}$$

24. The correct answer is D. The average rate of change of f over $[-1, 1]$ is given by

$$\frac{f(1)-f(-1)}{(1)-(-1)}=\frac{f(1)-0}{2}$$
$$=\frac{1}{2}f(1),$$

which you can estimate, by looking at the graph, as about

$$\frac{1}{2}(1.5)=0.75$$
$$>0.$$

Geometrically, the average rate of change of f over $[-1, 1]$ is the slope of the line connecting the points $(-1, 0)$ and $(1, f(1))$. Since the derivative, $f'(x)$, of f at a point x equals the slope of the tangent line at the point x, you can work this problem by comparing the slope of the tangent line at each of the indicated points to the slope of the line connecting the points $(-1, 0)$ and $(1, f(1))$.

You can eliminate point *C* from consideration because the slope of the tangent line at *C* is zero, so the derivative of f at *C* would not equal the average rate of change of f over $[-1, 1]$.

You can also eliminate points *A* and *B* from consideration because the slope of the tangent line at each of these points is negative, so the derivative of f evaluated at either of these points would not equal the average rate of change of f over $[-1, 1]$.

Looking at points *D* and *E* and considering their tangent lines, you can see that the tangent line at point *D* would more likely be parallel to the line connecting the points $(-1, 0)$ and $(1, f(1))$ than the tangent line at point *E*, which is too steep in comparison, indicating that the derivative at *D* could be equal to the average rate of change of f over $[-1, 1]$.

25. The correct answer is E. This question is an example of a multiple response set question. An efficient approach to working this type of question is to check Roman numeral options and eliminate answer choices, if possible, as you go along.
Checking Roman I, you have

$$
\begin{aligned}
y &= e^{-x} \\
\frac{dy}{dx} &= \frac{d\left(e^{-x}\right)}{dx} = -e^{-x} \\
\frac{d^2y}{dx^2} &= \frac{d\left(-e^{-x}\right)}{dx} = e^{-x} \\
\frac{d^3y}{dx^3} &= \frac{d\left(e^{-x}\right)}{dx} = -e^{-x} \\
\frac{d^4y}{dx^4} &= \frac{d\left(-e^{-x}\right)}{dx} = e^{-x}.
\end{aligned}
$$

Thus, $\frac{d^4y}{dx^4} \neq -\frac{d^2y}{dx^2}$, so eliminate choices A and D because these answer choices contain Roman I.

Checking Roman II, you have

$$
\begin{aligned}
y &= \cos x \\
\frac{dy}{dx} &= \frac{d(\cos x)}{dx} = -\sin x \\
\frac{d^2y}{dx^2} &= \frac{d(-\sin x)}{dx} = -\cos x \\
\frac{d^3y}{dx^3} &= \frac{d(-\cos x)}{dx} = \sin x \\
\frac{d^4y}{dx^4} &= \frac{d(\sin x)}{dx} = \cos x.
\end{aligned}
$$

Thus, $\frac{d^4y}{dx^4} = -\frac{d^2y}{dx^2}$, so eliminate choice C because this answer choice does not contain Roman II.

Checking Roman III, you have

$$
\begin{aligned}
y &= \sin x \\
\frac{dy}{dx} &= \frac{d(\sin)}{dx} = \cos x \\
\frac{d^2y}{dx^2} &= \frac{d(\cos)}{dx} = -\sin x \\
\frac{d^3y}{dx^3} &= \frac{d(-\sin x)}{dx} = -\cos x \\
\frac{d^4y}{dx^4} &= \frac{d(-\cos x)}{dx} = \sin x.
\end{aligned}
$$

Thus, $\frac{d^4y}{dx^4} = -\frac{d^2y}{dx^2}$, so eliminate choice B because this answer choice does not contain Roman III.

Choice E is correct because it includes every Roman numeral option that is correct and no incorrect Roman numeral options.

26. The correct answer is E. Since $s'(t)$ is the velocity, $v(t)$, you have

$$
\begin{aligned}
s'(t) &= v(t) \\
&= -32t + 256.
\end{aligned}
$$

Setting $v(t)$ equal to zero and solving for t you have

$$
\begin{aligned}
-32t + 256 &= 0 \\
-32t &= -256 \\
t &= 8
\end{aligned}
$$

seconds when the velocity is zero.
Evaluating $s(t)$ at $t = 8$ seconds, you obtain

$$
\begin{aligned}
s(8) &= -16(8)^2 + 256(8) + 50 \\
&= -1024 + 2048 + 50 \\
&= 1074
\end{aligned}
$$

feet, which is the height of the ball when the velocity is zero.

27. The correct answer is C. The function *f* is differentiable at $x = -2$ provided $\lim_{x \to -2} \frac{f(x) - f(-2)}{x - (-2)} = \frac{f(x) - f(-2)}{x + 2}$ exists. Since $f'(2) = 8,$ the statement given in *C* must be true.

The statement given in A is false.

The question stem fails to provide sufficient information for determining whether the statements given in the other answer choices are true or false.

28. The correct answer is A. Using the chain rule for the derivative of a composite function and letting $f(u) = \tan u$ and $u = e^x$, you have

$$\begin{aligned} \frac{d(f(u))}{dx} &= \frac{d(f(u))}{du} \times \frac{du}{dx} \\ &= \frac{d(\tan u)}{du} \times \frac{d(e^x)}{dx} \\ &= \left(\sec^2(e^x)\right)(e^x) \\ &= e^x \sec^2(e^x). \end{aligned}$$

29. The correct answer is D. To investigate relative extrema for *f*, first identify potential critical points for the function by solving $f'(x) = 0.$

Since $f'(x) = 12x^3 - 12x^2,$ you have:

$$\begin{aligned} 12x^3 - 12x^2 &= 0 \\ 12x^2(x-1) &= 0, \end{aligned}$$

and $x = 0$ and $x = 1$ are critical numbers for *f*.

The second derivative of *f* is given by $f''(x) = 36x^2 - 24x$. Since

$$\begin{aligned} f''(1) &= 36(1)^2 - 24(1) \\ &= 12 > 0, \end{aligned}$$

f has a minimum at $x = 1$.

Notice that you can eliminate choice A at this point. This is a smart test-taking strategy that you should use when taking the actual test.

Continuing with the analysis, evaluate $f''(x)$ at $x = 0$ to obtain

$$\begin{aligned} f''(0) &= 36(0)^2 - 24(0) \\ &= 0, \end{aligned}$$

and the second-derivative test fails at $x = 0$.

Since the second-derivative test fails at $x = 0$, using the first derivative test,

$$\begin{aligned} f'(0.5) &= 12(0.5)^3 - 12(0.5)^2 \\ &= -1.5 \end{aligned}$$

and

$$\begin{aligned} f'(-0.5) &= 12(-0.5)^3 - 12(-0.5)^2 \\ &= -4.5, \end{aligned}$$

and $f(0)$ is not a relative extrema for f. Eliminate choices B and C.

Now, investigate f for points of inflection. Points of inflection for f can occur only at numbers where $36x^2 - 24x = 12x(3x-2) = 0$; therefore $x = 0$ and $x = \frac{2}{3}$.

Thus, $(0,f(0))$ and $(\frac{2}{3}, f\left(\frac{2}{3}\right))$ are potential points of inflection.

Testing for concavity to the left of 0 and between 0 and $\frac{2}{3}$,

$$\begin{aligned} f''(-1) &= 36(-1)^2 - 24(-1) \\ &= 60 > 0 \end{aligned}$$

and

$$\begin{aligned} f''(\tfrac{1}{2}) &= 36\left(\tfrac{1}{2}\right)^2 - 24\left(\tfrac{1}{2}\right) \\ &= -3 < 0. \end{aligned}$$

Therefore $(0,f(0))$ is a point of inflection.

Testing for concavity between 0 and $\frac{2}{3}$ and to the right of $\frac{2}{3}$,

$$\begin{aligned} f''(\tfrac{1}{2}) &= 36\left(\tfrac{1}{2}\right)^2 - 24\left(\tfrac{1}{2}\right) \\ &= -3 < 0 \end{aligned}$$

and

$$\begin{aligned} f''(1) &= 36(1)^2 - 24(1) \\ &= 12 > 0. \end{aligned}$$

So $\left(\frac{2}{3}, f\left(\frac{2}{3}\right)\right)$ is a point of inflection.

Thus, f has one relative minimum value and two inflection points.

30. The correct answer is A. Since $\int \cot^2 x \csc^2 x dx$ cannot be integrated directly from commonly known formulas, use the technique of substitution to evaluate the integral. Let $u = \cot x$ and, thus, $du = -\csc^2 x dx$. Then, you have

$$\begin{aligned} \int \cot^2 x \csc^2 x dx &= -\int \cot^2 x\left(-\csc^2 x dx\right) \\ &= -\int u^2 du \\ &= -\frac{u^3}{3} + C \\ &= -\frac{\cot^3 x}{3} + C. \end{aligned}$$

31. The correct answer is A. The instantaneous rate of change of any function (usually called simply the rate of change) is given by the first derivative of the function. Since $V = s^3$, you have

$$\begin{aligned} V(t) &= \left(s(t)\right)^3 \\ \frac{dV}{dt} &= 3\left(s(t)\right)^2\left(s'(t)\right), \end{aligned}$$

which is the rate of change of V. The rate of change of $s(t)$ is $s'(t)$.

Now, setting $V'(t) = 48s'(t)$, you have $3\left(s(t)\right)^2\left(s'(t)\right) = 48s'(t)$. You can divide both sides of this equation by $s'(t)$ since $s'(t) \neq 0$, given $s(t)$ is a differentiable function that is positive and increasing. Thus,

$$\begin{aligned} 3\left(s(t)\right)^2 &= 48 \\ \left(s(t)\right)^2 &= 16 \\ s(t) &= 4. \end{aligned}$$

(The negative root is extraneous since this value would not make sense in the physical world.)

32. The correct answer is D. $\int_0^x \sin t\,dt$ is an integral with a variable upper limit. The following theorem about integrals with variable upper limits holds: If f is a function that is continuous on the closed interval $[a, b]$, and $F(x) = \int_a^x f(t)dt$, then

$$\begin{aligned} F'(x) &= \frac{d}{dx}\int_a^x f(t)dt \\ &= f(x) \end{aligned}$$

for all x in $[a, b]$. Thus,

$$\begin{aligned} F(x) &= \int_0^x \sin t\,dt \\ F'(x) &= \frac{d}{dx}\int_0^x \sin t\,dt = \sin x \\ F'\left(\frac{\pi}{6}\right) &= \sin\left(\frac{\pi}{6}\right) \\ &= \sin 30^\circ \\ &= \frac{1}{2}. \end{aligned}$$

33. The correct answer is D. This question is an example of a multiple response set question. An efficient approach to working this type of question is to check Roman numeral options and eliminate answer choices, if possible, as you go along.

Checking Roman I, you have

$$\begin{aligned}\lim_{x\to -8} f(x) &= \lim_{x\to -8}\frac{x^2-64}{x+8}\\ &= \lim_{x\to -8}\frac{(x+8)(x-8)}{x+8}\\ &= \lim_{x\to -8}(x-8)\\ &= -16.\end{aligned}$$

Thus, the statement in Roman I is true. Eliminate choices A and C because these answer choices do not contain Roman I.

Checking Roman II, you have by the definition of f that $f(-8) = 16$, so Roman II is true. Eliminate choice B because this answer choice does not contain Roman II.

Checking Roman III, since $\lim\limits_{x\to -8} f(x) = -16$ and $f(-8) = +16$, f is not continuous at $x = -8$. Thus, Roman III is false, so eliminate choice E because this answer choice contains Roman III.

This leaves choice D as the correct answer because it includes every Roman numeral option that is correct and no incorrect Roman numeral options.

34. The correct answer is E. If f is continuous on a closed interval $[a, b]$, a Riemann sum for $\int_a^b f(x)dx$, using n subintervals of equal length, has the form $\sum\limits_{i=1}^{n} f(\varepsilon_i)\Delta x_i$, where $\Delta x_i = \dfrac{b-a}{n}$ and $\{\varepsilon_i \mid x_{i-1} \le \varepsilon_i \le x_i, i = 1, 2, \ldots, n\}$ is an associated network of points such that the point ε_i can be any point in the subinterval; but, for convenience, can consist of the left endpoints of each subinterval, or the right endpoints of each subinterval, or the midpoints of each subinterval, or so on.
For $n = 2$ subintervals,

$$\begin{aligned}\Delta x_i &= \frac{2-0}{2}\\ &= 1.\end{aligned}$$

Thus, using ε_i = the right endpoint of the ith subinterval,

$$\begin{aligned}\int_0^2 f(x)dx &\doteq \left[f(1.0)(1) + f(2.0)(1)\right]\\ &= \left[6(1) + 7(1)\right]\\ &= 13,\end{aligned}$$

where $\doteq$ means "is approximately equal to."

35. The correct answer is A. The average rate of change of a function *f* over an interval [*a*, *b*] is given by $\frac{f(b)-f(a)}{b-a}$. Thus, the average rate of change of *f*(*x*) = cos *x* over the interval [0, π] equals

$$\begin{aligned}\frac{f(\pi)-f(0)}{\pi-0} &= \frac{\cos\pi-\cos 0}{\pi-0}\\ &= \frac{-1-1}{\pi}\\ &= -\frac{2}{\pi}.\end{aligned}$$

36. The correct answer is D. The statement given in choice D might be true for some functions *f* as defined in the question stem, but it is NOT necessarily true for <u>all</u> functions satisfying the conditions given. For instance, a counterexample is the function *f* defined by $f(x)=|x|$, which is continuous at *x* = 0. You have

$$\begin{aligned}\lim_{h\to 0}\frac{f(a+h)}{h} &= \lim_{h\to 0}\frac{|a+h|}{h}\\ &= \lim_{h\to 0}\frac{|h|}{h}.\end{aligned}$$

Consider the limit *h* as it approaches zero from each direction:

$$\begin{aligned}\lim_{h\to 0^-}\frac{|h|}{h} &= \lim_{h\to 0^-}\frac{-h}{h}\\ &= -1.\end{aligned}$$

But

$$\lim_{h\to 0^+}\frac{|h|}{h} = \lim_{h\to 0^+}\frac{h}{h} = 1$$

$$\lim_{h\to 0^-}\frac{|h|}{h} \neq \lim_{h\to 0^+}\frac{|h|}{h},$$

and $\lim_{h\to 0}\frac{|h|}{h}$ does not exist.

The statements in the other answer choices are always true for the function *f* as defined in the question stem.

37. The correct answer is D. Let the two sides of the field that are perpendicular to the river each have length of x yards. Then the side parallel to the river has length 100 yards – $2x$, since the sum of the lengths of the three sides is 100 yards. The area of the field is given by the function f, where

$$\begin{aligned} f(x) &= x(100-2x) \\ &= 100-2x^2 \end{aligned}$$

(omitting units for convenience). To identify potential relative extrema for this function, find values of x for which $f'(x)=0$. Thus, you have

$$\begin{aligned} f'(x) = 100-4x &= 0 \\ -4x &= -100 \\ x &= 25 \text{ yards.} \end{aligned}$$

Using the second-derivative test and $f''(x) = -4$, a relative maximum occurs at $x = 25$ yards because

$$\begin{aligned} f''(25) &= -4 \\ &< 0. \end{aligned}$$

The length of the side parallel to the river that will yield maximum area is

$$\begin{aligned} 100 \text{ yards} - 2(25) \text{ yards} &= 100 \text{ yards} - 50 \text{ yards} \\ &= 50 \text{ yards.} \end{aligned}$$

38. The correct answer is E. Using the chain rule and letting $f(u) = u^2$, $u + \tan v$, and $v + \cot x$, you have

$$\begin{aligned} \frac{d(f(u))}{dx} &= \frac{d(f(u))}{du} \times \frac{du}{dv} \times \frac{dv}{dx} \\ &= \frac{d(u^2)}{du} \times \frac{d(\tan v)}{dv} \times \frac{d(\cot x)}{dx} \\ &= (2u)(\sec^2 v)(-\csc^2 x) \\ &= (2\tan v)(\sec^2 v)(-\csc^2 x) \\ &= (2\tan(\cot x))(\sec^2(\cot x))(-\csc^2 x) \\ &= -2\tan(\cot x)(\sec^2(\cot x))\csc^2 x. \end{aligned}$$

39. The correct answer is D. By the first derivative test for relative extrema, *f* has a relative minimum value at *c*. The first derivative test for relative extrema states that if *f* is continuous and its first derivative changes sign from negative to positive as *x* increases through the number *c*, then *f* has a relative minimum at *c*.

Since *c* is a critical number for *f*, you know that *either* $f'(c)=0$ (choice A) *or* $f'(c)$ does not exist (choice B). However, the question stem does not provide sufficient information to determine which must be true.

The statements given in choices C and E are false.

40. The correct answer is C. The area of the rectangle is given by

$$\begin{aligned} A(x) &= (x-0.5)\left(\frac{1}{x^2}\right) \\ &= (x-0.5)x^{-2} \\ &= x^{-1}-0.5x^{-2}. \end{aligned}$$

To identify a maximum for this function, find values of *x* for which $A'(x)=0$. Thus, you have

$$\begin{aligned} A'(x) &= -x^{-2}+x^{-3} \\ &= 0 \end{aligned}$$

and

$$\begin{aligned} x^{-3}\left(-x^1+1\right) &= x^{-3}(-x+1) \\ &= 0. \end{aligned}$$

Therefore, $x = 1$ or $x - 3 = 0$ (no solution).

Using the second-derivative test and $A''(x)=2x^{-3}-3x^{-4}$, a relative maximum occurs at $x = 1$ because

$$\begin{aligned} A''(1) &= 2(1)^{-3}-3(1)^{-4} \\ &= -1<0. \end{aligned}$$

41. The correct answer is D. The standard formula for the derivative of an inverse is given by $\left(f^{-1}\right)'(x)=\dfrac{1}{f'\left(f^{-1}(x)\right)}$. Thus, for this problem, we have

$$h'(x) = \frac{1}{f'(h(x))}$$
$$h'(-9) = \frac{1}{f'(h(-9))}.$$

Since $f'(x) = 3x^2 + 4x + 10,$ we have $h'(-9) = \dfrac{1}{3(h(-9))^2 + 4(h(-9)) + 10}.$

Since h and f are inverses, then $f(h(-9)) = -9$.
Therefore, by solving $f(x) = -9$ for x, you can determine $h(-9)$.
Proceeding in this manner, you have

$$x^3 + 2x^2 + 10x = -9$$
$$x^3 + 2x^2 + 10x + 9 = 0$$
$$(x+1)(x^2 + x + 9) = 0,$$

which has one real solution: $x = -1$. Thus, $h(-9) = -1$, and

$$h'(-9) = \frac{1}{3(-1)^2 + 4(-1) + 10}$$
$$= \frac{1}{9}.$$

42. The correct answer is E. This question is an example of a multiple response set question with the added feature that you must consider two or more Roman numeral options at the same time.
Checking Romans I and II: the integrals given in these two options might have the same value for some continuous functions, but not for <u>all</u> continuous functions. For example, a counterexample is the function f defined by $f(x) = x$. For this function, you have

$$\int_{-a}^{a} x\,dx = \frac{1}{2}\left(a^2 - a^2\right)$$
$$= 0,$$

But

$$2\int_0^a xdx = 2\times\frac{1}{2}\left(a^2 - 0^2\right)$$
$$= a^2.$$

Thus, in general, $\int_{-a}^{a} f(x)dx \neq 2\int_{o}^{a} f(x)dx$, so you can eliminate choices A and D.

Checking Romans I and III: the integrals given in these two options might have the same value for some continuous functions, but not for <u>all</u> continuous functions. For instance, for $f(x) = x$, you have

$$\int_{-a}^{a} xdx = \frac{1}{2}\left(a^2 - a^2\right)$$
$$= 0,$$

But

$$\int_0^{2a} xdx = \frac{1}{2}\left(4a^2 - 0^2\right)$$
$$= 2a^2.$$

Thus, in general,

$$\int_{-a}^{a} f(x)dx \neq \int_0^{2a} f(x)dx$$

so you can eliminate choice B.

Checking Romans II and III: for $f(x) = x$, since

$$2\int_0^a xdx = a^2$$
$$\neq 2a^2$$
$$= \int_0^{2a} xdx,$$

so you can eliminate choice C.

43. The correct answer is D. You need to find an antiderivative for F that satisfies the conditions: $F'(t) = (1000)e^{0.25t}$ and $F(0) = 4000$. An efficient way to work this problem is to check the answer choices, keeping in mind that while a function has at most one derivative, it may have many antiderivatives.

Therefore, you should check all the answer choices to make sure you have the correct antiderivative. Since the math is easy to do (in fact, you can do it mentally when taking the test) start off by checking $F(0) = 4000$ for each of the answer choices.

Checking A: using $F(t) = (2000)e^{4t} + (2000)e^{4t}$, you have

$$\begin{aligned} F(0) &= (2,000)e^0 + (2,000)e^0 \\ &= (2,000)\times 1 + (2,000)\times 1 \\ &= 4000. \end{aligned}$$

Thus, the function given in A satisfies $F(0) = 4000$.

Checking B: using $F(t) = (4000)^{0.25t}$, you have

$$\begin{aligned} F(0) &= (4000)^0 \\ &= 1 \\ &\neq 4000. \end{aligned}$$

Thus, the function given in B fails to satisfy $F(0) = 4000$, so eliminate B.

Checking C: using $F'(t) = (1000)e^{0.25t} + 4000$, you have

$$\begin{aligned} F(0) &= (1000)e^0 + 4000 \\ F(0) &= (1000)\times 1 + 4000 \\ &= 5000 \\ &\neq 4000. \end{aligned}$$

Thus, the function given in C fails to satisfy $F(0) = 4000$, so eliminate C.

Checking D: using $F(t) = (4000)e^{0.25t}$ you have

$$\begin{aligned} F(0) &= (4000)e^0 \\ &= (4000)\times 1 \\ &= 4000. \end{aligned}$$

Thus, the function given in D satisfies $F(0) = 4000$.

Checking E: using $F(x) = (1000)e^{0.25t}$ you have

$$\begin{aligned} F(0) &= (1000)e^{0} \\ &= (1000)\times 1 \\ &= 1000 \\ &\neq 4000. \end{aligned}$$

Thus, the function given in E fails to satisfy $F(0) = 4000$, so eliminate choice E.

Now check each of the remaining answer choices for $F'(t) = (1000)e^{0.25t}$.
Checking A: using $F(t) = (2000)e^{4t} + (2000)e^{4t}$, you have

$$\begin{aligned} F'(t) &= (2000)4e^{4t} + (2000)4e^{4t} \\ &= (8000)e^{4t} + (8000)e^{4t} \\ &= (16,000)e^{4t} \\ &\neq 1000e^{0.25t}. \end{aligned}$$

Since the function given in A fails to satisfy the condition $F'(t) = (1000)e^{0.25t}$, you can eliminate A.
Of course, at this point, you know that choice D is the correct answer since it is the only remaining answer choice. In a test situation, you should move on to the next question since you have determined the correct answer. You would not have to continue working out the problem, but for your information, for $F(t) = (4000)e^{0.25t}$,

$$\begin{aligned} F'(t) &= (4000)0.25e^{0.25t} \\ &= (1000)e^{0.25t}. \end{aligned}$$

44. The correct answer is B. The volume of the cube is given by $V(t) = (s(t))^3$. The rate of change of the volume is given by $V'(t) = 3(s(t))^2 s'(t)$. Thus, $s'(t)$, the rate of change of the edge of the cube, is given by $s'(t) = \dfrac{V'(t)}{3(s(t))^2}$. When $V'(t) = 30\dfrac{\text{ft}^3}{\text{h}}$ and $s(t) = 5$ feet, you have

$$s'(t) = \frac{30\frac{\text{ft}^3}{\text{h}}}{3(5\text{ ft})^2}$$

$$= \frac{30\frac{\text{ft}^3}{\text{h}}}{75\text{ ft}^2}$$

$$= 0.40\frac{\text{ft}}{\text{h}}.$$

45. The correct answer is C. If f is continuous on a closed interval $[a, b]$, a Riemann sum for $\int_a^b f(x)dx$, using n subintervals of equal length, has the form $\sum_{i=1}^{n} f(\varepsilon_i)\Delta x_i$, where $\Delta x_i = \frac{b-a}{n}$ and $\{\varepsilon_i \mid x_{i-1} \le \varepsilon_i \le x_i, i = 1, 2, \ldots, n\}$ is an associated network of points such that the point ε_i can be any point in the subinterval; but, for convenience, can consist of the left endpoints of each subinterval, or the right endpoints of each subinterval, or the midpoints of each subinterval, or so on. Since you have to match the Riemann sum given in the question to one of the integrals given in the answer choices, an efficient way to work this problem is to check the answer choices—a smart test-taking strategy for multiple-choice math tests. Using this strategy, you can eliminate choices A and E because the limits for the definite integral on the closed interval [0, 1] are 0 and 1, not 1 and 100.

Checking B: using $f(x) = x^4$, for 100 subintervals,

$$\Delta x_i = \frac{1-0}{100}$$

$$= \frac{1}{100}.$$

Using the right endpoints of each subinterval for $\{\varepsilon_i\}$, you have

$$\begin{aligned}
\varepsilon_1 &= \frac{1}{100}\\
\varepsilon_2 &= 2\left(\frac{1}{100}\right) = \frac{2}{100}\\
\varepsilon_3 &= 3\left(\frac{1}{100}\right) = \frac{3}{100}\\
&\dots\\
\varepsilon_i &= i\left(\frac{1}{100}\right) = \frac{i}{100}\\
&\dots\\
\varepsilon_{100} &= 100\left(\frac{1}{100}\right) = 1.
\end{aligned}$$

Thus, $f(\varepsilon_i) = \left(\frac{i}{100}\right)^4$, yielding the Riemann sum $\sum_{i=1}^{100}\left(\frac{i}{100}\right)^4\frac{1}{100}$. You can see that changing $\{\varepsilon_i\}$ to use the left endpoints, the midpoints, or any other point in each subinterval will not result in $\sum_{i=1}^{100}\frac{1}{4}\left(\frac{i}{100}\right)^3\frac{1}{100}$, so you can eliminate B.

Checking C, using $f(x) = \frac{x^3}{4}$, for 100 subintervals,

$$\begin{aligned}
\Delta x_i &= \frac{1-0}{100}\\
&= \frac{1}{100}.
\end{aligned}$$

Using the right endpoints of each subinterval for $\{\varepsilon_i\}$,

$$\begin{aligned}
f(\varepsilon_i) &= \frac{\left(\frac{i}{100}\right)^3}{4}\\
&= \frac{1}{4}\left(\frac{i}{100}\right)^3,
\end{aligned}$$

yielding the Riemann sum $\sum_{i=1}^{100} \frac{1}{4}\left(\frac{i}{100}\right)^3 \frac{1}{100}$. Therefore, choice C is the correct answer. It is not necessary to check the other answer choice since you have obtained the correct answer.