Contents

About the Author

Kenneth H. Rosen has had a long career as a Distinguished Member of the Technical Staff at AT&T Laboratories in Monmouth County, New Jersey. He currently holds the position of visiting research professor at Monmouth University where he is working on the security and privacy aspects of the Rapid Response Database Project and where he is teaching a course on cryptographic applications.

Dr. Rosen received his B.S. in Mathematics from the University of Michigan, Ann Arbor (1972), and his Ph.D. in Mathematics from M.I.T. (1976), where he wrote his thesis in the area of number theory under the direction of Harold Stark. Before joining Bell Laboratories in 1982, he held positions at the University of Colorado, Boulder; The Ohio State University, Columbus; and the University of Maine, Orono, where he was an associate professor of mathematics. While working at AT&T Labs, he taught at Monmouth University, teaching courses in discrete mathematics, coding theory, and data security.

Dr. Rosen has published numerous articles in professional journals in the areas of number theory and mathematical modeling. He is the author of the textbooks *Elementary Number Theory and Its Applications*, published by Addison-Wesley and currently in its fifth edition, and *Discrete Mathematics and Its Applications*, published by McGraw-Hill and currently in its sixth edition. Both of these textbooks have been used extensively at hundreds of universities throughout the world. *Discrete Mathematics and Its Applications* has sold more than 300,000 copies in its lifetime with translations into Spanish, French, Chinese, and Korean. He is also co-author of *UNIX: The Complete Reference*; *UNIX System V Release 4: An Introduction*; and *Best UNIX Tips Ever*, each published by Osborne McGraw-Hill. These books have sold more than 150,000 copies with translations into Chinese, German, Spanish, and Italian. Dr. Rosen is also the editor of the *Handbook of Discrete and Combinatorial Mathematics*, published by CRC Press, and he is the advisory editor of the CRC series of books in discrete mathematics, consisting of more than 30 volumes on different aspects of discrete mathematics, most of which are introduced in this book. He is also interested in integrating mathematical software into the educational and professional environments, and has worked on projects with Waterloo Maple Inc.'s Maple software in both these areas.

At Bell Laboratories and AT&T Laboratories, Dr. Rosen worked on a wide range of projects, including operations research studies and product line planning for computers and data communications equipment. He helped plan AT&T's products and services in the area of multimedia, including video communications, speech recognition and synthesis, and image networking. He evaluated new technology for use by AT&T and did standards work in the area of image networking. He also invented many new services, and holds or has submitted more than 70 patents. One of his more interesting projects involved helping evaluate technology for the AT&T attraction at EPCOT Center.

Preface

In writing this book, I was guided by my long-standing experience and interest in teaching discrete mathematics. For the student, my purpose was to present material in a precise, readable manner, with the concepts and techniques of discrete mathematics clearly presented and demonstrated. My goal was to show the relevance and practicality of discrete mathematics to students, who are often skeptical. I wanted to give students studying computer science all of the mathematical foundations they need for their future studies. I wanted to give mathematics students an understanding of important mathematical concepts together with a sense of why these concepts are important for applications. And most importantly, I wanted to accomplish these goals without watering down the material.

For the instructor, my purpose was to design a flexible, comprehensive teaching tool using proven pedagogical techniques in mathematics. I wanted to provide instructors with a package of materials that they could use to teach discrete mathematics effectively and efficiently in the most appropriate manner for their particular set of students. I hope that I have achieved these goals.

I have been extremely gratified by the tremendous success of this text. The many improvements in the sixth edition have been made possible by the feedback and suggestions of a large number of instructors and students at many of the more than 600 schools where this book has been successfully used. There are many enhancements in this edition. The companion website has been substantially enhanced and more closely integrated with the text, providing helpful material to make it easier for students and instructors to achieve their goals.

This text is designed for a one- or two-term introductory discrete mathematics course taken by students in a wide variety of majors, including mathematics, computer science, and engineering. College algebra is the only explicit prerequisite, although a certain degree of mathematical maturity is needed to study discrete mathematics in a meaningful way.

Goals of a Discrete Mathematics Course

A discrete mathematics course has more than one purpose. Students should learn a particular set of mathematical facts and how to apply them; more importantly, such a course should teach students how to think logically and mathematically. To achieve these goals, this text stresses mathematical reasoning and the different ways problems are solved. Five important themes are interwoven in this text: mathematical reasoning, combinatorial analysis, discrete structures, algorithmic thinking, and applications and modeling. A successful discrete mathematics course should carefully blend and balance all five themes.

1. *Mathematical Reasoning:* Students must understand mathematical reasoning in order to read, comprehend, and construct mathematical arguments. This text starts with a discussion of mathematical logic, which serves as the foundation for the subsequent discussions of methods of proof. Both the science and the art of constructing proofs are addressed. The technique of mathematical induction is stressed through many different types of examples of such proofs and a careful explanation of why mathematical induction is a valid proof technique.
2. *Combinatorial Analysis:* An important problem-solving skill is the ability to count or enumerate objects. The discussion of enumeration in this book begins with the basic techniques of counting. The stress is on performing combinatorial analysis to solve counting problems and analyze algorithms, not on applying formulae.

3. *Discrete Structures:* A course in discrete mathematics should teach students how to work with discrete structures, which are the abstract mathematical structures used to represent discrete objects and relationships between these objects. These discrete structures include sets, permutations, relations, graphs, trees, and finite-state machines.
4. *Algorithmic Thinking:* Certain classes of problems are solved by the specification of an algorithm. After an algorithm has been described, a computer program can be constructed implementing it. The mathematical portions of this activity, which include the specification of the algorithm, the verification that it works properly, and the analysis of the computer memory and time required to perform it, are all covered in this text. Algorithms are described using both English and an easily understood form of pseudocode.
5. *Applications and Modeling:* Discrete mathematics has applications to almost every conceivable area of study. There are many applications to computer science and data networking in this text, as well as applications to such diverse areas as chemistry, botany, zoology, linguistics, geography, business, and the Internet. These applications are natural and important uses of discrete mathematics and are not contrived. Modeling with discrete mathematics is an extremely important problem-solving skill, which students have the opportunity to develop by constructing their own models in some of the exercises.

Changes in the Sixth Edition

The fifth edition of this book has been used successfully at over 600 schools in the United States, dozens of Canadian universities, and at universities throughout Europe, Asia, and Oceania. Although the fifth edition has been an extremely effective text, many instructors, including longtime users, have requested changes designed to make this book more effective. I have devoted a significant amount of time and energy to satisfy these requests and I have worked hard to find my own ways to make the book better.

The result is a sixth edition that offers both instructors and students much more than the fifth edition did. Most significantly, an improved organization of topics has been implemented in this sixth edition, making the book a more effective teaching tool. Changes have been implemented that make this book more effective for students who need as much help as possible, as well as for those students who want to be challenged to the maximum degree. Substantial enhancements to the material devoted to logic, method of proof, and proof strategies are designed to help students master mathematical reasoning. Additional explanations and examples have been added to clarify material where students often have difficulty. New exercises, both routine and challenging, have been inserted into the exercise sets. Highly relevant applications, including many related to the Internet and computer science, have been added. The MathZone companion website has benefited from extensive development activity and now provides tools students can use to master key concepts and explore the world of discrete mathematics.

Improved Organization

- The first part of the book has been restructured to present core topics in a more efficient, more effective, and more flexible way.
- Coverage of mathematical reasoning and proof is concentrated in Chapter 1, flowing from propositional and predicate logic, to rules of inference, to basic proof techniques, to more advanced proof techniques and proof strategies.
- A separate chapter on discrete structures—Chapter 2 in this new edition—covers sets, functions, sequence, and sums.
- Material on basic number theory, covered in one section in the fifth edition, is now covered in two sections, the first on divisibility and congruences and the second on primes.
- The new Chapter 4 is entirely devoted to induction and recursion.

Logic

- Coverage of logic has been amplified with key ideas explained in greater depth and with more care.
- Conditional statements and De Morgan's laws receive expanded coverage.
- The construction of truth tables is introduced earlier and in more detail.
- More care is devoted to introducing predicates and quantifiers, as well as to explaining how to use and work with them.
- The application of logic to system specifications—a topic of interest to system, hardware, and software engineers—has been expanded.
- Material on valid arguments and rules of inference is now presented in a separate section.

Writing and Understanding Proofs

- Proof methods and proof strategies are now treated in separate sections of Chapter 1.
- An appendix listing basic axioms for real numbers and for the integers, and how these axioms are used to prove new results, has been added. The use of these axioms and basic results that follow from them has been made explicit in many proofs in the text.
- The process of making conjectures, and then using different proof methods and strategies to attack these conjectures, is illustrated using the easily accessible topic of tilings of checkerboards.
- Separate and expanded sections on mathematical induction and on strong induction begin the new Chapter 4. These sections include more motivation and a rich collection of examples, providing many examples different than those usually seen.
- More proofs are displayed in a way that makes it possible to explicitly list the reason for each step in the proof.

Algorithms and Applications

- More coverage is devoted to the use of strong induction to prove that recursive algorithms are correct.
- How Bayes' Theorem can be used to construct spam filters is now described.
- Examples and exercises from computational geometry have been added, including triangulations of polygons.
- The application of bipartite graphs to matching problems has been introduced.

Number Theory, Combinatorics, and Probability Theory

- Coverage of number theory is now more flexible, with four sections covering different aspects of the subject and with coverage of the last three of these sections optional.
- The introduction of basic counting techniques, and permutations and combinations, has been enhanced.
- Coverage of counting techniques has been expanded; counting the ways in which objects can be distributed in boxes is now covered.
- Coverage of probability theory has been expanded with the introduction of a new section on Bayes' Theorem.

Graphs and Theory of Computation

- The introduction to graph theory has been streamlined and improved.
- A quicker introduction to terminology and applications is provided, with the stress on making the correct decisions when building a graph model rather than on terminology.
- Material on bipartite graphs and their applications has been expanded.
- Examples illustrating the construction of finite-state automata that recognize specified sets have been added.
- Minimization of finite-state machines is now mentioned and developed in a series of exercises.
- Coverage of Turing machines has been expanded with a brief introduction to how Turing machines arise in the study of computational complexity, decidability, and computability.

Exercises and Examples

- Many new routine exercises and examples have been added throughout, especially at spots where key concepts are introduced.
- Extra effort has been made to ensure that both odd-numbered and even-numbered exercises are provided for basic concepts and skills.
- A better correspondence has been made between examples introducing key concepts and routine exercises.
- Many new challenging exercises have been added.
- Over 400 new exercises have been added, with more on key concepts, as well as more introducing new topics.

Additional Biographies, Historical Notes, and New Discoveries

- Biographies have been added for Archimedes, Hopper, Stirling, and Bayes.
- Many biographies found in the previous edition have been enhanced, including the biography of Augusta Ada.
- The historical notes included in the main body of the book and in the footnotes have been enhanced.
- New discoveries made since the publication of the previous edition have been noted.

The MathZone Companion Website (*www.mhhe.com/rosen*)

- MathZone course management and online tutorial system now provides homework and testing questions tied directly to the text.
- Expanded annotated links to hundreds of Internet resources have been added to the Web Resources Guide.
- Additional Extra Examples are now hosted online, covering all chapters of the book. These Extra Examples have benefited from user review and feedback.
- Additional Self Assessments of key topics have been added, with 14 core topics now addressed.
- Existing Interactive Demonstration Applets supporting key algorithms are improved. Additional applets have also been developed and additional explanations are given for integrating them with the text and in the classroom.
- An updated and expanded *Exploring Discrete Mathematics with Maple* companion workbook is also hosted online.

Special Features

ACCESSIBILITY This text has proved to be easily read and understood by beginning students. There are no mathematical prerequisites beyond college algebra for almost all of this text. Students needing extra help will find tools on the MathZone companion website for bringing their mathematical maturity up to the level of the text. The few places in the book where calculus is referred to are explicitly noted. Most students should easily understand the pseudocode used in the text to express algorithms, regardless of whether they have formally studied programming languages. There is no formal computer science prerequisite.

Each chapter begins at an easily understood and accessible level. Once basic mathematical concepts have been carefully developed, more difficult material and applications to other areas of study are presented.

FLEXIBILITY This text has been carefully designed for flexible use. The dependence of chapters on previous material has been minimized. Each chapter is divided into sections of approximately the same length, and each section is divided into subsections that form natural blocks of material for teaching. Instructors can easily pace their lectures using these blocks.

WRITING STYLE The writing style in this book is direct and pragmatic. Precise mathematical language is used without excessive formalism and abstraction. Care has been taken to balance the mix of notation and words in mathematical statements.

MATHEMATICAL RIGOR AND PRECISION All definitions and theorems in this text are stated extremely carefully so that students will appreciate the precision of language and rigor needed in mathematics. Proofs are motivated and developed slowly; their steps are all carefully justified. The axioms used in proofs and the basic properties that follow from them are explicitly described in an appendix, giving students a clear idea of what they can assume in a proof. Recursive definitions are explained and used extensively.

WORKED EXAMPLES Over 750 examples are used to illustrate concepts, relate different topics, and introduce applications. In most examples, a question is first posed, then its solution is presented with the appropriate amount of detail.

APPLICATIONS The applications included in this text demonstrate the utility of discrete mathematics in the solution of real-world problems. This text includes applications to a wide variety of areas, including computer science, data networking, psychology, chemistry, engineering, linguistics, biology, business, and the Internet.

ALGORITHMS Results in discrete mathematics are often expressed in terms of algorithms; hence, key algorithms are introduced in each chapter of the book. These algorithms are expressed in words and in an easily understood form of structured pseudocode, which is described and specified in Appendix A.3. The computational complexity of the algorithms in the text is also analyzed at an elementary level.

HISTORICAL INFORMATION The background of many topics is succinctly described in the text. Brief biographies of more than 65 mathematicians and computer scientists, accompanied by photos or images, are included as footnotes. These biographies include information about the lives, careers, and accomplishments of these important contributors to discrete mathematics and images of these contributors are displayed. In addition, numerous historical footnotes are included that supplement the historical information in the main body of the text. Efforts have been made to keep the book up-to-date by reflecting the latest discoveries.

KEY TERMS AND RESULTS A list of key terms and results follows each chapter. The key terms include only the most important that students should learn, not every term defined in the chapter.

EXERCISES There are over 3800 exercises in the text, with many different types of questions posed. There is an ample supply of straightforward exercises that develop basic skills, a large number of intermediate exercises, and many challenging exercises. Exercises are stated clearly and unambiguously, and all are carefully graded for level of difficulty. Exercise sets contain special discussions that develop new concepts not covered in the text, enabling students to discover new ideas through their own work.

Exercises that are somewhat more difficult than average are marked with a single star *; those that are much more challenging are marked with two stars **. Exercises whose solutions require calculus are explicitly noted. Exercises that develop results used in the text are clearly identified with the symbol ☞. Answers or outlined solutions to all odd-numbered exercises are provided at the back of the text. The solutions include proofs in which most of the steps are clearly spelled out.

REVIEW QUESTIONS A set of review questions is provided at the end of each chapter. These questions are designed to help students focus their study on the most important concepts and techniques of that chapter. To answer these questions students need to write long answers, rather than just perform calculations or give short replies.

SUPPLEMENTARY EXERCISE SETS Each chapter is followed by a rich and varied set of supplementary exercises. These exercises are generally more difficult than those in the exercise sets following the sections. The supplementary exercises reinforce the concepts of the chapter and integrate different topics more effectively.

COMPUTER PROJECTS Each chapter is followed by a set of computer projects. The approximately 150 computer projects tie together what students may have learned in computing and in discrete mathematics. Computer projects that are more difficult than average, from both a mathematical and a programming point of view, are marked with a star, and those that are extremely challenging are marked with two stars.

COMPUTATIONS AND EXPLORATIONS A set of computations and explorations is included at the conclusion of each chapter. These exercises (approximately 100 in total) are designed to be completed using existing software tools, such as programs that students or instructors have written or mathematical computation packages such as Maple or Mathematica. Many of these exercises give students the opportunity to uncover new facts and ideas through computation. (Some of these exercises are discussed in the *Exploring Discrete Mathematics with Maple* companion workbook available online.)

WRITING PROJECTS Each chapter is followed by a set of writing projects. To do these projects students need to consult the mathematical literature. Some of these projects are historical in nature and may involve looking up original sources. Others are designed to serve as gateways to new topics and ideas. All are designed to expose students to ideas not covered in depth in the text. These projects tie mathematical concepts together with the writing process and help expose students to possible areas for future study. (Suggested references for these projects can be found online or in the printed *Student's Solutions Guide*.)

APPENDIXES There are three appendixes to the text. The first introduces axioms for real numbers and the integers, and illustrates how facts are proved directly from these axioms. The second covers exponential and logarithmic functions, reviewing some basic material used heavily in the course. The third specifies the pseudocode used to describe algorithms in this text.

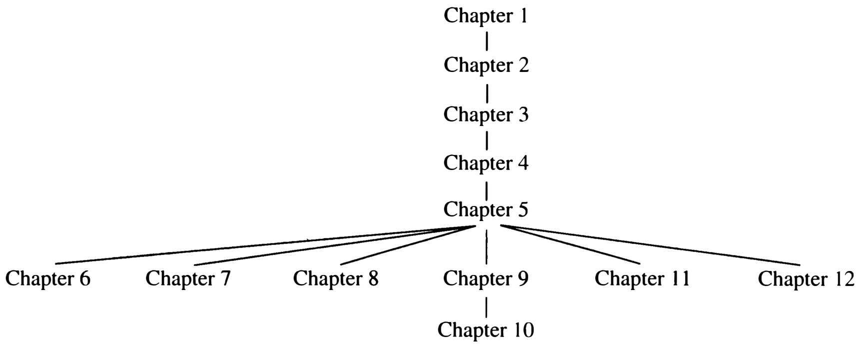
SUGGESTED READINGS A list of suggested readings for each chapter is provided in a section at the end of the text. These suggested readings include books at or below the level of this text, more difficult books, expository articles, and articles in which discoveries in discrete mathematics were originally published. Some of these publications are classics, published many years ago, while others have been published within the last few years.

How to Use This Book

This text has been carefully written and constructed to support discrete mathematics courses at several levels and with differing foci. The following table identifies the core and optional sections. An introductory one-term course in discrete mathematics at the sophomore level can be based on the core sections of the text, with other sections covered at the discretion of the instructor. A two-term introductory course can include all the optional mathematics sections in addition to the core sections. A course with a strong computer science emphasis can be taught by covering some or all of the optional computer science sections.

Chapter	*Core Sections*	*Optional Computer Science Sections*	*Optional Mathematics Sections*
1	1.1–1.7 (as needed)		
2	2.1–2.4 (as needed)		
3	3.1–3.5, 3.8 (as needed)	3.6	3.7
4	4.1–4.3	4.4, 4.5	
5	5.1–5.3	5.6	5.4, 5.5
6	6.1	6.4	6.2, 6.3
7	7.1, 7.5	7.3	7.2, 7.4, 7.6
8	8.1, 8.3, 8.5	8.2	8.4, 8.6
9	9.1–9.5		9.6–9.8
10	10.1	10.2, 10.3	10.4, 10.5
11		11.1–11.4	
12		12.1–12.5	

Instructors using this book can adjust the level of difficulty of their course by choosing either to cover or to omit the more challenging examples at the end of sections, as well as the more challenging exercises. The dependence of chapters on earlier chapters is shown in the following chart.



Ancillaries

STUDENT'S SOLUTIONS GUIDE This student manual, available separately, contains full solutions to all odd-numbered problems in the exercise sets. These solutions explain why a particular method is used and why it works. For some exercises, one or two other possible approaches are described to show that a problem can be solved in several different ways. Suggested references for the writing projects found at the end of each chapter are also included in this volume. Also included are a guide to writing proofs and an extensive description of common mistakes students make in discrete mathematics, plus sample tests and a sample crib sheet for each chapter designed to help students prepare for exams.

(ISBN-10: 0-07-310779-4) *(ISBN-13: 978-0-07-310779-0)*

INSTRUCTOR'S RESOURCE GUIDE This manual, available by request for instructors, contains full solutions to even-numbered exercises in the text. Suggestions on how to teach the material in each chapter of the book are provided, including the points to stress in each section and how to put the material into perspective. It also offers sample tests for each chapter and a test bank containing over 1300 exam questions to choose from. Answers to all sample tests and test bank questions are included. Finally, several sample syllabi are presented for courses with differing emphasis and student ability levels, and a complete section and exercise migration guide is included to help users of the fifth edition update their course materials to match the sixth edition.

(ISBN-10: 0-07-310781-6) *(ISBN-13: 978-0-07-310781-3)*

INSTRUCTOR'S TESTING AND RESOURCE CD An extensive test bank of more than 1300 questions using Brownstone Diploma testing software is available by request for use on Windows or Macintosh systems. Instructors can use this software to create their own tests by selecting questions of their choice or by random selection. They can also sort questions by section, difficulty level, and type; edit existing questions or add their own; add their own headings and instructions; print scrambled versions of the same test; export tests to word processors or the Web; and create and manage course grade books. A printed version of this test bank, including the questions and their answers, is included in the *Instructor's Resource Guide*.

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Acknowledgments

I would like to thank the many instructors and students at a variety of schools who have used this book and provided me with their valuable feedback and helpful suggestions. Their input has made this a much better book than it would have been otherwise. I especially want to thank Jerrold Grossman, John Michaels, and George Bergman for their technical reviews of the sixth edition and their "eagle eyes," which have helped ensure the accuracy of this book. I also appreciate the help provided by all those who have submitted comments via the website.

I thank the reviewers of this sixth and the five previous editions. These reviewers have provided much helpful criticism and encouragement to me. I hope this edition lives up to their high expectations.

Reviewers for the Sixth Edition

Charles Ashbacher,
Mount Mercy College

Ian Barland,
Rice University

George Bergman,
University of California, Berkeley

David Berman,
University of North Carolina, Wilmington

Miklós Bóna,
University of Florida

Prosenjit Bose,
Carleton University

Kirby Brown,
Polytechnic University

Michael Button,
The Master's College

Greg Cameron,
Brigham Young University

John Carter,
University of Toronto

Greg Chapman,
Cosumnes River College

Chao-Kun Cheng,
Virginia Commonwealth University

Thomas Cooper,
Georgia Perimeter College, Lawrenceville

Barbara Cortzen,
DePaul University

Daniel Cunningham,
Buffalo State College

George Davis,
University of Georgia

Beverly Diamond,
College of Charleston

Thomas Dunion,
Atlantic Union College

Bruce Elenbogen,
University of Michigan, Dearborn

Herbert Enderton,
University of California, Los Angeles

Anthony Evans,
Wright State University

Kim Factor,
Marquette University

William Farmer,
McMaster University

Li Feng,
Albany State University