PREFACE

> Τοῦ δὲ ποσοῦ τὸ μέν ἐστι διωρισμένον, τὸ δὲ συνεχες.
> *As to quantity, it can be either discrete or continuous.*
> —Aristotle, *Categories* (*ca.* 350 BCE)

This introductory text treats the discrete mathematics that computer scientists should know but generally do not learn in calculus and linear algebra courses. It aims to achieve breadth rather than depth and to teach reasoning as well as concepts and skills.

We stress the art of proof in the hope that computer scientists will learn to think formally and precisely. Almost every formula and theorem is proved in full. The text teaches the cumulative nature of mathematics; in spite of the breadth of topics covered, seemingly unrelated results in later chapters rest on concepts derived early on.

The text requires precalculus and occasionally uses a little bit of calculus. Chapter 21, on order notation, uses limits, but includes a quick summary of the needed basic facts. Proofs and exercises that use basic facts about derivatives and integrals, including l'Hôpital's rule, can be skipped without loss of continuity.

A fast-paced one-semester course at Harvard covers most of the material in this book. That course is typically taken by freshmen and sophomores as a prerequisite for courses on theory of computation (automata, computability, and algorithm analysis). The text is also suitable for use in secondary schools, for students of mathematics or computer science interested in topics that are mathematically accessible but off the beaten track of the standard curriculum.

The book is organized as a series of short chapters, each of which might be the subject of one or two class sessions. Each chapter ends with a brief summary and about ten problems, which can be used either as homework or as in-class exercises to be solved collaboratively in small groups.

Instructors who choose not to cover all topics can abridge the book in several ways. The spine of the book includes Chapters 1–8 on foundational concepts, Chapters 13–18 on digraphs and graphs, and Chapters 21–25 on order notation and counting. Four blocks of chapters are optional and can be included or omitted at the instructor's discretion and independently of each other:

- Chapters 9–12 on logic;
- Chapters 19–20 on automata and formal languages;

- Chapters 26–29 on discrete probability; and
- Chapters 30–31 on modular arithmetic and cryptography.

None of these blocks, if included at all, need be treated in full, since only later chapters in the same block rely on the content of chapters earlier in the block.

It has been our goal to provide a treatment that is generic in its tastes and therefore suitable for wide use, without the heft of an encyclopedic textbook. We have tried throughout to respect our students' eagerness to learn and also their limited budgets of time, attention, and money.

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