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PREFACE

THE EMERGENCE OF MATHEMATICS

Although the emergence of mathematics can be traced back to many regions of the world, there is an international mathematics that had its roots in Egypt and Babylonia and developed significantly in ancient Greece. The mathematics of Greece was translated into Arabic, as was some mathematics of India about the same time. This mathematics was later translated into Latin and became the mathematics of Western Europe and, centuries later, the mathematics of the world. Although mathematics developed in other parts of the world, particularly China, Japan and South India, this Asian mathematics did not have the impact that the international mathematics did.

Mathematical history has not followed the same path as other histories of the human spirit. Whether one is considering science, medicine or space exploration, nearly every human endeavor has seen its history marked by advances, corrections and reversals. Mathematics, however, has experienced only advancement. The theorems of yesterday are also theorems today. The accomplishments of the mathematicians of the past have become the foundation of present-day mathematics. With every new era in mathematics, a new peak has been created.

During the latter part of the 17th century and into the 18th century, much attention was paid to the development of calculus. During that period, there was great emphasis on real numbers and continuous mathematics. By the time the 19th century had arrived and early into the 20th century, logic and the emphasis placed on valid arguments to establish the truth of mathematical statements achieved ever-increasing importance.

THE ROAD TO DISCRETE MATHEMATICS

By the middle of the 20th century, another kind of mathematics, quite unlike the continuous mathematics of calculus, had become prominent: discrete mathematics. Instead of being concerned with real numbers, this mathematics was more involved with integers and other "similar" collections of objects, as well as with finite collections.

Discrete mathematics became primarily the study of the mathematical properties of sets, systems and structures having a "countable" collection of elements. The growing popularity of and interest in discrete mathematics was greatly influenced by its applications to areas such as computer science, engineering, communication and transportation as well as its emergence as a fascinating area of mathematics.

Like all mathematics, the validity and understanding of discrete mathematics rely on logic, sets, functions and methods of proof. More unique to discrete mathematics, however, are topics involving procedures (algorithms) for solving problems and the number of objects possessing certain properties of interest – the area of discrete mathematics called enumerative combinatorics. Also, discrete mathematics is concerned with the study of relations and graph theory, a growing area of discrete mathematics with a much shorter history.

In the middle of the 20th century, a beginning college student, even those with a strong mathematical background, would likely take courses in college algebra, trigonometry and analytic geometry, followed then by a sequence in calculus. At the same time, high school seniors were taking courses in Euclidean geometry, advanced algebra and trigonometry. As time went by, it became commonplace for beginning college students to take courses in precalculus and perhaps calculus, while high school students were taking courses dealing with functions, statistics and continuous mathematics (an introduction to calculus).

The passing years saw first the introduction of, then advances in and now the bombardment of technology into college courses and everyday life. Computer science became a major area of study, especially computer programming, development of software, data structures and analysis of algorithms. Discrete mathematics is the mathematics underlying computer science. Indeed, this mathematics has become important for many areas of study and has developed into a major area of mathematics of its own.

A COURSE IN DISCRETE MATHEMATICS

Developing an appreciation of and learning any area of mathematics requires a comprehension of the concepts occurring in the area (each of which requires a clear definition) and a knowledge of certain valid statements, each of which is either an axiom (whose truth is accepted without proof) or a theorem (whose truth can be established with the aid of concepts and other statements). This is best accomplished if we can understand proofs of theorems and are able to write some proofs of our own. We will be introduced to several methods of proof that can be used to establish the truth of theorems as well as ways to show that statements are false. These methods are based on logic, which allow us to use reasoning to show that a given statement is true or false. Although it is not our intention to go into any of this in great depth, it is our goal to present enough details and examples so that a sound introduction to proofs can be obtained. Developing a good understanding of proofs requires a great deal of practice and experience and comprehending how others prove theorems. These are the subjects of Chapters 1–3.

Chapter 4 is devoted to an important proof technique that can be used to establish the truth of special kinds of statements: mathematical induction. This proof technique is frequently employed in computer science. The related topic of recurrence is discussed in Chapter 4 as well.

Chapter 5 deals with concepts that are fundamental to all areas of mathematics: relations and functions. The important subject of equivalence relations is introduced in this chapter. Functions that are both one-to-one and onto are especially important and are emphasized in the chapter.

Algorithms are discussed in Chapter 6. The concept of algorithms is well known to and encountered often in computer science. There are problems in which it is convenient to have a solution consisting of an efficient step-by-step set of instructions from which a computer program can be written. The number of steps needed to use some algorithms to solve certain kinds of problems is analyzed. Particular emphasis is paid to search and sort algorithms.

The numbers encountered most often in discrete mathematics are the integers. Chapter 7 is devoted to the study of integers, particularly their divisibility properties. Primes and their properties are discussed, as is the important topic of congruence. The Division Algorithm and Euclidean Algorithm are introduced and illustrated in this chapter as well. Integer representations and a discussion of the bases 2, 8 and 16 occur here.

Chapters 8–10 deal with the area of discrete mathematics called enumerative combinatorics: the study of counting. The basic principles of counting are introduced in Chapter 8: multiplication, addition, pigeonhole, inclusion-exclusion. In Chapter 8, permutations and combinations are also introduced. These two concepts give rise to a wide variety of counting problems. Chapter 9 discusses the concept of Pascal triangles and the related binomial theorem and presents more advanced counting problems, some through the subject of generating functions. Chapter 10 gives an introduction to discrete probability. Random variables and expected values are also presented in this chapter.

There are many ways in which a pair of elements can be compared or related. Particularly when elements belong to the same set, there are numerous occurrences of this idea in mathematics. One of these relations has the same basic properties of the equality of numbers and is called an equivalence relation (discussed in Chapter 5). Another relation has the same basic properties of the "less than or equal to" comparison of two numbers and is called a partial ordering. Partially ordered sets are discussed in Chapter 11. This gives rise to the structures referred to as lattices and Boolean algebras, which are also discussed in Chapter 11.

Chapters 12–15 deal with the area of graph theory, a major subject within discrete mathematics. Chapter 12 introduces the fundamental concepts of graph theory and concepts involving paths and cycles. This gives rise to the study of Eulerian graphs and Hamiltonian graphs. Chapter 13 is devoted to the important class of graphs called trees. Minimum spanning trees are discussed and two algorithms, Kruskal's algorithm and Prim's algorithm, are presented and illustrated. Depth-first search and breadth-first search are also discussed in Chapter 13. One of the most famous problems in graph theory (in fact, in mathematics) was the Four Color Problem. This problem involves the concepts of planar graphs and graph colorings, which are the major topics in Chapter 14. Chapter 15 deals with directed graphs, the most studied class being the tournaments. Directed graphs are useful in the study of finite-state machines, another concept discussed in Chapter 15.

TO THE STUDENT

- **Prerequisites**

  A background in algebra and precalculus (and recalling the major topics and techniques in these areas) is an important prerequisite for a course in discrete mathematics. It would also be helpful if the student had taken a course or two in calculus. It is probably most important, however, that a student enters a course in discrete mathematics with a desire for learning and a goal of doing the best that he or she can do. This certainly means attending class regularly, being an attentive student and doing homework faithfully. Being a successful student in discrete mathematics takes a certain amount of dedication. Discrete mathematics, unlike much of calculus and precalculus, may not build on other courses the student has already taken. For most students, this is an exciting new subject whose topics will be encountered often again if there are more mathematics, computer science or engineering courses in a student's future.

- **Concepts and Theorems**

  As with all areas of mathematics, discrete mathematics deals with the understanding of concepts and theorems. The definitions of all concepts are presented and, if the concept is considered a major concept, it is prominently displayed. Almost all theorems are proved. Care has been taken to give clear proofs. Time will not permit your instructor to prove all theorems in class but, especially if your instructor emphasizes proofs in class, it is a good idea to read the proofs of any theorems discussed in class.

- **Examples**

  This textbook contains a large number of examples that illustrate the concepts, theorems and various methods introduced. These include methods for solving certain kinds of problems and methods for proving the truth of statements. Before attempting any exercises that have been assigned and as part of a study plan to prepare for a forthcoming quiz or exam, it is useful to read and understand the examples presented. Prior to reading the examples, it is useful to review the concepts and their definitions and theorems that the exercises deal with.

- **Exercises**

  Each chapter contains a large number of exercises. There are exercises at the end of each section and additional exercises at the end of each chapter. The exercises range from routine (which are often similar to examples presented in the text that require a basic understanding of the concepts or theorems presented) to moderately challenging (which will require some thought on the student's part) to more challenging and innovative (which require coming up with ideas for solving the problem).

  Some students spend little time, if any, reading a textbook. This is not a good idea. Only when an exercise is assigned that they don't know how to do (and no classmate or the instructor is available for help) do such students search through the book, looking primarily for examples that are similar to the exercise assigned. It is far better to read the textbook and understand the examples in the textbook before attempting to do the exercises assigned. The authors have spent much time and effort to make this textbook clear and informative (and hopefully interesting as well). It's good to take advantage of this.

Answers or hints to odd-numbered exercises (both section exercises and chapter exercises) are given at the end of the textbook.

- **Chapter Highlights**

  At the end of each chapter are Chapter Highlights. This includes a list of the major concepts introduced in the chapter (arranged alphabetically) with a brief definition of each concept. Also included are the major results introduced in the chapter (arranged in the order presented). When the chapter has been completed, it is probably a good idea to review the concepts and theorems covered in class to refresh your memory. It is also good to review these again prior to a quiz or an exam on this material.

TO THE INSTRUCTOR

- **Construction of Chapters**

  Each chapter is divided into sections (and sometimes even further into topics) for the purpose of making it easier to decide which portions of a chapter to discuss each class period. With few exceptions, proofs of the theorems are provided. Even if it is decided not to present a proof in class, proofs can be read by students. Each chapter contains many examples. The instructor could present some of these in class. If the instructor prefers not to do this, other examples can be given. Possibly exercises could be presented to serve as examples.

- **Concepts and Theorems**

  Each chapter contains several concepts and theorems central to the chapter. The definition of each concept is presented and definitions of the major concepts are prominently displayed. Proofs of most theorems are given. Care has been taken to give clear proofs. Many theorems are preceded by a "Proof Strategy," which provides a discussion of a plan of what needs to be proved and how we might be able to do this. Many proofs of theorems are followed by a "Proof Analysis," which is a discussion that reflects on a proof just given and emphasizes certain details that might have gone unnoticed. Occasionally, an example is followed by an "Analysis," which serves a similar purpose.

- **Exercises**

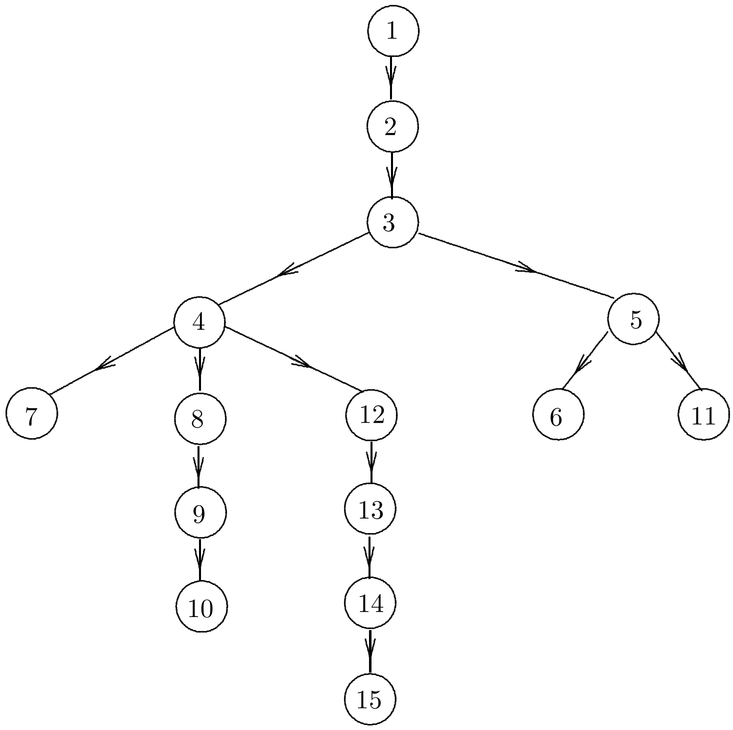
  Each section is followed by a set of exercises dealing with the material in that section. In addition, each chapter is followed by a supplemental set of exercises dealing with the material in that chapter. There are a number of exercises that range from routine to moderately challenging so that it can be determined if the students have an understanding of the concepts, examples and theorems presented in the section or chapter. In addition, there are more innovative exercises to challenge students to think for themselves and come up with ideas of their own.

- **Solutions Manual for Instructors**

  There is a *Solutions Manual for Instructors* (prepared by the authors) that contains detailed solutions of all exercises in the textbook (both section exercises and chapter exercises), including complete proofs of all exercises requesting a proof (not simply hints) and often explanations of solutions of exercises having a numerical answer (not simply the answer).

TEACHING A COURSE FROM THIS TEXTBOOK

There is enough material in this textbook for a 2-semester sequence in discrete mathematics but considerably more material than one would attempt to teach in a 1-semester course. What portions of the textbook to be covered depends, to a great degree, on what the instructor's goals are. If the instructor wants to emphasize logic and proofs, the first four chapters can be covered in some detail. It is likely that some of Chapter 5 (Relations and Functions) would be covered. In many instances, algorithms are covered in Computer Science courses. If this is the case, Chapter 6 can be omitted. How each chapter in this text relies on other chapters is indicated in the diagram (digraph) below.



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