

# LAB2 MATLAB Programming

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## Introduction

MATLAB is a powerful tool to handle on the problem of signal and system.

After completing this lab, we are able to,

1. Use "conv" to convolve x and y to get the convolution result.
2. Use "fliter" to remove the echos which are caused by time delay.
3. Use "lsim" to simulate the output of continuous-time, causal LTI systems described by linear constant-

$$\sum_{k=0}^N a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^M b_m \frac{d^m x(t)}{dt^m}.$$

coefficient differential equations of the form

4. Prove several propertites of convolution operation.
5. Compare the difference of the infinite signal and the finite signal.

## Results and Analysis

### 2.4

#### (a)

(a). Many of the problems in this exercise will use the following three signals:

$$x_1[n] = \begin{cases} 1, & 0 \leq n \leq 4, \\ 0, & \text{otherwise,} \end{cases}$$

$$h_1[n] = \begin{cases} 1, & n = 0, \\ -1, & n = 1, \\ 3, & n = 2, \\ 1, & n = 4, \\ 0, & \text{otherwise,} \end{cases}$$

$$h_2[n] = \begin{cases} 2, & n = 1, \\ 5, & n = 2, \\ 4, & n = 3, \\ -1, & n = 4, \\ 0, & \text{otherwise.} \end{cases}$$

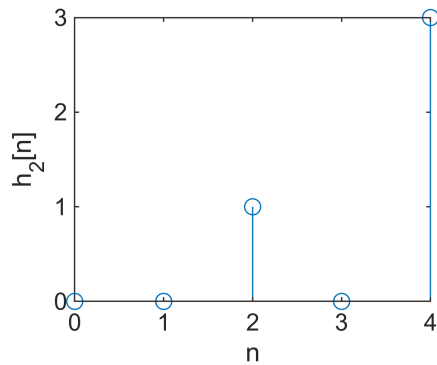
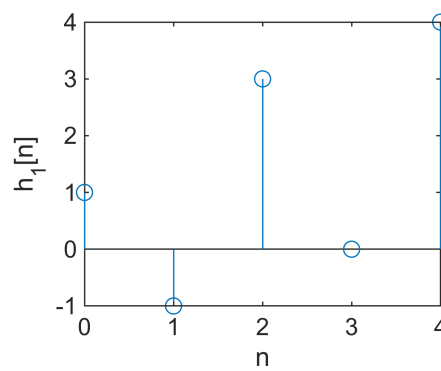
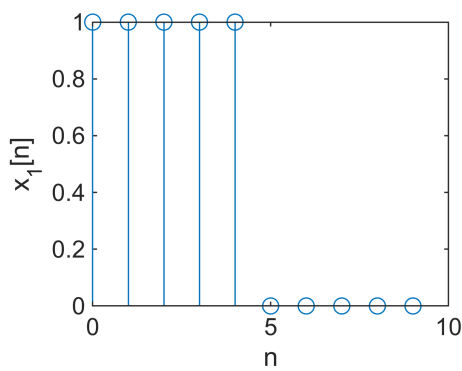
Define the MATLAB vector **x1** to represent  $x_1[n]$  on the interval  $0 \leq n \leq 9$ , and the vectors **h1** and **h2** to represent  $h_1[n]$  and  $h_2[n]$  for  $0 \leq n \leq 4$ . Also define **nx1** and **nh1** to be appropriate index vectors for these signals. Make appropriately labeled plots of all the signals using **stem**.

Initiate the three signals  $x_1[n]$ ,  $h_1[n]$ ,  $h_2[n]$ .

```
clear;clc;
x1 = [1 1 1 1 1 zeros(1,5)];
nx1 = 0:9;
h1 = [1 -1 3 0 4];
h2 = [0 0 1 0 3];
nh1 = 0:4;
```

Then stem the three signals.

```
figure;
tiledlayout(2,2);
nexttile;stem(nx1,x1);
ylabel('x_1[n]');xlabel('n');
nexttile;stem(nh1,h1);
ylabel('h_1[n]');xlabel('n');
nexttile;stem(nh1,h2);
ylabel('h_2[n]');xlabel('n');
```



(b)

- (b). The commutative property states that the result of a convolution is the same regardless of the order of the operands. This implies that the output of an LTI system with impulse response  $h[n]$  and input  $x[n]$  will be the same as the output of an LTI system with impulse response  $x[n]$  and input  $h[n]$ . Use `conv` with `h1` and `x1` to verify this property. Is the output of `conv` the same regardless of the order of the input arguments?

Introduce the three signals  $x_1[n]$ ,  $h_1[n]$ ,  $h_2[n]$ .

```
x1 = [1 1 1 1 1 zeros(1,5)];
nx1 = 0:9;
h1 = [1 -1 3 0 4];
h2 = [0 0 1 0 3];
nh1 = 0:4;
```

Let  $h_1[n]$  be the impulse response and  $x_1[n]$  be the input signal, then  $y_1[n] = h_1[n] * x_1[n]$ .

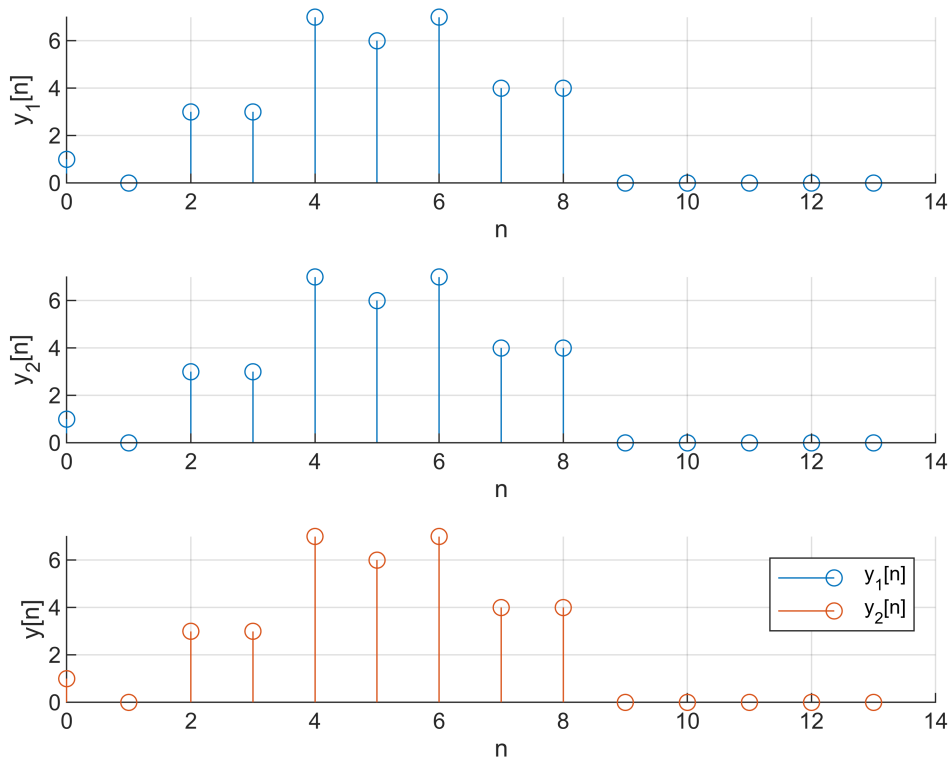
Then Let  $x_1[n]$  be the impulse response and  $h_1[n]$  be the input signal, then  $y_2[n] = x_1[n] * h_1[n]$ .

```
y1 = conv(h1, x1);
figure;
ny = (nx1(1) + nh1(1)):(nx1(end) + nh1(end));
```

```

tiledlayout(3,1);
nexttile;stem(ny,y1);
ylabel('y_1[n]');xlabel('n');grid on;box off;
y2 = conv(x1, h1);
nexttile;stem(ny,y2);
ylabel('y_2[n]');xlabel('n');grid on;box off;
nexttile;hold on;
stem(ny,y1);stem(ny,y2);
grid on;
xlabel('n');
ylabel('y[n]');
legend('y_1[n]', 'y_2[n]');

```



And now, we can use `isequal()` to compare the  $y_1[n]$  and  $y_2[n]$ .

```
isequal(y1,y2)
```

```
ans = logical
      1
```

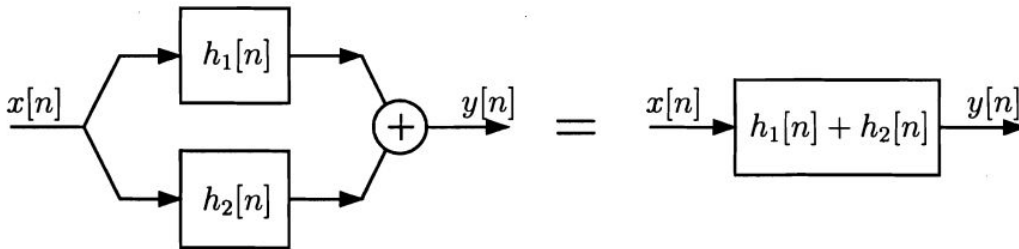
Since they are the same, the output of `conv` regardless of the order of the input arguments is the same.

(c)

(c). Convolution is also distributive. This means that

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n].$$

This implies that the output of two LTI systems connected in parallel is the same as one system whose impulse response is the sum of the impulse responses of the parallel systems. Figure 2.8 illustrates this property.



**Figure 2.8.** Distributive property of convolution.

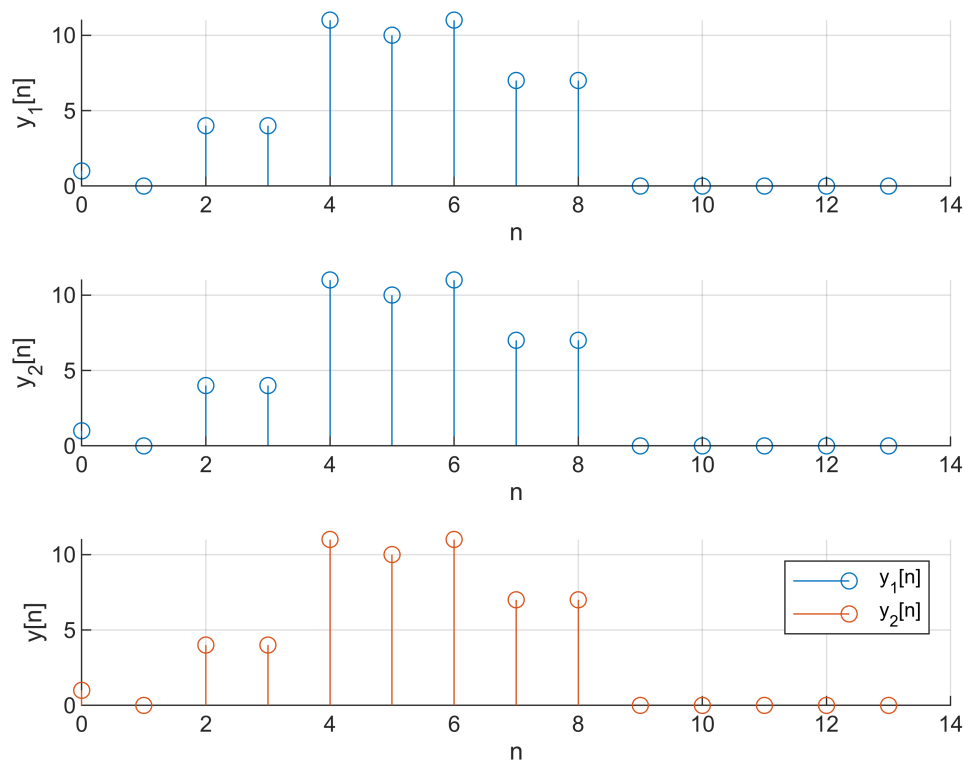
Verify the distributive property using `x1`, `h1` and `h2`. Compute the sum of the outputs of LTI systems with impulse responses  $h_1[n]$  and  $h_2[n]$  when  $x_1[n]$  is the input. Compare this with the output of the LTI system whose impulse response is  $h_1[n] + h_2[n]$  when the input is  $x_1[n]$ . Do these two methods of computing the output give the same result?

Assume  $y_1[n] = x[n] * h_1[n] + x[n] * h_2[n]$  and  $y_2[n] = x[n] * (h_1[n] + h_2[n])$ .

```
y1 = conv(x1,h1)+conv(x1,h2);
y2 = conv(x1,h1+h2);
ny = nx1(1)+nh1(1):nx1(end)+nh1(end);
```

Plot  $y_1$  and  $y_2$ .

```
figure;
tiledlayout(3,1);
nexttile;
stem(ny,y1);
xlabel('n');ylabel('y_1[n]');grid on;box off;
nexttile;
stem(ny,y2);
xlabel('n');ylabel('y_2[n]');grid on;box off;
nexttile;
hold on;
stem(ny,y1);
stem(ny,y2);
grid on;
xlabel('n');
ylabel('y[n]');
legend('y_1[n]', 'y_2[n]');
```



```
isequal(y1,y2)
```

```
ans = logical
      1
```

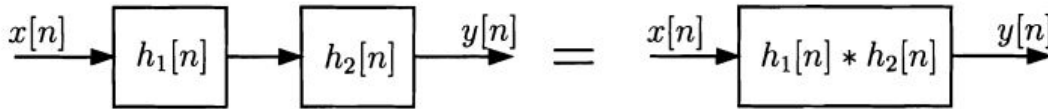
They're overlapped in the figure and isequal. So  $x[n] * h_1[n] + x[n] * h_2[n] = x[n] * (h_1[n] + h_2[n])$ .

(d)

(d). Convolution also possesses the associative property, i.e.,

$$(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n]).$$

This property implies that the result of processing a signal with a series of LTI systems is equivalent to processing the signal with a single LTI system whose impulse response is the convolution of all the individual impulse responses of the connected systems. Figure 2.9 illustrates this property for the case of two LTI systems connected in series.



**Figure 2.9.** Associative property of convolution.

Use the following steps to verify the associative property using **x1**, **h1** and **h2**:

- Let  $w[n]$  be the output of the LTI system with impulse response  $h_1[n]$  shown in Figure 2.9. Compute  $w[n]$  by convolving  $x_1[n]$  and  $h_1[n]$ .
- Compute the output  $y_{d1}[n]$  of the whole system by convolving  $w[n]$  with  $h_2[n]$ .
- Find the impulse response  $h_{\text{series}}[n] = h_1[n] * h_2[n]$ .
- Convolve  $x_1[n]$  with  $h_{\text{series}}[n]$  to get the output  $y_{d2}[n]$ .

Compare  $y_{d1}[n]$  and  $y_{d2}[n]$ . Did you get the same results when you process  $x_1[n]$  with the individual impulse responses as when you process it with  $h_{\text{series}}[n]$ ?

Initiate the three signals  $x_1[n]$ ,  $h_1[n]$ ,  $h_2[n]$  and  $nx1$ ,  $nx2$ .

```
x1 = [ones(1,5) zeros(1,5)];
h1 = [1 -1 3 0 4];
h2 = [0 0 1 0 3];
nx1 = 0:9;
nh1 = 0:4;
```

Calculate  $w[n]$ ,  $y_{d1}[n]$ ,  $h_{\text{series}}[n]$ ,  $y_{d2}[n]$ .

```
w = conv(x1,h1);
nw = nx1(1)+nh1(1):nx1(end)+nh1(end);
yd1 = conv(w,h2);
nyd1 = nw(1)+nh1(1):nw(end)+nh1(end);
hseries = conv(h1,h2);
nhseries = nh1(1)+nh1(1):nh1(end)+nh1(end);
yd2 = conv(x1,hseries);
nyd2 = nx1(1)+nhseries(1):nx1(end)+nhseries(end);
```

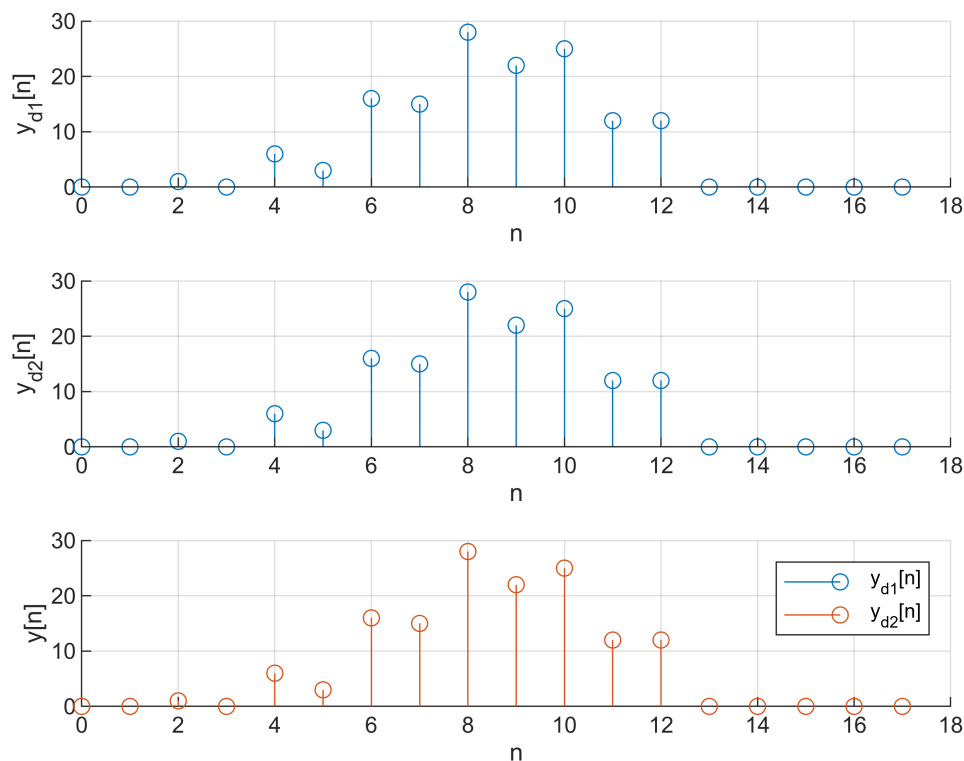
Plot  $y_{d1}$  and  $y_{d2}$ .

```
figure;
```

```

tiledlayout(3,1);
nexttile;
stem(nyd1,yd1);
xlabel('n');ylabel('y_{d1}[n]');grid on;box off;
nexttile;
stem(nyd2,yd2);
xlabel('n');ylabel('y_{d2}[n]');grid on;box off;
nexttile;
hold on;
stem(nyd1,yd1);
stem(nyd2,yd2);
grid on;
xlabel('n');
ylabel('y[n]');
legend('y_{d1}[n]', 'y_{d2}[n]');

```



```

isequal(yd1,yd2)

```

```

ans = logical
     1

```

They're overlapped in the figure and isequal. So  $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$ .

## 2.10

(a)



In this exercise, you will consider the problem of removing an echo from a recording of a speech signal. This project will use the audio capabilities of MATLAB to play recordings of both the original speech and the result of your processing. To begin this exercise you will need to load the speech file `lineup.mat`, which is contained in the Computer Explorations Toolbox. The Computer Explorations Toolbox can be obtained from The MathWorks at the address provided in the Preface. If this speech file is already somewhere in your MATLABPATH, then you can load the data into MATLAB by typing

```
>> load lineup.mat
```

You can check your MATLABPATH, which is a list of all the directories which are currently accessible by MATLAB, by typing `path`. The file `lineup.mat` must be in one of these directories.

Once you have loaded the data into MATLAB, the speech waveform will be stored in the variable `y`. Since the speech was recorded with a sampling rate of 8192 Hz, you can hear the speech by typing

```
>> sound(y,8192)
```

You should hear the phrase “line up” with an echo. The signal  $y[n]$ , represented by the vector `y`, is of the form

$$y[n] = x[n] + \alpha x[n - N], \quad (2.21)$$

where  $x[n]$  is the uncorrupted speech signal, which has been delayed by  $N$  samples and added back in with its amplitude decreased by  $\alpha < 1$ . This is a reasonable model for an echo resulting from the signal reflecting off of an absorbing surface like a wall. If a microphone is placed in the center of a room, and a person is speaking at one end of the room, the recording will contain the speech which travels directly to the microphone, as well as an echo which traveled across the room, reflected off of the far wall, and then into the microphone. Since the echo must travel further, it will be delayed in time. Also, since the speech is partially absorbed by the wall, it will be decreased in amplitude. For simplicity ignore any further reflections or other sources of echo.

For the problems in this exercise, you will use the value of the echo time,  $N = 1000$ , and the echo amplitude,  $\alpha = 0.5$ .

### Basic Problems

- (a). In this exercise you will remove the echo by linear filtering. Since the echo can be represented by a linear system of the form Eq. (2.21), determine and plot the impulse response of the echo system Eq. (2.21). Store this impulse response in the vector `he` for  $0 \leq n \leq 1000$ .

Initiate the environment.

```
clear;clc;
load lineup.mat
n = 0:1000;
alpha = 0.5;
N = 1000;
```

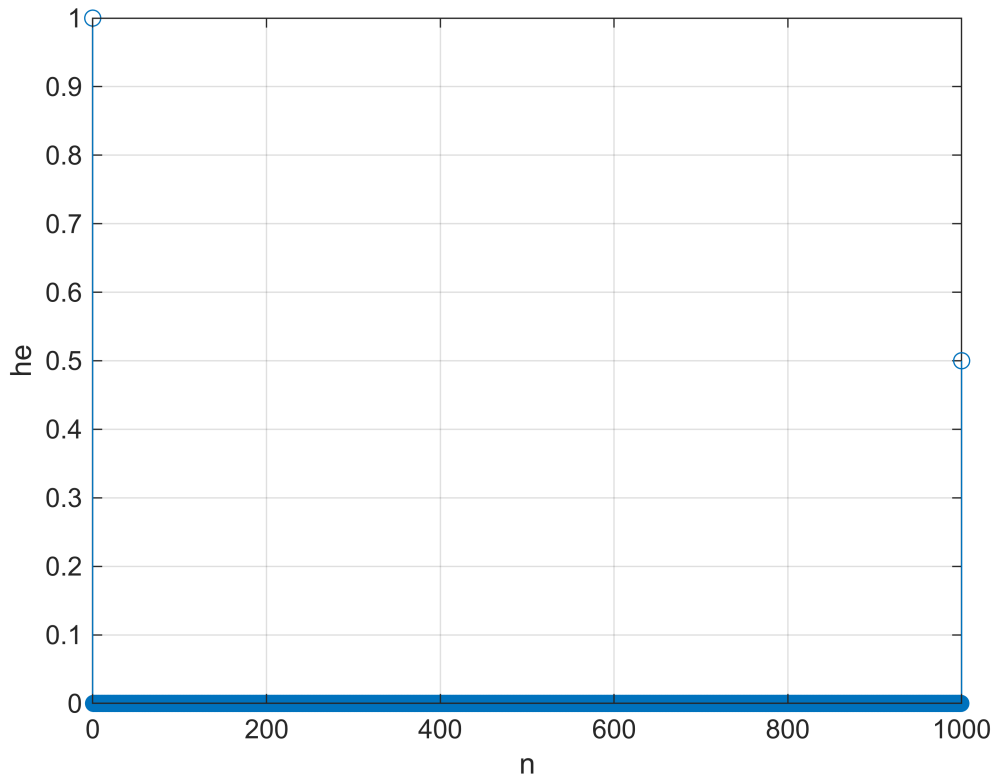
Use filter to get the vector `he`.

```
he = unitimpulse(n)+alpha*unitimpulse(n-N);
```

Plot `he`.

```
figure;
```

```
stem(n,he);
grid on;
xlabel('n');
ylabel('he');
```



**(b)**

(b). Consider an echo removal system described by the difference equation

$$z[n] + \alpha z[n - N] = y[n], \quad (2.22)$$

where  $y[n]$  is the input and  $z[n]$  is the output which has the echo removed. Show that Eq. (2.22) is indeed an inverse of Eq. (2.21) by deriving the overall difference equation relating  $z[n]$  to  $x[n]$ . Is  $z[n] = x[n]$  a valid solution to the overall difference equation?

the overall difference equation:

$$y[n] - (z[n] + \alpha z[n - N]) = (x[n] + \alpha x[n - N]) - y[n]$$

$$0 = 0$$

So that Eq.(2.22) is indeed an inverse of Eq.(2.21).

$z[n] = x[n]$  is a valid solution to the overall difference equation.

**(c)**

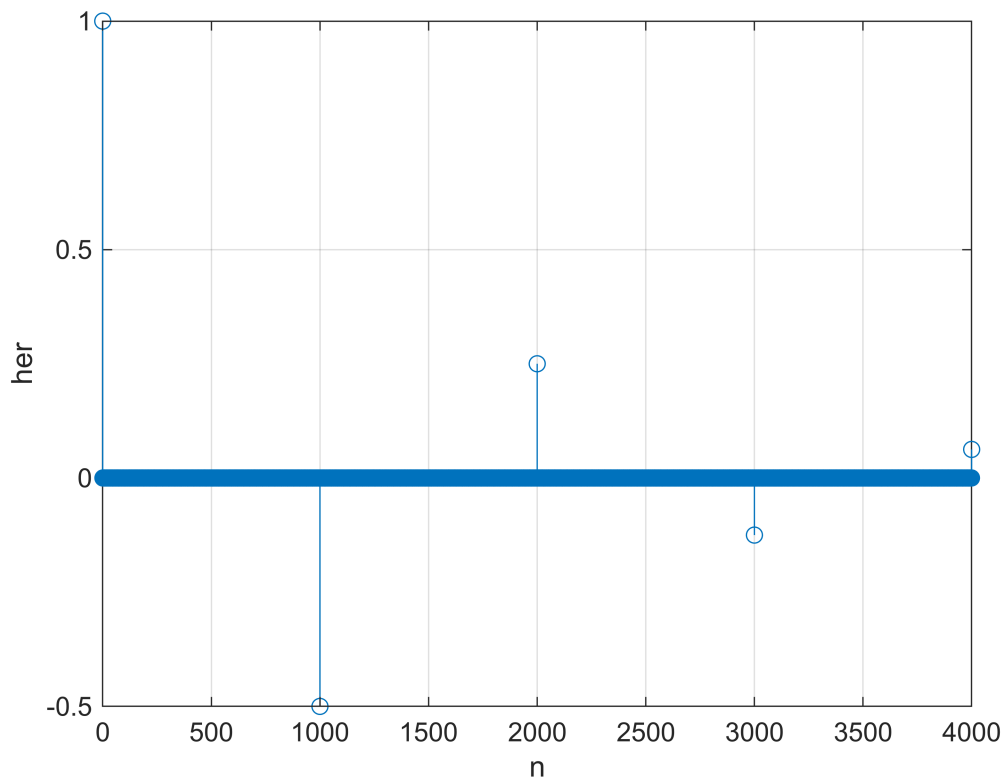
- (c). The echo removal system Eq. (2.22) will have an infinite-length impulse response. Assuming that  $N = 1000$ , and  $\alpha = 0.5$ , compute the impulse response using `filter` with an input that is an impulse given by `d=[1 zeros(1,4000)]`. Store this 4001 sample approximation to the impulse response in the vector `her`.

Initiate the impulse.

```
d = [1 zeros(1,4000)];
```

Use `filter()` to get the vector `her`.

```
a = [1 zeros(1,999) 0.5];  
her = filter(1,a,d);  
n = 0:length(her)-1;  
figure;  
stem(n,her);  
grid on;  
xlabel('n');  
ylabel('her');
```



(d)

- (d). Implement the echo removal system using `z=filter(1,a,y)`, where `a` is the appropriate coefficient vector derived from Eq. (2.22). Plot the output using `plot`. Also, listen to the output using `sound`. You should no longer hear the echo.

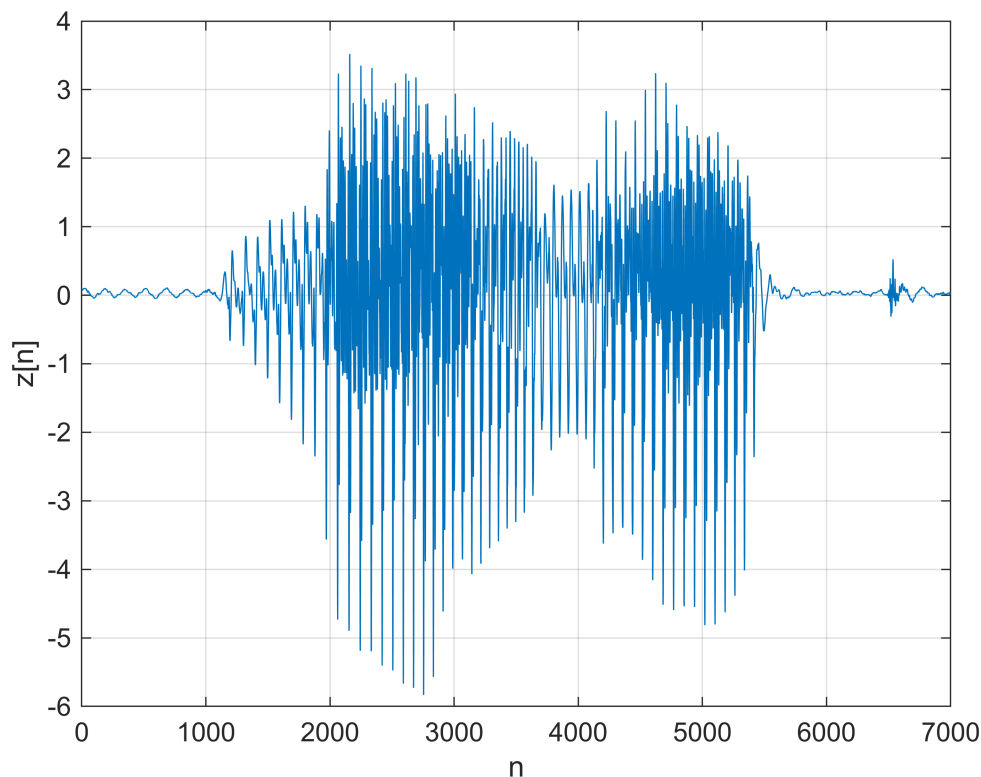
Load the speech file firstly.

```
load("lineup.mat")
```

Known from Eq.(2.22), the `a` should be `[1 zeros(1, 999) 0.5]` for  $z[n] + 0.5 * z[n - 1000] = y[n]$ .

So we can use remove the echo by using `filter()` to get  $z[n]$ .

```
a = [1 zeros(1,999) 0.5];  
z = filter(1,a,y);  
nz = 1:length(z);  
figure;  
plot(nz,z);  
xlabel('n');ylabel('z[n]');grid on;
```



Then use `sound()` to listen the signal  $z[n]$  with the frequency of 8192 Hz.

```
sound(z,8192);
```

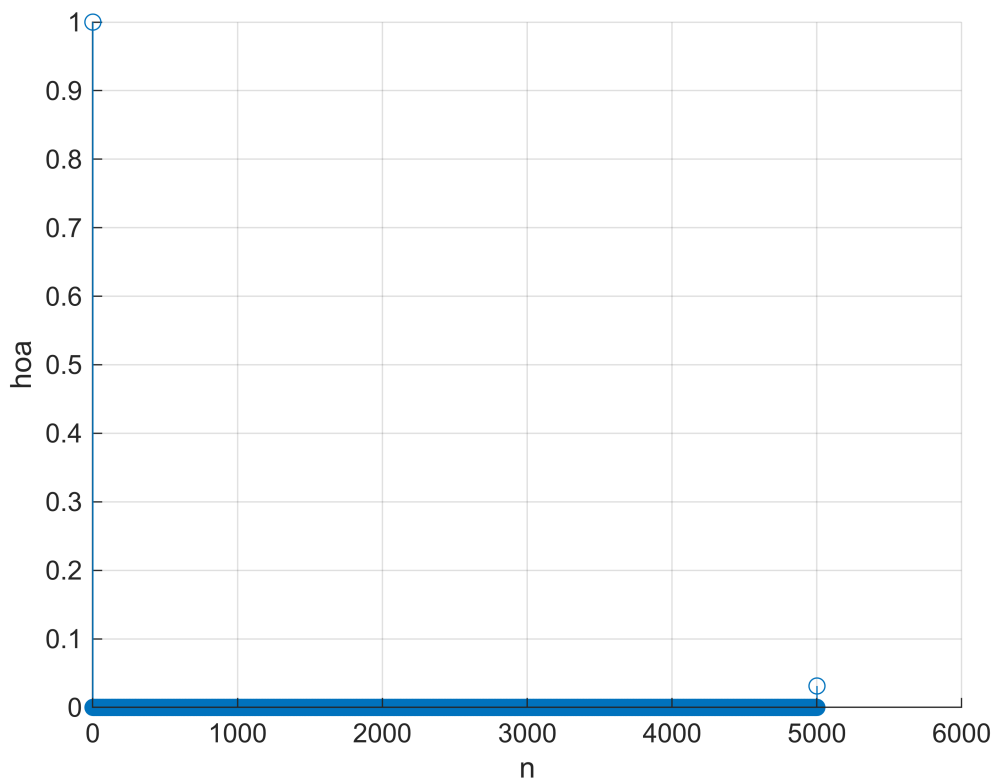
We can hear that the echo is removed and we get a more clear sound.

(e)

- (e). Calculate the overall impulse response of the cascaded echo system, Eq. (2.21), and echo removal system, Eq. (2.22), by convolving  $h_e$  with  $h_{er}$  and store the result in  $h_{oa}$ . Plot the overall impulse response. You should notice that the result is not a unit impulse. Given that you have computed  $h_{er}$  to be the inverse of  $h_e$ , why is this the case?

Calculate the result  $h_{oa} = h_e * h_{er}$ .

```
hoa = conv(he,her);  
figure;  
stem(hoa);grid on;box off;  
ylabel('hoa');xlabel('n');
```



Since the  $d$  is not an infinite-length unit impulse, the output  $h_{er}[n]$  generated by  $filter(1, a, d)$  is not totally equal to  $filter(1, a, \sigma[n])$ , which loses the values behind  $N \geq 4000$ .

So convolving  $h_e$  and  $h_{er}$  is not a unit impulse here.

(f)

- (f). Suppose that you were given  $y[n]$  but did not know the value of the echo time,  $N$ , or the amplitude of the echo,  $\alpha$ . Based on Eq. (2.21), can you determine a method of estimating these values? Hint: Consider the output  $y$  of the echo system to be of the form:

$$y[n] = x[n] * (\delta[n] + \alpha\delta[n - N])$$

and consider the signal,

$$R_{yy}[n] = y[n] * y[-n].$$

This is called the autocorrelation of the signal  $y[n]$  and is often used in applications of echo-time estimation. Write  $R_{yy}[n]$  in terms of  $R_{xx}[n]$  and also plot  $R_{yy}[n]$ . You will have to truncate  $y[n]$  before your calculations to keep  $R_{yy}[n]$  within the limitations of the Student Edition of MATLAB. You will find that many of the properties of the autocorrelation will still hold when  $y$  is truncated. Also try experimenting with simple echo problems such as

```
>> NX=100;
>> x=randn(1,NX);
>> N=50;
>> alpha=0.9;
>> y=filter([1 zeros(1,N) alpha],1,x);
>> Ryy=conv(y,flip(y));
>> plot([-NX+1:NX-1],Ryy)
```

by varying  $N$ ,  $\alpha$ , and  $NX$ . Also, when you loaded `lineup.mat`, you loaded in two additional vectors. The vector `y2` contains the phrase “line up” with a different echo time  $N$  and different echo amplitude  $\alpha$ . The vector `y3` contains the same phrase with two echoes, each with different times and amplitudes. Can you estimate  $N$  and  $\alpha$  for `y2`, and  $N_1, \alpha_1, N_2$ , and  $\alpha_2$  for `y3`?

$$\begin{aligned} R_{yy}[n] &= y[n] * y[-n] \\ &= x[n] * (\delta[n] + \alpha\delta[n - N]) * x[-n] * (\delta[-n] + \alpha\delta[-n - N]) \\ &= R_{xx}[n] * ((1 + \alpha^2)\delta[n] + \alpha\delta[n - N] + \alpha\delta[-n - N]) \\ &= (1 + \alpha^2)R_{xx}[n] + \alpha R_{xx}[n - N] + \alpha R_{xx}[n + N] \end{aligned}$$

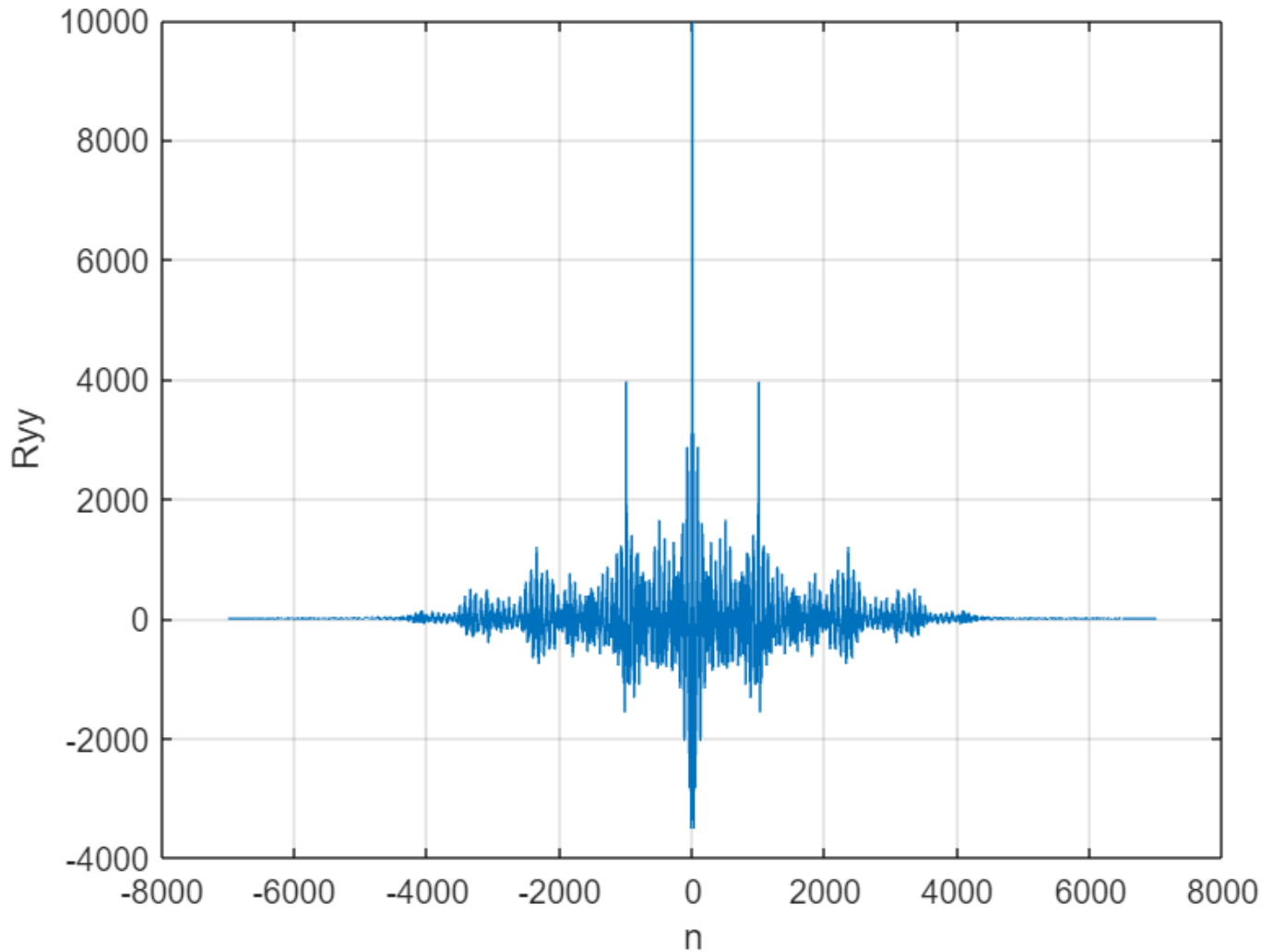
Calculate  $R_{yy}[n]$ .

```
ny = 1:length(y);
Ryy = conv(y,flip(y));
nRyy = ny(1)-ny(end):ny(end)-ny(1);
```

Plot  $R_{yy}[n]$ .

```
figure;
plot(nRyy,Ryy);
```

```
grid on;
xlabel('n');
ylabel('Ryy');
```



According to the property of Auto-Correlation signal,  $R_{xx}[n]$  has the maximum when  $n = 0$  since  $x[n]$  isn't a period signal, and the value when  $|n|$  is large is so small compared to  $R_{xx}[0]$  that it can be ignored.

Therefore, according to the equation  $R_{yy}[n] = (1 + \alpha^2)R_{xx}[n] + \alpha R_{xx}[n - N] + \alpha R_{xx}[n + N]$ , the max local maximum within local maximum of  $R_{yy}[n]$  is at  $n = 0$ . But because when  $n$  is close to 0, it also can be very big, so judging where the  $R_{yy}[N]$  is need to take both it's value and it's  $|n|$  into consideration. In this report, we simply use the first two values in the decreasing array of local maximums which is at  $n \geq 0$  as  $R_{yy}[0]$  and  $R_{yy}[n]$ .

Since  $R_{xx}[N]$ ,  $R_{xx}[-N]$ ,  $R_{xx}[2N]$ ,  $R_{xx}[-2N]$  can be ignored compared to  $R_{xx}[0]$ ,  $\frac{R_{yy}[N]}{R_{yy}[0]}$  can be seen as the

same as  $\frac{\alpha}{1+\alpha^2}$ . Solve and get  $\alpha = \frac{R_{yy}[0] \pm \sqrt{R_{yy}[0]^2 - 4R_{yy}[N]^2}}{2R_{yy}[N]}$

As above, calculate  $N$  and  $\alpha$  of  $y[n]$ .

```
% only care about n>=0
[local_maximum,local_maximum_idx] = all_local_maximum(Ryy((length(Ryy)+1)/2:end));
decreasing_local_maximum_length = 1;
for i = 2:length(local_maximum)
    if local_maximum(i)>local_maximum(decreasing_local_maximum_length)
        while local_maximum(i)>local_maximum(decreasing_local_maximum_length)
            decreasing_local_maximum_length = decreasing_local_maximum_length-1;
        end
        decreasing_local_maximum_length = decreasing_local_maximum_length+1;
        local_maximum(decreasing_local_maximum_length) = local_maximum(i);
        local_maximum_idx(decreasing_local_maximum_length) = local_maximum_idx(i);
    end
end
N = abs(local_maximum_idx(2)-local_maximum_idx(1))
```

N = 1000

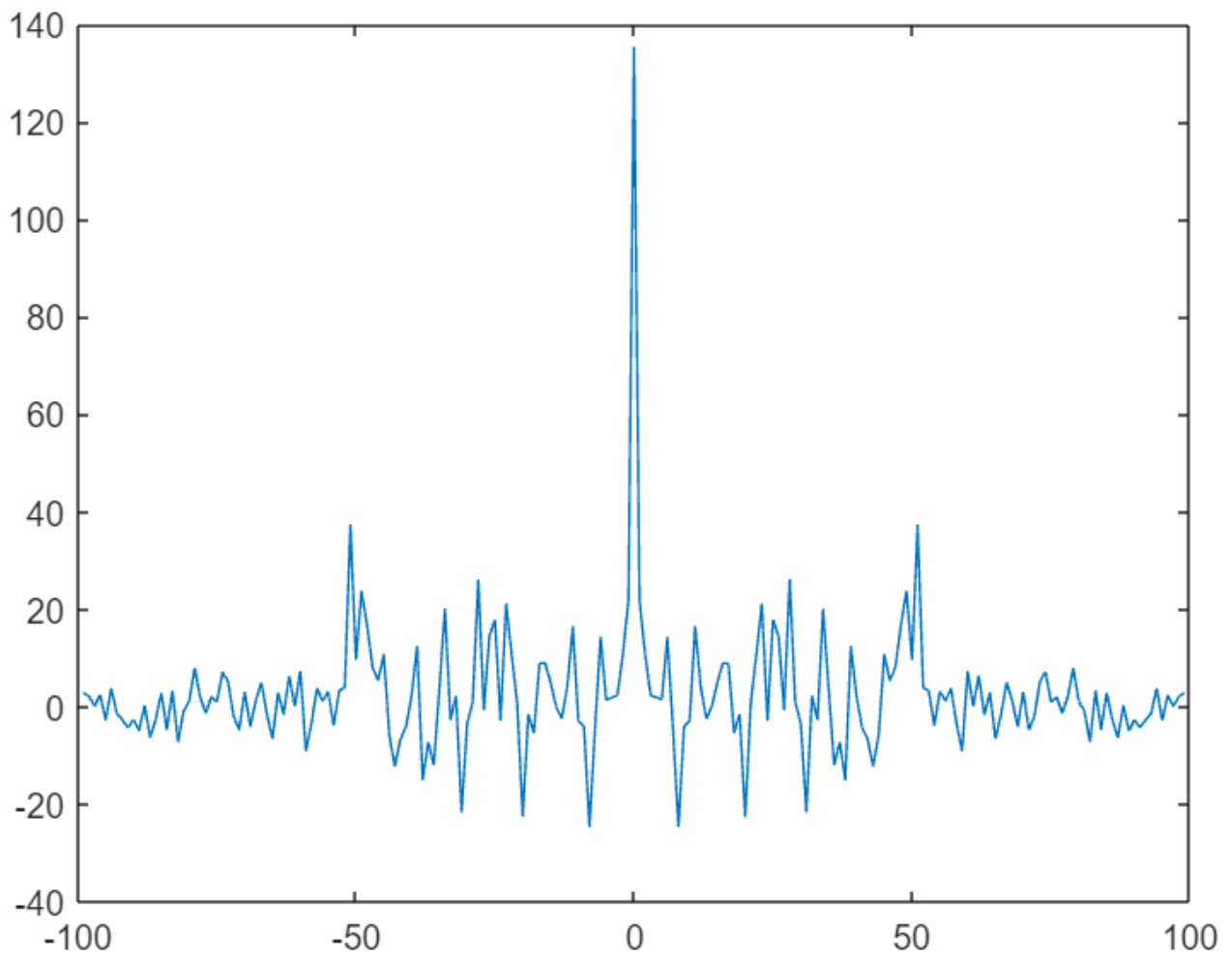
```
alpha = [(local_maximum(1)+sqrt(local_maximum(1)^2-4*local_maximum(2)^2))/
(2*local_maximum(2));(local_maximum(1)-
sqrt(local_maximum(1)^2-4*local_maximum(2)^2))/(2*local_maximum(2))]
```

```
alpha = 2×1
    2.0249
    0.4938
```

Then try the experiment in the question.

```
NX = 100;
x = randn(1,NX);
N = 50;
alpha = 0.9;
y = filter([1 zeros(1,N) alpha],1,x);
Ryy = conv(y,fliplr(y));
plot(-NX+1:NX-1,Ryy);
```





```
% only care about n>=0
[local_maximum,local_maximum_idx] = all_local_maximum(Ryy((length(Ryy)+1)/2:end));
decreasing_local_maximum_length = 1;
for i = 2:length(local_maximum)
    if local_maximum(i)>local_maximum(decreasing_local_maximum_length)
        while local_maximum(i)>local_maximum(decreasing_local_maximum_length)
            decreasing_local_maximum_length = decreasing_local_maximum_length-1;
        end
    end
    decreasing_local_maximum_length = decreasing_local_maximum_length+1;
    local_maximum(decreasing_local_maximum_length) = local_maximum(i);
    local_maximum_idx(decreasing_local_maximum_length) = local_maximum_idx(i);
end
N = abs(local_maximum_idx(2)-local_maximum_idx(1))
```

N = 51

```
alpha = [(local_maximum(1)+sqrt(local_maximum(1)^2-4*local_maximum(2)^2))/
(2*local_maximum(2));(local_maximum(1)-
sqrt(local_maximum(1)^2-4*local_maximum(2)^2))/(2*local_maximum(2))]
```

```
alpha = 2×1
 3.3182
 0.3014
```

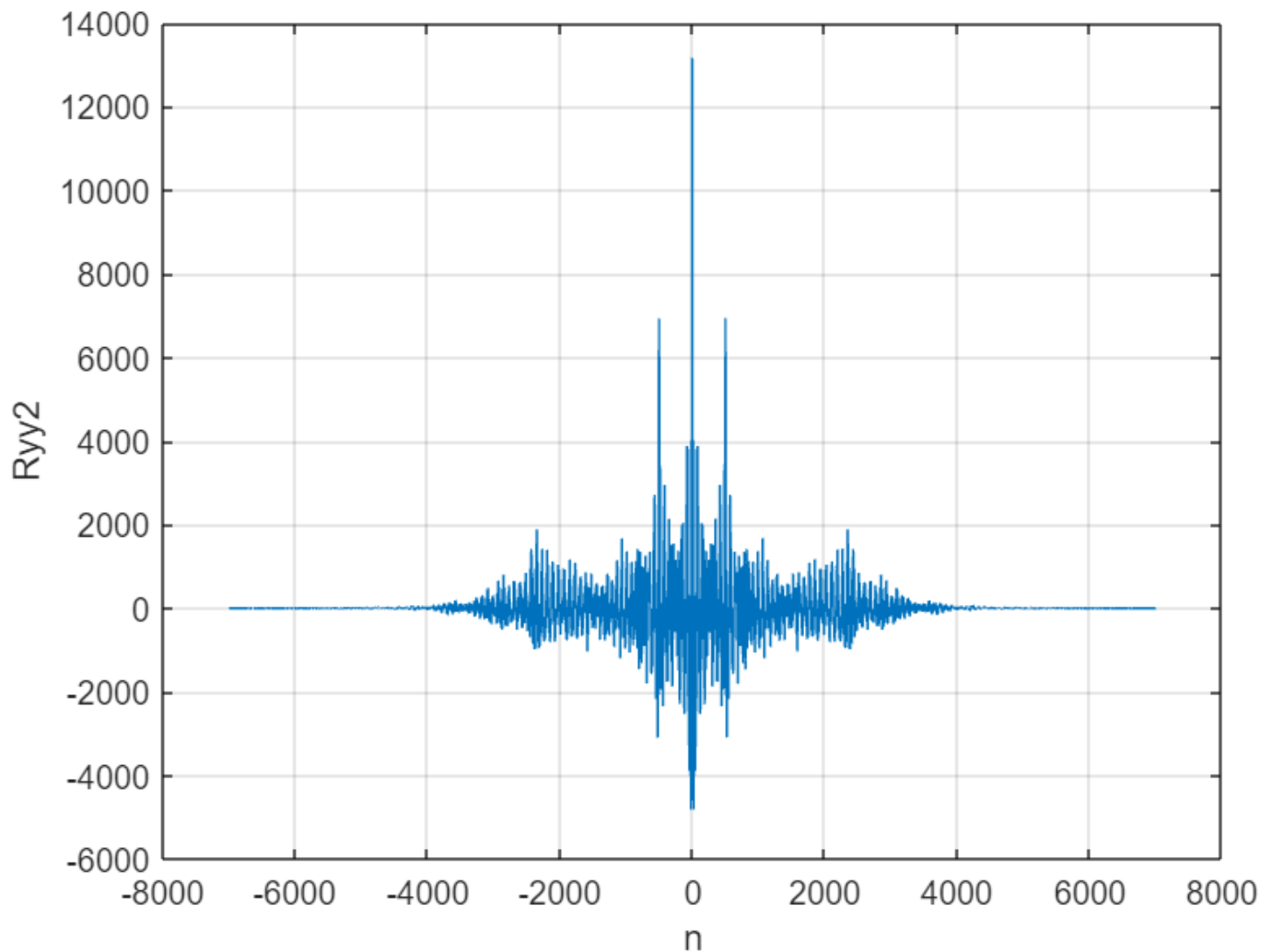
In this case, since the  $y$  is generated by filter and is incomplete, so the  $R_{yy}$  isn't the real  $R_{yy}$ , which leads to more inaccurate  $\alpha$  and may leads to the second local maximum being not  $R_{yy}[N]$  and the  $N$  going wrong too.

Then deal with  $y_2[n]$ .

```
ny2 = 1:length(y2);
Ryy2 = conv(y2,flip(y2));
nRyy2 = ny2(1)-ny2(end):ny2(end)-ny2(1);
```

Plot  $y_2[n]$ .

```
figure;
plot(nRyy2,Ryy2);
grid on;
xlabel('n');
ylabel('Ryy2');
```



Calculate  $N$  and  $\alpha$ .

```
% only care about n>=0
[local_maximum,local_maximum_idx] = all_local_maximum(Ryy2((length(Ryy2)+1)/2:end));
decreasing_local_maximum_length = 1;
for i = 2:length(local_maximum)
    if local_maximum(i)>local_maximum(decreasing_local_maximum_length)
        while local_maximum(i)>local_maximum(decreasing_local_maximum_length)
            decreasing_local_maximum_length = decreasing_local_maximum_length-1;
        end
    end
    decreasing_local_maximum_length = decreasing_local_maximum_length+1;
    local_maximum(decreasing_local_maximum_length) = local_maximum(i);
    local_maximum_idx(decreasing_local_maximum_length) = local_maximum_idx(i);
end
N = abs(local_maximum_idx(2)-local_maximum_idx(1))
```

$N = 501$

```
alpha = [(local_maximum(1)+sqrt(local_maximum(1)^2-4*local_maximum(2)^2))/
(2*local_maximum(2));(local_maximum(1)-
sqrt(local_maximum(1)^2-4*local_maximum(2)^2))/(2*local_maximum(2))]
```

```
alpha = 2x1 complex
0.9487 + 0.3163i
0.9487 - 0.3163i
```

Then deal with  $y_3[n]$ .

$$\begin{aligned}
R_{yy}[n] &= y[n] * y[-n] \\
&= x[n] * (\delta[n] + \alpha_1 \delta[n - N_1] + \alpha_2 \delta[n - N_2]) * x[-n] * (\delta[-n] + \alpha_1 \delta[-n - N_1] + \alpha_2 \delta[-n - N_2]) \\
&= R_{xx}[n] * ((1 + \alpha_1^2 + \alpha_2^2) \delta[n] + \alpha_1 \delta[n - N_1] + \alpha_1 \delta[-n - N_1] + \alpha_2 \delta[n - N_2] + \alpha_2 \delta[-n - N_2] + \alpha_1 \alpha_2 \delta[-n - N_2 + N_1] + \alpha_1 \alpha_2 \delta[n + N_2 - N_1]) \\
&= (1 + \alpha_1^2 + \alpha_2^2) R_{xx}[n] + \alpha_1 R_{xx}[n - N_1] + \alpha_1 R_{xx}[n + N_1] + \alpha_2 R_{xx}[n - N_2] + \alpha_2 R_{xx}[n + N_2] + \alpha_1 \alpha_2 R_{xx}[n + N_2 - N_1] + \alpha_1 \alpha_2 R_{xx}[n - N_2 + N_1]
\end{aligned}$$

As mentioned above, if  $N_2 \neq 2N_1$ , we define the first three values in the decreasing array of local maximums which is at  $n \geq 0$  as "peak", The highest one is  $R_{yy}[0]$ . The fastest is  $R_{yy}[N_2]$ . The rest is  $R_{yy}[N_1]$ . There is no  $R_{yy}[N_2 - N_1]$  because it's between  $R_{yy}[N_1]$  and  $R_{yy}[N_2]$  and it's smaller than both of them. But when we determined  $R_{yy}[N_1]$  and  $R_{yy}[N_2]$ , we also get  $N_1$  and  $N_2$  so we can get the value of  $R_{yy}[N_2 - N_1]$ .

$$\alpha_1 = \frac{R_{yy}[N_2 - N_1]}{R_{yy}[N_2]} \quad \alpha_2 = \frac{R_{yy}[N_2 - N_1]}{R_{yy}[N_1]}$$

If  $N_2 = 2N_1$ , the  $R_{yy}[N_2 - N_1]$  is overlapped by  $R_{yy}[N_1]$  so we can't get the value of  $R_{yy}[N_2 - N_1]$  and the computation will be more complex.

$$\frac{R_{yy}[N_2]}{R_{yy}[0]} = \frac{\alpha_2}{1 + \alpha_1^2 + \alpha_2^2}, \quad \alpha_1 = \sqrt{-\alpha_2^2 + \frac{R_{yy}[0]}{R_{yy}[N_2]} \alpha_2 - 1}$$

$$\frac{R_{yy}[N_1]}{R_{yy}[N_2]} = \frac{\alpha_1}{\alpha_2} = \frac{\sqrt{-\alpha_2^2 + \frac{R_{yy}[0]}{R_{yy}[N_2]} \alpha_2 - 1}}{\alpha_2}$$

Solve and get  $\alpha_2$  and substitute to get  $\alpha_1$ .

```
ny3 = 1:length(y3);
Ryy3 = conv(y3,flip(y3));
nRyy3 = ny3(1)-ny3(end):ny3(end)-ny3(1);
```

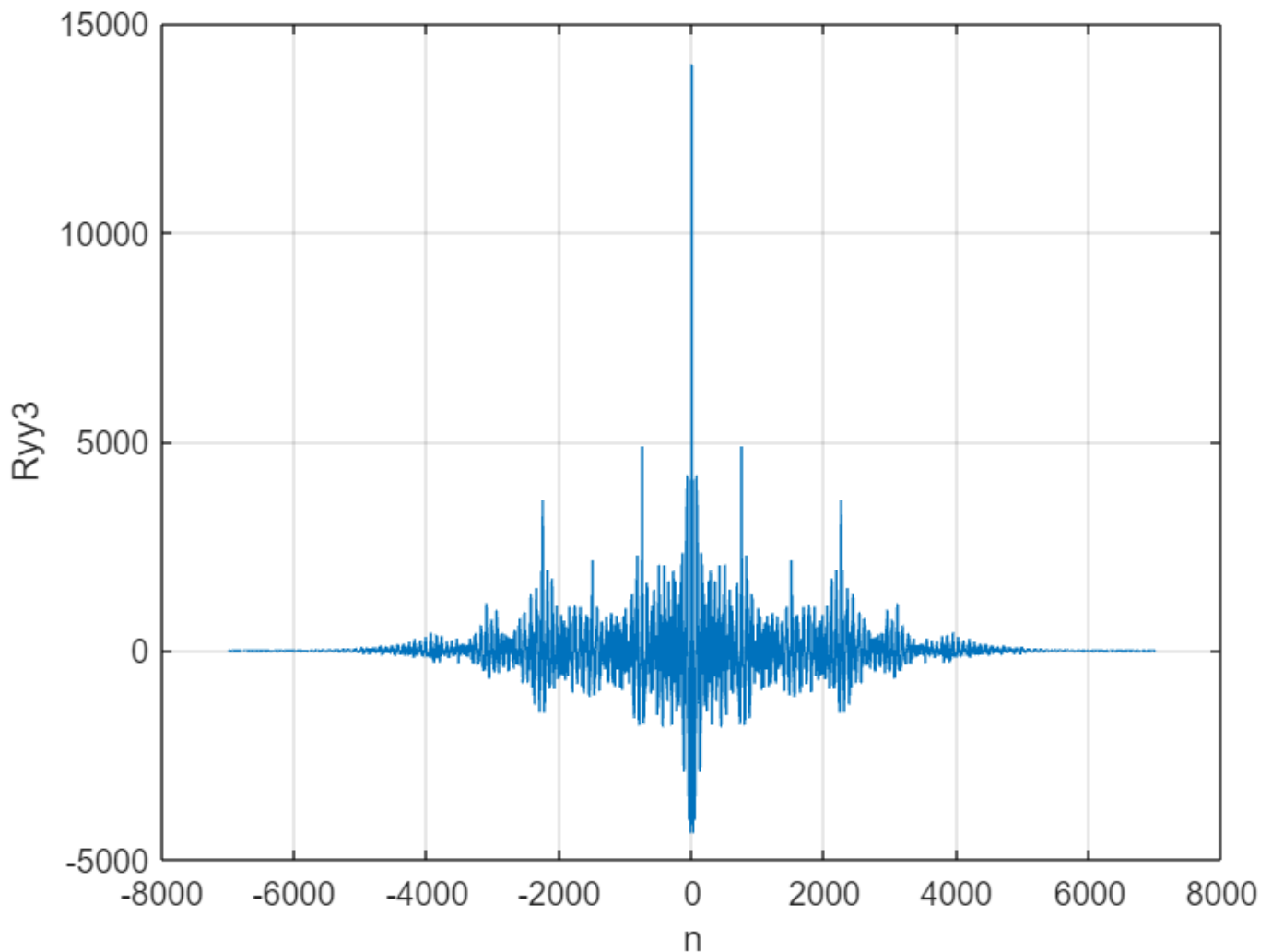
Plot  $y_3[n]$ .

```
figure;
```

```

plot(nRyy3,Ryy3);
grid on;
xlabel('n');
ylabel('Ryy3');

```



Calculate  $N$  and  $\alpha$ . To figure out how many peak there are, we decide to judge whether the 6th and 7th largest local maximum more closer to 5th one than 8th.

```

% only care about n>=0
[local_maximum,local_maximum_idx] = all_local_maximum(Ryy3((length(Ryy3)+1)/2:end));
decreasing_local_maximum_length = 1;
for i = 2:length(local_maximum)
    if local_maximum(i)>local_maximum(decreasing_local_maximum_length)
        while local_maximum(i)>local_maximum(decreasing_local_maximum_length)
            decreasing_local_maximum_length = decreasing_local_maximum_length-1;
        end
    end
    decreasing_local_maximum_length = decreasing_local_maximum_length+1;
    local_maximum(decreasing_local_maximum_length) = local_maximum(i);
end

```

```

    local_maximum_idx(decresing_local_maximum_length) = local_maximum_idx(i);
end
syms a;
alpha2 = double(solve(sqrt(-a^2+local_maximum(1)/local_maximum(3)*a-1)/
a==local_maximum(2)/local_maximum(3),a));
N1 = local_maximum_idx(2)-local_maximum_idx(1)

```

```
N1 = 751
```

```
N2 = local_maximum_idx(3)-local_maximum_idx(1)
```

```
N2 = 2252
```

```
alpha1 = local_maximum(2)/local_maximum(3)*alpha2
```

```

alpha1 = 2×1
    0.4648
    1.3947

```

```
alpha2
```

```

alpha2 = 2×1
    0.3424
    1.0274

```

Since  $\alpha_1$  and  $\alpha_2$  smaller than 1.

```
alpha1 = alpha1(1)
```

```
alpha1 = 0.4648
```

```
alpha2 = alpha2(1)
```

```
alpha2 = 0.3424
```

## Functions

Unit impulse function

```

function y = unitimpulse (x)
    y = double(x==0);
end

```

Find next local maximum and it's index

```

function [max,idx] = next_local_maximum (array,idx)
    if idx >= length(array)
        idx = -1;
        max = -inf;
        return;
    end
    while idx ~= length(array) && array(idx+1) <= array(idx)
        idx = idx+1;
    end

```

```

if idx== length(array)
    idx = -1;
    max = -inf;
    return;
end
while idx ~= length(array) && array(idx+1) >= array(idx)
    idx = idx+1;
end
max = array(idx);
end

```

Find all local maximum and its index

```

function [res,res_idx] = all_local_maximum (array)
if array(1)>array(2)
    res = array(1);
    res_idx = 1;
    res_num = 1;
else
    res = [];
    res_idx = [];
    res_num = 0;
end
[value,idx] = next_local_maximum(array,2);
while idx ~= -1
    if res_num == length(res)
        res = [res zeros(1,100)];
        res_idx = [res_idx zeros(1,100)];
    end
    res_num = res_num+1;
    res(res_num) = value;
    res_idx(res_num) = idx;
    [value,idx] = next_local_maximum(array,idx+1);
end
res = res(1:res_num);
res_idx = res_idx(1:res_num);
end

```

## Expeience

Through Matlab work, I have a preliminary understanding of Matlab simulation tools in the signal and system of the application of the discipline. I'm more famililar with the usage of the function and and to form echo system and echo removal system. I also learned the properties of Auto-Correlation signal and how to use it to figure out the echo from a signal containing both original signal, by which we can know the echo time to get the distance or even approximately remove the echo to get a more clear signal. What I learned in this chapter is very useful in our daily life. And I will continuously improve myself.

## Score

涂峻陵 (100) , 欧阳安男 (100)

