COMPARING NONLINEAR DIMENSIONALITY REDUCTION METHODS FOR DATA-DRIVEN UNSTEADY FLUID FLOW MODELING

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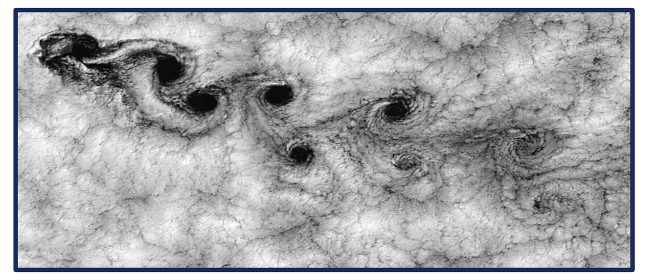
Introduction



- CFD data is high-dimensional both in space and time, required for the numerical schemes to converge
- Natural systems tend to live on low-dimensional manifolds
- Finding these manifolds is a crucial first step towards reduced order models (ROMs). This can be used for evolving the time dynamics, extracting flow features or denoising
- Objective: Identify a new set of low-dimensional coordinates that provide a good representation of the data and extract underlying coherent structures, flow patterns.

Methods

fenics project, SimVascular, PyTorch, learn

PCA/POD/SVD

- $\mathbf{X} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}$, columns of $\mathbf{U}$ contain spatial modes – "eigenflows"
- Non-parametric, computationally cheap but inherently linear and sensitive to outliers

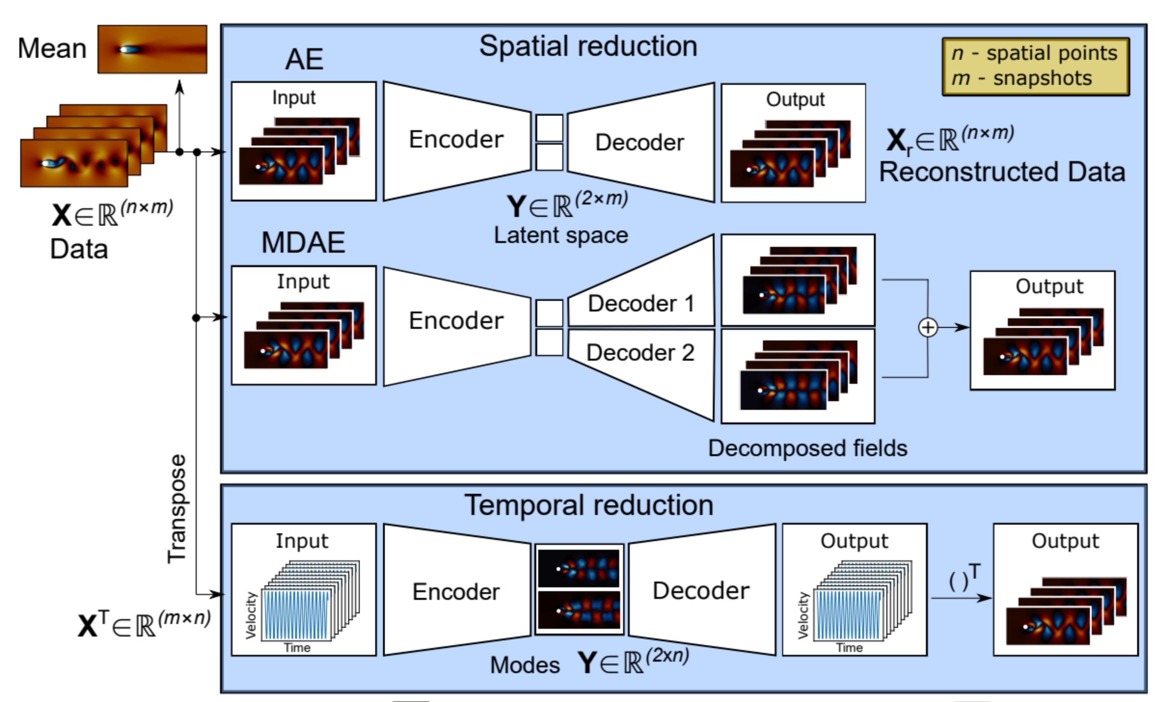
Nonlinear dimensionality reduction (NDR)

- Manifold learning: kernel PCA (**KPCA**), Locally Linear Embedding (**LLE**), Laplacian Eigenmaps (**LEM**), isometric mapping (**Isomap**)
- Can handle nonlinearities, but needs hyperparameter tuning, slower than PCA and reconstruction is not straightforward
- Autoencoder (**AE**), Mode-decomposing autoencoder (**MDAE**)

Fig.1 Data matrix

$$\mathbf{X}_{(n\times m)} = \begin{bmatrix} | & | & \dots & | \\ \boldsymbol{x}_1 & \boldsymbol{x}_2 & \dots & \boldsymbol{x}_m \\ | & | & \dots & | \end{bmatrix}$$

Column Row



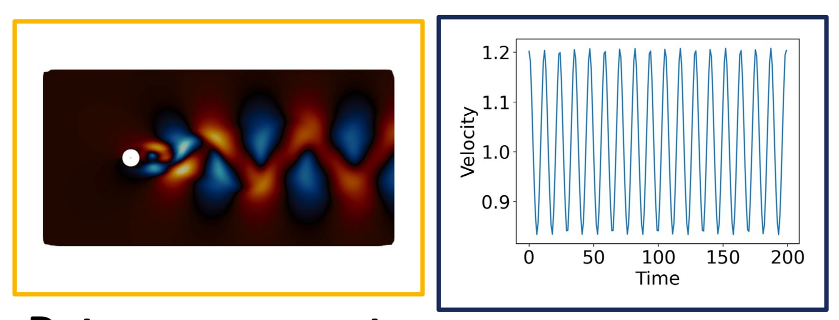
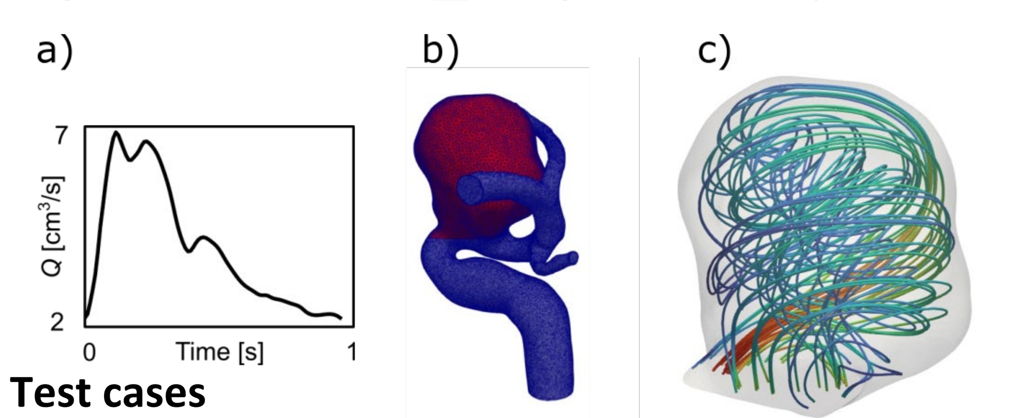


Fig.2 AE architecture Fig.3 ICA aneurysm case



Data arrangement: spatial vs temporal reduction

- Flattened velocity snapshots stacked column-wise into $\mathbf{X}$
- Using the columns of $\mathbf{X}$ as inputs leads to spatial reduction, useful for obtaining low dimensional embeddings suitable for ROMs
- Using the rows of $\mathbf{X}$ as inputs leads to temporal reduction, useful for obtaining visualizable velocity modes that can uncover the underlying flow features

Test cases

- 2D flow over cylinder at $Re = 100$, leading to periodic vortex shedding, known as the von Kármán vortex street
- Pulsatile blood flow in an internal carotid artery (ICA) aneurysm, using a population averaged inlet flow waveform (Fig. 3a). Domain is cropped and downsampled to contain only the aneurysmal region for the dimensionality reduction (Fig. 3b)

Results

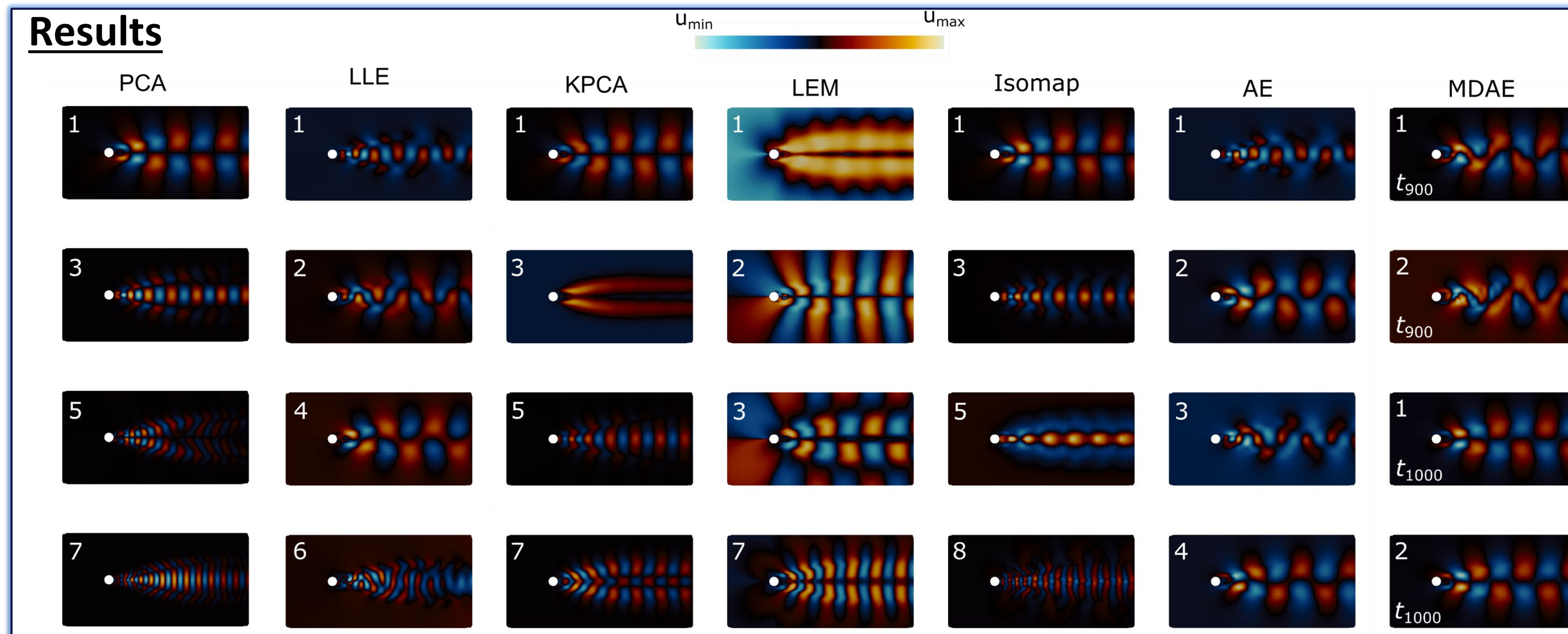
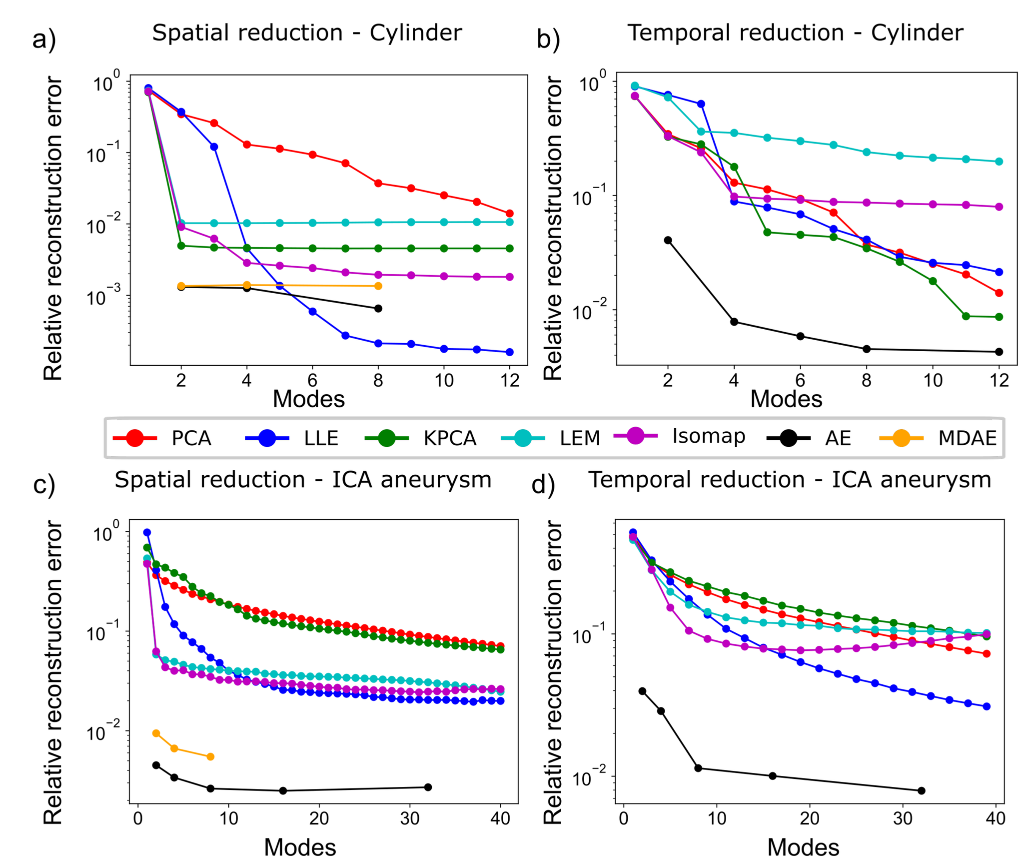


Fig.4 Cylinder modes Fig.5 Errors



- NDR methods have smaller error for spatial reduction than PCA, but for temporal reduction the results are less decisive

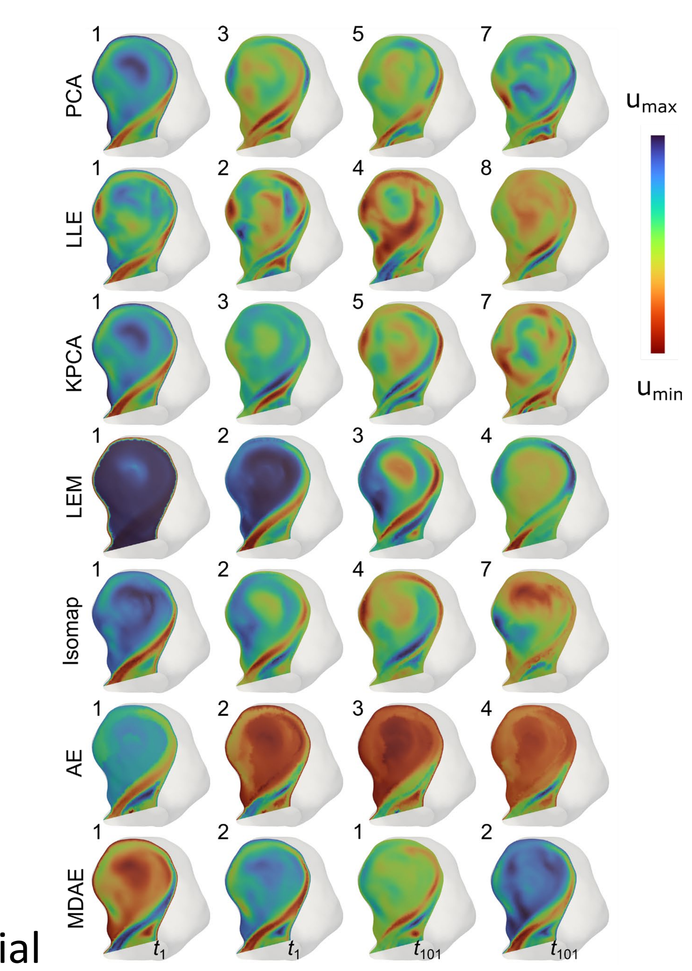


Fig.6 ICA modes

- PCA – spatial and temporal are the same: $\mathbf{X}$ vs. $\mathbf{X}^{\mathrm{T}}$
- AE is clearly better than all others for both space & time
- Dominant mode structures appear in all methods
- For NDR error does not decrease monotonically

Conclusions

- For spatial dimensionality reduction nonlinear methods have a smaller error than PCA
- Spatial reduction → ROMs; Temporal reduction → visualizable modes, coherent structures
- Overall, AE has the best performance and most flexible framework for NDR
- Several advantageous properties of PCA are not inherited by the other techniques

References

H. Csala, S. T. M. Dawson, A. Arzani; **Comparing different nonlinear dimensionality reduction techniques for data-driven unsteady fluid flow modeling**. *Physics of Fluids* 1 November 2022; 34 (11): 117119. https://doi.org/10.1063/5.0127284

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