COMPARING NONLINEAR DIMENSIONALITY REDUCTION METHODS FOR DATA-DRIVEN UNSTEADY FLUID FLOW MODELING

Cardiovascular Biomechanics Laboratory
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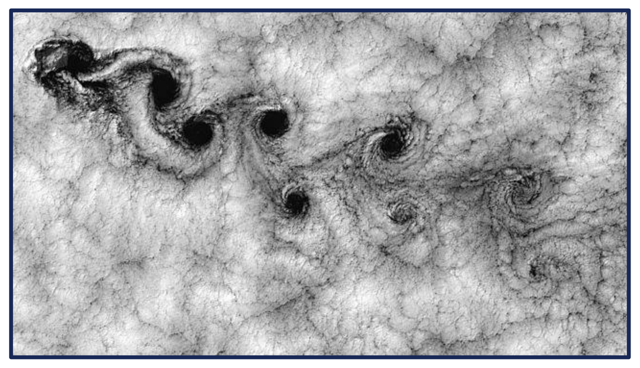
Introduction

- CFD data is high-dimensional both in space and time, required for the numerical schemes to converge
- Natural systems tend to live on low-dimensional manifolds
- Finding these manifolds is a crucial first step towards reduced order models (ROMs). This can be used for evolving the time dynamics, extracting flow features or denoising



- **Objective:** Identify a new set of low-dimensional coordinates that provide a good representation of the data and extract underlying coherent structures, flow patterns.

Methods

FEniCS Project SimVascular PyTorch learn

PCA/POD/SVD

- $\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathbf{T}}$, columns of $\mathbf{U}$ contain spatial modes – "eigenflows"
- Non-parametric, computationally cheap but inherently linear and sensitive to outliers

Nonlinear dimensionality reduction (NDR)

- Manifold learning: kernel PCA (**KPCA**), Locally Linear Embedding (**LLE**), Laplacian Eigenmaps (**LEM**), isometric mapping (**Isomap**)
- Can handle nonlinearities, but needs hyperparameter tuning, slower than PCA and reconstruction is not straightforward
- Autoencoder (**AE**): neural networks with special architecture
- Mode-decomposing autoencoder (**MDAE**) based on [1]

$$X_{(n\times m)} = \begin{bmatrix} | & | & \dots & | \\ x_1 & x_2 & \dots & x_m \\ | & | & \dots & | \end{bmatrix}$$

Column Row

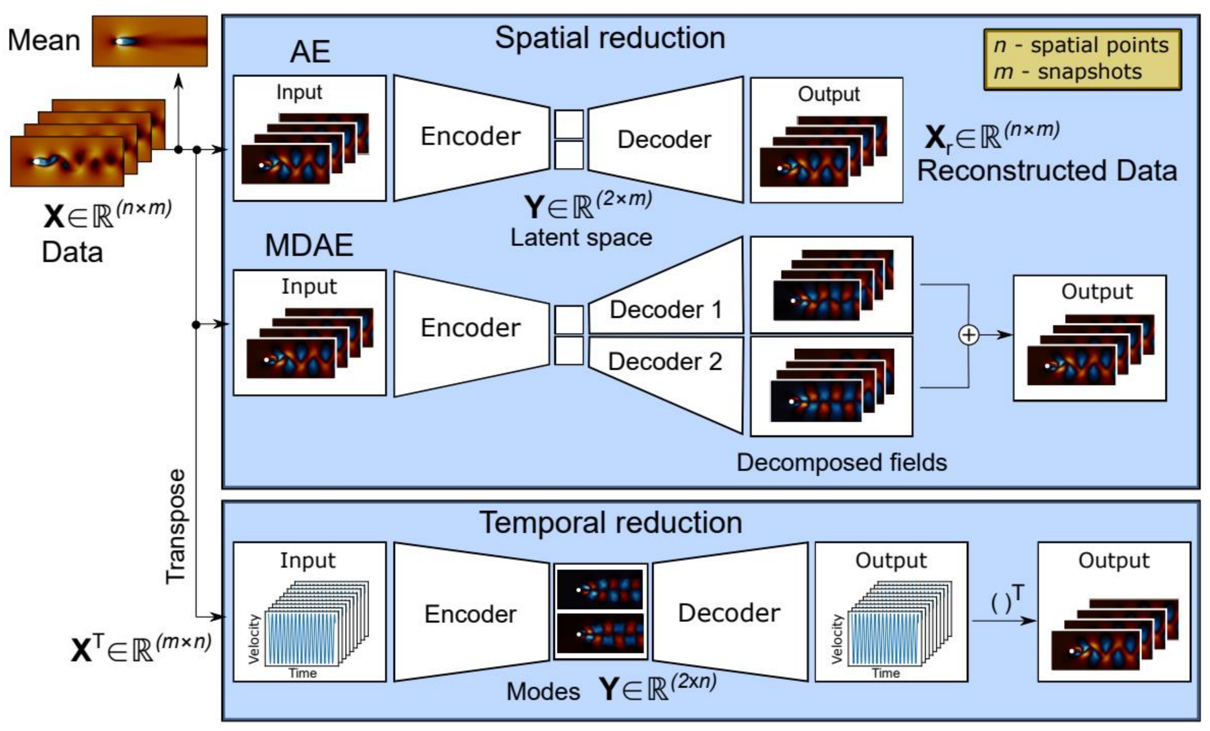


Fig 2. Autoencoder architectures.

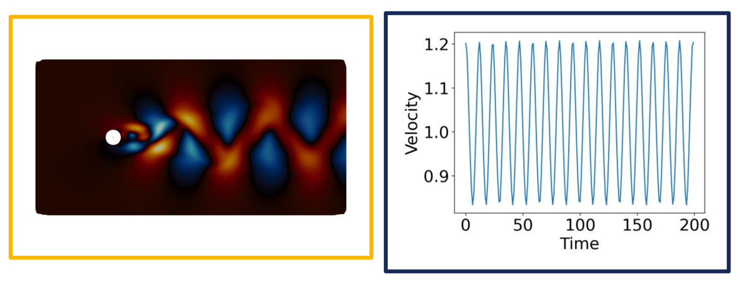


Fig 1. Data arrangement matrix.

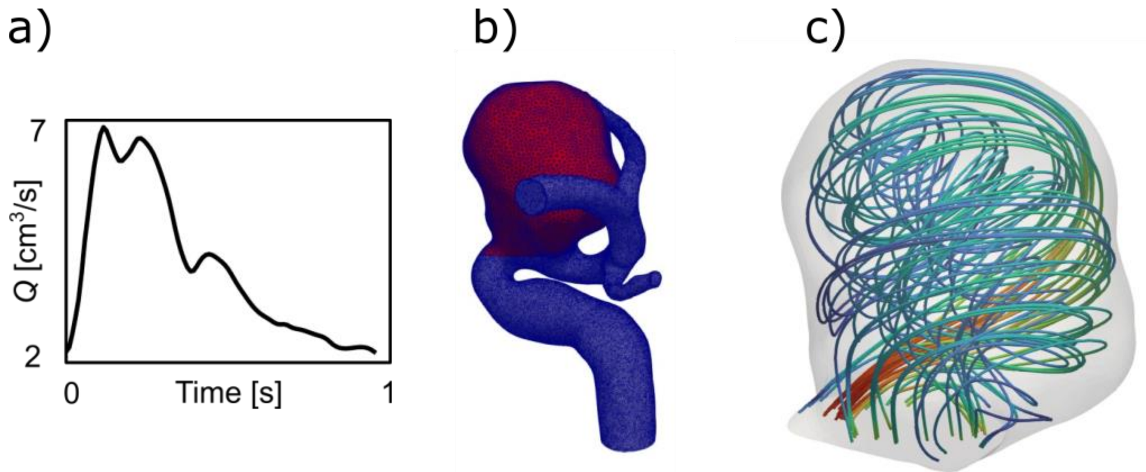


Fig 3. Brain aneurysm – inlet waveform, simulation domain, streamlines in cropped region.

Data arrangement: spatial vs temporal reduction

- Flattened velocity snapshots stacked column-wise into $\mathbf{X}$
- Using the columns of $\mathbf{X}$ as inputs leads to spatial reduction, useful for obtaining low dimensional embeddings suitable for ROMs
- Using the rows of $\mathbf{X}$ as inputs leads to temporal reduction, useful for obtaining visualizable velocity modes that can uncover the underlying flow features

Test cases

- 2D flow over cylinder at $Re = 100$, leading to periodic vortex shedding, known as the von Kármán vortex street
- Pulsatile blood flow in an internal carotid artery (ICA) aneurysm, using a population averaged inlet flow waveform from [2]. Domain is cropped and downsampled to contain only the aneurysmal region for the dimensionality reduction

Results

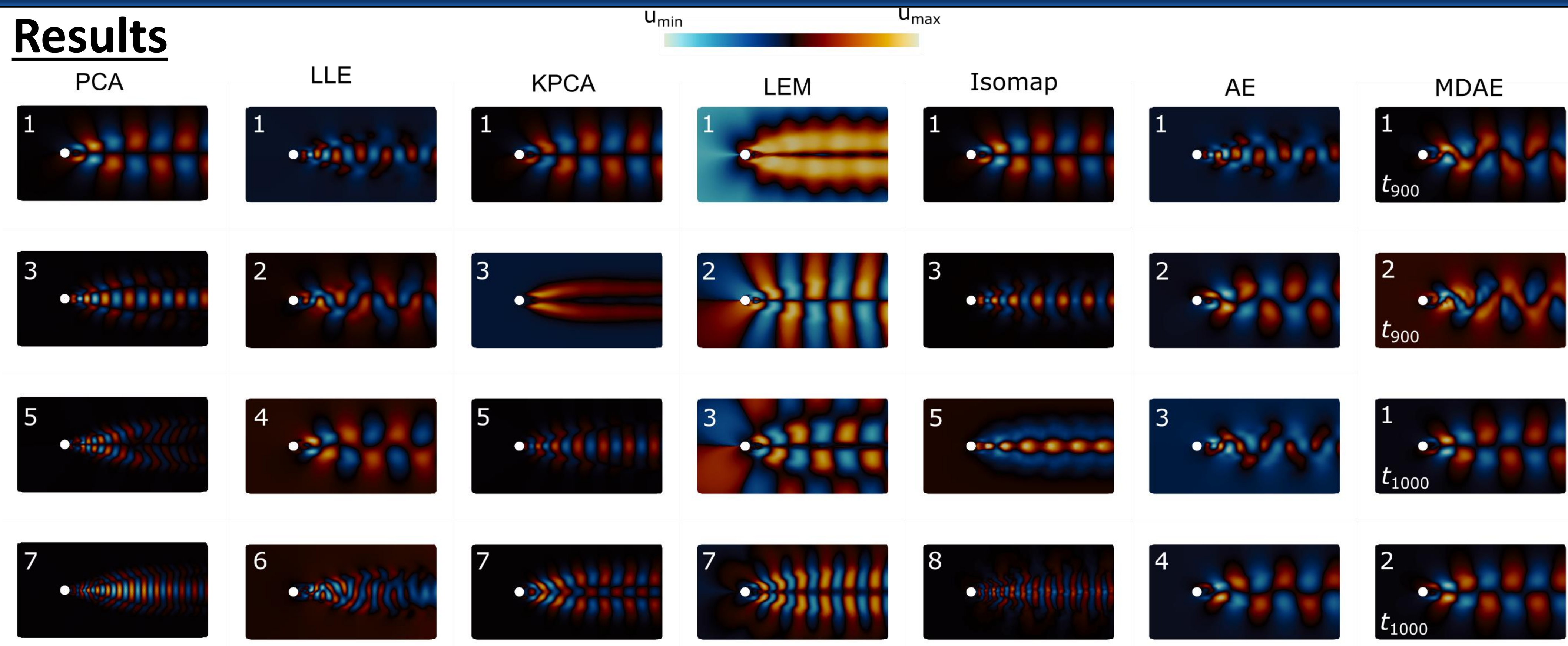


Fig 4. Flow over cylinder modes.

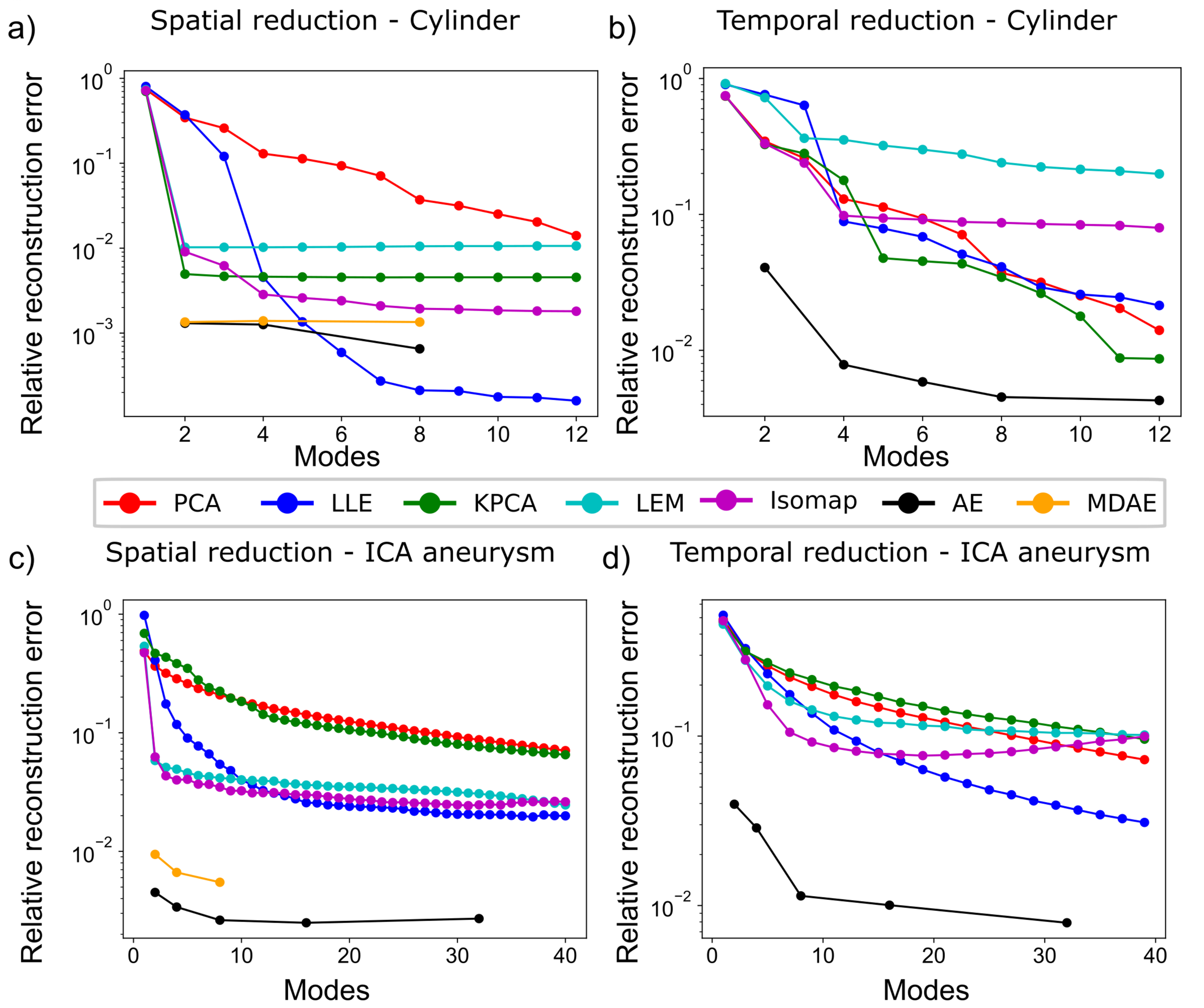


Fig 5. Relative reconstruction error.

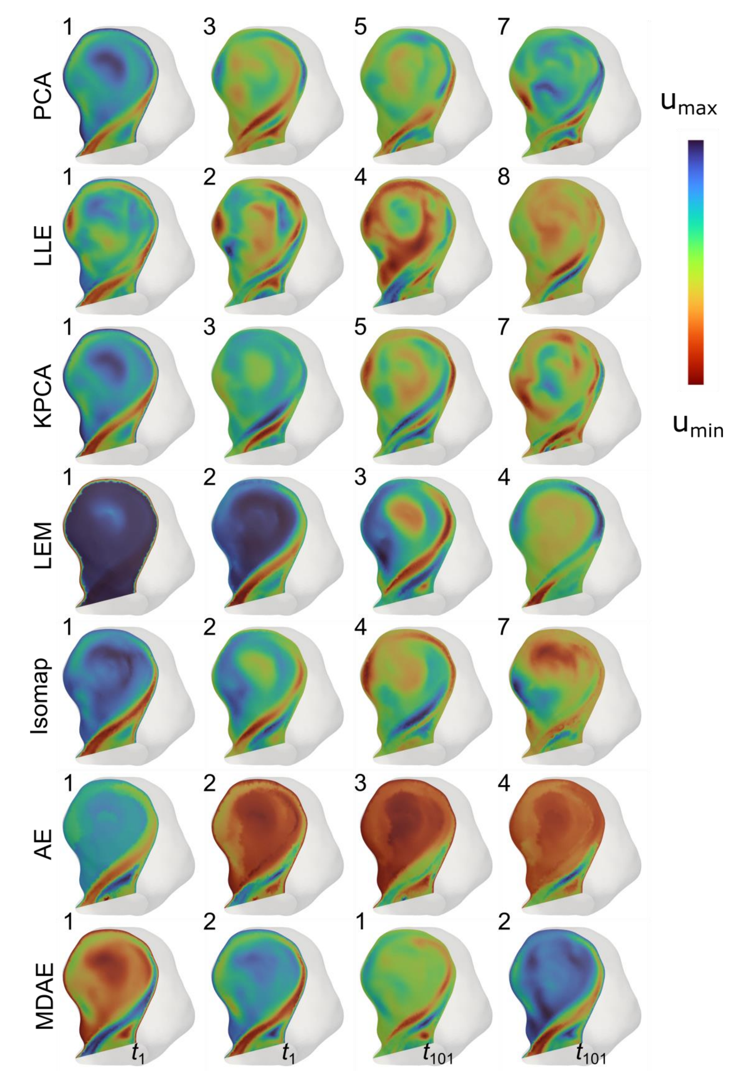


Fig 6. ICA brain aneurysm modes.

- PCA – spatial and temporal are the same: $\mathbf{X}$ vs. $\mathbf{X}^{\mathbf{T}}$
- NDR methods have smaller error for spatial reduction than PCA, but for temporal reduction the results are less decisive
- For NDR error does not decrease monotonically
- AE is clearly better than all others for both space & time
- Dominant mode structures appear in all methods

Conclusions

- For spatial dimensionality reduction nonlinear methods have a smaller error than PCA
- Spatial reduction → ROMs
- Temporal reduction → visualizable modes, coherent structures
- Overall, AE has the best performance and most flexible framework for NDR
- Several advantageous properties of PCA are not inherited by the other techniques

References

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[2] Hoi, Y., Wasserman, B.A., Xie, Y.J., Najjar, S.S., Ferruci, L., Lakatta, E.G., Gerstenblith, G. and Steinman, D.A., 2010. Characterization of volumetric flow rate waveforms at the carotid bifurcations of older adults. Physiological measurement, 31(3), p.291.

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