1. Outline and Lecture Notes

Session 08: Laplace Transform and System Stability

Learning Outcomes:

- Understand the fundamentals of the Laplace Transform and its inverse.
- Apply the Laplace Transform for solving differential equations in system analysis.
- Analyse system poles and zeros for determining stability.

1.1 What is the Laplace Transform?

The Laplace Transform converts a time-domain function f(t) into a complex frequency-domain function F(s):

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

Where:

- $s=\sigma+j\omega$ is a complex variable
- Used for analysing LTI systems, especially differential equations.

1.2 Inverse Laplace Transform

$$f(t) = \mathcal{L}^{-1}\{F(s)\}$$

- Often retrieved using tables or partial fraction expansion.

1.3 Laplace Transform Properties

- Linearity
- Time Shifting
- Frequency Shifting
- Differentiation in Time Domain
- Convolution Theorem

1.4 Application in System Analysis

- Use Laplace to convert differential equations into algebraic equations.
- Solve for output Y(s) given X(s) and transfer function H(s):

$$H(s) = \frac{Y(s)}{X(s)}$$

1.5 Poles and Zeros

- **Poles**: Values of s that make the denominator zero.
- **Zeros**: Values of s that make the numerator zero.
- **System Stability**: A system is stable if all poles of H(s) have negative real parts (in the Left Half Plane).

1.6 Example: First Order System

$$H(s) = \frac{1}{s + a}$$

- Pole at $s=-a$ → stable if $a>0$

1.7 Visualization

- Use pole-zero plots and step/impulse responses to analyze system behavior.
