Msc in Statistics
Exercises for having fun

Consider the untitled distribution with density

$$f(x;\theta) = \frac{\theta^2}{\theta+1}(1+x)\exp(-\theta x), \quad x,\theta > 0$$

To start with give a name to this distribution. Derive and write `R` code to simulate based on

a Inversion method (hint, you need to find the cdf and invert it)

b Rejection method for $\theta = 2$, need to select some envelope

c Composition method (hint: look the density and see if you can split it in two parts)

You can cheat if you like, but for your practice assume that you have only uniforms with `runif`.

Since now you have at least an algorithm to simulate from this distribution, use the idea of Monte Carlo for

- Take a sample of size $n = 100$ and find the sampling distribution of the sample mean and the sample median. Compare also their variances and their correlation. For your convenience use $\theta = 5$. Repeat with size $n = 1000$.
- Estimate the quantity $P(X > 5)$ where X follows the above distribution with $\theta = 5$.

Important The exercise is not obligatory, it does not give any credits and we will not solve it in detail unless we find sufficient time at the end of the course. For your info, partially this was part of the exams at the past. If you like you can hand me solutions to discuss in person if you like

Bonus round: Consider the following bivariate distribution defined in the unit squared

$$f(x,y) = \frac{\Gamma(\alpha+\beta+\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)} x^{\alpha-1} y^{\beta-1} (1-x-y)^{\gamma-1}$$

for $x, y \geq 0$ with $x + y \leq 1$, and $\alpha, \beta, \gamma > 2$. Derive a rejection algorithm to simulate from this distribution with $\alpha = \beta = \gamma = 3$. How can you check that you did it?