

Stochastic finance: Mid term

1. Consider the trinomial model in which the risky asset has an up, a middle and a down state.
 - (a) Does there exist equivalent martingale measures (EMM) Q for this model? What do you observe?
 - (b) Are there any arbitrage opportunities in the trinomial model? Justify your answer
 - (c) Write a code to simulate paths from the trinomial model and use it to calculate $\mathbb{E}[S(t)]$ and $Var(S(t))$. See if your results are in accordance with the analytic results.
2. Write a code to perform simulation of the Garch(1,1) model. Use your simulation code to verify the theoretical results concerning the behaviour of the unconditional variance and the predictions (e.g. $\mathbb{E}[X(t+n) | \mathcal{F}_t]$) we obtained in class.
3. Show that the binomial model in the right limit converges in distribution to the Black-Scholes model

$$S(t) = S(0) \exp((\mu - \sigma^2/2)t + \sigma B(t)),$$

where $\{B(t) : t \geq 0\}$ is the Wiener process (Brownian motion), defined by

- $B(t+s) - B(s) \sim N(0, t)$,
- $B(t_3) - B(t_2), B(t_2) - B(t_1)$ independent for any choice $t_1 < t_2 < t_3$,
- $t \mapsto B(t)$ continuous a.s.
- $B(0) = 0$.

Ideally you should be able to provide a mixture of theoretical arguments (based on central limit type arguments) and numerical indications based on simulation.