# HW1. Steepest edge

資工四 408410098 蔡嘉祥

## Meaning of Steepest edge

$$X = \begin{bmatrix} X_B \\ — \\ X_N \end{bmatrix} = \begin{bmatrix} B^{-1}b \\ — \\ 0 \end{bmatrix} = \begin{bmatrix} x_{B1} \\ x_{B2} \\ \dots \\ x_{Bm} \\ \hline x_{N1}(0) \\ \dots \\ x_{N_{n-m}}(0) \end{bmatrix}$$

$$z = \mathbf{c}_B B^{-1} N - \sum_{j \in Nonbasic} (\underbrace{\mathbf{c}_B B^{-1} A_j}_{z_j} - c_j) x_j$$

### Gradiant of $z$

$$\nabla z_j = \frac{\partial z}{\partial x_j} = -(z_j - c_j)$$

### Length of $\Delta X$ if changing $x_j$

Minimum ratio test :

- $\beta = B^{-1}b = X_B$
- $\alpha_k = B^{-1}A_k$
- entering variable :
  - $\theta_k = \min(\frac{\beta_r}{\alpha_{r,k}} | \forall \alpha_{r,k} > 0, k \in \text{Nonbasic})$

if entering $x_r$ to $\theta_k$ by minimum ratio test:

- $X^* = \begin{bmatrix} x_{B_1} - \theta_k * \alpha_{1,k} \\ x_{B_2} - \theta_k * \alpha_{2,k} \\ \dots \\ x_{B_m} - \theta_k * \alpha_{m,k} \\ \hline x_{N1}(0) \\ \dots \\ \theta_k \\ \dots \\ x_{N_{n-m}}(0) \end{bmatrix}$

different (distance, In $L_2$ norm) between 2 solution vectors:

- $\Delta X = |X - X^*|_2 = \left| \begin{bmatrix} \theta_k * \alpha_{1,k} \\ \theta_k * \alpha_{2,k} \\ \dots \\ \theta_k * \alpha_{m,k} \\ \hline 0 \\ \dots \\ -\theta_k \\ \dots \\ 0 \end{bmatrix} \right|_2$
  $= \sqrt{\theta_k^2 + \sum_{j \in Nonbasic} (\theta_k * \alpha_{j,k})^2} = \theta_k \sqrt{1 + \sum_{j \in Nonbasic} (\alpha_{j,k})^2} = \theta_k \sqrt{1 + |\alpha_k|_2^2}$

### Steepest edge

if change $x_{j_k}$ to $\theta_k$,

$\Delta z = \nabla z_k \theta_k$ and hence **change per unit length** = $\frac{\nabla z_k \theta_k}{\theta_k \sqrt{1+|\alpha_k|_2^2}}$

Entering the most steepest one:

(i.e. argument maximum of **change per unit length**)

$\mathrm{argmax}_k(\frac{\nabla z_k \theta_k}{\theta_k \sqrt{1+|\alpha_k|_2^2}}) = \mathrm{argmax}_k(\frac{\nabla z_k}{|\alpha_k|_2}) = \mathrm{argmin}_k(\frac{z_k - c_k}{|B^{-1} A_k|_2})$

## Comparison

- disadvantage:
  - Computationally expensive. But still, less than `greatest improvement` method.
- advantage:

- In practice, it works well.
  - Leading to fewer pivots overall than `greatest improvement` method.

## Reference:

Steepest-edge rule and its number of simplex iterations for a nondegenerate LP (https://www.sciencedirect.com/science/article/pii/S0167637718304346)

https://people.orie.cornell.edu/dpw/orie6300/Lectures/lec13.pdf