Normalized Adjacency and Laplacian Matrixes (II)

$$A_N = D^{-\frac{1}{2}} A D^{-\frac{1}{2}}$$

Normalized Laplacian Matrix :

$$L_{N} = I - A_{N}$$

$$L_{N} = I - A_{N} = D^{-\frac{1}{2}}(D - A)D^{-\frac{1}{2}} = D^{-\frac{1}{2}}LD^{-\frac{1}{2}}$$

■ Claim: let $\alpha_1 \ge \cdots \ge \alpha_n$ be the eigenvalues of A_N and $\lambda_1 \le \lambda_2 \ldots \le \lambda_n$ are eigenvalues of L_N

$$1 = \alpha_1 \ge \dots \ge \alpha_n \ge -1$$
$$0 \le \lambda_1 \le \lambda_2 \dots \le \lambda_n \le 2$$