Potential Outcome Framework and Estimate of Treatment Effect

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Purpose

- The origin of causal inference is field experiment. The study of yield from a particular variety of plot turns out the principle of potential outcome model (POM).
- The goal of causal inference is to assess the difference caused by one and only one differing circumstance. We want to investigate how much our treatment causes the observed phenomenon, assuming that the observed and unobserved ones occur randomly.

Examples

- Post cards to boost turn out (Green and Gerber, 2008)
- Minimum wage on employment (Card and Krueger, 1994)
- Draft on civic duty (Erikson, 2011)
- Change of positions of newspaper on public opinion (Lenz and Ladd, 2009)
- Incumbency status on vote choice (Lee, 2008)
- Artillery fire incites insurgence (Lyall, 2009)

Example of Plots

- Suppose that there are i urns and each urn has m plots. We pick one plot (ex. plot 1) and the rest of plot 1 in other urns are taken away. In other words, we assume that every plot 1 is the same and we don't need to know which urn that we choose.
- Imagine that there are only two urns. The first urn is called treatment and the second one is called control. We pick certain number of cards from each urn but we immediately forget which urn the cards come from. In doing so, we make sure that every post-treatment effect would be randomized.

Treatment, Outcome, Potential Outcome

Treatment

D_i : Indicator of treatment for *unit i*

$$D_i = \begin{cases} 1, \text{if unit } i \text{ receives the treatment} \\ 0, \text{otherwise} \end{cases}$$

Outcome

Y_i : Observed outcome of interest for unit i

Potential Outcomes

$$Y_{di} = \begin{cases} Y_{0i}, \text{Potential outcome for unit } i \text{ without treatment} \\ Y_{1i}, \text{Potential outcome for unit } i \text{ with treatment} \end{cases}$$

Definition and Assumption

Assumption

Observed outcomes are realized as

$\mathrm{Y}_i = D_i \cdot \mathrm{Y}_{1i} + (1 - D_i) \cdot \mathrm{Y}_{0i}$

so $\mathrm{Y}_i = \begin{cases} Y_{1i}, \text{if } D_i = 1 \\ Y_{0i}, \text{if } D_i = 0 \end{cases}$

Definition

Causal effect is the difference between its two potential outcomes:

$\alpha_i = Y_{1i} - Y_{0i}$

SUTVA

Potential outcomes for unit i are unaffected by unit j

Potential Outcome Framework

- The quantity Y_{1i} means the unit i have outcome as variable Y. Actually, *it may or may not receive the treatment*, even it is from the treated group ($D_i = 1$). Y_{0i} represents the quantity of outcome for unit i of the not-treatment group, whether or not it receives the treatment.
- That involves missing data because we cannot observe both *potential outcomes*, namely Y_{0i} and Y_{1i}. Remember we only observe *realized outcomes*, Y_i.
- For example, we may observe students' performance who actually come to this talk, but we may not observe these students who would stay home. In the same way, we can evaluate students' performance who are not in this room, but we may not be able to observe performance of those students if they were in this room. That is the idea of *counterfactual*.

Identification Problem

Problem

Provided that we cannot observe unit i's potential outcome, either treated or not-treated, how can we compute the difference due to the treatment?

$$\alpha_i = Y_{1i} - Y_{0i}$$

homogeneity

Most individuals are different, so we cannot assume that their responses to treatment D is the same, ie. Y_{1i} or Y_{0i} is the same for every unit even some of them do not receive the treatment.

The random assignment process ensures that who receives the treatment is random, so we can assume that one group of unit *is* that receives the treatment is the same with another group that receives the treatment.

If so, we can estimate average treatment effect (ATE) as:

$$\begin{aligned} ATE &= E[Y_1 - Y_0] \\ &= E[Y_1] - E[Y_0] \\ &= E[Y_1 \mid D = 1] - E[Y_0 \mid D = 0] \end{aligned}$$

ATE

ATE=E$[Y_1 \mid D=1] - E[Y_0 \mid D=0] = E[Y_1 - Y_0] = E[Y_1] - E[Y_0]$

ATET

We care about average treatment effect among the treated

ATET=E$[Y_1 - Y_0 \mid D=1]$

ATE=ATET

When $Y_1 \perp D, E[Y_1] = E[Y_1 \mid D=1]$

When $Y_0 \perp D, E[Y_0] = E[Y_0 \mid D=0]$

Subgroup ATE

When there are x subgroups, each subgroup has its ATE:

$\text{ATE}_x = E[Y_1 - Y_0 \mid X = x]$

$= \text{E}[\text{Y}_1 \mid X = x, D = 1] - E[Y_0 \mid X = x, D = 0]$

ATE example

Suppose that there are 4 units in our study:

i	Y_{1i}	Y_{0i}	Y_i	D_i	α_i
1	3	0	3	1	3
2	1	1	1	1	0
3	1	0	0	0	1
4	1	1	1	0	0
$E[Y_1]$	1.5				
$E[Y_0]$		0.5			
$E[Y_1 - Y0]$					1

$$\alpha_{ATE} = E[Y_1 - Y0] = \frac{1}{4} \cdot (3 + 0 + 1 + 0) = 1$$

Selection Problem

ATE can be expressed as:

$$E[Y_1 \mid D=1] - E[Y_0 \mid D=0] = \underbrace{E[Y_1 \mid D=1] - E[Y_0 \mid D=1]}_{ATET} + \underbrace{E[Y_0 \mid D=1] - E[Y_0 \mid D=0]}_{Bias}$$

The selection problem, ATE $\neq$ ATET, arises when the bias term is not zero. Apart from the difference in treatment, units who receive the treatment may be different from units who do not receive the treatment *even this treatment has no effect*.

Selection Problem

For instance, education of parents may have nothing to do with children's intelligence, but their social status may encourage their children to receive similar education.

People who participate in the job training program may be more aggressive than who do not get it. Simple comparison of earnings of people who receive job training or not may give us the wrong answer.

Whether or not the bias term is zero depends on Y_0.Unfortunately, we cannot observe Y_0 whenever $D = 1$. We should keep in mind of the bias term and avoid it.

Let's run a real example.

Description

Olken (2007) reported that increasing participation would reduce corruption. His field experiment invites villages to monitor public projects. The dependent variable is *pct_missing*, which means the percent of expenditure missing. The treatment variable is *treat_invite*, which means whether villages receive the intervention (participation in monitoring). The covariates are:

Variable	**Definition**
head_ edu	Village head education
mosques	Mosques per 1,000
pct_ poor	Percent of households below the poverty line
total_ budget	Total budget (Rp. million)

Benjamin A. Olken, 2007. "Monitoring Corruption: Evidence from a Field Experiment in Indonesia" *Journal of Political Economy*, 115,2:200-249.

Balance Check

Variable	Levels	n	Min	Max	x	s	#NA
Education	control	191	6.0	20.0	11.5	2.7	0
	treatment	371	6.0	20.0	11.4	2.7	5
$p = 0.78$	all	562	6.0	20.0	11.5	2.7	5
Mosques	control	191	0.1	4.5	1.5	0.8	0
	treatment	374	0.0	6.9	1.4	0.8	2
$p = 0.41$	all	565	0.0	6.9	1.4	0.8	2
Poor	control	190	0.0	0.9	0.4	0.2	1
	treatment	370	0.0	0.9	0.4	0.2	6
$p = 0.64$		560	0.0	0.9	0.4	0.2	7
Budget	control	191	19.1	273.5	82.0	41.2	0
	treatment	374	8.8	890.2	80.2	56.7	2
$p = 0.70$		565	8.8	890.2	80.8	51.9	2

Table: Balance Check of Covariates in Olken's data.

Covariate Distribution

Figure: Distributions of 4 Covariates under Treatment and Control

ATE

ATE=E[$Y_1 - Y_0] = E[Y_1] - E[Y_0]$

```
T<-subset(olken, Treat=="treatment")
C<-subset(olken, Treat=="control")
ATE<-mean(T$Missing,na.rm=T)-mean(C$Missing,na.rm=T)
> ATE
[1] -0.02314737
```

Regression

Without covariates, we have the model

$$y = \beta_0 + \beta_1 D + \epsilon_1$$

where D is the treatment variable and y is the vector of outcomes.
When D=0, the model becomes:

$$\begin{aligned} \hat{y} &= \hat{\beta}_0 + \hat{\beta}_1 0 \\ &= \hat{\beta}_0 \end{aligned} \tag{1}$$

When D=1, the model becomes:

$$\begin{aligned} \hat{y} &= \hat{\beta}_0 + \hat{\beta}_1 1 \\ &= \hat{\beta}_0 + \hat{\beta}_1 \end{aligned} \tag{2}$$

Treatment Effect

Eq. 1: $\hat{y} = \hat{\beta}_0$

Eq. 2: $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1$

Because linear equation assumes linear parametric form for the conditional expectation form:
$E[Y \mid X] = \beta_0 + \beta_1 X$

Therefore, Eq. 2 - Eq. 1=
$E[Y \mid D = 1] - E[Y \mid D = 0] = \hat{\beta}_1$, which is the effect of D.

Treatment Effect

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It means the estimate of D represents the average treatment effect.

Single Regression Model

Table: Binary Explanatory Variable OLS Model

	Model 1
(Intercept)	0.25^{***}
	(0.03)
Treatment	-0.02
	(0.03)
R^2	0.00
Adj. R^2	0.00
Num. obs.	477

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$, $^{\cdot}p < 0.1$

Multiple Regression Model

	Model 2
(Intercept)	0.39^{***}
	(0.09)
Treatment	-0.03
	(0.03)
Budget	0.00^{*}
	(0.00)
Mosques	-0.05^{**}
	(0.02)
Education	-0.01
	(0.01)
Poor	-0.12
	(0.07)
R^2	0.03
Adj. R^2	0.02
Num. obs.	472

*** ** *

Standard Error

We assume that the treated and un-treated groups are independent. Y_T is unrelated to Y_C. Therefore, the standard error of ATE is the squared root of sum of both variances.

Pooled S.E.

$$var(\hat{Y}_{treated} - \hat{Y}_{control}) = var(\hat{Y}_{treated}) - var(\hat{Y}_{control}) = \frac{\sigma_T^2}{N_T} + \frac{\sigma_C^2}{N_C}$$

```
PoolSE=sqrt((var(T$Missing,na.rm=T)/nrow(T)) +
    (var(C$Missing,na.rm=T)/nrow(C)))
> PoolSE
[1] 0.03019662
```

Results

- The analysis shows the small treatment effect; only .02 decrease in corruption.
- Including covariates will influence the variance of the coefficient of D. If the covariates effectively explain the variation of the outcome variable, then the overall variance of the coefficients becomes smaller. If the treatment can predict the covariates, however, including those covariates may increase the variance estimate.
- The standard error of Model 1 is smaller than the pooled standard error. It is because the OLS model assumes the equal variance of each level of the covariate, which is D is this case. Pooled S.E. allows unequal variance, which makes the standard error larger. Our inference will be more conservative if we use pooled S.E.

SUTVA

- One of assumptions of potential outcome model is **stable unit treatment value assumption**.
- In this study, SUTVA assumption means villages of two groups are independent of each other. If people in the villages that receive no treatment heard about the experiment may alter their behavior. In this case, the treatment effect may become even smaller.

More Topics

- What if we estimate the subgroup ATE? For example, we estimate ATE of villages above and under the poverty line, say .5.
- What if we only include one or two more covariates? Will the size of standard error change?