



## Model : Oblivious routing

Tags	112-2 Oblivious Routing
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Präsentation	<a href="https://docs.google.com/presentation/d/1RmGI7sELMFbk2IOxzNBs6EziNTCBfE1dtSw2Sp2Es1o/edit#slide=id.g1122_1_1">https://docs.google.com/presentation/d/1RmGI7sELMFbk2IOxzNBs6EziNTCBfE1dtSw2Sp2Es1o/edit#slide=id.g1122_1_1</a>
Status	Es kann geändert werden

## Der Thikung von dem Lehrer

**Input :** Given a Demand Matrix  $DM_1$

**Output :** Routing  $\phi$  (Fraction of demand entering vertex)

1. Find the best routing  $\phi_1$  of  $DM_1$
2. Find the worst demand matrix  $DM_2$  of the routing  $\phi_1$
3. Find the best routing  $\phi_2$  of  $DM_1 + DM_2$
4. Iteration until it meet the cutoff condition



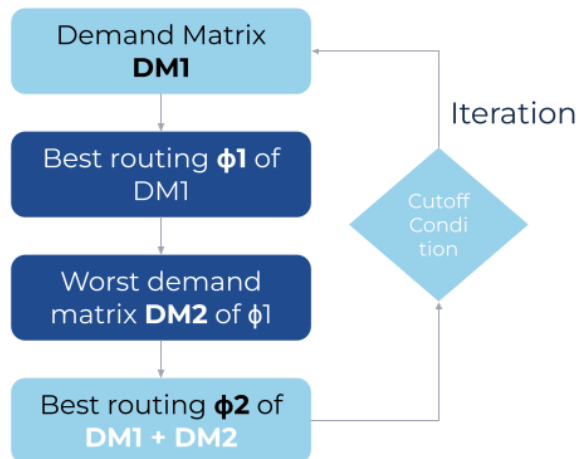
## Approach

**Input** : A demand Matrix

**Output** : Routing  
(Fraction of demand entering vertex)

**Best routing** : Minimize maximum link utilization

**Worst demand matrix**: Maximum maximum link utilization



## Pseudo code

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### Algorithm 1: main algorithm

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**Input:** Demand matrix DM

**Output:** Routing  $\phi_{best}$  (Fraction of demand)

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1 Initialization :  $DM_{worst} = \emptyset$ ;  
2 while not meet the cutoff condition do  
3    $DM = DM + DM_{worst}$ ;  
4   Find the best routing  $\phi$  of DM; // The First model  
5    $\phi_{best} = \phi$ ;  
6   Find the worst demand matrix  $DM_{worst}$  of  $\phi_{best}$  // The  
   second model  
7 end  
8 return  $\phi_{best}$ 
```

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## Finden Sie größte Routing $\phi$

Finding the link weight and traffic splitting ratio to minimize the maximum link utilization



## The first model

### Indicate

DM	Demand matrix
$e=(u,v) \in E$	Edges form vertex $u$ to vertex $v$
$d_{st}$	Demand form $s$ to $t$
$f_{st}(u)$	Fraction of demand entering vertex $u$
$c_e$	Link capacity

Given demand matrix DM, find the fraction of demands that minimizing the maximum link utilization.

### Variable

$\alpha$	The maximum link utilization of the demand matrix DM
$\phi_{st}(e)$	Routing fraction for edge $e=(u, v)$

### Indicate

DM	Demand matrix
$e = (u, v) \in E$	Edges
$d_{st}$	Demand from $s$ to $t$
$f_{st}(u)$	Fraction of demand entering vertex $u$
$c_e$	Link capacity

### Variable

$\alpha$  The maximum link utilization of the demand matrix DM

$\phi_{st}(e)$  Routing fraction for edge  $e = (u, v)$

### Objective function

Minimize the maximum link utilization  $\alpha$

$$\min \alpha$$

### Constraints

$$\begin{aligned}
 &0 \leq \alpha \leq 1 \text{ (Decision variable constraint)} \\
 &0 \leq \phi_{st}(e) \leq 1, \forall e \in E \text{ (Decision variable constraint)} \\
 &\sum_{(s,t)} \frac{d_{st} f_{st}(u) \phi_{st}(e)}{c(e)} \leq \alpha \text{ (Link capacity constraint)} \\
 &\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t, \forall (s,t) \in D \text{ (Flow conservation)} \\ 0 & \text{others} \end{cases}
 \end{aligned}$$

**Model : Demand matrix has no constrained**

min  $\alpha$  (Minimize maximum link utilization)

$$\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t, \forall (s,t) \in D, \forall u \in V \\ 0 & \text{others} \end{cases} \quad (\text{Flow conservation})$$

$$\sum_{(s,t)} \frac{d_{st} f_{st}(u) \phi_{st}(e)}{c(e)} \leq \alpha \quad (\text{Link capacity constraint})$$

$$0 \leq \phi_{st}(e) \leq 1, \forall e \in E \quad (\text{Decision variable constraint})$$

$$0 \leq \alpha \leq 1 \quad (\text{Decision variable constraint})$$

min  $\alpha$  (Minimize maximum link utilization)

$$\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t, \forall (s,t) \in D, \forall u \in V \\ 0 & \text{others} \end{cases} \quad (\text{Flow conservation})$$

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$$0 \leq \phi_{st}(e) \leq 1, \forall e \in E \quad (\text{Decision variable constraint})$$

$$0 \leq \alpha \leq 1 \quad (\text{Decision variable constraint})$$

## Find the worst demand matrix of routing $\phi$

Find the demand matrix maximizing the maximum link utilization with given routing  $\phi$



## The second model

### Indicate

$\phi_{st}(e)$  Given routing fraction for edge  $e=(u, v)$

$e=(u,v) \in E$  Edges from vertex  $u$  to vertex  $v$

$d_{st}$  Demand from  $s$  to  $t$

$f_{st}(u)$  Fraction of demand entering vertex  $u$

$c_e$  Link capacity

Given routing (Fraction of demand)  $\phi$ , find the demand matrix maximizing maximum link utilization with routing  $\phi$ .

### Variable

$\alpha$  The maximum link utilization of the demand matrix DM

DM The worstcase demand matrix of given routing  $\phi$

### Indicate

$\phi_{st}(e)$	Given routing
$e = (u, v) \in E$	Edges
$d_{st}$	Demand from $s$ to $t$
$f_{st}(u)$	Fraction of demand entering vertex $u$

$c_e$	Link capacity
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## Variable

$DM$  The worstcase demand matrix of given routing  $\phi$

$\alpha$  The maximum link utilization of the demand matrix  $DM$

## Objective function

Maximize the maximum link utilization  $\alpha$

$$\max \alpha$$

## Constraints

### Demand matrix is unconstrained

$$\begin{aligned} 0 \leq \alpha \leq 1 & \text{ (Decision variable constraint)} \\ \sum_{(s,t)} \frac{d_{st} f_{st}(u) \phi_{st}(e)}{c(e)} \leq \alpha & \text{ (Link capacity constraint)} \\ \sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t, \forall (s,t) \in D \text{ (Flow conservation)} \\ 0 & \text{others} \end{cases} \end{aligned}$$

### Demand matrix must in a set $D$

$$\begin{aligned} 0 \leq \alpha \leq 1 & \text{ (Decision variable constraint)} \\ DM \in D & \text{ (Decision variable constraint)} \\ \sum_{(s,t)} \frac{d_{st} f_{st}(u) \phi_{st}(e)}{c(e)} \leq \alpha & \text{ (Link capacity constraint)} \\ \sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t, \forall (s,t) \in D \text{ (Flow conservation)} \\ 0 & \text{others} \end{cases} \end{aligned}$$

## Model

### Demand matrix is unconstrained

$\max \alpha$  (Maximize maximum link utilization)

$$\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t, \forall (s,t) \in D, \forall u \in V \\ 0 & \text{others} \end{cases} \quad \text{(Flow conservation)}$$

$$\sum_{(s,t)} \frac{d_{st} f_{st}(u) \phi_{st}(e)}{c(e)} \leq \alpha \quad \text{(Link capacity constraint)}$$

$$0 \leq \alpha \leq 1 \quad \text{(Decision variable constraint)}$$

$\max \alpha$  (Maximize maximum link utilization)

$$\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t, \forall (s,t) \in D, \forall u \in V \\ 0 & \text{others} \end{cases} \quad \text{(Flow conservation)}$$

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## Demand matrix must in a set $D$

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$$DM \in D \quad (\text{Decision variable constraint})$$

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$\max \alpha$  (Maximize maximum link utilization)

$$\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t, \forall (s,t) \in D, \forall u \in V \\ 0 & \text{others} \end{cases} \quad (\text{Flow conservation})$$

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## Etwas Fragen

### ? Some question

#### About model 1 :

What happens if **total traffic is over than the total capacity** of the network?

#### About model 2 :

1. How to defined the constraints of the OD pairs with **continuous interval constraints** ?
2. In practice, how to **define the first demand matrix**?
3. In model 2, there is high probability **that several feasible solution**.

## Über Modell Eins

What happens if total traffic is over than the total capacity of the network?

Antwort

## Über Modell Zwei

1. How to defined the constraints of the OD pairs with continuous interval constraints ?

**Antwort**

2. In practice, how to define the first demand matrix?

**Antwort**

3. In model 2, there is high probability that several feasible solution.

**Antwort**

## ? Some question

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### About cutoff condition :

1. How to set the **cutoff condition** ?
2. About the routing will **converge with interaction**, how can I prove it?

### Else :

How I change model for **PD routing**?

## Über die Abschaltbedingung

1. How to set the **cutoff condition** ?

**Antwort**

2. About the routing will **converge with interaction**, how can I prove it?

**Antwort**

### Andere

How I change model for **PD routing**?

**Antwort**