0.1 Main algorithm

Algorithm 1: main algorithm

Input: Demand matrix DM

Output: Routing ϕ_{best} (Fraction of demand)

1 Initialization : $D = \{DM\}$, $DM_{worst} = 0$;

    // D is the set of demand matrix

2 **while** *not meet the cutoff condition* **do**

3     $D = D \cup DM_{worst}$;

4   Find the best routing ϕ of D; // The First model

5     $\phi_{best} = \phi$;

6   Find the worst demand matrix DM_{worst} of ϕ_{best} // The second model

7 **end**

8 return ϕ_{best}

0.2 Original model in the paper

$$\min \alpha$$

$$\phi \text{ is a PD routing}$$
$$\forall \text{ edges } e = (u, v)$$
$$\forall \text{ DMs D} \in D \text{ with } OPTU(D) = r : \tag{1}$$
$$\sum_{(s,t)} \frac{d_{st} f_{st}(u) \phi_t(e)}{c(e)} \leq \alpha r$$

$$OPTU(D) = \min_{\phi | \phi \text{ is a PD routing}} MxLU(\phi, D) \tag{2}$$

0.3   Model 1 : The first model

Find the best routing ϕ of the given set of demand matrix to minimize maximum link ultilization

$\min \alpha$ (Minimize maximum link ultilization)

$$\sum_{e=(v,u)\in E} \phi_{st}(e) - f_{st}^i(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t \\ 0 & \text{others} \end{cases}$$

$,\forall (s,t) \in V \times V, 1 \leq i \leq |D|, \forall u \in V$ \hfill (Flow conservation)

$\sum_{(s,t)} \dfrac{d_{st}^i f_{st}^i(u) \phi_{st}(e)}{c(e)} \leq \alpha$, $\forall e \in E, 1 \leq i \leq |D|$ \hfill (Link capacity constraint)

$0 \leq \phi_{st}(e) \leq 1$, $\forall e \in E$ \hfill (Decision variable constraint)

$0 \leq \alpha \leq 1$ \hfill (Decision variable constraint)

\hfill (3)

0.4 Model 1 v2.1.0 : The first model Kommen Sie Einschränkung $\frac{\sum_{s \in V, s \neq t} d_{st}^i \phi_{st}(e)}{\sum_{s \in V, s \neq t} d_{st}^i} = \phi_t(e)$ dazu

Find the best routing ϕ of the given set of demand matrix to minimize maximum link ultilization

$\min \alpha$ (Minimize maximum link ultilization)

$$\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}^i(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t \\ 0 & \text{others} \end{cases}$$

$, \forall (s,t) \in V \times V, 1 \leq i \leq |D|, \forall u \in V$ \hfill (Flow conservation)

$$\sum_{(s,t)} \frac{d_{st}^i f_{st}^i(u) \phi_{st}(e)}{c(e)} \leq \alpha \ , \ \forall e \in E, 1 \leq i \leq |D|$$ \hfill (Link capacity constraint)

$0 \leq \phi_{st}(e) \leq 1 \ , \forall e \in E$ \hfill (Decision variable constraint)

$0 \leq \alpha \leq 1$ \hfill (Decision variable constraint)

$$\frac{\sum_{s \in V, s \neq t} d_{st}^i \phi_{st}(e)}{\sum_{s \in V, s \neq t} d_{st}^i} = \phi_t(e), \ \forall t \in V, 1 \leq i \leq |D|$$ \hfill (Decision variable constraint)

$$(4)$$

0.5   Model 1 v2.1.1 : The first model Versuchen Sie, $\phi_t(e)$ durch $\phi_{st}(e)$ zu ersetzen

Find the best routing ϕ of the given set of demand matrix to minimize maximum link ultilization

$\min \alpha$ (Minimize maximum link ultilization)

$$\sum_{e=(u,v)\in E} \sum_{(s,t)} d_{st}^i f_{st}^i(v)\phi_t(e) - \sum_{(s,t)} d_{st}^i f_{st}^i(u) \qquad = \begin{cases} d_{st} & \text{if } u = s \\ -\sum_{(s,t)} d_{st} & \text{if } u = t \\ 0 & \text{others} \end{cases}$$

$$, \forall (s,t) \in V \times V \qquad\qquad\qquad \text{(Flow conservation)}$$

$$\sum_{(s,t)} \frac{d_{st}^i f_{st}^i(u)\phi_t(e)}{c(e)} \leq \alpha \ , \ \forall e \in E, 1 \leq i \leq |D| \qquad \text{(Link capacity constraint)}$$

$$0 \leq \phi_t(e) \leq 1 \ , \forall e \in E \qquad\qquad \text{(Decision variable constraint)}$$

$$0 \leq \alpha \leq 1 \qquad\qquad \text{(Decision variable constraint)}$$

$$\tag{5}$$

0.6   Model 2 : The second model

Find the worst demand matrix with given routing ϕ to maximize maximum link ultilization

$\max \rho$ (Maximize minimum link ultilization)

$$\sum_{e=(v,u)\in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t \ , \forall s,t,u \in V \\ 0 & \text{others} \end{cases} \qquad \text{(Flow conservation)}$$

$$\rho \leq \sum_{(s,t)} \frac{d_{st} f_{st}(u)\phi_{st}(e)}{c(e)} \leq 1 \qquad\qquad \text{(Link capacity constraint)}$$

$$0 \leq \rho \leq 1 \qquad\qquad \text{(Decision variable constraint)}$$

$$\tag{6}$$

0.7 Model 2 v2.1.0 : The second model Kommen Sie Einschränkung $\frac{\sum_{s \in V, s \neq t} d_{st}^i \phi_{st}(e)}{\sum_{s \in V, s \neq t} d_{st}^i} = \phi_t(e)$ dazu

Find the worst demand matrix with given routing ϕ to maximize
maximum link ultilization

$\max \rho$ (Maximize minimum link ultilization)

$$\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t \\ 0 & \text{others} \end{cases}, \forall s, t, u \in V \qquad \text{(Flow conservation)}$$

$$\rho \le \sum_{(s,t)} \frac{d_{st} f_{st}(u) \phi_{st}(e)}{c(e)} \le 1 \qquad \text{(Link capacity constraint)}$$

$$0 \le \rho \le 1 \qquad \text{(Decision variable constraint)}$$

$$\frac{\sum_{s \in V, s \neq t} d_{st}^i \phi_{st}(e)}{\sum_{s \in V, s \neq t} d_{st}^i} = \phi_t(e), \ \forall t \in V, 1 \le i \le |D|, e \in E \qquad \text{(Decision variable constraint)}$$

$$(7)$$

0.8 Model 2 v2.1.1 : The second model Versuchen Sie, $\phi_t(e)$ durch $\phi_{st}(e)$ zu ersetzen

Find the worst demand matrix with given routing ϕ to maximize
maximum link ultilization

$\max \rho$ (Maximize minimum link ultilization)

$$\sum_{e=(u,v) \in E} \sum_{(s,t)} d_{st}^i f_{st}^i(v) \phi_t(e) - \sum_{(s,t)} d_{st}^i f_{st}^i(u) = \begin{cases} d_{st} & \text{if } u = s \\ -\sum_{(s,t)} d_{st} & \text{if } u = t \\ 0 & \text{others} \end{cases}$$

$$, \forall (s,t) \in V \times V, s \neq t \qquad \text{(Flow conservation)}$$

$$\rho \le \sum_{(s,t)} \frac{d_{st} f_{st}(u) \phi_t(e)}{c(e)} \le 1 \qquad \text{(Link capacity constraint)}$$

$$0 \le \rho \le 1 \qquad \text{(Decision variable constraint)}$$

$$(8)$$