# Model V3.2

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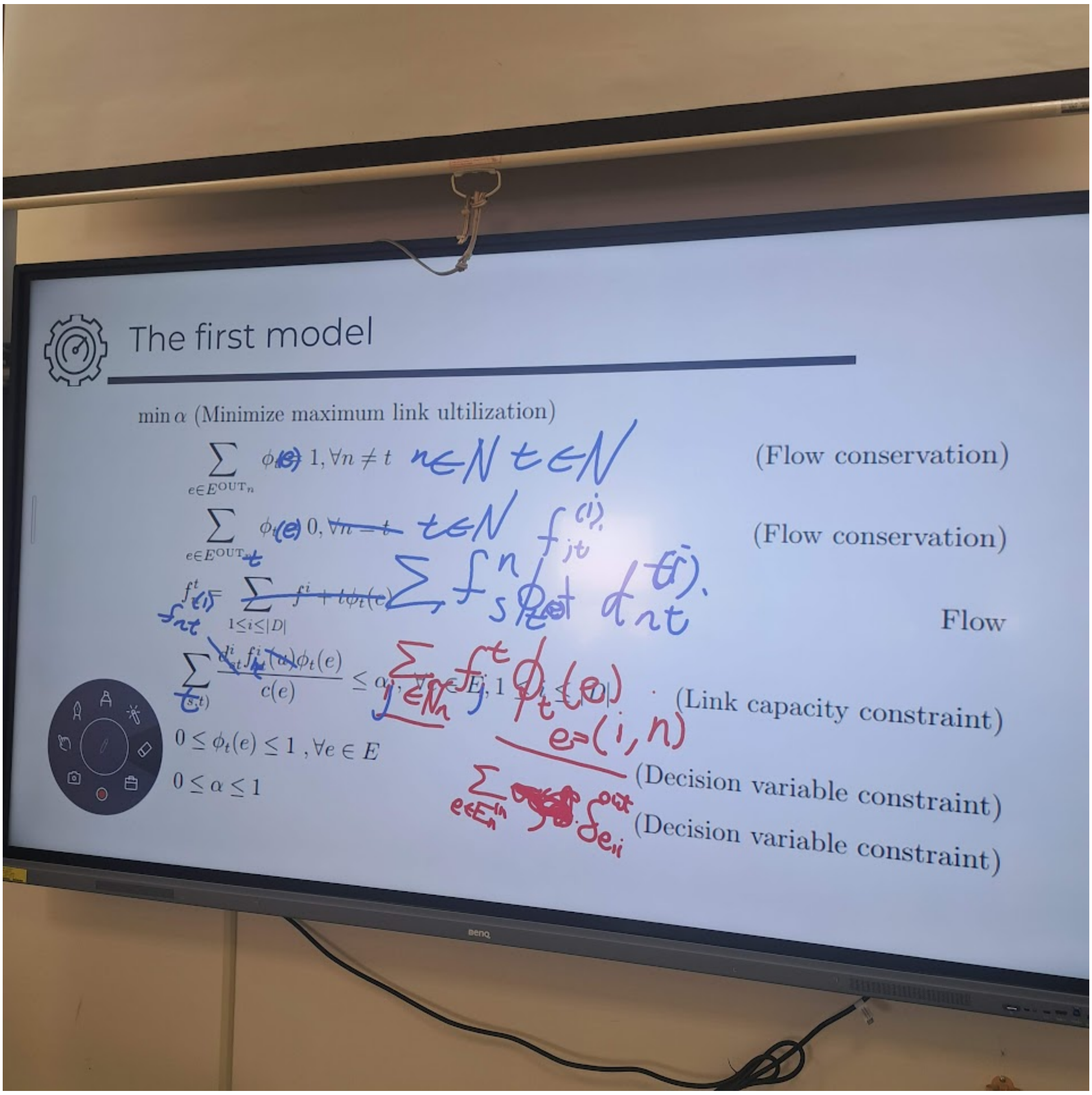
|---|---|
| Tags | 112-2 Oblivious Routing |
| Datum | @4. März 2024 |
| Präsentation | https://docs.google.com/presentation/d/1SNR6h9RAr6sjFPT4PI4MXJaHZKR5mn1nka4y3PGCEOM/edit# |
| Status | Im Gange |

## Aussage des Lehrers

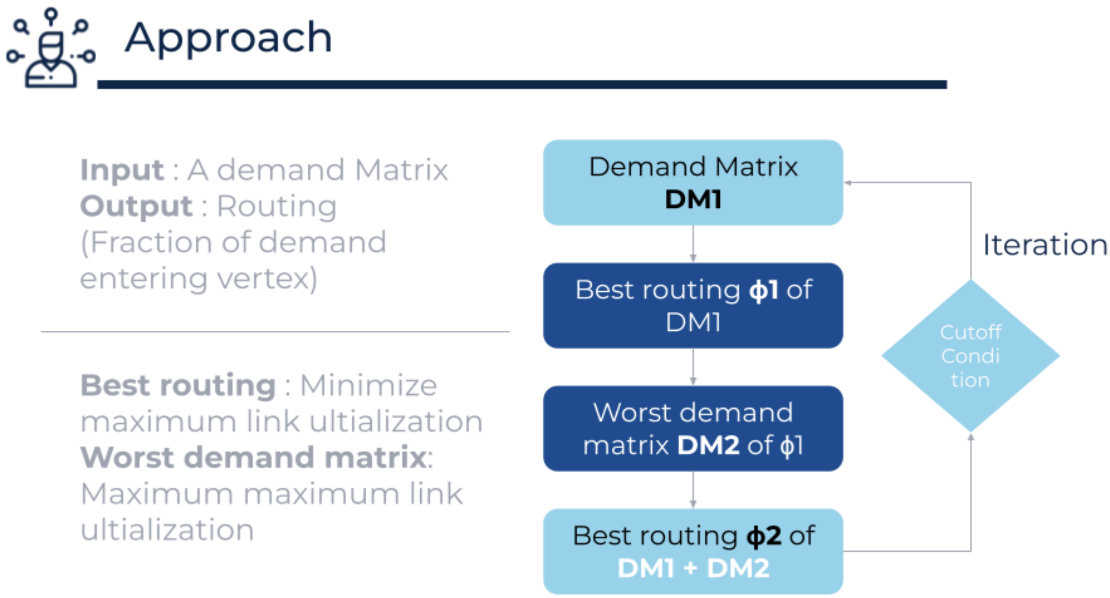



**Input** : Given a Demand Matrix $\mathrm{DM}_1$

**Output** : Routing $\phi$ (Fraction of demand entering vertex)

1. Find the best routing $\phi_1$ of $\mathrm{DM}_1$
2. Find the worst demand matrix $\mathrm{DM}_2$ of the routing $\phi_1$

3. Find the best routing $\phi_2$ of $\mathrm{DM}_1 \bigcup \mathrm{DM}_2$
4. Iteration until it meet the cutoff condition




### Pseudo code

**Algorithm 1:** main algorithm

**Input:** Demand matrix DM

**Output:** Routing $\phi_{best}$ (Fraction of demand)

Initialization : $D = \{\mathrm{DM}\}$, $\mathrm{DM}_{worst} = 0$;

`// D is the set of demand matrix`

**while** *not meet the cutoff condition* **do**

- $D = D \cup \mathrm{DM}_{worst}$;
- Find the best routing $\phi$ of $D$; `// The First model`
- $\phi_{best} = \phi$;
- Find the worst demand matrix $\mathrm{DM}_{worst}$ of $\phi_{best}$ `// The second model`

**end**

return $\phi_{best}$

## Finden Sie größe Routing $\phi$

Finding the link weight and traffic splitting ratio to minimize the maximum link ultilization

## The first model

Given set of demand matrix *D*, find the fraction of demands that minimizing the maximum link ultilization.

### Variable

| | |
|---|---|
| $\alpha$ | **The maximum link ultilization of all the demand matrix in *D*** |
| $\phi_t(e)$ | **Routing fraction for edge e=(u, v) to vertex t** |

For PD(Per-destination) routing in IP network

## The first model

### Indicate

| | |
|---|---|
| $D$ | **The set of demand matrix** |
| $e=(u,v) \in E$ | **Edges form vertex u to vertex v** |
| $d^i_{st}$ | **Demand form s to t of DM(i)** |
| $f^i_{st}$ | **Traffic from vertex s to t of DM(i)** |
| $c_e$ | **Link capacity of the edge e** |
| $h_e$ | **Header node of the edge e** |
| $E_n^{\mathrm{OUT}}$ | **Set includes all the edge from vertex n** |
| $\widetilde{N}_n$ | **Set includes all the neighbor vertexs of vertex n** |

### Indicate

| | |
|---|---|
| *D* | The set of demand matrix |
| $e=(u,v) \in E$ | Edges form vertex $u$ to vertex $v$ |
| $d^i_{st}$ | Demand from $s$ to $t$ of $i-th$ DM in *D* |
| $f^i_{st}$ | Traffic from vertex $s$ to $t$ of $i-th$ DM in *D* |
| $c_e$ | Link capacity of the edge $e$ |
| $h_e$ | Header node of the edge $e$ |
| $E_n^{\mathrm{OUT}}$ | Set includes all the edge from vertex $n$ |
| $\widetilde{N}_n$ | Set includes all the neighbor vertexs of vertex $n$ |

### Variable

$\alpha$ The maximum link ultilization of all the demand matrix in $D$

$\phi_t(e)$ Routing fraction for edge $e = (u, v)$ to vertex $t$

### Objective function

Minimize the maximum link ultilization $\alpha$

$$\min \alpha$$

### Constraints

$$0 \le \alpha \le 1 \text{ (Decision variable constraint)}$$
$$0 \le \phi_{st}(e) \le 1 \text{ },\forall e \in E \text{ (Decision variable constraint)}$$
$$\sum_{t \in N} \frac{f^i_{h_e t}\phi_t(e)}{c(e)} \le \alpha \text{ }, \text{ } \forall e \in E, 1 \le i \le |D| \text{(Link capacity constraint)}$$
$$\sum_{e \in E^{\text{OUT}}_n} \phi_t(e) = 1, \forall n, t \in N, n \ne t \text{(Flow conservation)}$$
$$\sum_{e \in E^{\text{OUT}}_t} \phi_t(e) = 1, \forall t \in N \text{(Flow conservation)}$$
$$f^i_{nt} = \sum_{j \in \widetilde{N}_n} f^i_{jt}\phi_t((j, n)) + d^i_{nt}, \text{ } \forall n, t \in N, n \ne t, 1 \le i \le |D| \text{(Flow conservation)}$$

## Model

$$\begin{aligned}
&\min \alpha \text{ (Minimize maximum link ultilization)} \\
&\quad \sum_{e \in E^{\text{OUT}}_n} \phi_t(e) = 1, \forall n, t \in N, n \ne t \\
&\quad \sum_{e \in E^{\text{OUT}}_t} \phi_t(e) = 1, \forall t \in N \\
&\quad f^i_{nt} = \sum_{j \in \widetilde{N}_n} f^i_{jt}\phi_t((j, n)) + d^i_{nt}, \forall n, t \in N, n \ne t, 1 \le i \le |D| && \text{(Flow conservation)} \\
&\quad \sum_{t \in N} \frac{f^i_{h_e t}\phi_t(e)}{c(e)} \le \alpha \text{ }, \text{ } \forall e \in E, 1 \le i \le |D| && \text{(Link capacity constraint)} \\
&\quad 0 \le \phi_t(e) \le 1 \text{ },\forall e \in E && \text{(Decision variable constraint)} \\
&\quad 0 \le \alpha \le 1 && \text{(Decision variable constraint)}
\end{aligned}$$

$$\begin{aligned}
&\min \alpha \text{ (Minimize maximum link ultilization)} \\
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&\quad 0 \le \phi_t(e) \le 1 \text{ },\forall e \in E && \text{(Decision variable constraint)} \\
&\quad 0 \le \alpha \le 1 && \text{(Decision variable constraint)}
\end{aligned}$$

# Find the worst demand matrix of routing $\phi$

Find the demand matrix maximizing the maximum link ultilization with given routing $\phi$

## The second model

Given routing (Fracttion of demand) $\phi$, find the **demand matrix maximizing maximum link ultilization** with routing $\phi$.

### Variable

| | |
|---|---|
| $\rho$ | **The minimum link ultilization of the demand matrix DM** |
| DM | **The worstcase demand matrix of given routing $\phi$** |
| $d_{st}$ | **Demand form s to t** |

For **PD**(Per-destination) routing in IP network

## The second model

### Indicate

| | |
|---|---|
| $\phi_t(e)$ | **Given routing / fraction of traffic to vertex t on the edge e** |
| $e=(u,v)\in E$ | **Edges form vertex u to vertex v** |
| $f_{st}$ | **Traffic from vertex s to t** |
| $c_e$ | **Link capacity of the edge e** |
| $h_e$ | **Header node of the edge e** |
| $E_n^{OUT}$ | **Set includes all the edge from vertex n** |
| $\check{N}_n$ | **Set includes all the neighbor vertexs of vertex n** |

### Indicate

| | |
|---|---|
| $\phi_t(e)$ | Given routing |
| $e=(u,v)\in E$ | Edges form vertex $u$ to vertex $v$ |
| $f_{st}$ | Traffic from vertex $s$ to $t$ |
| $c_e$ | Link capacity of the edge $e$ |
| $h_e$ | Header node of the edge $e$ |
| $E_n^{\mathrm{OUT}}$ | Set includes all the edge from vertex $n$ |
| $\widetilde{N}_n$ | Set includes all the neighbor vertexs of vertex $n$ |

### Variable

$DM$ The worstcase demand matrix of given routing $\phi$

$\rho$ The mnimum link ultilization of the demand matrix $\mathrm{DM}$

$d_{st}$  Demand from $s$ to $t$

### Objective function

Maximize the maximum link ultilization $\alpha$

$$\max \rho$$

### Constraints

$$0 \leq \rho \leq 1 \text{ (Decision variable constraint)}$$
$$\rho \leq \sum_{t \in N} \frac{f_{h_e t} \phi_t(e)}{c(e)} \leq \alpha \text{ , } \forall e \in E \text{(Link capacity constraint)}$$
$$f_{nt} = \sum_{j \in \widetilde{N}_n} f_{jt} \phi_t((j,n)) + d_{nt}, \ \forall n, t \in N, n \neq t \text{(Flow conservation)}$$

## Model

$$\begin{aligned}
&\max \rho \text{ (Maximize minimum link ultilization)} \\
&\quad f_{nt} = \sum_{j \in \widetilde{N}_n} f_{jt} \phi_t((j,n)) + d_{nt}, \forall n, t \in N, n \neq t && \text{(Flow conservation)} \\
&\quad \rho \leq \sum_{t \in N} \frac{f_{h_e t} \phi_t(e)}{c(e)} \leq \alpha, \forall e \in E && \text{(Link capacity constraint)} \\
&\quad 0 \leq \rho \leq 1 && \text{(Decision variable constraint)}
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&\quad 0 \leq \rho \leq 1 && \text{(Decision variable constraint)}
\end{aligned}$$