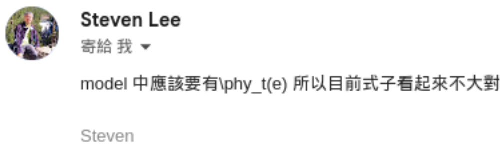
Model V2.1

Tags	112-2 Oblivious Routing
Datum	@26. Februar 2024
Präsentation	https://docs.google.com/presentation/d/1nGoEsjfNGFfNEGCVg3400Why6oGlugcIs2TvDghK-XM/edit#slide=id.p
Status	Im Gange

Aussage des Lehrers



model 中應該要有 $\phi_t(e)$ 所以目前式子看起來不大對

Lösung

1. Kommen Sie Einschränkung $\frac{\sum_{s\in V, s\neq t} d^i_{st}\phi_{st}(e)}{\sum_{s\in V, s\neq t} d^i_{st}} = \phi_t(e)$ dazu (v2.1.0)
2. Versuchen Sie, $\phi_t(e)$ durch $\phi_{st}(e)$ zu ersetzen (v2.1.1)

Der Thikung von dem Lehrer

Input : Given a Demand Matrix DM_1

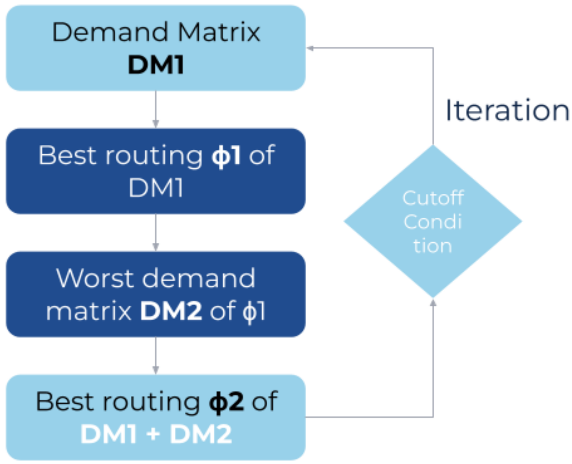
Output : Routing ϕ (Fraction of demand entering vertex)

1. Find the best routing ϕ_1 of DM_1
2. Find the worst demand matrix DM_2 of the routing ϕ_1
3. Find the best routing ϕ_2 of $\mathrm{DM}_1 \bigcup \mathrm{DM}_2$
4. Iteration until it meet the cutoff condition

Approach

Input : A demand Matrix
Output : Routing
(Fraction of demand entering vertex)

Best routing : Minimize maximum link ultialization
Worst demand matrix: Maximum maximum link ultialization



Pseudo code

Algorithm 1: main algorithm

Input: Demand matrix DM

Output: Routing ϕ_{best} (Fraction of demand)

Initialization : $D = \{\mathrm{DM}\}$, $\mathrm{DM}_{worst} = 0$;

`// D is the set of demand matrix`

while *not meet the cutoff condition* **do**

  $D = D \cup \mathrm{DM}_{worst}$;

  Find the best routing ϕ of D; `// The First model`

  $\phi_{best} = \phi$;

  Find the worst demand matrix DM_{worst} of ϕ_{best} `// The second model`

end

return ϕ_{best}

Finden Sie größe Routing ϕ

Finding the link weight and traffic splitting ratio to minimize the maximum link ultilization

Kommen Sie Einschränkung $\frac{\sum_{s \in V, s \neq t} d^i_{st} \phi_{st}(e)}{\sum_{s \in V, s \neq t} d^i_{st}} = \phi_t(e)$ dazu

The first model

Indicate

D	The set of demand matrix
e=(u,v)∈E	Edges form vertex u to vertex v
d^i_{st}	Demand form s to t of DM(i)
$f^i_{st}(u)$	Fraction of demand entering vertex u of DM(i)
c_e	Link capacity of the edge e

Given set of demand matrix *D*, find the fraction of demands that minimizing the maximum link ultilization.

Variable

α	The maximum link ultilization of all the demand matrix in *D*
$\phi_{st}(e)$	Routing fraction for edge e=(u, v) for OD pair (s,t)
$\phi_t(e)$	Routing fraction for edge e=(u, v)

Indicate

D	The set of demand matrix
$e = (u, v) \in E$	Edges form vertex u to vertex v
d^i_{st}	Demand from s to t of $i - th$ DM of *D*
$f_{st}(u)^i$	Fraction of demand entering vertex u of $i - th$ DM of *D*
c_e	Link capacity of the edge e

Variable

α The maximum link ultilization of all the demand matrix in *D*

$\phi_{st}(e)$ Routing fraction for edge $e = (u, v)$ for OD pair (s, t)

$\phi_t(e)$ Routing fraction for edge $e = (u, v)$

Objective function

Minimize the maximum link ultilization α

$$\min \alpha$$

Constraints

$$0 \le \alpha \le 1 \text{ (Decision variable constraint)}$$
$$0 \le \phi_{st}(e) \le 1 \text{ ,} \forall e \in E \text{ (Decision variable constraint)}$$
$$\sum_{(s,t)} \frac{d^i_{st} f^i_{st}(u) \phi_{st}(e)}{c(e)} \le \alpha \text{ , } \forall e \in E, 1 \le i \le |D| \text{(Link capacity constraint)}$$
$$\sum_{e=(v,u)\in E} \phi_{st}(e) - f^i_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t \\ 0 & \text{others} \end{cases} \text{, } \forall (s,t) \in V \times V, \ 1 \le i \le |D| \text{(Flow conservation)}$$
$$\frac{\sum_{s \in V, s \neq t} d^i_{st} \phi_{st}(e)}{\sum_{s \in V, s \neq t} d^i_{st}} = \phi_t(e), \ \forall t \in V, 1 \le i \le |D| \text{ (Decision variable constraint)}$$

Model

$$
\begin{aligned}
&\min \alpha \text{ (Minimize maximum link ultilization)} \\
&\sum_{e=(v,u)\in E} \phi_{st}(e) - f^i_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t \\ 0 & \text{others} \end{cases} \\
&, \forall (s,t) \in V \times V, 1 \le i \le |D|, \forall u \in V && \text{(Flow conservation)} \\
&\sum_{(s,t)} \frac{d^i_{st} f^i_{st}(u) \phi_{st}(e)}{c(e)} \le \alpha \text{ , } \forall e \in E, 1 \le i \le |D| && \text{(Link capacity constraint)} \\
&0 \le \phi_{st}(e) \le 1 \text{ , } \forall e \in E && \text{(Decision variable constraint)} \\
&0 \le \alpha \le 1 && \text{(Decision variable constraint)} \\
&\frac{\sum_{s\in V, s\neq t} d^i_{st} \phi_{st}(e)}{\sum_{s\in V, s\neq t} d^i_{st}} = \phi_t(e), \ \forall t \in V, 1 \le i \le |D| && \text{(Decision variable constraint)}
\end{aligned}
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&, \forall (s,t) \in V \times V, 1 \le i \le |D|, \forall u \in V && \text{(Flow conservation)} \\
&\sum_{(s,t)} \frac{d^i_{st} f^i_{st}(u) \phi_{st}(e)}{c(e)} \le \alpha \text{ , } \forall e \in E, 1 \le i \le |D| && \text{(Link capacity constraint)} \\
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&\frac{\sum_{s\in V, s\neq t} d^i_{st} \phi_{st}(e)}{\sum_{s\in V, s\neq t} d^i_{st}} = \phi_t(e), \ \forall t \in V, 1 \le i \le |D| && \text{(Decision variable constraint)}
\end{aligned}
$$

Versuchen Sie, $\phi_t(e)$ durch $\phi_{st}(e)$ zu ersetzen

The first model

Indicate

D	The set of demand matrix
e=(u,v)∈E	Edges form vertex u to vertex v
d^i_{st}	Demand form s to t of DM(i)
$f^i_{st}(u)$	Fraction of demand entering vertex u of DM(i)
c_e	Link capacity of the edge e

Given set of demand matrix *D*, find the fraction of demands that minimizing the maximum link ultilization.

Variable

α	The maximum link ultilization of all the demand matrix in *D*
$\phi_t(e)$	Routing fraction for edge e=(u, v)

Indicate

D	The set of demand matrix
$e=(u,v)\in E$	Edges form vertex u to vertex v
d_{st}^{i}	Demand from s to t of $i-th$ DM of D
$f_{st}(u)^{i}$	Fraction of demand entering vertex u of $i-th$ DM of D
c_e	Link capacity of the edge e

Variable

α The maximum link ultilization of all the demand matrix in D

$\phi_t(e)$ Routing fraction for edge $e=(u,v)$

Objective function

Minimize the maximum link ultilization α

$$\min \alpha$$

Constraints

$$0 \le \alpha \le 1 \text{ (Decision variable constraint)}$$
$$0 \le \phi_t(e) \le 1 \text{ ,} \forall e \in E \text{ (Decision variable constraint)}$$
$$\sum_{(s,t)} \frac{d_{st}^{i} f_{st}^{i}(u)\phi_t(e)}{c(e)} \le \alpha \text{ , } \forall e \in E, 1 \le i \le |D| \text{(Link capacity constraint)}$$
$$\sum_{e=(u,v)\in E} \sum_{(s,t)} d_{st}^{i} f_{st}^{i}(v)\phi_t(e) - \sum_{(s,t)} d_{st}^{i} f_{st}^{i}(u) = \begin{cases} d_{st} & \text{if } u=s \\ -\sum_{(s,t)} d_{st} & \text{if } u=t \\ 0 & \text{others} \end{cases} \text{ , } \forall (s,t) \in V\times V \text{(Flow conservation)}$$

Model

$$\begin{aligned}
&\min \alpha \text{ (Minimize maximum link ultilization)} \\
&\qquad \sum_{e=(u,v)\in E} \sum_{(s,t)} d_{st}^{i} f_{st}^{i}(v)\phi_t(e) - \sum_{(s,t)} d_{st}^{i} f_{st}^{i}(u) = \begin{cases} d_{st} & \text{if } u=s \\ -\sum_{(s,t)} d_{st} & \text{if } u=t \\ 0 & \text{others} \end{cases} \\
&\qquad ,\forall (s,t) \in V\times V && \text{(Flow conservation)} \\
&\qquad \sum_{(s,t)} \frac{d_{st}^{i} f_{st}^{i}(u)\phi_t(e)}{c(e)} \le \alpha \text{ , } \forall e \in E, 1 \le i \le |D| && \text{(Link capacity constraint)} \\
&\qquad 0 \le \phi_t(e) \le 1 \text{ ,} \forall e \in E && \text{(Decision variable constraint)} \\
&\qquad 0 \le \alpha \le 1 && \text{(Decision variable constraint)}
\end{aligned}$$

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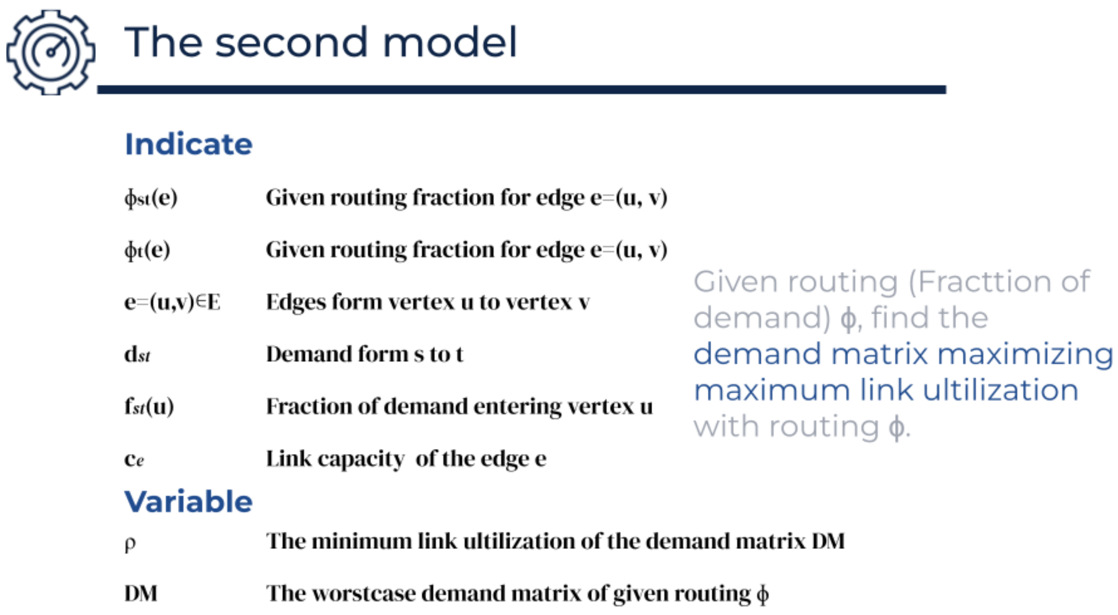
&\qquad \sum_{e=(u,v)\in E} \sum_{(s,t)} d_{st}^{i} f_{st}^{i}(v)\phi_t(e) - \sum_{(s,t)} d_{st}^{i} f_{st}^{i}(u) \qquad = \begin{cases} d_{st} & \text{if } u=s \\ -\sum_{(s,t)} d_{st} & \text{if } u=t \\ 0 & \text{others} \end{cases} \\
&\qquad ,\forall (s,t) \in V\times V && \text{(Flow conservation)} \\
&\qquad \sum_{(s,t)} \frac{d_{st}^{i} f_{st}^{i}(u)\phi_t(e)}{c(e)} \le \alpha \text{ , } \forall e \in E, 1 \le i \le |D| && \text{(Link capacity constraint)} \\
&\qquad 0 \le \phi_t(e) \le 1 \text{ ,} \forall e \in E && \text{(Decision variable constraint)} \\
&\qquad 0 \le \alpha \le 1 && \text{(Decision variable constraint)}
\end{aligned}$$

Find the worst demand matrix of routing ϕ

Find the demand matrix maximizing the maximum link ultilization with given routing ϕ

Kommen Sie Einschränkung $\frac{\sum_{s \in V, s \neq t} d_{st}^{i} \phi_{st}(e)}{\sum_{s \in V, s \neq t} d_{st}^{i}} = \phi_t(e)$ dazu



Indicate

$\phi_{st}(e)$	Given routing
$\phi_t(e)$	Given routing
$e=(u,v) \in E$	Edges
d_{st}	Demand from s to t
$f_{st}(u)$	Fraction of demand entering vertex u
c_e	Link capacity of the edge e

Variable

DM The worstcase demand matrix of given routing ϕ

ρ The mnimum link ultilization of the demand matrix DM

Objective function

Maximize the maximum link ultilization α

$$\max \rho$$

Constraints

Demand matrix is unconstrained

$$0 \le \rho \le 1 \text{ (Decision variable constraint)}$$

$$\frac{\sum_{s\in V, s\neq t} d_{st}^{i}\phi_{st}(e)}{\sum_{s\in V, s\neq t} d_{st}^{i}} = \phi_t(e),\ \forall t \in V, 1 \le i \le |D|, e \in E \text{ (Decision variable constraint)}$$

$$\rho \le \sum_{(s,t)} \frac{d_{st} f_{st}(u)\phi_{st}(e)}{c(e)} \le 1 \text{ (Link capacity constraint)}$$

$$\sum_{e=(v,u)\in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t \\ 0 & \text{others} \end{cases},\ \forall (s,t) \in V \times V \text{(Flow conservation)}$$

Model

$$\max \rho \text{ (Maximize minimum link ultilization)}$$

$$\sum_{e=(v,u)\in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t \\ 0 & \text{others} \end{cases}, \forall s,t,u \in V \qquad \text{(Flow conservation)}$$

$$\rho \le \sum_{(s,t)} \frac{d_{st} f_{st}(u)\phi_{st}(e)}{c(e)} \le 1 \qquad \text{(Link capacity constraint)}$$

$$0 \le \rho \le 1 \qquad \text{(Decision variable constraint)}$$

$$\frac{\sum_{s\in V, s\neq t} d_{st}^{i}\phi_{st}(e)}{\sum_{s\in V, s\neq t} d_{st}^{i}} = \phi_t(e),\ \forall t \in V, 1 \le i \le |D|, e \in E \qquad \text{(Decision variable constraint)}$$

$$\min \alpha \text{ (Minimize maximum link ultilization)}$$

$$\sum_{e=(v,u)\in E} \phi_{st}(e) - f_{st}^{i}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t \\ 0 & \text{others} \end{cases}$$

$$, \forall (s,t) \in V \times V, 1 \le i \le |D|, \forall u \in V \qquad \text{(Flow conservation)}$$

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$$0 \le \phi_{st}(e) \le 1\ , \forall e \in E \qquad \text{(Decision variable constraint)}$$

$$0 \le \alpha \le 1 \qquad \text{(Decision variable constraint)}$$

$$\frac{\sum_{s\in V, s\neq t} d_{st}^{i}\phi_{st}(e)}{\sum_{s\in V, s\neq t} d_{st}^{i}} = \phi_t(e),\ \forall t \in V, 1 \le i \le |D| \qquad \text{(Decision variable constraint)}$$

Versuchen Sie, $\phi_t(e)$ durch $\phi_{st}(e)$ zu ersetzen

The second model

Indicate

$\phi_t(e)$	Given routing fraction for edge e=(u, v)
e=(u,v)∈E	Edges form vertex u to vertex v
d_{st}	Demand form s to t
$f_{st}(u)$	Fraction of demand entering vertex u
c_e	Link capacity of the edge e

Given routing (Fracttion of demand) ϕ, find the demand matrix maximizing maximum link ultilization with routing ϕ.

Variable

ρ	The minimum link ultilization of the demand matrix DM
DM	The worstcase demand matrix of given routing ϕ

Indicate

$\phi_t(e)$	Given routing
$e=(u,v)\in E$	Edges
d_{st}	Demand from s to t
$f_{st}(u)$	Fraction of demand entering vertex u
c_e	Link capacity of the edge e

Variable

DM The worstcase demand matrix of given routing ϕ

ρ The mnimum link ultilization of the demand matrix DM

Objective function

Maximize the maximum link ultilization α

$$\max \rho$$

Constraints

Demand matrix is unconstrained

$$0 \le \rho \le 1 \text{ (Decision variable constraint)}$$
$$\rho \le \sum_{(s,t)} \frac{d_{st} f_{st}(u)\phi_t(e)}{c(e)} \le 1 \text{ (Link capacity constraint)}$$
$$\sum_{e=(u,v)\in E} \sum_{(s,t)} d^i_{st} f^i_{st}(v)\phi_t(e) - \sum_{(s,t)} d^i_{st} f^i_{st}(u) = \begin{cases} d_{st} & \text{if } u = s \\ -\sum_{(s,t)} d_{st} & \text{if } u = t \\ 0 & \text{others} \end{cases}, \forall (s,t) \in V \times V, s \neq t \text{(Flow conservation}$$

Model

$\max \rho$ (Maximize minimum link ultilization)

$$\sum_{e=(u,v)\in E}\sum_{(s,t)} d^i_{st} f^i_{st}(v)\phi_t(e) - \sum_{(s,t)} d^i_{st} f^i_{st}(u) = \begin{cases} d_{st} & \text{if } u = s \\ -\sum_{(s,t)} d_{st} & \text{if } u = t \\ 0 & \text{others} \end{cases}$$

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$\max \rho$ (Maximize minimum link ultilization)

$$\sum_{e=(u,v)\in E}\sum_{(s,t)} d^i_{st} f^i_{st}(v)\phi_t(e) - \sum_{(s,t)} d^i_{st} f^i_{st}(u) = \begin{cases} d_{st} & \text{if } u = s \\ -\sum_{(s,t)} d_{st} & \text{if } u = t \\ 0 & \text{others} \end{cases}$$

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