0.1 Main algorithm

Algorithm 1: main algorithm

Input: Demand matrix DM

Output: Routing ϕ_{best} (Fraction of demand)

Initialization : $D = \{\mathrm{DM}\}$, $\mathrm{DM}_{worst} = 0$;

`//` D `is the set of demand matrix`

while *not meet the cutoff condition* **do**

    $D = D \cup \mathrm{DM}_{worst}$;

    Find the best routing ϕ of D; `// The First model`

    $\phi_{best} = \phi$;

    Find the worst demand matrix DM_{worst} of ϕ_{best} `// The second model`

end

return ϕ_{best}

0.2 Original model in the paper

$$
\begin{aligned}
&\min \alpha \\
&\qquad \phi \text{ is a PD routing} \\
&\qquad \forall \text{ edges } e = (u, v) \\
&\qquad \forall \text{ DMs D} \in D \text{ with } OPTU(D) = r : \\
&\qquad \sum_{(s,t)} \frac{d_{st} f_{st}(u) \phi_t(e)}{c(e)} \leq \alpha r
\end{aligned}
\tag{1}
$$

$$
OPTU(D) = \min_{\phi | \phi \text{ is a PD routing}} MxLU(\phi, D) \tag{2}
$$

0.3 Model 1 : The first model

`Find the best routing` ϕ `of the given set of demand matrix to minimize maximum link ultilization`

$$
\begin{aligned}
&\min \alpha \text{ (Minimize maximum link ultilization)} \\
&\quad \sum_{e \in E_n^{\mathrm{OUT}}} \phi_t(e) = 1, \forall n, t \in N, n \neq t \\
&\quad \sum_{e \in E_t^{\mathrm{OUT}}} \phi_t(e) = 1, \forall t \in N \\
&\quad f_{nt}^i = \sum_{j \in \widetilde{N}_n} f_{jt}^i \phi_t((j,n)) + d_{nt}^i, \forall n, t \in N, n \neq t, 1 \leq i \leq |D| && \text{(Flow conservation)} \\
&\quad \sum_{t \in N} \Big(\frac{f_{h_c t}^i \phi_t^{(0)}(e)}{c(e)} + \frac{f_{h_c t}^i \phi_t^{(1)}(e)}{c(e)} (\phi_t(e) - \phi_t^{(0)}(e)) \Big) \leq \alpha, \\
&\quad \forall e \in E, 1 \leq i \leq |D| && \text{(Link capacity constraint)} \\
&\quad 0 \leq \phi_t(e) \leq 1 \;, \forall e \in E && \text{(Decision variable constraint)} \\
&\quad 0 \leq \alpha \leq 1 && \text{(Decision variable constraint)}
\end{aligned} \tag{3}
$$

0.4 Model 2 : The second model

`Find the worst demand matrix with given routing` ϕ `to maximize maximum link ultilization`

$$
\begin{aligned}
&\max \rho \text{ (Maximize minimum link ultilization)} \\
&\quad f_{nt} = \sum_{j \in \widetilde{N}_n} f_{jt} \phi_t((j,n)) + d_{nt}, \forall n, t \in N, n \neq t && \text{(Flow conservation)} \\
&\quad \rho \leq \sum_{t \in N} \Big(\frac{f_{h_c t} \phi_t^{(0)}(e)}{c(e)} + \frac{f_{h_c t} \phi_t^{(1)}(e)}{c(e)} (\phi_t(e) - \phi_t^{(0)}(e)) \Big) \leq \alpha, \\
&\quad \forall e \in E, 1 \leq i \leq |D| && \text{(Link capacity constraint)} \\
&\quad 0 \leq \rho \leq 1 && \text{(Decision variable constraint)}
\end{aligned} \tag{4}
$$

0.5 Linearization

Find the function $f(x)$ around α to formalize $f(x) \approx f(\alpha) + f'(\alpha)(x - \alpha)$

$$
\begin{aligned}
& f(x) \approx f(\alpha) + f'(\alpha)(x-\alpha) && \text{Formalize} \\
& \frac{f^i_{h_c t}\phi_t(e)}{c(e)} \approx \frac{f^i_{h_c t}\phi_t^{(0)}(e)}{c(e)} + \frac{f^i_{h_c t}\phi_t^{(1)}(e)}{c(e)}\Big|_{\phi_t(e)=\phi_t^{(0)}(e)}(\phi_t(e) - \phi_t^{(0)}(e)) && \\
& \frac{d}{dx}\left(\frac{f^i_{h_c t}\phi_t(e)}{c(e)}\right) = \frac{f^i_{h_c t}}{c(e)} && \text{Deriative} \\
& \frac{f^i_{h_c t}\phi_t(e)}{c(e)} \approx \frac{f^i_{h_c t}\phi_t^{(0)}(e)}{c(e)} + \frac{f^i_{h_c t}\phi_t^{(1)}(e)}{c(e)}(\phi_t(e) - \phi_t^{(0)}(e)) && \text{Final form}
\end{aligned} \tag{5}
$$