



## Model V2

☰ Tags	112-2 Oblivious Routing
📅 Datum	@26. Februar 2024
🔗 Präsentation	<a href="https://docs.google.com/presentation/d/1nGoEsjfNGFfNEGCVg3400Why6oGlugcIs2TvDghK-XM/edit#slide=id.p">https://docs.google.com/presentation/d/1nGoEsjfNGFfNEGCVg3400Why6oGlugcIs2TvDghK-XM/edit#slide=id.p</a>
⚙️ Status	Im Gange

## Der Thikung von dem Lehrer

**Input** : Given a Demand Matrix  $DM_1$

**Output** : Routing  $\phi$  (Fraction of demand entering vertex)

1. Find the best routing  $\phi_1$  of  $DM_1$
2. Find the worst demand matrix  $DM_2$  of the routing  $\phi_1$
3. Find the best routing  $\phi_2$  of  $DM_1 \cup DM_2$
4. Iteration until it meet the cutoff condition



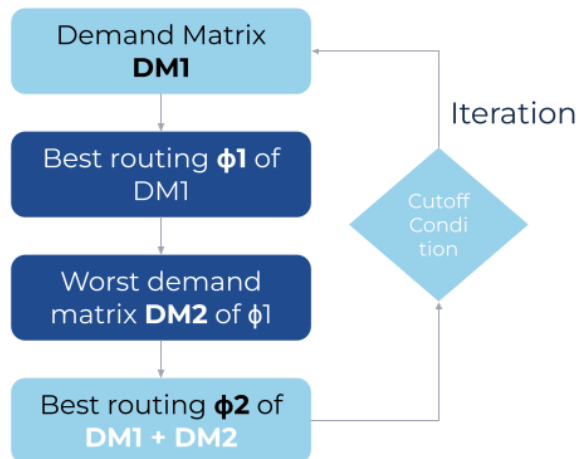
## Approach

**Input** : A demand Matrix

**Output** : Routing  
(Fraction of demand entering vertex)

**Best routing** : Minimize maximum link utilization

**Worst demand matrix**: Maximum maximum link utilization



## Pseudo code

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**Algorithm 1:** main algorithm

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**Input:** Demand matrix DM

**Output:** Routing  $\phi_{best}$  (Fraction of demand)

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1 Initialization :  $D = \{DM\}$ ,  $DM_{worst} = 0$ ;  
  //  $D$  is the set of demand matrix  
2 while not meet the cutoff condition do  
3    $D = D \cup DM_{worst}$ ;  
4   Find the best routing  $\phi$  of  $D$ ; // The First model  
5    $\phi_{best} = \phi$ ;  
6   Find the worst demand matrix  $DM_{worst}$  of  $\phi_{best}$  // The  
    second model  
7 end  
8 return  $\phi_{best}$ 
```

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## Finden Sie größte Routing $\phi$

Finding the link weight and traffic splitting ratio to minimize the maximum link utilization



## The first model

### Indicate

$D$  The set of demand matrix

$e=(u,v) \in E$  Edges form vertex  $u$  to vertex  $v$

$d_{st}^i$  Demand form  $s$  to  $t$  of  $DM(i)$

$f_{st}^i(u)$  Fraction of demand entering vertex  $u$  of  $DM(i)$

$c_e$  Link capacity

Given demand matrix  $DM$ , find the fraction of demands that minimizing the maximum link utilization.

### Variable

$\alpha$  The maximum link utilization of the demand matrix  $DM$

$\phi_{st}(e)$  Routing fraction for edge  $e=(u, v)$

### Indicate

$D$	The set of demand matrix
$e = (u, v) \in E$	Edges form vertex $u$ to vertex $v$
$d_{st}^i$	Demand from $s$ to $t$ of $i$ - th DM of $D$
$f_{st}(u)^i$	Fraction of demand entering vertex $u$ of $i$ - th DM of $D$
$c_e$	Link capacity of the edge $e$

### Variable

$\alpha$  The maximum link utilization of all the demand matrix in  $D$

$\phi_{st}(e)$  Routing fraction for edge  $e = (u, v)$

### Objective function

Minimize the maximum link utilization  $\alpha$

$$\min \alpha$$

### Constraints

$$0 \leq \alpha \leq 1 \text{ (Decision variable constraint)}$$

$$0 \leq \phi_{st}(e) \leq 1, \forall e \in E \text{ (Decision variable constraint)}$$

$$\sum_{(s,t)} \frac{d_{st}^i f_{st}^i(u) \phi_{st}(e)}{c(e)} \leq \alpha, \forall e \in E, 1 \leq i \leq |D| \text{ (Link capacity constraint)}$$

$$\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}^i(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t, \forall (s,t) \in V \times V, 1 \leq i \leq |D| \\ 0 & \text{others} \end{cases} \text{ (Flow conservation)}$$

### Model : Demand matrix has no constrained

min  $\alpha$  (Minimize maximum link utilization)

$$\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}^i(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t \\ 0 & \text{others} \end{cases}$$

,  $\forall (s, t) \in V \times V, 1 \leq i \leq |D|, \forall u \in V$  (Flow conservation)

$$\sum_{(s,t)} \frac{d_{st}^i f_{st}^i(u) \phi_{st}(e)}{c(e)} \leq \alpha, \forall e \in E, 1 \leq i \leq |D|$$
 (Link capacity constraint)

$0 \leq \phi_{st}(e) \leq 1, \forall e \in E$  (Decision variable constraint)

$0 \leq \alpha \leq 1$  (Decision variable constraint)

min  $\alpha$  (Minimize maximum link utilization)

$$\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}^i(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t \\ 0 & \text{others} \end{cases}$$

,  $\forall (s, t) \in V \times V, 1 \leq i \leq |D|, \forall u \in V$  (Flow conservation)

$$\sum_{(s,t)} \frac{d_{st}^i f_{st}^i(u) \phi_{st}(e)}{c(e)} \leq \alpha, \forall e \in E, 1 \leq i \leq |D|$$
 (Link capacity constraint)

$0 \leq \phi_{st}(e) \leq 1, \forall e \in E$  (Decision variable constraint)

$0 \leq \alpha \leq 1$  (Decision variable constraint)

## Find the worst demand matrix of routing $\phi$

Find the demand matrix maximizing the maximum link utilization with given routing  $\phi$



### The second model

#### Indicate

$\phi_{st}(e)$  Given routing fraction for edge  $e=(u, v)$

$e=(u,v) \in E$  Edges from vertex  $u$  to vertex  $v$

$d_{st}$  Demand from  $s$  to  $t$

$f_{st}(u)$  Fraction of demand entering vertex  $u$

$c_e$  Link capacity of the edge  $e$

Given routing (Fraction of demand)  $\phi$ , find the demand matrix maximizing maximum link utilization with routing  $\phi$ .

#### Variable

$\rho$  The minimum link utilization of the demand matrix DM

DM The worstcase demand matrix of given routing  $\phi$

Indicate

$\phi_{st}(e)$	Given routing
$e = (u, v) \in E$	Edges
$d_{st}$	Demand from $s$ to $t$
$f_{st}(u)$	Fraction of demand entering vertex $u$
$c_e$	Link capacity of the edge $e$

## Variable

$DM$  The worstcase demand matrix of given routing  $\phi$

$\rho$  The minimum link utilization of the demand matrix  $DM$

## Objective function

Maximize the maximum link utilization  $\alpha$

$$\max \rho$$

## Constraints

### Demand matrix is unconstrained

$$\begin{aligned}
& 0 \leq \rho \leq 1 \text{ (Decision variable constraint)} \\
& \rho \leq \sum_{(s,t)} \frac{d_{st} f_{st}(u) \phi_{st}(e)}{c(e)} \leq 1 \text{ (Link capacity constraint)} \\
& \sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t, \forall (s,t) \in V \times V \text{ (Flow conservation)} \\ 0 & \text{others} \end{cases}
\end{aligned}$$

## Model

$\max \rho$  (Maximize maximum link utilization)

$$\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t, \forall s, t, u \in V \\ 0 & \text{others} \end{cases} \quad \text{(Flow conservation)}$$

$$\rho \leq \sum_{(s,t)} \frac{d_{st} f_{st}(u) \phi_{st}(e)}{c(e)} \leq 1 \quad \text{(Link capacity constraint)}$$

$$0 \leq \rho \leq 1 \quad \text{(Decision variable constraint)}$$

$\max \rho$  (Maximize minimum link utilization)

$$\sum_{e=(v,u) \in E} \phi_{st}(e) - f_{st}(u) = \begin{cases} 1 & \text{if } u = s \\ -1 & \text{if } u = t, \forall s, t, u \in V \\ 0 & \text{others} \end{cases} \quad \text{(Flow conservation)}$$

$$\rho \leq \sum_{(s,t)} \frac{d_{st} f_{st}(u) \phi_{st}(e)}{c(e)} \leq 1 \quad \text{(Link capacity constraint)}$$

$$0 \leq \rho \leq 1 \quad \text{(Decision variable constraint)}$$