CSED601
Dependable Computing
Lecture 10

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POSTECH

Review of Previous Lecture

- Time Redundancy
- Methods
  - Retry
  - Alternating logic
  - Re-computing with shifted operands (RESO)
  - Re-computing with swapped operands (RWSO)
  - Re-computing with duplication

Evaluation

- Concept
  - Need a way to evaluate fault-tolerance methods
- Methods
  - Fault distributions
  - Evaluation models
  - Modeling techniques

Fault distributions

- Concept
  - Observed behavior ➔ express it by probabilities and averages
  - Probabilities must be determined as a function of time
  - Hazard function $Z(t)$
    - A description of how the instantaneous probability of failure evolves in time
    - $Z(t) = f(t) / [\, 1 - F(t)]$
    - $Z(t - \tau)\delta t = [F(t - \tau + \delta t) - F(t - \tau)] / [1 - F(t - \tau)]$
    - $F(t)$: Cumulative distribution function (CDF) of failure

      $F(t) = P\ (T <= t)$
    - $f(t)$: Probability mass (density) function (PMF)

      $f(t) = P\ (T = t) = dF / dt$
    - $R(t) = 1 - F(t)$: probability of not observing any failure before time t

Hazard function details

- Concept
  - $R(t) = N_o(t) / N = N_o(t) / [N_o(t) + N_f(t)]$
  - $Q(t) = N_f(t) / N = N_f(t) / [N_o(t) + N_f(t)]$
  - $R(t) = 1 – Q(t) = 1 - N_f(t) / N$
  - $dR(t)/dt = -1/N\ dN_f(t)/dt$
  - $dN_f(t)/dt = - N\ dR(t)/dt$

    the change in the number of faulty units in instantaneous time

    or instantaneous failing rate
  - $Z(t) = 1 / N_o(t)\ dN_f(t)/dt$

    $= 1 / N_o(t)\ [- N\ dR(t)/dt] = - [dR(t)/dt] / R(t)$

    $= [dQ(t)/dt] / [1 – Q(t)]$
  - $dR(t)/dt = -Z(t)R(t)$
  - If $Z(t) = \lambda$, then $R(t) = \exp(- \lambda\ t)$

Failure distribution models

- Exponential distribution
  - Most encountered in reliability models
  - PDF = f(t) = λ exp(- λ t)
  - CDF = F(t) = 1 – exp(-λ t)
  - Reliability function = R(t) = exp (- λ t)
  - Hazard function = Z(t) = λ

    λ is referred to as the failure rate
  - In reality, λ is a function of time. But use a constant failure rate to model the steady-state condition of bathtub curve
  - Mean = 1 / λ, standard deviation = σx = 1 / λ

Failure distribution models

- Weibull distribution
  - Has two parameters
  - α : the shape parameter, λ : the scale parameter
  - PDF = f(t) = $\alpha \lambda (\lambda t)^{(\alpha - 1)} \exp(-(\lambda t)^{\alpha})$
  - CDF = F(t) = $1 - \exp(-(\lambda t)^{\alpha})$
  - Reliability function = R(t) = $\exp(-(\lambda t)^{\alpha})$
  - Hazard function = Z(t) = $\alpha \lambda (\lambda t)^{(\alpha - 1)}$
  - If $\alpha < 1$, the failure rate is decreasing with time
  - If $\alpha = 1$, the failure rate is constant
  - If $\alpha > 1$, the failure rate is increasing with time
  - Mean = $\Gamma(\alpha + 1) / \alpha \lambda$

Failure distribution models

- Geometric distribution
  - If t takes only the discrete times
  - PMF = f(n) = q^n – q^(n+1) = q^n (1 – q)
  - CDF = F(n) = 1 – q^n
  - Reliability function = R(n) = q^n

    where q is the probability that the system works during this time interval
  - Mean = 1 / (1 – q)
  - Variance = q^(1/2) / (1 – q)

Failure distribution models

- Discrete weibull distribution
  - PMF = $f(n) = q^{(n^\alpha)}[1 - q^{((n+1)^a - n^\alpha)}]$
  - CDF = $F(n) = 1 - q^{(n^\alpha)}$
  - Reliability function = $R(n) = q^{(n^\alpha)}$
  - Hazard function = $Z(n) = 1 - q^{((n+1)^a - n^\alpha)}$
  - Mean = $\Sigma\, q \wedge k \wedge \alpha$ no closed-form equation

Distribution model of permanent fault

- DOD reliability analysis center has extensively studied statistics on electronic component failures.
- Come up with MIL-HDBK-217 model
- Assumed to have an exponential distribution model
- The failure rate for a single chip taking the form

$$\lambda = \pi_l \pi_q (c_1 \pi_t \pi_v + c_2 \pi_e)$$

  – Affecting parameters:

    Learning, Quality, Temperature, Voltage stress, Environmental factor, complexity

Evaluation models

- Two approaches
  - Deterministic modeling
    - The minimum number of component failures that can be tolerated without system failure
    - Disadvantage: waste resources and unbalanced system design

      highly reliable components must be replicated as many times as the low reliability components.
  - Probabilistic modeling
    - Based on relative component failure and repair rates

Single parameter probabilistic models

- MTTF : Mean time to failure
  - MTTF = integrate(0, inf) R(t) dt = integrate(0,inf) t f(t)dt
  - A non-redundant system with n components each with individual constant failure rate λ_i

    MTTF = 1 / [Σ λ_i]
- MTTR : Mean time to repair
  - Used to measure the repairability of a system
  - If repair rate is μ, then MTTR = 1/ μ as like MTTF
  - Availability at steady-state : Ass

    Ass = MTTF / (MTTF + MTTR) = μ / μ + λ

Single parameter probabilistic models

- MTBF : Mean time between failures
  - MTBF = MTTF + MTTR
- Coverage
  - The fault coverage available in a system can have a tremendous impact on safety, reliability, and other attributes of the system.
  - The intuitive definition of coverage is a measure of a system's ability to perform fault detection, fault location, fault containment, and/or fault recovery
  - Fault coverage is mathematically defined as the conditional probability that, given the existence of a fault, the system recovers

    c = P (fault recovery | fault existence)
  - How to decide fault coverage

    List all faults that can occur in a system and form a list of faults that can be detected.

Probabilistic comparative models

- Comparative models
  - Reliability difference : $R_{new}(t) – R_{old}(t)$
  - Reliability gain : $R_{new}(t) / R_{old}(t)$
  - Mission time improvement : $MT_{new}(r) / MT_{old}(r)$
  - Reliability Improvement Index : $\log R_{new}(t) / \log R_{old}(t)$

Probabilistic model functions

- Concept
  - Represent as a function of time
- Types
  - Hazard functions
  - Reliability functions
  - Mission time functions
  - Repair functions
  - Availability functions

Mission time & Reliability

- Mission time
  - MT(r) gives the time at which system reliability falls below the level r.
  - The mission time function is particularly well studied for applications with a minimum lifetime requirement.
  - The relationship between MT(r) and R(t)
    - $R(MT(r)) = r$
    - $MT(R(t)) = t$
    - If $Z(t) = \lambda$, $MT(r) = - \ln(r) / \lambda$
    - For non-redundant system with n components

      $MT(r) = -\ln \lambda / \Sigma \lambda_i$