CSED601
Dependable Computing
Lecture 11

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POSTECH

Review of Previous Lecture

- Evaluation of fault-tolerance
- Fault models
  - Continuous time distribution
    - Exponential distribution
    - Weibull distribution
  - Discrete time distribution
    - Geometric distribution
    - Discrete weibull distribution
- Evaluation models
  - Deterministic modeling
  - Probabilistic modeling
    - Single parameter probabilistic modeling
    - Probabilistic model functions

Reliability modeling techniques

- Concept
  - Need a systematic way to evaluate reliability
- Modeling methods
  - Combinatorial approach
  - Markov model
  - Fault-tree model
  - Hybrid technique

Combinatorial model

- Concept
  - Goal: derive the probability or function $R_{sys}(t)$ of correct system operation
  - A failure to exhaustion approach
- Assumption
  - Module failures are independent
  - Once a module has failed, it is assumed always to yield incorrect results
  - The system is considered failed if it does not satisfy the minimal set of functioning modules
  - Once the system enters a failed state, subsequent failures cannot return the system to a functional state.

Coherency property

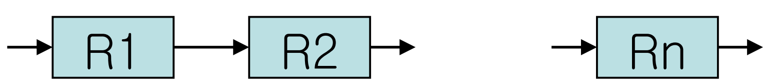
- Definition
  - Let $\Phi(x)$ be a structure function.
  - X be the vector composed of elements $x_1, x_2, \ldots, x_n$, where each x_i is one if module i is functional and zero if module i is failed.
  - A coherent system satisfies the following property.
    - $\Phi(1,1,\ldots,1) = 1$ when all modules function, the system must function.
    - $\Phi(0,0,\ldots,0) = 0$ when all modules fail, the system fails.
    - $\Phi(x) >= \Phi(y)$, whenever $x_i >= y_i$ for all i

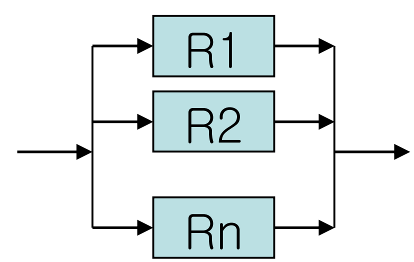
Combinatorial model

- Methods
  - Series/Parallel model
  - M-out-of-N model
  - Non-series/Non-parallel model

Series / Parallel Models

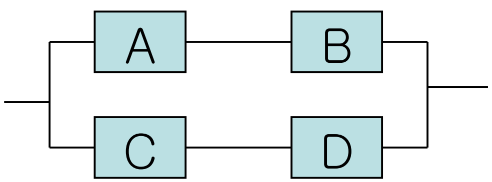
- Concept
  - A series and parallel combination of independent systems
  - Series combination
    - $R_{sys}(t) = R_{series}(t) = \Pi\ R_i(t)$
    - $Q_{series}(t) = 1 – R_{series}(t) = 1 – \Pi\ R_i(t) = 1 – \Pi\ (1 – Q_i(t))$
  - Parallel combination
    - $Q_{parallel}(t) = \Pi\ Q_i(t)$
    - $R_{parallel}(t) = 1 – Q_{parallel}(t) = 1 – \Pi\ Q_i(t) = 1 – \Pi\ (1 – R_i(t))$

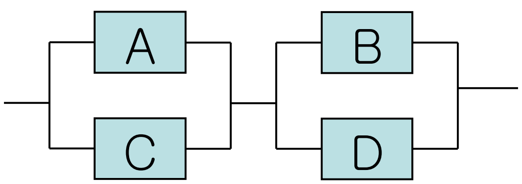




Reliability Block Diagrams

- Concept
  - Can be thought of as a flow diagram from the input of the system to the output of the system
- Two different connections





  - Assume (A,C) are processors and (B,D) are memories, and one processor-memory pair is required for operation
  - Rsys(t) = 1 – (1 – RaRb)(1-RcRd)
  - Rsys(t) = [1 – (1 – Ra)(1 – Rb)][1 – (1 – Rc)(1 – Rd)]
- For n parallel system (stand-by spare)
  - If Qparallel = 10^(-6), Qm = 0.1 then n = ln e / ln Q ~= 6

Coverage effect

- No perfect coverage
- For two modules
  - $R_{sys} = R_1 + CR_2(1-R_1)$
- If generalize with $R_1=R_2=..=R_n=R_m$
  - $R_{sys}= R_m \Sigma\ C^{\wedge}i\ (1 – R_m)^{\wedge}i = R_m\ (\ (1-C^{\wedge}nQ_m{}^{\wedge}n)/(1-CQ_m))$
  - If e=10^-6, R_m=0.9, C=1.0, then n=6
  - If C=0.99 and n -> inf, then R=0.99889

MTTF of n modules

- MTTF of n parallel systems
  - MTTF (n modules) = int $R_m \Sigma C^{\wedge i}(1-R_m)^{\wedge i}$ dt

    = MTTF(n-1) + int $R_m C^{\wedge(n-1)} (1-R_m)^{\wedge(n-1)}$ dt

    = MTTF(n-1) + $C^{\wedge(n-1)} /(n \lambda) = 1/ (\lambda C) \Sigma C^{\wedge}i / I$
  - If C is not 1.0, then the effect of nth addition is negligible.

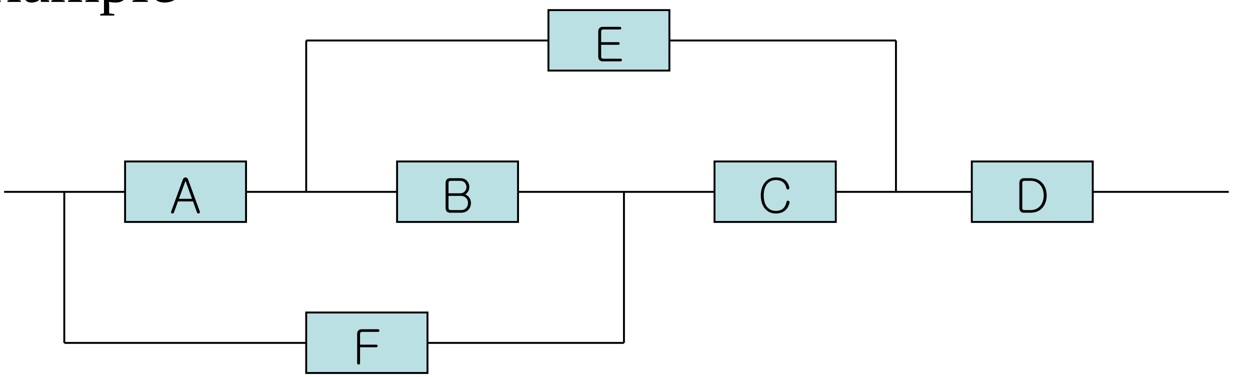
M-out-of-N model

- Assumption
  - M modules are required to function correctly
  - Example : TMR
- Approach
  - Enumerate all the working states
  - $R_{TMR} = R_m^3 + (3 \text{ choose } 2) R_m^2 (1 - R_m)$
  - Generalization

    $R = \Sigma \, (N \text{ choose } i) \, R_m^{(N-i)} \, (1 - R_m)^i$

Non-series / Non-parallel model

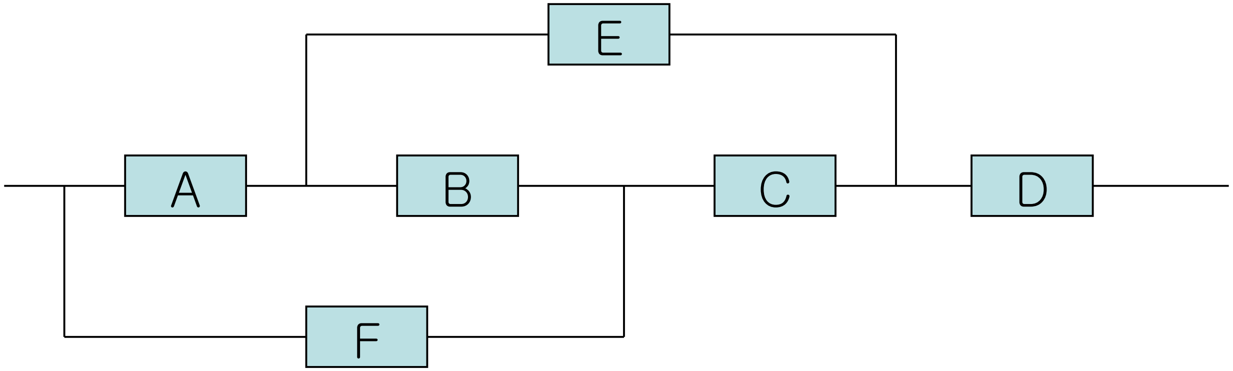
- Approach
  - Success diagram is used to describe the operational modes of a system
  - Example



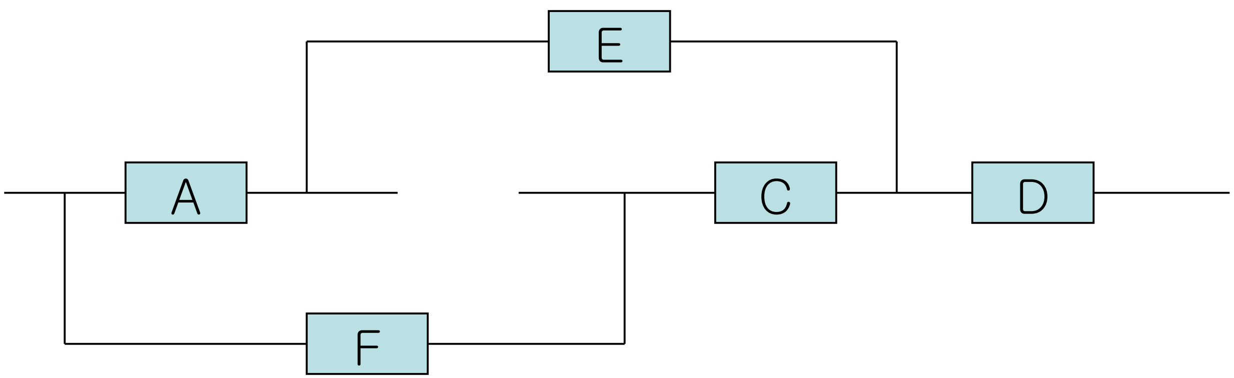
  - Exact reliability can be derived by expanding around a single module
  - $R_{sys} = R_m$ P(system works| m works)

    $+ (1 – R_m)$ P(system works| m fails)

Non-series / Non-parallel model

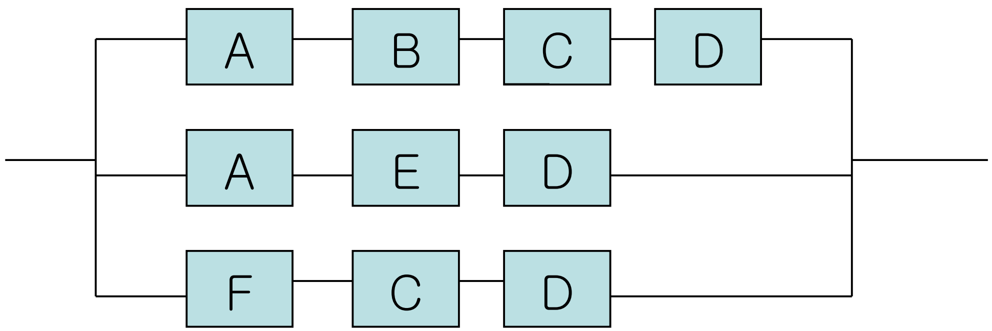


– Example extension
  - Module B works → represent it as short
  - Module B fails → represent it as open



Non-series / Non-parallel model

- Upper bound using path theory
  - Find RBD



- Lower bound using the minimal cut theory
  - D, AC, AF, CE, BEF
  - Not working for different components

Markov modeling

- Concept
  - A family of random variables that is indexed by a parameter such as time is known as a stochastic process
  - Definition (Stochastic process) : A stochastic process is a family of random variables {X(t)| t in T}, defined on a given probability space, indexed by the parameter t, where t varies over an index set T.
  - The value assumed by the random variable X(t) are called “states”, and the set of all possible values forms the state space of the process.

Markov modeling

- Types of stochastic process

	Index set (time)	T
State space	Discrete	Continuous
Discrete	Discrete parameter stochastic chain	Continuous parameter stochastic chain
Continuous	Discrete parameter continuous state process	Continuous parameter continuous state process

  – Discrete parameter process =

    a stochastic sequence {Xn| n in T}

Queuing equivalence

- Concept
  - The successive interarrival times between jobs are independent and identically distributed random variables having a distribution Fy.
  - The service times are assumed to be independent and identically distributed random variables having a distribution Fs.
  - Let m denote the number of servers then queue is represented as Fy/Fs/m