

詳 解

單元 3

結構式問題

建議題解

$$\begin{aligned} 1. \quad (a) \quad & 9a^2 - 4b^2 \\ &= (3a)^2 - (2b)^2 \\ &= \underline{(3a - 2b)(3a + 2b)} \end{aligned}$$

$$\begin{aligned} (b) \quad & 9a^2 - 4b^2 - 12a + 8b \\ &= (3a - 2b)(3a + 2b) - 12a + 8b \\ &= (3a - 2b)(3a + 2b) - 4(3a - 2b) \\ &= \underline{(3a - 2b)(3a + 2b - 4)} \end{aligned}$$

$$\begin{aligned} 2. \quad (a) \quad & 4m^2 - 16n^2 \\ &= 4(m^2 - 4n^2) \\ &= 4[m^2 - (2n)^2] \\ &= \underline{4(m - 2n)(m + 2n)} \end{aligned}$$

$$\begin{aligned} (b) \quad & 4m^2 - 16n^2 - 4n + 2m \\ &= 4(m - 2n)(m + 2n) - 4n + 2m \\ &= 4(m - 2n)(m + 2n) + 2(m - 2n) \\ &= 2(m - 2n)[2(m + 2n) + 1] \\ &= \underline{2(m - 2n)(2m + 4n + 1)} \end{aligned}$$

$$\begin{aligned} 3. \quad (a) \quad & x^2 + 10xy + 25y^2 \\ &= \underline{(x + 5y)^2} \end{aligned}$$

$$\begin{aligned} (b) \quad & x^2 + 10xy + 25y^2 - 2x - 10y \\ &= (x + 5y)^2 - 2x - 10y \\ &= (x + 5y)^2 - 2(x + 5y) \\ &= \underline{(x + 5y)(x + 5y - 2)} \end{aligned}$$

$$\begin{aligned} 4. \quad (a) \quad & 5x^2 - 125y^2 \\ & = 5(x^2 - 25y^2) \\ & = 5[x^2 - (5y)^2] \\ & = \underline{\underline{5(x-5y)(x+5y)}} \end{aligned}$$

$$\begin{aligned} (b) \quad & 5x^2 - 125y^2 - x + 5y \\ & = 5(x-5y)(x+5y) - x + 5y \\ & = 5(x-5y)(x+5y) - (x-5y) \\ & = (x-5y)[5(x+5y) - 1] \\ & = \underline{\underline{(x-5y)(5x+25y-1)}} \end{aligned}$$

$$\begin{aligned} 5. \quad (a) \quad & 4xy + 6x^2 \\ & = 2x(2y) + 2x(3x) \\ & = \underline{\underline{2x(2y+3x)}} \end{aligned}$$

$$\begin{aligned} (b) \quad & 4xy + 6x^2 - 12x - 8y \\ & = 2x(2y+3x) - 4(2y+3x) \\ & = \underline{\underline{(2x-4)(2y+3x)}} \end{aligned}$$

$$\begin{aligned} 6. \quad (a) \quad & 4xy^2 + 4x^2y \\ & = 4xy(y) + 4xy(x) \\ & = \underline{\underline{4xy(y+x)}} \end{aligned}$$

$$\begin{aligned} (b) \quad & 4xy^2 + 4x^2y - x - y \\ & = 4xy(y+x) - (x+y) \\ & = \underline{\underline{(y+x)(4xy-1)}} \end{aligned}$$

$$\begin{aligned} 7. \quad (a) \quad & a^2 + 4a - 5 \\ & = \underline{\underline{(a+5)(a-1)}} \end{aligned}$$

$$\begin{aligned} (b) \quad & a^2 + 4a - 5 + ab + 5b \\ & = (a+5)(a-1) + ab + 5b \\ & = (a+5)(a-1) + b(a+5) \\ & = \underline{\underline{(a+5)(a+b-1)}} \end{aligned}$$

$$\begin{aligned} 8. \quad (a) \quad & x^2 - 9xy + 14y^2 \\ &= \underline{(x-2y)(x-7y)} \end{aligned}$$

$$\begin{aligned} (b) \quad & x^2 - 9xy + 14y^2 + 21y - 3x \\ &= (x-2y)(x-7y) + 21y - 3x \\ &= (x-2y)(x-7y) - 3(x-7y) \\ &= \underline{(x-7y)(x-2y-3)} \end{aligned}$$

$$\begin{aligned} 9. \quad (a) \quad & 3x^2 + 10xy - 8y^2 \\ &= \underline{(x+4y)(3x-2y)} \end{aligned}$$

$$\begin{aligned} (b) \quad & 3x^2 + 10xy - 8y^2 + 9x - 6y \\ &= (x+4y)(3x-2y) + 9x - 6y \\ &= (x+4y)(3x-2y) + 3(3x-2y) \\ &= \underline{(3x-2y)(x+4y+3)} \end{aligned}$$

$$\begin{aligned} 10. \quad (a) \quad & x^2 - 8xy + 16y^2 \\ &= \underline{(x-4y)^2} \end{aligned}$$

$$\begin{aligned} (b) \quad & x^2 - 8xy + 16y^2 - 16 \\ &= (x-4y)^2 - 16 \\ &= (x-4y)^2 - 4^2 \\ &= \underline{(x-4y-4)(x-4y+4)} \end{aligned}$$

$$\begin{aligned} 11. \quad (a) \quad & 4m^2 + 12mn + 9n^2 \\ &= \underline{(2m+3n)^2} \end{aligned}$$

$$\begin{aligned} (b) \quad & 4m^2 + 12mn + 9n^2 - k^2 \\ &= (2m+3n)^2 - k^2 \\ &= \underline{(2m+3n-k)(2m+3n+k)} \end{aligned}$$

$$\begin{aligned} 12. \quad (a) \quad & 4x^2 - 4x + 1 \\ &= \underline{(2x-1)^2} \end{aligned}$$

$$(b) \quad 4x^2 - 4x + 1 - (y+3)^2$$

$$\begin{aligned}
 &= (2x-1)^2 - (y+3)^2 \\
 &= [(2x-1) - (y+3)][(2x-1) + (y+3)] \\
 &= \underline{\underline{(2x-y-4)(2x+y+2)}}
 \end{aligned}$$

13. (a) $4m^2 + 4mn + n^2$
 $= \underline{\underline{(2m+n)^2}}$

(b) $8m^2 + 8mn + 2n^2 - 8$
 $= 2(4m^2 + 4mn + n^2 - 4)$
 $= 2[(2m+n)^2 - 2^2]$
 $= \underline{\underline{2(2m+n-2)(2m+n+2)}}$

14. (a) $a^2 + 4a - 12$
 $= \underline{\underline{(a+6)(a-2)}}$

(b) $(b^2 - b)^2 + 4(b^2 - b) - 12$
 $= [(b^2 - b) + 6][(b^2 - b) - 2]$
 $= (b^2 - b + 6)(b^2 - b - 2)$
 $= \underline{\underline{(b^2 - b + 6)(b-2)(b+1)}}$

多項選擇題

1. C

$$\begin{aligned}
 &ac - bc - ad + bd \\
 &= c(a-b) - d(a-b) \\
 &= \underline{\underline{(a-b)(c-d)}}
 \end{aligned}$$

2. B

$$\begin{aligned}
 &pr - qr - ps + qs + pt - qt \\
 &= r(p-q) - s(p-q) + t(p-q) \\
 &= \underline{\underline{(p-q)(r-s+t)}}
 \end{aligned}$$

3. A

$$ac - b^2 + ab - cb$$

$$\begin{aligned} &= ab + ac - b^2 - cb \\ &= a(b+c) - b(b+c) \\ &= \underline{\underline{(a-b)(b+c)}} \end{aligned}$$

4. A

$$\begin{aligned} &m^2 + 2m + 1 - 4n^2 \\ &= (m+1)^2 - (2n)^2 \\ &= (m+1-2n)(m+1+2n) \\ &= \underline{\underline{(m-2n+1)(m+2n+1)}} \end{aligned}$$

5. C

$$\begin{aligned} &2x^2 - 2y^2 - x - y \\ &= 2(x^2 - y^2) - (x+y) \\ &= 2(x-y)(x+y) - (x+y) \\ &= \underline{\underline{(x+y)(2x-2y-1)}} \end{aligned}$$

6. D

$$\begin{aligned} &(3m+2n)^2 - (3m-2n)^2 \\ &= [(3m+2n) - (3m-2n)][(3m+2n) + (3m-2n)] \\ &= (3m+2n-3m+2n)(3m+2n+3m-2n) \\ &= (4n)(6m) \\ &= \underline{\underline{24mn}} \end{aligned}$$

7. C

$$5x^2 + (p+1)x + 2 \equiv 5x(x+2) + q(x+1) - 1$$

$$\text{左方} = 5x^2 + (p+1)x + 2$$

$$\text{右方} = 5x(x+2) + q(x+1) - 1$$

$$= 5x^2 + 10x + qx + q - 1$$

$$= 5x^2 + (10+q)x + q - 1$$

比較常數項，可得：

$$2 = q - 1$$

$$q = 3$$

比較 x 的係數，可得：

$$p + 1 = 10 + q$$

$$p = q + 9$$

$$p = (3) + 9$$

$$p = \underline{\underline{12}}$$

8. D

$$(x+3)(x-h)-3 \equiv x^2-3k$$

$$\text{左方} = (x+3)(x-h)-3$$

$$= x^2 + 3x - hx - 3h - 3$$

$$= x^2 + (3-h)x - 3h - 3$$

$$\text{右方} = x^2 - 3k$$

比較 x 的係數，可得：

$$3-h=0$$

$$h=3$$

比較常數項，可得：

$$-3h-3=-3k$$

$$k=h+1$$

$$k=(3)+1$$

$$k = \underline{\underline{4}}$$

9. A

$$x^2 + 2a(x+1) \equiv (x+1)(x+b) - (x+b-2c)$$

$$\text{左方} = x^2 + 2a(x+1)$$

$$= x^2 + 2ax + 2a$$

$$\text{右方} = (x+1)(x+b) - (x+b-2c)$$

$$= x^2 + x + bx + b - x - b + 2c$$

$$= x^2 + bx + 2c$$

比較 x 的係數，可得：

$$2a = b$$

$$\frac{a}{b} = \frac{1}{2}$$

比較常數項，可得：

$$2a = 2c$$

$$a = c$$

$$\therefore a:b:c = \underline{\underline{1:2:1}}$$