詳 解

單元 3

結構式問題

<u>建議題解</u>

1. **(a)** $9a^2 - 4b^2$
$= (3a)^2 - (2b)^2$
$= \underline{\underline{(3a - 2b)(3a + 2b)}}$

(b) $9a^2 - 4b^2 - 12a + 8b$
$= (3a - 2b)(3a + 2b) - 12a + 8b$
$= (3a - 2b)(3a + 2b) - 4(3a - 2b)$
$= \underline{\underline{(3a - 2b)(3a + 2b - 4)}}$

2. **(a)** $4m^2 - 16n^2$
$= 4(m^2 - 4n^2)$
$= 4[m^2 - (2n)^2]$
$= \underline{\underline{4(m - 2n)(m + 2n)}}$

(b) $4m^2 - 16n^2 - 4n + 2m$
$= 4(m - 2n)(m + 2n) - 4n + 2m$
$= 4(m - 2n)(m + 2n) + 2(m - 2n)$
$= 2(m - 2n)[2(m + 2n) + 1]$
$= \underline{\underline{2(m - 2n)(2m + 4n + 1)}}$

3. **(a)** $x^2 + 10xy + 25y^2$
$= \underline{\underline{(x + 5y)^2}}$

(b) $x^2 + 10xy + 25y^2 - 2x - 10y$
$= (x + 5y)^2 - 2x - 10y$
$= (x + 5y)^2 - 2(x + 5y)$
$= \underline{\underline{(x + 5y)(x + 5y - 2)}}$

4. **(a)** $5x^2 - 125y^2$

$= 5(x^2 - 25y^2)$

$= 5[x^2 - (5y)^2]$

$= \underline{\underline{5(x-5y)(x+5y)}}$

(b) $5x^2 - 125y^2 - x + 5y$

$= 5(x-5y)(x+5y) - x + 5y$

$= 5(x-5y)(x+5y) - (x-5y)$

$= (x-5y)[5(x+5y) - 1]$

$= \underline{\underline{(x-5y)(5x+25y-1)}}$

5. **(a)** $4xy + 6x^2$

$= 2x(2y) + 2x(3x)$

$= \underline{\underline{2x(2y+3x)}}$

(b) $4xy + 6x^2 - 12x - 8y$

$= 2x(2y+3x) - 4(2y+3x)$

$= \underline{\underline{(2x-4)(2y+3x)}}$

6. **(a)** $4xy^2 + 4x^2y$

$= 4xy(y) + 4xy(x)$

$= \underline{\underline{4xy(y+x)}}$

(b) $4xy^2 + 4x^2y - x - y$

$= 4xy(y+x) - (x+y)$

$= \underline{\underline{(y+x)(4xy-1)}}$

7. **(a)** $a^2 + 4a - 5$

$= \underline{\underline{(a+5)(a-1)}}$

(b) $a^2 + 4a - 5 + ab + 5b$

$= (a+5)(a-1) + ab + 5b$

$= (a+5)(a-1) + b(a+5)$

$= \underline{\underline{(a+5)(a+b-1)}}$

8. **(a)** $x^2 - 9xy + 14y^2$
$= \underline{\underline{(x - 2y)(x - 7y)}}$

(b) $x^2 - 9xy + 14y^2 + 21y - 3x$
$= (x - 2y)(x - 7y) + 21y - 3x$
$= (x - 2y)(x - 7y) - 3(x - 7y)$
$= \underline{\underline{(x - 7y)(x - 2y - 3)}}$

9. **(a)** $3x^2 + 10xy - 8y^2$
$= \underline{\underline{(x + 4y)(3x - 2y)}}$

(b) $3x^2 + 10xy - 8y^2 + 9x - 6y$
$= (x + 4y)(3x - 2y) + 9x - 6y$
$= (x + 4y)(3x - 2y) + 3(3x - 2y)$
$= \underline{\underline{(3x - 2y)(x + 4y + 3)}}$

10. **(a)** $x^2 - 8xy + 16y^2$
$= \underline{\underline{(x - 4y)^2}}$

(b) $x^2 - 8xy + 16y^2 - 16$
$= (x - 4y)^2 - 16$
$= (x - 4y)^2 - 4^2$
$= \underline{\underline{(x - 4y - 4)(x - 4y + 4)}}$

11. **(a)** $4m^2 + 12mn + 9n^2$
$= \underline{\underline{(2m + 3n)^2}}$

(b) $4m^2 + 12mn + 9n^2 - k^2$
$= (2m + 3n)^2 - k^2$
$= \underline{\underline{(2m + 3n - k)(2m + 3n + k)}}$

12. **(a)** $4x^2 - 4x + 1$
$= \underline{\underline{(2x - 1)^2}}$

(b) $4x^2 - 4x + 1 - (y + 3)^2$

$= (2x-1)^2 - (y+3)^2$
$= [(2x-1)-(y+3)][(2x-1)+(y+3)]$
$= \underline{\underline{(2x-y-4)(2x+y+2)}}$

13. (a) $4m^2 + 4mn + n^2$
$= \underline{\underline{(2m+n)^2}}$

(b) $8m^2 + 8mn + 2n^2 - 8$
$= 2(4m^2 + 4mn + n^2 - 4)$
$= 2[(2m+n)^2 - 2^2]$
$= \underline{\underline{2(2m+n-2)(2m+n+2)}}$

14. (a) $a^2 + 4a - 12$
$= \underline{\underline{(a+6)(a-2)}}$

(b) $(b^2-b)^2 + 4(b^2-b) - 12$
$= [(b^2-b)+6][(b^2-b)-2]$
$= (b^2-b+6)(b^2-b-2)$
$= \underline{\underline{(b^2-b+6)(b-2)(b+1)}}$

多項選擇題

1. C

$ac - bc - ad + bd$
$= c(a-b) - d(a-b)$
$= \underline{\underline{(a-b)(c-d)}}$

2. B

$pr - qr - ps + qs + pt - qt$
$= r(p-q) - s(p-q) + t(p-q)$
$= \underline{\underline{(p-q)(r-s+t)}}$

3. A

$ac - b^2 + ab - cb$

$= ab + ac - b^2 - cb$
$= a(b + c) - b(b + c)$
$= \underline{\underline{(a - b)(b + c)}}$

4. A

$m^2 + 2m + 1 - 4n^2$
$= (m + 1)^2 - (2n)^2$
$= (m + 1 - 2n)(m + 1 + 2n)$
$= \underline{\underline{(m - 2n + 1)(m + 2n + 1)}}$

5. C

$2x^2 - 2y^2 - x - y$
$= 2(x^2 - y^2) - (x + y)$
$= 2(x - y)(x + y) - (x + y)$
$= \underline{\underline{(x + y)(2x - 2y - 1)}}$

6. D

$(3m + 2n)^2 - (3m - 2n)^2$
$= [(3m + 2n) - (3m - 2n)][(3m + 2n) + (3m - 2n)]$
$= (3m + 2n - 3m + 2n)(3m + 2n + 3m - 2n)$
$= (4n)(6m)$
$= \underline{\underline{24mn}}$

7. C

$5x^2 + (p + 1)x + 2 \equiv 5x(x + 2) + q(x + 1) - 1$
左方 $= 5x^2 + (p + 1)x + 2$
右方 $= 5x(x + 2) + q(x + 1) - 1$
$= 5x^2 + 10x + qx + q - 1$
$= 5x^2 + (10 + q)x + q - 1$
比較常數項，可得：
$2 = q - 1$
$q = 3$
比較 x 的係數，可得：

$$p+1=10+q$$
$$p=q+9$$
$$p=(3)+9$$
$$p=\underline{\underline{12}}$$

8. D

$(x+3)(x-h)-3\equiv x^2-3k$

左方 $=(x+3)(x-h)-3$

$$=x^2+3x-hx-3h-3$$
$$=x^2+(3-h)x-3h-3$$

右方 $=x^2-3k$

比較 x 的係數，可得：

$$3-h=0$$
$$h=3$$

比較常數項，可得：

$$-3h-3=-3k$$
$$k=h+1$$
$$k=(3)+1$$
$$k=\underline{\underline{4}}$$

9. A

$x^2+2a(x+1)\equiv(x+1)(x+b)-(x+b-2c)$

左方 $=x^2+2a(x+1)$

$$=x^2+2ax+2a$$

右方 $=(x+1)(x+b)-(x+b-2c)$

$$=x^2+x+bx+b-x-b+2c$$
$$=x^2+bx+2c$$

比較 x 的係數，可得：

$$2a=b$$
$$\frac{a}{b}=\frac{1}{2}$$

比較常數項，可得：

$$2a=2c$$
$$a=c$$

$\therefore\quad a:b:c=\underline{\underline{1:2:1}}$