Compiler Design - Notes Week 6

Ruben Schenk, ruben.schenk@inf.ethz.ch

October 26, 2021

8. LL & LR Parsing

8.1 LL(1) Grammars

One problem with grammars is that not all grammars can be parsed “top-down” with a *single lookahead*. *Top-down* means that we start from the start symbol, i.e. the root of the parse tree, and go down.

LL(1) means:

- Left-to-right scanning
- Left-most derivation
- 1 lookahead symbol

8.1.1 Making a Grammar LL(1)

The main problem is that we can't decide which S production to apply until we see the symbol after the first expression. The solution is to *left-factor* the grammar. There is a common S prefix for each choice, so add a new non-terminal S' at the decision point:

This means that we transform our example grammar

$$S \to E + S \,|\, E E \to \text{number} \,|\, (S)$$

to the following “left-factor” grammar:

$$S \to ES' S' \to \epsilon S' \to +S E \to \text{number} \,|\, (S)$$

However, we also need to *eliminate left-recursion* somehow. In general, this is done by rewriting the following left-recursive rule

$$S \to S\alpha_1 \,|\, \cdots \,|\, S\alpha_n \,|\, \beta_1 \,|\, \cdots \,|\, \beta_m$$

to a rule of the form:

$$S \to \beta_1 S' \,|\, \cdots \,|\, \beta_m S' S' \to \alpha_1 S' \,|\, \cdots \,|\, \alpha_n S' \,|\, \epsilon$$

In our running example, this would mean to rewrite

$$S \to S + E \,|\, E E \to \text{number} \,|\, (S)$$

to the following left-recursion-eliminating grammar:

$$S \to ES' S' \to +ES' \,|\, \epsilon E \to \text{number} \,|\, (S)$$

8.1.2 Predictive Parsing

Given an LL(1) grammar:

- For a given non-terminal, the lookahead symbol uniquely determines the production to apply
- The parsing is driven by a predictive parsing table
- It is convenient to add a special *end-of-file token* \$ and a start symbol T that requires \$

Example: Let us look at the following LL(1) grammar:

$$T \to S\$ S \to ES' S' \to \epsilon S' \to +S E \to \text{number} \mid (S)$$

We then propose the following **predictive parsing table:**

	number	**+**	**(**	**)**	**\$ (EOF)**
T	$\to S$		$\to S$		
S	$\to ES'$		$\to ES'$		
S'		$\to +S$		$\to \epsilon$	$\to \epsilon$
E	$\to$ number		$\to (S)$		

8.1.3 Construction of Parse Table

How do we construct the parse table? We examine two possible cases by considering the following production: $A \to \gamma$

Case 1 Construct the set of all input tokens that may appear *first* in strings that can be derived from γ. Then add the production $\to \gamma$ to the entry (A, token) for each such token.

Case 2 If γ can derive ϵ, then we construct the set of all input tokens that may *follow* the nonterminal A in the grammar. We then add the production $\to \gamma$ to the entry (A, token) for each such token.

Note: If there are two different productions for a given entry, then the grammar is not LL(1).

Example:

- First(T) = First(S)
- First(S) = First(E)
- First(S') = { +, ε }
- First(E) = { number, '(' }

- Follow(S') = Follow(S)
- Follow(S) = { \$, ')' } ∪ Follow(S')

Note: we want the *least* solution to this system of set equations... a *fixpoint* computation. More on these later in the course.

T ⟼ S\$
S ⟼ ES'
S' ⟼ ε
S' ⟼ + S
E ⟼ number | (S)

	number	+	(	)	\$ (EOF)
T	⟼ S\$		⟼S\$		
S	⟼ E S'		⟼E S'		
S'		⟼ + S		⟼ ε	⟼ ε
E	⟼ number		⟼ (S)		

8.1.4 Converting The Parsing Table to Code

When we want to convert a parsing table to code, we proceed as follows:

- Define N mutually recursive functions, one for each non-terminal A: `parse_A`
- `parse_A` is of type `unit -> ast` if £A\$ is not an auxiliary non-terminal
- Otherwise, `parse_A` takes additional ast's as inputs, one for each non-terminal in the "factored" prefix

Then, each function `parse_A`:

- "Peeks" at the lookahead token
- Follows the production rule in the corresponding entry
- Consumes the terminal tokens from the input stream
- Calls `parse_X` to create the sub-tree for the non-terminal X
- If the rule ends in an auxiliary non-terminal, it is called with the appropriate ast's
- Otherwise, this function builds the ast tree itself and returns it

8.1.5 LL(1) Summary

LL(1) is top-down based parsing that finds the left-most derivation. The process proceeds with the following steps:

1. Language grammar =>
2. LL(1) grammar =>
3. Prediction table =>
4. Recursive-descent parser

8.2 LR Grammars

8.2.1 Bottom-up Parsing

LR(k) parser are *bottom-up parser*:

- Left-to-right scanning
- Right-most derivation
- k lookahead symbols

LR grammars are *more expressive* than LL grammars:

- They can handle left-recursive (and right-recursive) grammars (i.e. virtually all programming languages)
- They make it easier to express programming language syntax (e.g. no left factoring)

The most common technique are **Shift-Reduce parsers:**

- Work bottom up instead of top down
- Construct the right-most derivation of a program in the grammar
- Better error detection/recovery

8.2.2 Shift/Reduce Parsing

In shift/reduce parsing, the parser has a **parser state** described as follows:

- Stack of terminals and non-terminals
- Unconsumed input is a string of terminals
- The current derivation step is `stack + input`

Parsing is a sequence of `shif` and `reduce` operations:

- Shift: Move lookahead token to the stack
- Reduce: Replace symbols γ at the top of the stack with a non-terminal X s.t. $X \to \gamma$ is a production, i.e. `pop gamma, push X`

Example: We consider our previous example

$$S \to S + E \,|\, EE \to \text{number} \,|\, (S)$$

Stack	Input	Action
	(1 + 2 + (3 + 4)) + 5	shift (
(	1 + 2 + (3 + 4)) + 5	shift 1
(1	+ 2 + (3 + 4)) + 5	reduce: E ⟼ number
(E	+ 2 + (3 + 4)) + 5	reduce: S ⟼ E
(S	+ 2 + (3 + 4)) + 5	shift +
(S +	2 + (3 + 4)) + 5	shift 2
(S + 2	+ (3 + 4)) + 5	reduce: E ⟼ number

8.3 LR(0) Grammars

8.3.1 LR Parser States

Our goal it is to know *what set of reductions are legal* at any given point. The idea to solve this problem is to summarize all possible stack prefixes α as a finite parser state:

- The parser state is computed by a DFA that reads the stack σ
- Accept states of the DFA correspond to unique reductions that apply

8.3.2 Example LR(0) Grammar: Tuples

The following grammar is an example grammar for non-empty tuples and identifiers:

$$S \to (L) \,|\, \mathrm{id} L \to S \,|\, L,\, S$$

Now, if we apply parsing as a sequence of shift and reduce operations, we end up with the following parse operation:

- Shift: Move look-ahead token to the stack

Stack	Input	Action
	(x, (y, z), w)	shift (
(	x, (y, z), w)	shift x

- Reduce: Replace symbols γ at top of stack with nonterminal X s.t. X ⟼ γ is a production, i.e., pop γ, push X

Stack	Input	Action
(x	, (y, z), w)	reduce S ⟼ id
(S	, (y, z), w)	reduce L ⟼ S

8.3.3 Action Selection Problem

Given a stack σ and a lookahead symbol b, should the parser either:

- Shift b onto the stack (new stack is σb), or
- Reduce a production $X \to \gamma$, assuming that $\sigma = \alpha\gamma$ (new stack is αX) ?

The main idea to solve this problem is to decide based on a prefix α of the stack plus the lookahead. The prefix α is different for different possible reductions since in productions $X \to \gamma$ and $Y \to \beta$, γ and β might have different lengths.

8.3.4 LR(0) States

An **LR(0) state** consists of items to track progress on possible upcoming reductions. An **LR(0) item** is a production with an extra separator **.** in the RHS. Example items could be: $S \to .(L)$ or $S \to (.L)$ or $L \to S.$

The intuition for the meaning of the dot is:

- Stuff before the **.** is already on the stack
- Stuff after the **.** is what might be seen next

8.3.5 Constructing The DFA

We will consider the following grammar:

$$S' \to S\$S \to (L) \,|\, \mathrm{id} L \to S \,|\, L,\, S$$

The first step when creating the DFA is to add a new production $S' \to S$ to the grammar. The *start state* of the DFA is the empty stack, so it contains the item $S' \to .S$. We then proceed to add the **closure of the state** to our DFA:

- Add items for all productions whose LHS non-terminal occurs in an item in the state just after the **.** (e.g. S in $S' \to .S$)

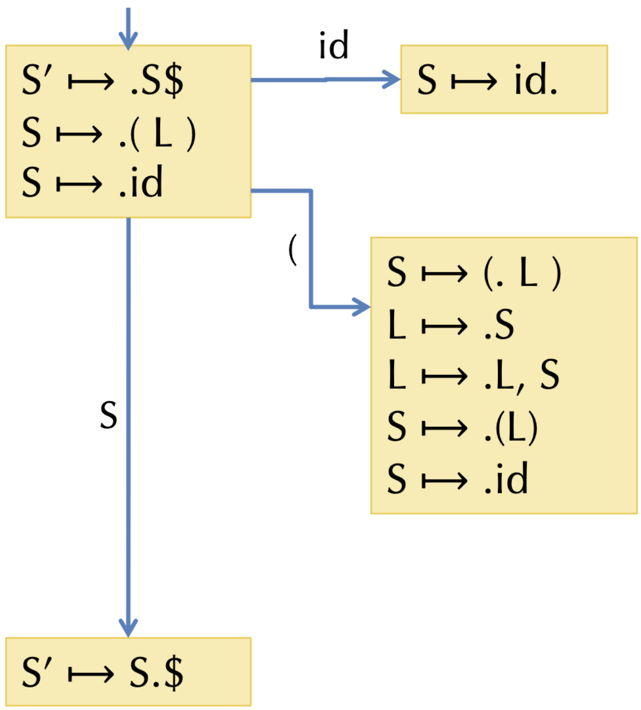
- The added items have the . located at the beginning
- Note that newly added items may cause yet more items to be added to the state, we keep iterating until a *fixed point* is reached

Example:

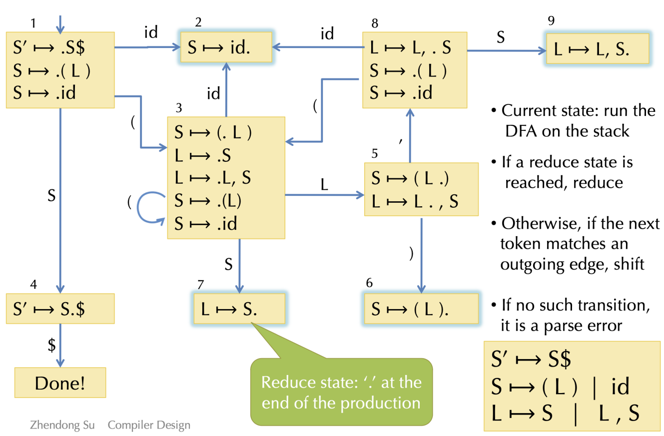
$$\text{Closure}(\{S' \to .S\$\}) = \{S' \to .S\$,\ S \to .(L),\ S \to .\text{id}\}$$

Next we need to add the *transitions:*

1. First, we see what terminals and non-terminals can appear after the . in the source state.
2. The target state initially includes all items from the source state that have the edge-label symbol after the ., but we advance the . to simulate shifting the item onto the stack.
3. Finally, for each new state, we again take the closure of it.



By continuing the above approach, we will reach the following **full DFA** for our example:



8.3.6 Using The DFA

To use our DFA now, we run the parser stack σ through the DFA. The resulting state tells us what productions may be reduced next:

- If not in a reduce state, we shift the next symbol and transition w.r.t. the DFA
- If in a reduce state, $X \to \gamma$ with stack $\alpha\gamma$, we `pop gamma` and `push X`

Optimization: There is no need to rerun the DFA from the beginning at each step. We might simply store the state with each symbol on the stack, e.g. $_1(_3)_3L_5)_6$. Then:

- On a reduction $X \to \gamma$, we `pop` the stack to reveal the state too, e.g. from stack $_1(_3)_3L_5)_6$ we reduce $S \to (L)$ to reach stack $_1(_3$
- Next, we push the reduction symbol, e.g. to reach the stack $_1(_3S$
- Then we take just one step in the DFA to find the next state $_1(_3S_7$

8.3.7 Implementing The Parsing Table

We represent the DFA as a table of shape `state * (terminals + nonterminals)`.

Entries for the **action table** specify two kinds of actions:

- Shift and go to state n
- Reduce using the reduction $X \to \gamma$: First, `pop gamma` of the stack to reveal the state, second, look up X in the **goto table** and go to that state

Example:

	(	)	id	,	$	S	L
1	s3		s2			g4	
2	S↦id	S↦id	S↦id	S↦id	S↦id		
3	s3		s2			g7	g5
4					DONE		
5		s6		s8			
6	S ↦ (L)	S ↦ (L)	S ↦ (L)	S ↦ (L)	S ↦ (L)		
7	L ↦ S	L ↦ S	L ↦ S	L ↦ S	L ↦ S		
8	s3		s2			g9	
9	L ↦ L,S	L ↦ L,S	L ↦ L,S	L ↦ L,S	L ↦ L,S		

sx = shift and go to state x
gx = go to state x

8.3.8 LR(0) Limitations

An LR(0) machine only works if states with reduce actions have a *single* reduce action. In such states, the machine always reduces, ignoring lookahead.

With more complex grammars, the DFA construction will yield states with *sift/reduce* and *reduce/reduce* problems:

OK	shift/reduce	reduce/reduce
S ↦ (L).	S ↦ (L). L ↦ .L , S	S ↦ L ,S. S ↦ ,S.

8.4 LR(1) Parsing

The algorithm for **LR(1) parsing** is similar to LR(0) DFA construction:

- LR(1) state is the set of all LR(1) items
- An LR(1) item is an LR(0) item plus a set of lookahead symbols, i.e. $A \to \alpha.\beta,\ \mathcal{L}$

However, the **LR(1) closure** is a little more complex:

1. We first form the set of items just as we did in the LR(0) algorithm
2. Whenever a new item $C \to .\gamma$ is added, because the item $A \to \beta.C\delta,\ \mathcal{L}$ is already in the set, we need to compute its lookahead set $\mathcal{M}$:
   1. The lookahead set $\mathcal{M}$ includes FIRST(δ), i.e. the set of terminals that may start strings derived from δ

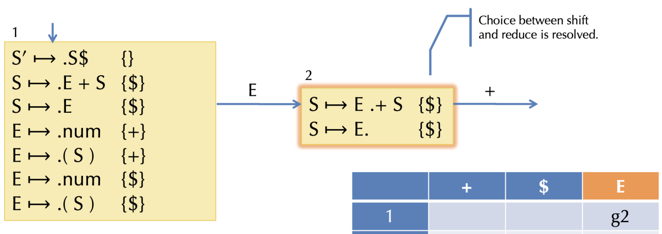
2. If δ is or can derive ϵ, then the lookahead $\mathcal{M}$ also contains $\mathcal{L}$

8.4.1 Example Closure in LR(1)

S′ ⟼ S$
S ⟼ E + S | E
E ⟼ number | (S)

- Start item: S′ ⟼ .S$, {}
- Since S is to the right of a '.', add
  S ⟼ .E + S , {\$} Note: {\$} is FIRST(\$)
  S ⟼ .E , {\$}
- Need to keep closing, since E appears to the right of a '.' in '.E + S'
  E ⟼ .number , {+} Note: + added for reason 1
  E ⟼ .(S) , {+} FIRST(+ S) = {+}
- Because E also appears to the right of '.' in '.E' we get:
  E ⟼ .number , {\$} Note: \$ added for reason 2
  E ⟼ .(S) , {\$} δ is ε
- All items are distinct, so we're done

8.4.2 Using The DFA



	+	\$	E
1			g2
2	s3	S ⟼ E	

Fragment of the Action & Goto tables

The behavior is determined if:

- There is no overlap among the lookahead sets for each reduce item, and
- None of the lookahead symbols appear to the right a .

8.4.3 LR Variant: LALR(1)

Consider for example the following two LR(1) states:

$$S_1 : \quad \{[X \to \alpha., a], [Y \to \beta., c]\} S_2 : \quad \{[X \to \alpha., b], [Y \to \beta., d]\}$$

They have the same core and can therefore be *merged*. The merged state contains:

$$\{[X \to \alpha., a/b], [Y \to \beta., c/d]\}$$

These are so-called **LALR(1)** states. Typically, there are 10 times fewer LALR(1) states than LR(1). However, LALR(1) may introduce new reduce/reduce conflicts (but not new shift/reduce conflicts).