$$\mathcal{M} : (\mathrm{Sets}) \rightleftarrows (\mathrm{Mon}) : \mathcal{U}$$

$$\mathrm{Hom}_{\mathrm{Mon}}(\mathcal{M}(X), M) \cong \mathrm{Hom}_{\mathrm{Set}}(X, \mathcal{U}(M))$$

$$\mathcal{C} : (\mathrm{Quiv}) \rightleftarrows (\mathrm{Cats}) : \mathcal{U}$$

$$\mathrm{Hom}_{\mathrm{Cats}}(\mathcal{C}(q), D) \cong \mathrm{Hom}_{\mathrm{Quiv}}(q, \mathcal{U}(D)) \qquad D = k\text{-mat}$$

$$\begin{tikzcd}
\mathrm{Quiv} \arrow[r, shift left, "\mathrm{CatClosure}"] \arrow[rr, bend right] & \mathrm{Cats} \arrow[l, shift left, "\mathcal{U}"] \arrow[r, shift left, "k\text{-Algebroid}"] & k\text{-Cats} \arrow[l, shift left, "\mathcal{U}"] \arrow[r, shift left, "\mathrm{AdditiveClosure}"] & k\text{-Cats}^{\oplus} \arrow[l, shift left, "\mathcal{U}"]
\end{tikzcd}$$

$$k \cdot q = \bigoplus_{i,j} \underbrace{e_i (k \cdot q) \cdot e_j}_{\mathrm{Hom}(e_i, e_j)}$$

$A \quad (\sum e_i = 1) \qquad e_i e_j = \delta_{ij} e_i$

$$A = 1 \cdot A \cdot 1 = \bigoplus_{i,j} e_i A e_j$$

The commutative ring / field k is a pre-additive category on one object.

$\mathrm{Obj}\ k = \{*\}$

$\mathrm{Mor}\ k = k$

It also follows that k is as a pre-additive category k-linear.

Corollary: Each commutative ring is a k-linear category over itself.

$$k\text{-mat} := \overset{\mathrm{MatrixCategory}(k)}{\mathrm{AdditiveClosure}(k)}$$

Prop: k-mat is by construction an k-linear additive category, which has weak kernel & cokernels. If k is a field then it has kernels & cokernels and is Abelian.

A two-sided ideal I in a category $\mathcal{C}$ (with zero object) is a subset of morphisms which is closed under composed with all other morphism in $\mathcal{C}$: $\mathcal{C} I \mathcal{C} \subset I$.

A congruence relation on category $\mathcal{C}$, is an equivalence relation on the set of morphisms such that $a \sim b$, $c, d \Rightarrow \begin{matrix} ca \sim cb \\ ad \sim bd \end{matrix}$

$\Rightarrow$ $\mathcal{C}/\sim$ is again a category.

$$\begin{tikzcd}
\mathcal{C}(q) \arrow[r, "\mathcal{R}"] \arrow[dr] & \mathrm{FinSets} \\
& \langle g_1, \dots, g_r \rangle \arrow[u, phantom, "\subset" rotate=90]
\end{tikzcd}$$

induziert eine Kongruenzrelation of $\mathcal{C}(q)$

$$a \underset{\mathcal{R}}{\sim} b \Leftrightarrow \mathcal{R}(a) = \mathcal{R}(b)$$

IsCongruentForMorphisms

$$\mathcal{C}(q) / \underset{\mathcal{R}}{\sim} \; \cong \underbrace{\langle g_1 \to g_r \rangle}_{\text{Concrete category}} \subseteq \mathrm{FinSets}$$

Dein Algorithmus beschreibt diese Kongruenzrelation (in Wahrheit als Ideal in k-LinClosure(CatClosure(q)))