CatReps

Representations and cohomology of finite categories

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Contents

Chapter 1

Finite concrete categories

1.1 Example

1.1.1 A category of module homomorphisms

Example

```
gap> qc3c3 := RightQuiver( "q(2)[a:1->1,b:1->2,c:2->2]" );
q(2)[a:1->1,b:1->2,c:2->2]
gap> HOMALG_MATRICES.PreferDenseMatrices := true;
true
gap> GF3 := HomalgRingOfIntegers( 3 );
GF(3)
gap> GF3q := PathAlgebra( GF3, qc3c3 );
GF(3) * q
gap> rel := [GF3q.a^3-GF3q.1, GF3q.c^3-GF3q.2, GF3q.a*GF3q.b-GF3q.b*GF3q.c];;
gap> kq := Algebroid( GF3q, rel );
Algebroid generated by the right quiver q(2)[a:1->1,b:1->2,c:2->2]
gap> SetIsLinearClosureOfACategory( kq, true );
```

A representation of the category c3c3 is another way to encode a module homomorphism between two modules over the cyclic group C_3 of order 3: The vector space underlying the first module is the given by the value of (1). The action of C3 on the first module is given by the value of (a). The vector space underlying the second module is the given by the value of (2). The action on the second module is given by the value of (c). The above relation of the quiver states that the value of (b) is a module homomorphism from the first to the second C_3-module.

Example

```
gap> CatReps := Hom( kq, GF3 );
The category of functors: Algebroid generated by the right quiver
q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices over GF(3)
gap> InfoOfInstalledOperationsOfCategory( CatReps );
106 primitive operations were used to derive 236 operations for this category which
* IsLinearCategoryOverCommutativeRing
* IsSymmetricMonoidalCategory
* IsAbelianCategory
gap> CommutativeRingOfLinearCategory( CatReps );
GF(3)
gap> zero := ZeroObject( CatReps );
<(1)->0, (2)->0; (a)->0x0, (b)->0x0, (c)->0x0>
```

```
gap> Display( zero );
An object in The category of functors: Algebroid generated by the
right quiver q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices
over GF(3) defined by the following data:


Image of <(1)>:
A vector space object over GF(3) of dimension 0

Image of <(2)>:
A vector space object over GF(3) of dimension 0

Image of (1)-[{ Z(3)^0*(a) }]->(1):
(an empty 0 x 0 matrix)

A zero, identity morphism in Category of matrices over GF(3)


Image of (1)-[{ Z(3)^0*(b) }]->(2):
(an empty 0 x 0 matrix)

A zero, identity morphism in Category of matrices over GF(3)


Image of (2)-[{ Z(3)^0*(c) }]->(2):
(an empty 0 x 0 matrix)

A zero, identity morphism in Category of matrices over GF(3)
gap> const := TensorUnit( CatReps );
<(1)->1, (2)->1; (a)->1x1, (b)->1x1, (c)->1x1>
gap> Display( const );
An object in The category of functors: Algebroid generated by the
right quiver q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices
over GF(3) defined by the following data:


Image of <(1)>:
A vector space object over GF(3) of dimension 1

Image of <(2)>:
A vector space object over GF(3) of dimension 1

Image of (1)-[{ Z(3)^0*(a) }]->(1):
 1

An identity morphism in Category of matrices over GF(3)


Image of (1)-[{ Z(3)^0*(b) }]->(2):
 1

An identity morphism in Category of matrices over GF(3)
```

```
Image of (2)-[{ Z(3)^0*(c) }]->(2):
 1

An identity morphism in Category of matrices over GF(3)
gap> d := [[1,1,0,0,0],[0,1,1,0,0],[0,0,1,0,0],[0,0,0,1,1],[0,0,0,0,1]];;
gap> e := [[0,1,0,0],[0,0,1,0],[0,0,0,0],[0,1,0,1],[0,0,1,0]];;
gap> f := [[1,1,0,0],[0,1,1,0],[0,0,1,0],[0,0,0,1]];;
gap> nine := AsObjectInHomCategory( kq, [ 5, 4 ], [ d, e, f ] );
<(1)->5, (2)->4; (a)->5x5, (b)->5x4, (c)->4x4>
gap> Display( nine );
An object in The category of functors: Algebroid generated by the
right quiver q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices
over GF(3) defined by the following data:


Image of <(1)>:
A vector space object over GF(3) of dimension 5

Image of <(2)>:
A vector space object over GF(3) of dimension 4

Image of (1)-[{ Z(3)^0*(a) }]->(1):
 1 1 . . .
 . 1 1 . .
 . . 1 . .
 . . . 1 1
 . . . . 1

A morphism in Category of matrices over GF(3)


Image of (1)-[{ Z(3)^0*(b) }]->(2):
 . 1 . .
 . . 1 .
 . . . .
 . 1 . 1
 . . 1 .

A morphism in Category of matrices over GF(3)


Image of (2)-[{ Z(3)^0*(c) }]->(2):
 1 1 . .
 . 1 1 .
 . . 1 .
 . . . 1

A morphism in Category of matrices over GF(3)
gap> nine(kq.1);
<A vector space object over GF(3) of dimension 5>
```

```
gap> nine(kq.2);
<A vector space object over GF(3) of dimension 4>
gap> nine(kq.b);
<A morphism in Category of matrices over GF(3)>
gap> Display( nine(kq.b) );
 . 1 . .
 . . 1 .
 . . . .
 . 1 . 1
 . . 1 .

A morphism in Category of matrices over GF(3)
gap> IsWellDefined( nine );
true
gap> fortyone := TensorProductOnObjects( nine, nine );
<(1)->25, (2)->16; (a)->25x25, (b)->25x16, (c)->16x16>
gap> IsWellDefined( fortyone );
true
gap> fortyone( kq.1 );
<A vector space object over GF(3) of dimension 25>
gap> fortyone( kq.2 );
<A vector space object over GF(3) of dimension 16>
gap> fortyone(kq.a) = TensorProductOnMorphisms( nine(kq.a), nine(kq.a) );
true
gap> fortyone(kq.b) = TensorProductOnMorphisms( nine(kq.b), nine(kq.b) );
true
gap> fortyone(kq.c) = TensorProductOnMorphisms( nine(kq.c), nine(kq.c) );
true
gap> Display( fortyone );
An object in The category of functors: Algebroid generated by the
right quiver q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices
over GF(3) defined by the following data:


Image of <(1)>:
A vector space object over GF(3) of dimension 25

Image of <(2)>:
A vector space object over GF(3) of dimension 16

Image of (1)-[{ Z(3)^0*(a) }]->(1):
 1 1 . . . 1 1 . . . . . . . . . . . . . . . . . .
 . 1 1 . . . 1 1 . . . . . . . . . . . . . . . . .
 . . 1 . . . . 1 . . . . . . . . . . . . . . . . .
 . . . 1 1 . . . 1 1 . . . . . . . . . . . . . . .
 . . . . 1 . . . . 1 . . . . . . . . . . . . . . .
 . . . . . 1 1 . . . 1 1 . . . . . . . . . . . . .
 . . . . . . 1 1 . . . 1 1 . . . . . . . . . . . .
 . . . . . . . 1 . . . . 1 . . . . . . . . . . . .
 . . . . . . . . 1 1 . . . 1 1 . . . . . . . . . .
 . . . . . . . . . 1 . . . . 1 . . . . . . . . . .
 . . . . . . . . . . 1 1 . . . . . . . . . . . . .
```

```
 . . . . . . . . . . . 1 1 . . . . . . . . . . . .
 . . . . . . . . . . . . 1 . . . . . . . . . . . .
 . . . . . . . . . . . . . 1 1 . . . . . . . . . .
 . . . . . . . . . . . . . . 1 . . . . . . . . . .
 . . . . . . . . . . . . . . . 1 1 . . . 1 1 . . .
 . . . . . . . . . . . . . . . . 1 1 . . . 1 1 . .
 . . . . . . . . . . . . . . . . . 1 . . . . 1 . .
 . . . . . . . . . . . . . . . . . . 1 1 . . . 1 1
 . . . . . . . . . . . . . . . . . . . 1 . . . . 1
 . . . . . . . . . . . . . . . . . . . . 1 1 . . .
 . . . . . . . . . . . . . . . . . . . . . 1 1 . .
 . . . . . . . . . . . . . . . . . . . . . . 1 . .
 . . . . . . . . . . . . . . . . . . . . . . . 1 1
 . . . . . . . . . . . . . . . . . . . . . . . . 1

A morphism in Category of matrices over GF(3)


Image of (1)-[{ Z(3)^0*(b) }]->(2):
 . . . . . 1 . . . . . . . . . .
 . . . . . . 1 . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . 1 . 1 . . . . . . . .
 . . . . . . 1 . . . . . . . . .
 . . . . . . . . . 1 . . . . . .
 . . . . . . . . . . 1 . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . 1 . 1 . . . .
 . . . . . . . . . . 1 . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . 1 . . . . . . . 1 . .
 . . . . . . 1 . . . . . . . 1 .
 . . . . . . . . . . . . . . . .
 . . . . . 1 . 1 . . . . . 1 . 1
 . . . . . . 1 . . . . . . . 1 .
 . . . . . . . . . 1 . . . . . .
 . . . . . . . . . . 1 . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . 1 . 1 . . . .
 . . . . . . . . . . 1 . . . . .

A morphism in Category of matrices over GF(3)


Image of (2)-[{ Z(3)^0*(c) }]->(2):
 1 1 . . 1 1 . . . . . . . . . .
 . 1 1 . . 1 1 . . . . . . . . .
 . . 1 . . . 1 . . . . . . . . .
```

```
 . . . 1 . . . 1 . . . . . . . .
 . . . . 1 1 . . 1 1 . . . . . .
 . . . . . 1 1 . . 1 1 . . . . .
 . . . . . . 1 . . . 1 . . . . .
 . . . . . . . 1 . . . 1 . . . .
 . . . . . . . . 1 1 . . . . . .
 . . . . . . . . . 1 1 . . . . .
 . . . . . . . . . . 1 . . . . .
 . . . . . . . . . . . 1 . . . .
 . . . . . . . . . . . . 1 1 . .
 . . . . . . . . . . . . . 1 1 .
 . . . . . . . . . . . . . . 1 .
 . . . . . . . . . . . . . . . 1

A morphism in Category of matrices over GF(3)
gap> etas := WeakDirectSumDecomposition( fortyone );;
gap> dec := List( etas, eta -> List( SetOfObjects( kq ),
>               o -> Dimension( Source( UnderlyingCapTwoCategoryCell( eta )( o ) ) ) ) );
[ [ 3, 0 ], [ 3, 1 ], [ 3, 3 ], [ 3, 3 ], [ 0, 3 ],
  [ 3, 0 ], [ 3, 0 ], [ 3, 0 ], [ 1, 3 ], [ 3, 3 ] ]
gap> iso := UniversalMorphismFromDirectSum( etas );
<(1)->25x25, (2)->16x16>
gap> IsIsomorphism( iso );
true
gap> iso;
<(1)->25x25, (2)->16x16>
gap> Display( Source( iso ) );
An object in The category of functors: Algebroid generated by the
right quiver q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices
over GF(3) defined by the following data:


Image of <(1)>:
A vector space object over GF(3) of dimension 25

Image of <(2)>:
A vector space object over GF(3) of dimension 16

Image of (1)-[{ Z(3)^0*(a) }]->(1):
 1 1 . . . . . . . . . . . . . . . . . . . . . . .
 . 1 . . . . . . . . . . . . . . . . . . . . . . .
 1 1 1 . . . . . . . . . . . . . . . . . . . . . .
 . . . 1 1 2 . . . . . . . . . . . . . . . . . . .
 . . . . 1 1 . . . . . . . . . . . . . . . . . . .
 . . . . . 1 . . . . . . . . . . . . . . . . . . .
 . . . . . . 1 1 . . . . . . . . . . . . . . . . .
 . . . . . . . 1 1 . . . . . . . . . . . . . . . .
 . . . . . . . . 1 . . . . . . . . . . . . . . . .
 . . . . . . . . . 1 . . . . . . . . . . . . . . .
 . . . . . . . . . 1 1 2 . . . . . . . . . . . . .
 . . . . . . . . . 2 . 1 . . . . . . . . . . . . .
 . . . . . . . . . . . . 2 1 . . . . . . . . . . .
```

```
 . . . . . . . . . . . . 2 . 2 . . . . . . . . . .
 . . . . . . . . . . . . . . 1 . . . . . . . . . .
 . . . . . . . . . . . . . . . 2 1 . . . . . . . .
 . . . . . . . . . . . . . . . 2 . . . . . . . . .
 . . . . . . . . . . . . . . . 1 2 1 . . . . . . .
 . . . . . . . . . . . . . . . . . . 1 . . . . . .
 . . . . . . . . . . . . . . . . . . . 1 1 . . . .
 . . . . . . . . . . . . . . . . . . 1 . 1 . . . .
 . . . . . . . . . . . . . . . . . . . . . 1 . . .
 . . . . . . . . . . . . . . . . . . . . . . 1 1 2
 . . . . . . . . . . . . . . . . . . . . . . . 1 1
 . . . . . . . . . . . . . . . . . . . . . . . . 1

A morphism in Category of matrices over GF(3)


Image of (1)-[{ Z(3)^0*(b) }]->(2):
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 1 . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . 2 . . . . . . . . . . . . .
 . . . 2 . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . 2 . . . . . . . . .
 . . . . 1 . 1 . . . . . . . . .
 . . . . . 2 . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . . . . . .
 . . . . . . . . . . . 1 . . . .
 . . . . . . . . . . . . . 1 . .
 . . . . . . . . . . . . . . . 1
 . . . . . . . . . . . . . . . .

A morphism in Category of matrices over GF(3)


Image of (2)-[{ Z(3)^0*(c) }]->(2):
 1 . . . . . . . . . . . . . . .
 . 1 1 . . . . . . . . . . . . .
 . . 1 1 . . . . . . . . . . . .
 . . . 1 . . . . . . . . . . . .
 . . . . 1 1 2 . . . . . . . . .
```

```
 . . . . . 1 2 . . . . . . . . .
 . . . . . . 1 . . . . . . . . .
 . . . . . . . 1 . . . . . . . .
 . . . . . . . . 1 1 . . . . . .
 . . . . . . . 1 . 1 . . . . . .
 . . . . . . . . . . 1 1 . . . .
 . . . . . . . . . . . 1 . . . .
 . . . . . . . . . . 2 . 1 . . .
 . . . . . . . . . . . . . 1 . 1
 . . . . . . . . . . . . . 1 1 .
 . . . . . . . . . . . . . . . 1

A morphism in Category of matrices over GF(3)
gap> Display( iso );
A morphism in The category of functors: Algebroid generated by the
right quiver q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices
over GF(3) defined by the following data:


Image of <(1)>:
 . . 1 . . . 2 1 . 1 1 . . 2 . . . 1 . . . . . . .
 . . . . . . . . . . . . . . . . . . . . . . 1 . .
 . 1 2 . 2 1 1 2 2 2 1 . . 2 . . 1 . . 2 2 . . 1 .
 1 . 2 2 . 2 1 2 1 2 . . . . . 2 2 1 1 1 1 . . 2 .
 . 1 1 . 2 1 2 1 2 1 . . . . . . 2 1 . 1 2 . . 1 .
 . . 1 . . . 2 1 . 1 1 . . 2 . . . 2 . . . 1 . . 2
 1 1 1 . 2 . 2 1 2 1 1 . . . . 2 1 2 . 2 . . . 1 .
 . 1 1 . . 1 2 1 . 1 . . . 2 . . 2 1 . . 2 . . . .
 . . 1 . . . 2 1 . . 1 . . . . . . 2 . . . 1 . . .
 . . . . . . . . . . . . . . . . . 1 . . . 2 . . .
 . 1 2 . 2 1 1 2 2 2 1 . . 2 . 1 . 2 . 2 1 1 . 1 .
 . . 2 . . . 1 2 . 2 2 . . 1 . . 2 1 . . 2 . . . .
 . . . . . . . . 1 1 2 2 2 1 . . 2 2 . 1 . 2 . 2 .
 . . . . . . . . 2 . 1 . . . . . 1 . . 2 . . . 1 .
 . . . . . . . . . . . . 1 . 1 . . . . . . . 2 . .
 . . 2 . . . 2 1 . 2 . 1 . 1 . . . 2 . . . 2 1 . .
 . . 1 . . . 1 . . 1 . 1 1 2 . . . 1 . . . 1 . . .
 . 1 1 . 2 . 2 . 2 1 1 . . . . . 1 . . 2 . . . 1 .
 . . . . . . . 2 . . . 1 1 . . . . . . . . . 2 . .
 . 1 2 . 2 . 1 2 2 2 1 . . 2 . . 1 . . 2 . . . 1 .
 . . 1 . . . 1 1 . 1 . 2 . 2 . . . 1 . . . 1 1 . .
 . . . . . . . . . . . . . . . . . 1 . 2 . 2 . 1 .
 2 2 1 1 1 . 2 1 . . 1 . . . . . . 2 . . . 1 . . .
 . 2 1 . 1 2 2 1 1 1 2 . . 1 . . . . . . . . . . .
 . . 2 . . . 1 2 . 2 2 . . 1 . . . . . . . . . . .

An isomorphism in Category of matrices over GF(3)


Image of <(2)>:
 . . . . . . . . . . . . . . . 1
 . . . . . . . . . . . . 1 . . .
```

```
 . . . . . . . . . . . . . 1 . .
 . . . . . . . . . . . . . . 1 .
 . . . . . 1 2 . . 2 . . . 1 2 .
 . . . . . . 1 . . 1 . . . . 1 .
 . . . . . . . . . . 1 . . . . .
 . . 2 . . 1 . . 2 2 1 . . . . .
 2 1 . . . . . . . . . . . 1 . .
 . 2 1 . 2 . 1 . . . . . . . 1 .
 . . 1 . . 1 1 1 . . . . . 2 1 .
 . . . . . . 2 . . 1 2 1 . . 2 .
 . 2 . 2 . . . . . . . . 1 2 . .
 . . . . . . . 1 . . . . . . . .
 . . . 1 . . . . . . . . . . . .
 . . . . . . . . . . . 1 . . . .

An isomorphism in Category of matrices over GF(3)
gap> eta := etas[3];
<(1)->3x25, (2)->3x16>
gap> TensorProductOnMorphisms( eta, eta );
<(1)->9x625, (2)->9x256>
gap> six := Source( eta );
<(1)->3, (2)->3; (a)->3x3, (b)->3x3, (c)->3x3>
gap> Display( six );
An object in The category of functors: Algebroid generated by the
right quiver q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices
over GF(3) defined by the following data:


Image of <(1)>:
A vector space object over GF(3) of dimension 3

Image of <(2)>:
A vector space object over GF(3) of dimension 3

Image of (1)-[{ Z(3)^0*(a) }]->(1):
 1 1 .
 . 1 1
 . . 1

A morphism in Category of matrices over GF(3)


Image of (1)-[{ Z(3)^0*(b) }]->(2):
 . 2 .
 . . 2
 . . .

A morphism in Category of matrices over GF(3)


Image of (2)-[{ Z(3)^0*(c) }]->(2):
 1 1 .
```

```
 . 1 1
 . . 1

A morphism in Category of matrices over GF(3)
gap> emb := EmbeddingOfSumOfImagesOfAllMorphisms( const, six );
<(1)->1x3, (2)->0x3>
gap> Display( emb );
A morphism in The category of functors: Algebroid generated by the
right quiver q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices
over GF(3) defined by the following data:


Image of <(1)>:
 . . 1

A morphism in Category of matrices over GF(3)


Image of <(2)>:
(an empty 0 x 3 matrix)

A zero, split monomorphism in Category of matrices over GF(3)
gap> s1 := Source( emb );
<(1)->1, (2)->0; (a)->1x1, (b)->1x0, (c)->0x0>
gap> Display( s1 );
An object in The category of functors: Algebroid generated by the
right quiver q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices
over GF(3) defined by the following data:


Image of <(1)>:
A vector space object over GF(3) of dimension 1

Image of <(2)>:
A vector space object over GF(3) of dimension 0

Image of (1)-[{ Z(3)^0*(a) }]->(1):
 1

A morphism in Category of matrices over GF(3)


Image of (1)-[{ Z(3)^0*(b) }]->(2):
(an empty 1 x 0 matrix)

A zero, split epimorphism in Category of matrices over GF(3)


Image of (2)-[{ Z(3)^0*(c) }]->(2):
(an empty 0 x 0 matrix)

A zero, isomorphism in Category of matrices over GF(3)
```

```
gap> proj1 := YonedaProjective( CatReps, kq.1 );
<(1)->3, (2)->3; (a)->3x3, (b)->3x3, (c)->3x3>
gap> Display( proj1 );
An object in The category of functors: Algebroid generated by the
right quiver q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices
over GF(3) defined by the following data:


Image of <(1)>:
A vector space object over GF(3) of dimension 3

Image of <(2)>:
A vector space object over GF(3) of dimension 3

Image of (1)-[{ Z(3)^0*(a) }]->(1):
 . 1 .
 . . 1
 1 . .

A morphism in Category of matrices over GF(3)


Image of (1)-[{ Z(3)^0*(b) }]->(2):
 1 . .
 . 1 .
 . . 1

A morphism in Category of matrices over GF(3)


Image of (2)-[{ Z(3)^0*(c) }]->(2):
 . 1 .
 . . 1
 1 . .

A morphism in Category of matrices over GF(3)
gap> e1 := EmbeddingOfSumOfImagesOfAllMorphisms( const, proj1 );
<(1)->1x3, (2)->1x3>
gap> Source( e1 );
<(1)->1, (2)->1; (a)->1x1, (b)->1x1, (c)->1x1>
gap> IsEpimorphism( EmbeddingOfSumOfImagesOfAllMorphisms( proj1, six ) );
false
gap> five := CokernelObject( emb );
<(1)->2, (2)->3; (a)->2x2, (b)->2x3, (c)->3x3>
gap> Display( five );
An object in The category of functors: Algebroid generated by the
right quiver q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices
over GF(3) defined by the following data:


Image of <(1)>:
A vector space object over GF(3) of dimension 2
```

```
Image of <(2)>:
A vector space object over GF(3) of dimension 3

Image of (1)-[{ Z(3)^0*(a) }]->(1):
 1 1
 . 1

A morphism in Category of matrices over GF(3)


Image of (1)-[{ Z(3)^0*(b) }]->(2):
 . 2 .
 . . 2

A morphism in Category of matrices over GF(3)


Image of (2)-[{ Z(3)^0*(c) }]->(2):
 1 1 .
 . 1 1
 . . 1

A morphism in Category of matrices over GF(3)
gap> SumOfImagesOfAllMorphisms( s1, six );
<(1)->1, (2)->0; (a)->1x1, (b)->1x0, (c)->0x0>
gap> SumOfImagesOfAllMorphisms( s1, five );
<(1)->0, (2)->0; (a)->0x0, (b)->0x0, (c)->0x0>
gap> SumOfImagesOfAllMorphisms( const, five );
<(1)->1, (2)->1; (a)->1x1, (b)->1x1, (c)->1x1>
gap> SumOfImagesOfAllMorphisms( five, const );
<(1)->0, (2)->1; (a)->0x0, (b)->0x1, (c)->1x1>
gap> SumOfImagesOfAllMorphisms( six, const );
<(1)->0, (2)->1; (a)->0x0, (b)->0x1, (c)->1x1>
gap> proj2 := YonedaProjective( CatReps, kq.2 );
<(1)->0, (2)->3; (a)->0x0, (b)->0x3, (c)->3x3>
gap> Display( proj2 );
An object in The category of functors: Algebroid generated by the
right quiver q(2)[a:1->1,b:1->2,c:2->2] -> Category of matrices
over GF(3) defined by the following data:


Image of <(1)>:
A vector space object over GF(3) of dimension 0

Image of <(2)>:
A vector space object over GF(3) of dimension 3

Image of (1)-[{ Z(3)^0*(a) }]->(1):
(an empty 0 x 0 matrix)

A zero, isomorphism in Category of matrices over GF(3)
```

```
Image of (1)-[{ Z(3)^0*(b) }]->(2):
(an empty 0 x 3 matrix)

A zero, split monomorphism in Category of matrices over GF(3)


Image of (2)-[{ Z(3)^0*(c) }]->(2):
 . 1 .
 . . 1
 1 . .

A morphism in Category of matrices over GF(3)
```

1.2 GAP categories

1.2.1 IsFiniteConcreteCategory (for IsCapCategory)

▷ `IsFiniteConcreteCategory(`*object*`)` (filter)

Returns: `true` or `false`

The GAP category of a finite concrete category

1.2.2 IsCellInAFiniteConcreteCategory (for IsCapCategoryCell)

▷ `IsCellInAFiniteConcreteCategory(`*object*`)` (filter)

Returns: `true` or `false`

The GAP category of cell in a finite concrete category

1.2.3 IsObjectInAFiniteConcreteCategory (for IsCellInAFiniteConcreteCategory and IsCapCategoryObject)

▷ `IsObjectInAFiniteConcreteCategory(`*object*`)` (filter)

Returns: `true` or `false`

The GAP category of objects in a finite concrete category

1.2.4 IsMorphismInAFiniteConcreteCategory (for IsCellInAFiniteConcreteCategory and IsCapCategoryMorphism)

▷ `IsMorphismInAFiniteConcreteCategory(`*object*`)` (filter)

Returns: `true` or `false`

The GAP category of morphisms in a finite concrete category

1.3 Attributes

1.3.1 SetOfObjects (for IsCapSubcategory)

▷ `SetOfObjects(C)` (attribute)

Returns: a list

The set of objects of the concrete category *C*.

1.3.2 SetOfGeneratingMorphisms (for IsCapSubcategory)

▷ `SetOfGeneratingMorphisms(C)` (attribute)

Returns: a list

The set of generating morphisms of the concrete category *C*.

1.3.3 SetOfMorphisms (for IsCapSubcategory)

▷ `SetOfMorphisms(C)` (attribute)

Returns: a list

The set of morphisms of the concrete category *C*.

1.4 Constructors

1.4.1 ConcreteCategoryForCAP (for IsList)

▷ `ConcreteCategoryForCAP(L)` (operation)

Returns: a CAP object

Construct finite concrete category out of the list *L* of morphisms given by images.

Example

```
gap> c3c3 := ConcreteCategoryForCAP( [ [2,3,1], [4,5,6], [,,,5,6,4] ] );
A finite concrete category
gap> objects := SetOfObjects( c3c3 );
[ An object in subcategory given by: <An object in FinSets>,
  An object in subcategory given by: <An object in FinSets> ]
gap> Perform( objects, Display );
An object in subcategory given by: [ 1, 2, 3 ]
An object in subcategory given by: [ 4, 5, 6 ]
gap> gmorphisms := SetOfGeneratingMorphisms( c3c3 );
[ A morphism in subcategory given by: <A morphism in FinSets>,
  A morphism in subcategory given by: <A morphism in FinSets>,
  A morphism in subcategory given by: <A morphism in FinSets> ]
gap> Perform( gmorphisms, Display );
A morphism in subcategory given by:
[ [ 1, 2, 3 ], [ [ 1, 2 ], [ 2, 3 ], [ 3, 1 ] ], [ 1, 2, 3 ] ]
A morphism in subcategory given by:
[ [ 1, 2, 3 ], [ [ 1, 4 ], [ 2, 5 ], [ 3, 6 ] ], [ 4, 5, 6 ] ]
A morphism in subcategory given by:
[ [ 4, 5, 6 ], [ [ 4, 5 ], [ 5, 6 ], [ 6, 4 ] ], [ 4, 5, 6 ] ]
```

1.4.2 Algebroid (for IsHomalgRing, IsCapCategory)

▷ `Algebroid(k, C)` (operation)

Returns: a k-linear category

Return the k-linear closure of the category C over the commutative ring k.

1.4.3 EmbeddingOfSubRepresentation (for IsList, IsCapCategoryObjectInHomCategory)

▷ `EmbeddingOfSubRepresentation(eta, F)` (operation)

Returns: an morphism in a Hom-category

Concstruct the embedding of a subrepresentation S of F by a list `eta` of morphisms, where the image embeddings thereof are the components of the natural monomorphism from S into F.

1.5 Operations

1.5.1 WeakDirectSumDecomposition (for IsCapCategoryObjectInHomCategory)

▷ `WeakDirectSumDecomposition(F)` (operation)

Returns: a list

Return a list of monomorphisms describing the embeddings of a list of direct summands of the representation F, the direct sum thereof is isomorphic to F.

1.6 Tools

1.6.1 ConvertToMapOfFinSets

▷ `ConvertToMapOfFinSets(objects_list, generator)` (function)

Returns: a morphism of finite sets

Construct the map of finite sets corresponding to the list of images in the convention of catreps.

Example

```
gap> c3c3 := ConcreteCategory([[2,3,1],[4,5,6],[,,,5,6,4]]);
rec( codomain := [ 1, 2, 2 ], domain := [ 1, 1, 2 ],
     generators := [ [ 2, 3, 1 ], [ 4, 5, 6 ], [ ,,, 5, 6, 4 ] ],
     objects := [ [ 1, 2, 3 ], [ 4, 5, 6 ] ], operations := rec(  ) )
gap> g1 := ConvertToMapOfFinSets( c3c3.objects, c3c3.generators[1] );
<A morphism in FinSets>
gap> Display( g1 );
[ [ 1, 2, 3 ], [ [ 1, 2 ], [ 2, 3 ], [ 3, 1 ] ], [ 1, 2, 3 ] ]
gap> g2 := ConvertToMapOfFinSets( c3c3.objects, c3c3.generators[2] );
<A morphism in FinSets>
gap> Display( g2 );
[ [ 1, 2, 3 ], [ [ 1, 4 ], [ 2, 5 ], [ 3, 6 ] ], [ 4, 5, 6 ] ]
gap> g3 := ConvertToMapOfFinSets( c3c3.objects, c3c3.generators[3] );
<A morphism in FinSets>
gap> Display( g3 );
[ [ 4, 5, 6 ], [ [ 4, 5 ], [ 5, 6 ], [ 6, 4 ] ], [ 4, 5, 6 ] ]
gap> g := ConvertToMapOfFinSets( c3c3.objects, [,,,1,1,1] );
<A morphism in FinSets>
```

```
gap> Display( g );
[ [ 4, 5, 6 ], [ [ 4, 1 ], [ 5, 1 ], [ 6, 1 ] ], [ 1, 2, 3 ] ]
```

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