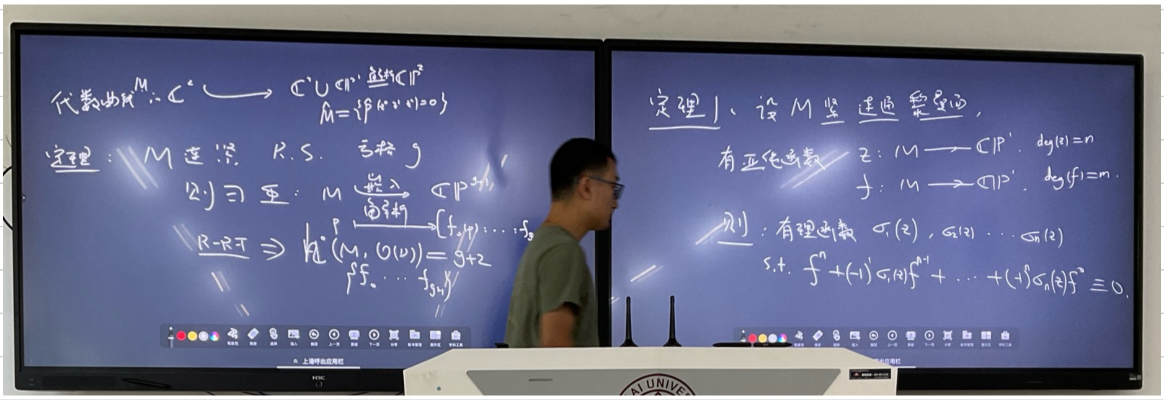
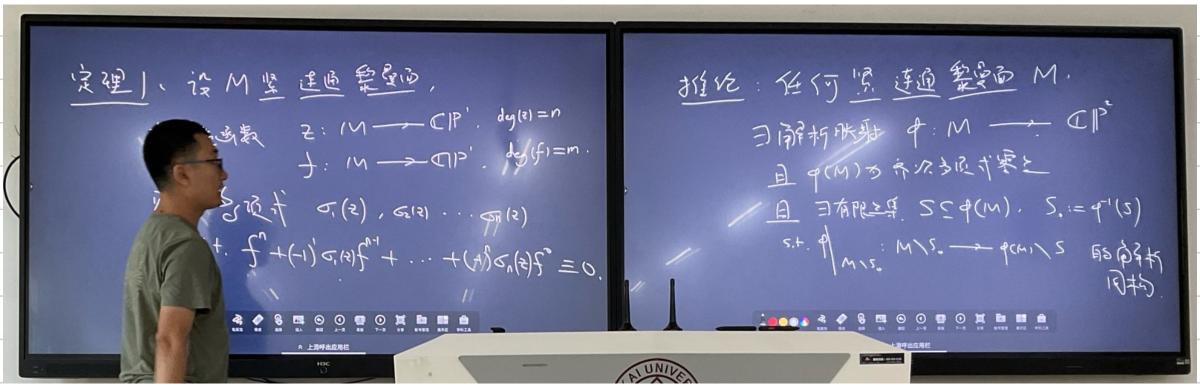
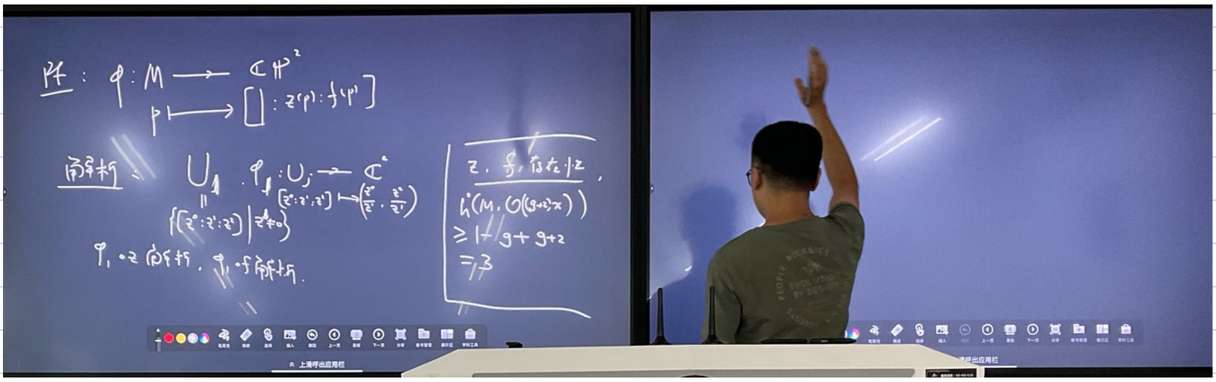
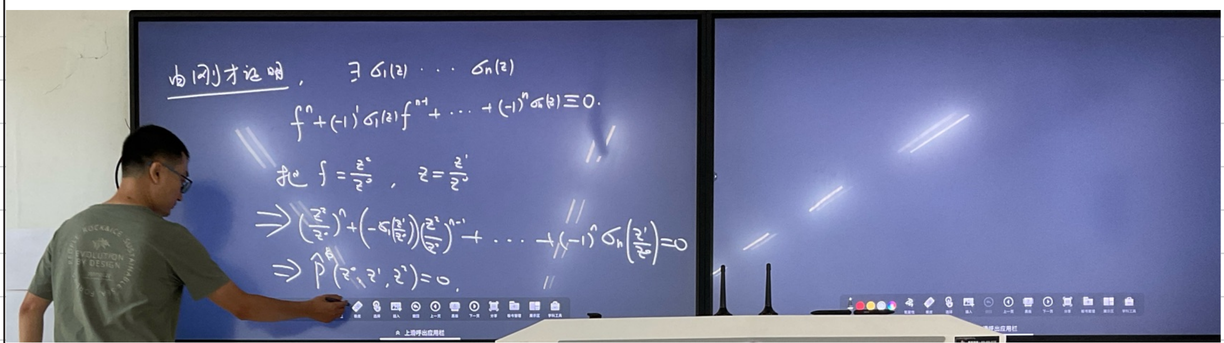
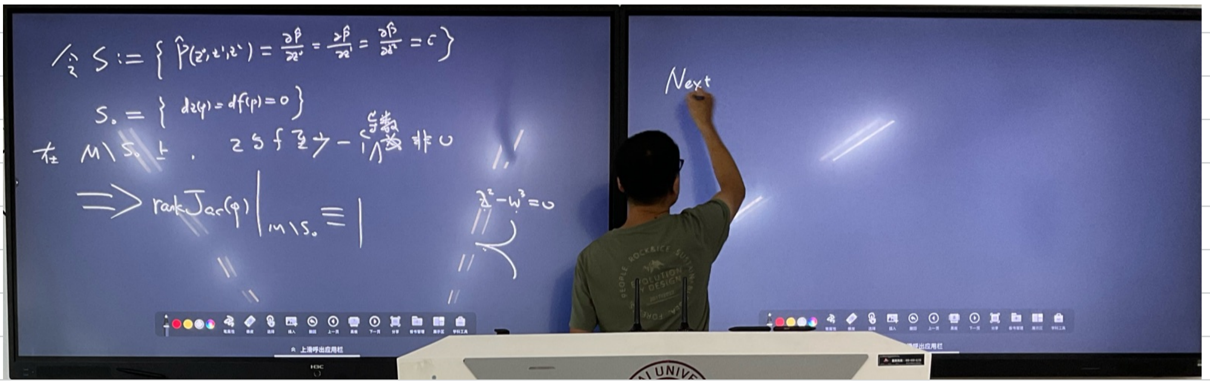
黎曼面解析映为射影代数曲线



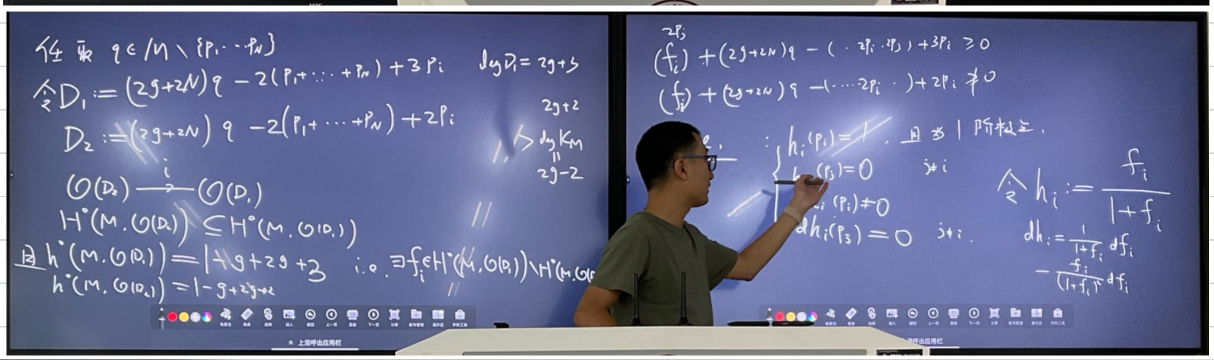


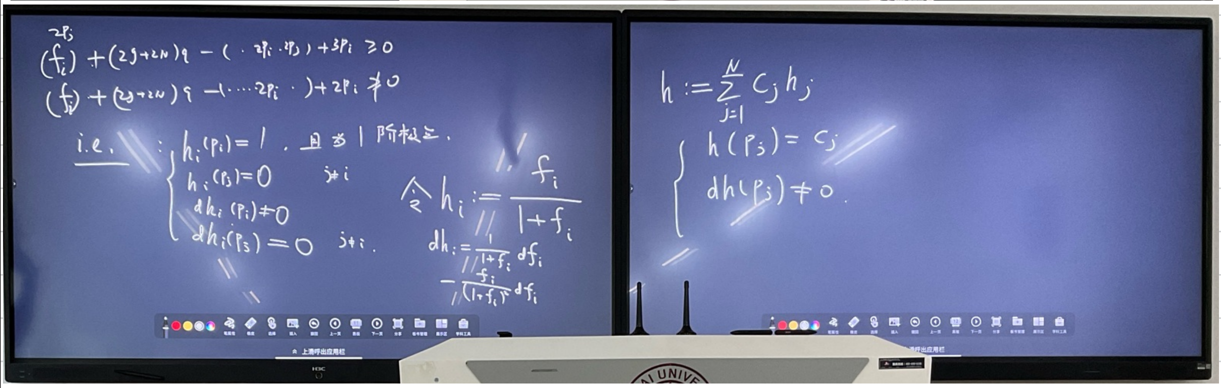


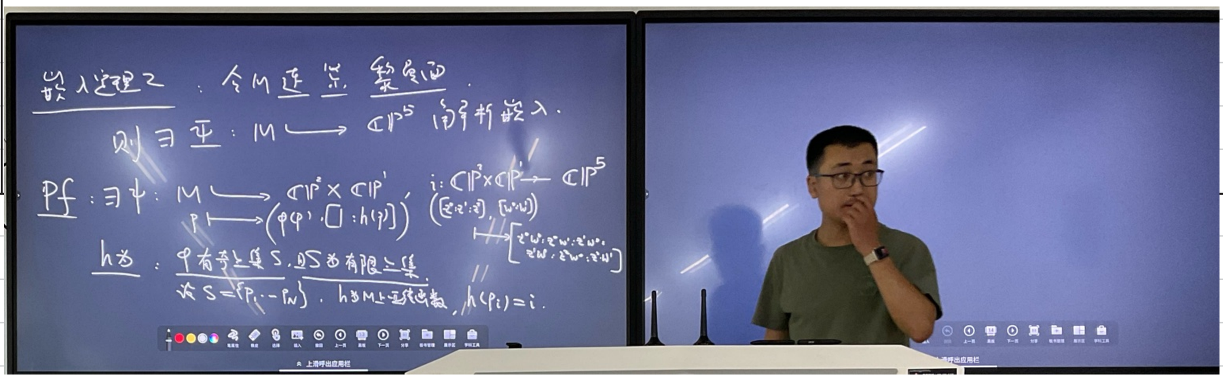


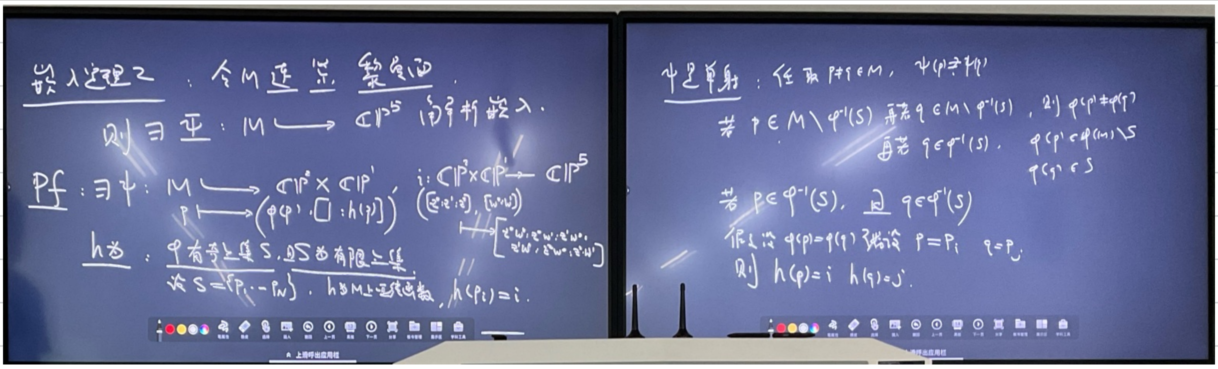

令 S₀ = { dz(φ) = df(φ) = 0 }
在 M\S₀ 上
⟹ rankJac(φ)|M\S₀ ≡ 1
Next


Next: ∃ Φ: M → CP^5 的解析嵌入.
准备工作:
Riemann Existence Theorem
设{p1…pN}是连紧黎曼面M上N个互异点
则∀c1…cN ∈ C*
必∃亚纯函数 h: M → CP^1
满足 (1) h(pi) = ci, i=1…N.
(2) dh(pi) ≠ 0 i=1…N
(3) h ∈ O({pi})

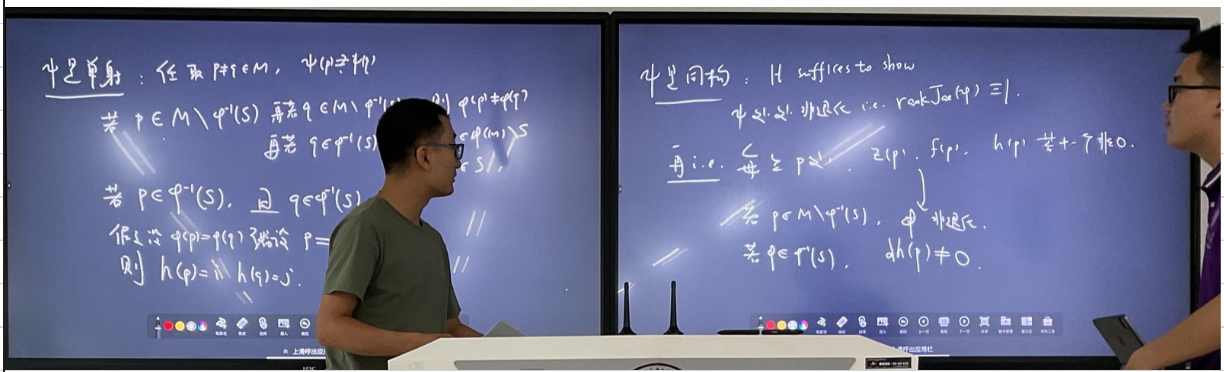

任取 q ∈ M \ {p1…pN}
令 D1 := (2g+2N)q − 2(p1+…+pN) + 3pi
D2 := (2g+2N)q − 2(p1+…+pN) + 2pi
O(D2) → O(D1)
H^0(M,O(D2)) ⊆ H^0(M,O(D1))
令 hi := fi/(1+fi)


i.e. hi(pi) = 1, 且为1阶极点.
hi(pj) = 0 j≠i
dhi(pi) ≠ 0
dhi(pj) = 0 j≠i.
令 hi := fi/(1+fi)
h := Σ_{j=1}^N cj hj
h(pj) = cj
dh(pj) ≠ 0.


嵌入定理2: 令M连紧黎曼面.
则∃Φ: M ↪ CP^5 的解析嵌入.
Pf: ∃φ: M ↪ CP^2 × CP^1
i: CP^2×CP^1 → CP^5
h为: φ有奇点集S, 且S为有限点集
设 S={p1…pN}. h为M上亚纯函数, h(pi)=i.



嵌入定理2：令M连紧黎曼面.
则∃Φ：M ↪ CP^5 的解析嵌入.
Pf：∃ψ：M ↪ CP^2 × CP^1
p ↦ (φ(p), [1 : h(p)])
i: CP^2×CP^1 → CP^5
h为：φ有奇点集S，且S为有限点集
设S={p_1, …, p_N}，h为M上亚纯函数，h(p_i)=i.
ψ是单射：任取p≠q∈M，ψ(p)≠ψ(q)
若p∈M\φ^{-1}(S) 再若q∈M\φ^{-1}(S)，则φ(p)≠φ(q)
再若q∈φ^{-1}(S)，φ(p)∈φ(M)\S
φ(q)∈S
若p∈φ^{-1}(S)，且q∈φ^{-1}(S)
假设φ(p)=φ(q) 不妨设p=p_i q=p_j
则h(p)=i h(q)=j.



ψ是单射：任取p≠q∈M，ψ(p)≠ψ(q)
若p∈φ^{-1}(S)，且q∈φ^{-1}(S)
则h(p)=i h(q)=j.
ψ是同构：It suffices to show
rank Jac(ψ) ≡ 1.
h(p) 若有一个非0.
若p∈M\φ^{-1}(S)，φ 非退化.
若p∈φ^{-1}(S)，dh(p)≠0.