18:

The *Pareto distribution* with parameter a>0 has PDF f(x)=a/x^{a+1} for x>=1 (and 0 otherwise).

(a) Find the CDF of a Pareto r.v. with parameter a; check that it is a valid CDF

CDF (*F(x)*) is the integral of PDF:

$F(x) = \int_1^x \frac{a}{t^{a+1}}\, dx$ for $x \geq 1$

$\Rightarrow F(x) = 1 - \frac{1}{x^a}$ for $x \geq 1$

When $x < 1$ the probability is zero $F(x) = 0$.

The final solution of CDF: $F(x) = \begin{cases} 0 & x < 0 \\ 1 - \frac{1}{x^a} \text{ for } & x \geq 1 \end{cases}$

To check if $F(x)$ is valid:

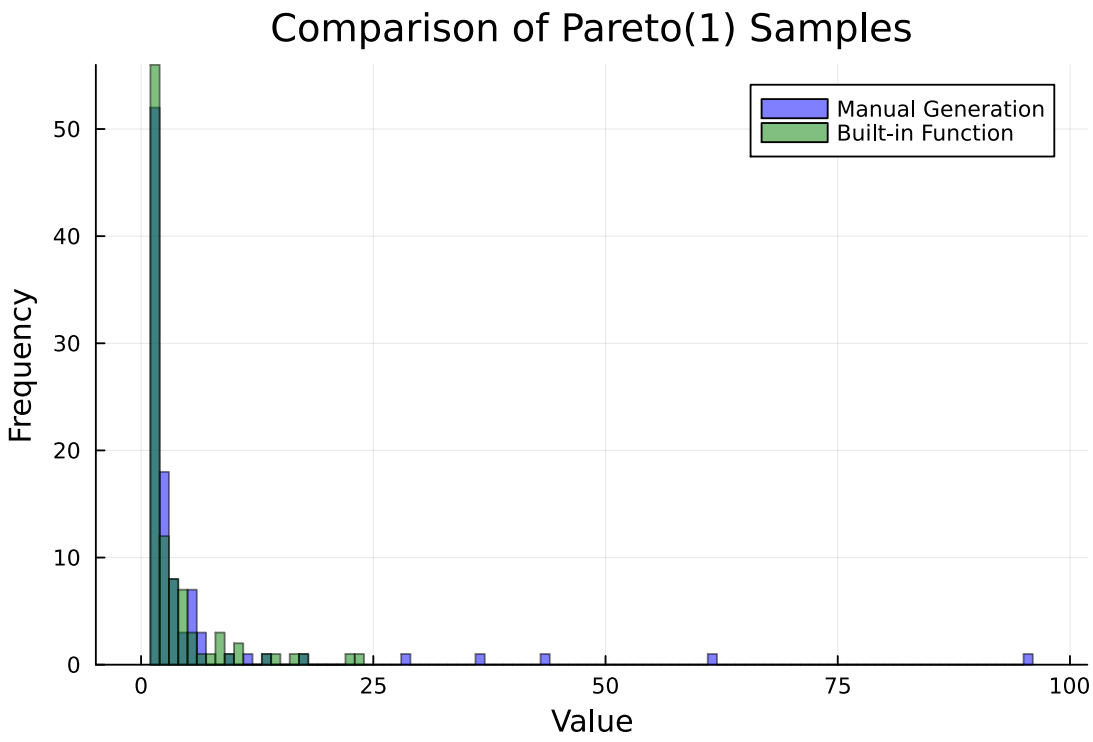
1. $F(x)$ is monotonic increasing because as monotonic increasing because as x increase, $\frac{1}{x^a}$ decrease, thus $F(x)$ increase.
2. When $x \to \infty$, $F(x)$ approach 1; when $x = 1$, $F(x) = 0$.
3. The values of $F(x)$ range from 0 to 1, satisfying the requirement of CDF.

(b) Suppose that for a simulation you want to run, you need to generate i.i.d. Pareto(a) r.v.s. You have a computer that knows how to generate i.i.d. Unif(0,1) r.v.s but does not know how to generate Pareto r.v.s. Show how to do this

Assuming Pareto r.v.s. is X and Unif(0,1) r.v.s is U. According to *inverse transform sampling*:

$$F(X) = U = 1 - \frac{1}{X^a}$$

The Pareto r.v.s. $X = \left(\frac{1}{1-U}\right)^{\frac{1}{a}}$.


Comparison of Pareto(1) Samples
Manual Generation
Built-in Function
Frequency
Value
0
10
20
30
40
50
0
25
50
75
100