The RC Circuit: The Charging and Discharging of a Capacitor

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1 Introduction and Theory

1.1 The Capacitor and Capacitance

The capacitor is described as a device which usually has two conducting plates or foils that are positioned some distance (d) apart, When a potential difference is applied across the plates, Charge accumulates on the plates and a separation of charge centers is induced in the dielectric (A substance or layer between the plates). The applied potential difference is related to the capacitance of the capacitor by the equation:

$$C = QV \tag{1}$$

The charge that accumulates on each plate (Q and $-Q$) is denoted by Q and the capacitance is denoted by C, the voltage is denoted by V.The capacitance can be further denoted in terms of the characteristics of the capacitor: the plate area, the distance between the plates and the permittivity of a vaccum or whatever dielectric that is placed between the plates, the following equation denotes the capacitance in terms of the characteristics mentioned:

$$C = \epsilon A/d \tag{2}$$

Where C is the capacitance, ϵ is the permittivity, A is the area of each plates and d is the distance between the plates. The following is a depiction of a basic capacitor:.

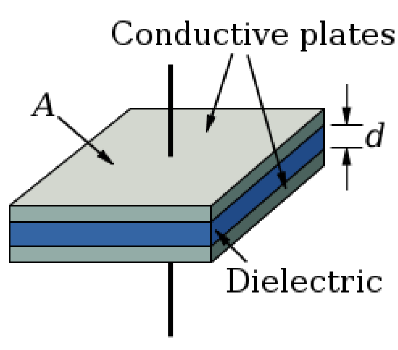

Figure 1: A diagram of a simple parallel plate capacitor, showing the plates, the plate area, A, the dielectric and the plate separation, d.

The values of the area (A) and distance (d) will affect the the charge on the plates in the following way: As area is increased there will be more charge on each plate due to the larger area and hence a greater capacitance. ($Q \propto F_Q$) where (F_Q) denotes the electrostatic force between the plates, the capacitance is proportional to F_Q , the distance between the plates is inversely proportional to the electrostatic force between the plates ($F_Q \propto \frac{1}{d^2}$) and hence an increase in distance will result in a decrease in capacitance. Since $C = QV$, $Q \propto A$ and $Q \propto \frac{1}{d}$.

1.2 Charging

In the charging phase of the circuit, when the switch is closed in a circuit connected in series with a resistor, a capacitor and a power supply with DC voltage, the current will flow towards the capacitor and charge will collect on the capacitor's plates:

$$q(t) = q_0(1 - e^{\frac{-t}{RC}}) \tag{3}$$

We have a situation where the charge on each capacitor plate increases with time at a rate determined by the value of:

$$\tau = RC \tag{4}$$

The dimensions of τ can be found in the following way:

$$\begin{aligned}
\tau &= RC \\
Dimension(\tau) &= Dimension(R) \times Dimension(C) \\
&= \Omega \times F \\
R = V/I \quad & \& \quad C = q/V \\
\tau &= \frac{q}{I} \\
Dimension(\tau) &= \frac{Dimension(q)}{Dimension(I)} \\
&= \frac{c}{A} \\
I = dq/dt & \\
Dimension(\tau) &= \frac{Dimension(q)}{Dimension(q)/Dimension(t)} \\
Dimension(\tau) &= Dimension(t) \\
Dimension(\tau) &= s
\end{aligned}$$

From equation 3, will can use the relation $q = CV$ to get the voltage across the capacitor as a function of time.

$$V_C(t) = V_\epsilon[1 - e^{\frac{-t}{RC}}] \tag{5}$$

1.3 Discharging

When the capacitor is discharged, the charge on the capacitor will decrease as a function of time according to the equation:

$$q(t) = -RC\frac{dq(t)}{dt} \tag{6}$$

Using the relation: $q = CV$, we get:

$$V_c(t) = V_\epsilon e^{\frac{-t}{RC}} \tag{7}$$

1.4 AC exitation of an RC circuit

When an alternating current:

$$V_\epsilon = V_0 cos(\omega t) \tag{8}$$

is applied to the RC circuit, the charge on the capacitor as a function of time is the following:

$$q(t) = q_0 cos(\omega t + \phi) \tag{9}$$

We find that the current can be shown as the following:

$$I(t) cos(\omega t + \theta) \tag{10}$$

This gives us the following phase shift of θ between the current $(I(t))$and the voltage $(V_\epsilon(t))$:

$$\theta = \phi + \frac{\pi}{2} \tag{11}$$

1.5 Impedence

The resistance that a circuit demonstrates against an alternating voltage with angular frequency ω is called the impedence and it is denoted by $Z(\omega)$, It can be written as:

$$Z(\omega) = R + iX(\omega) \tag{12}$$

The reactance of the circuit is denoted by $iX(\omega)$ in equation 12. The reactance of the circuit is written in terms of ω in the following formula:

$$X(\omega) = \frac{1}{\omega C} \tag{13}$$

.As the frequency increases, according to the formula $\omega = 2\pi f$, the ω value will increase and so the reactance $(X(\omega))$ will decrease, if the frequency is decreased, the reactance will increase.

1.6 Aim

The aim of this experiment is to study how a capacitor charges and discharges; we will also look into the aspects of the RC circuit including the time constant(τ) that is associated with the charging and the discharging phases. The response of a capacitor to the change in frequency of an applied voltage will also be investigated along with the relationship between the current and the voltage of the circuit.

1.7 Apparatus

The following are the components of the apparatus used to carry out the experiment: resistors with the resistance of 5.6 $k\Omega$,8.2$k\Omega$,10$k\Omega$ and 15$k\Omega$, a 100nF capacitor, a National Instrument myDAQ, a screw driver, a function generator, a digital oscilloscope, a breadboard and wire(pieces of wire).
The apparatus was assembled as shown in Figure 3.

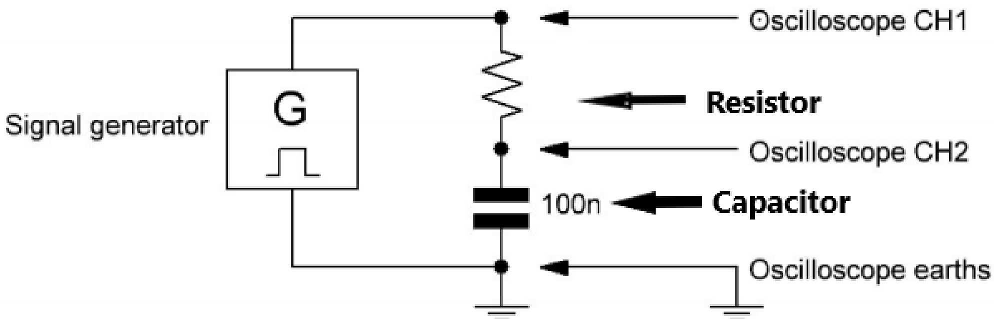

Figure 2: Diagram of the apparatus setup

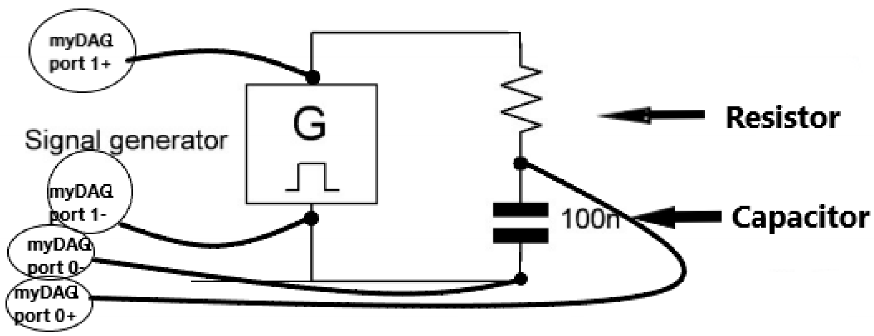

Figure 3: Diagram of the apparatus setup connected to myDAQ

1.8 Method

An RC circuit was constructed on a breadboard and a function generator was connected across the circuit. We connected the AI ports 1+ and 1- (of the myDAQ unit) to the positive and negative terminals of the function generator respectively. Another connection to the myDAQ unit was made on the AI ports 0+ and 0- to the positive and negative terminals of the capacitor respectively. The Cap.exe software was used to interact with the myDAQ unit so that data could be saved for analysis. Starting with a minimum frequency of $100Hz$, different frequencies were selected on the function generator and the effect of these frequencies on the signal being measured across the capacitor was noted. The circuit was then driven with a square wave and the peak to peak voltage of the generator was set to $8V_{pp}$; different frequencies were selected along with different resistances and this data was collected in order to determine the time constant of the circuit. The next step that was carried out was measuring the current that passes through the resistor; this was done by chosing a $15k\Omega$ resistor and connecting the myDAQ ports on the positive and negative terminals of the resistor. The circuit was driven by a sinusoidal wave and the data was collected.

2 Graphs and Best-fit line

2.1 Frequency and Resistor Response

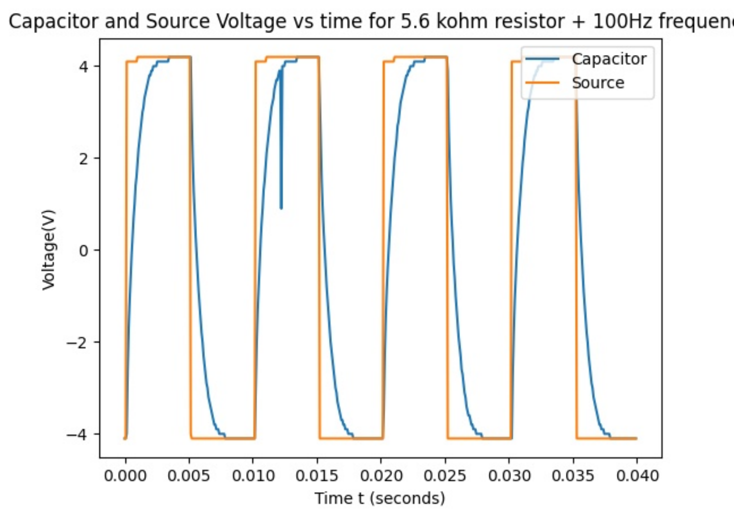

Figure 4: Capacitor and Source Voltage vs time for 5.6 Ω resistor and 100Hz frequency

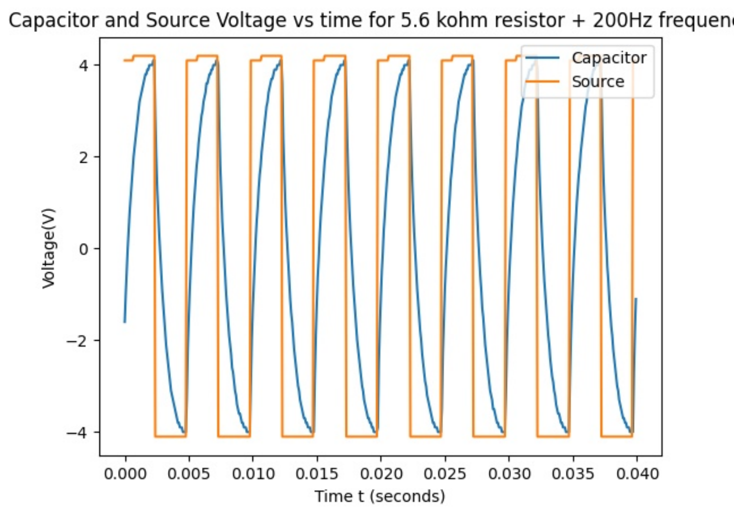

Figure 5: Capacitor and Source Voltage vs time for 5.6 kΩ resistor and 200Hz frequency

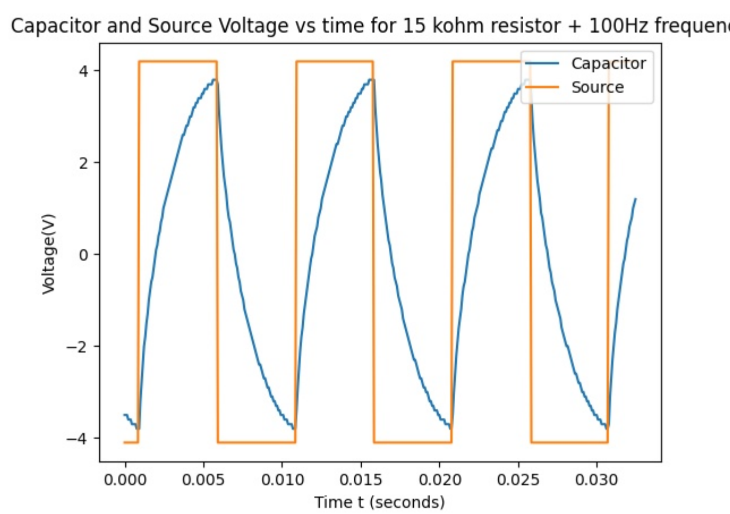

Figure 6: Capacitor and Source Voltage vs time for 15 kΩ resistor and 100Hz frequency

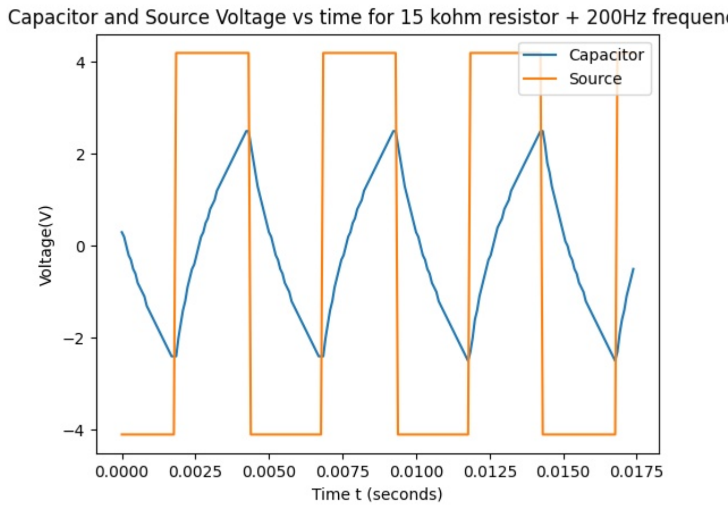

Figure 7: Capacitor and Source Voltage vs time for 15 kΩ resistor and 200Hz frequency

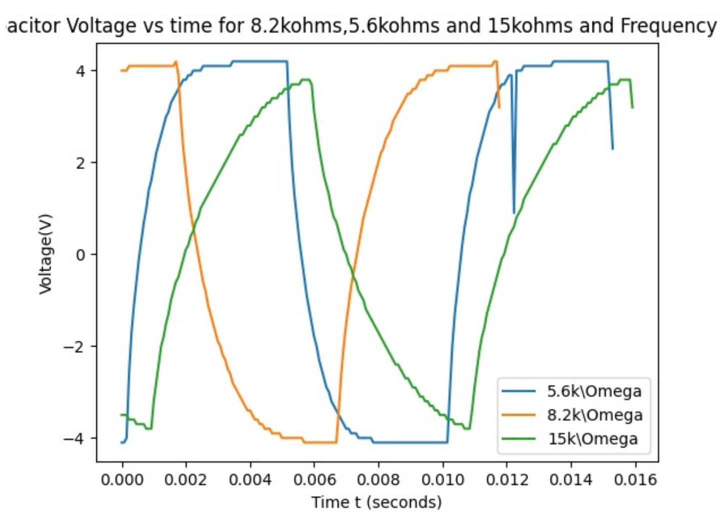

Figure 8: Capacitor Voltage vs time for 8.2kΩ, 5.6kΩ and 15kΩ and Frequency 100Hz

2.2 Time Constant

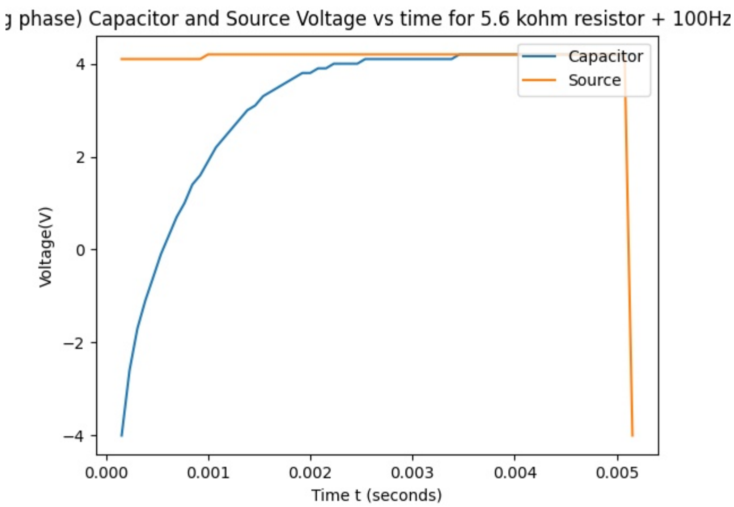

Figure 9: (Charging phase)Capacitor and Source Voltage vs time for 5.6 kΩ resistor and 100Hz frequency

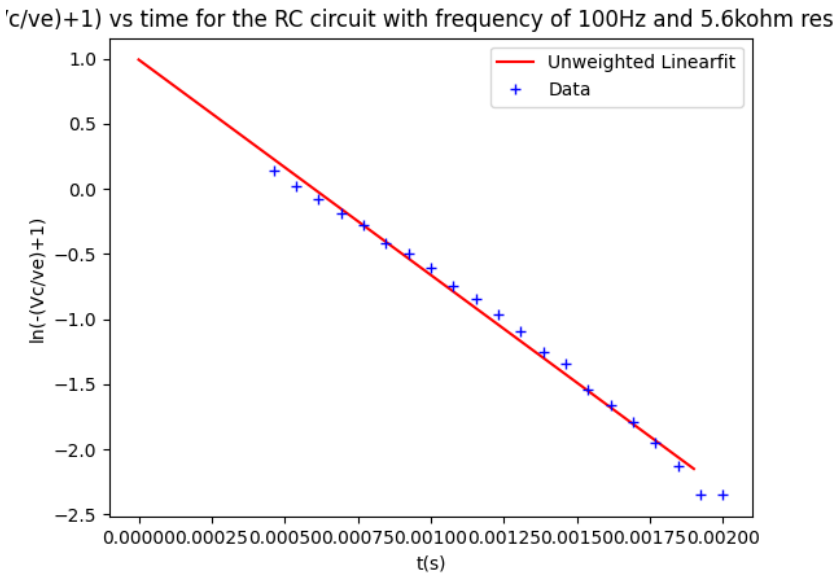

Figure 10: $ln(\frac{-V_C}{V_\epsilon}+1)$versus time for the circuit with 5.6kΩ resistance and $100Hz$ frequency during the charging phase

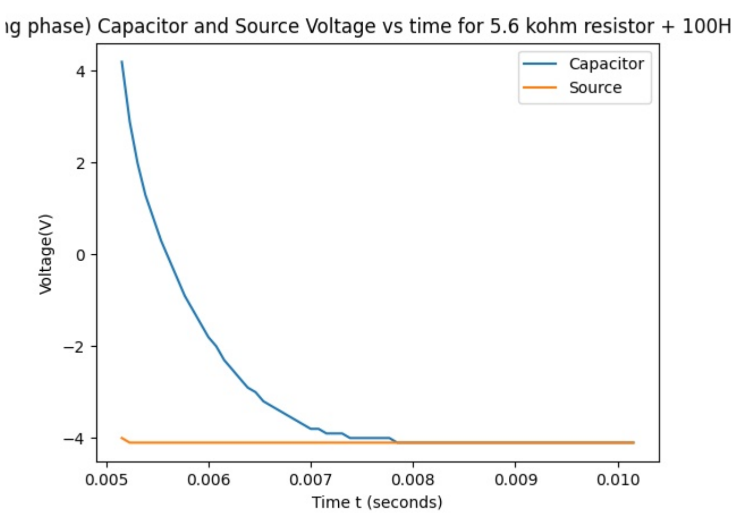

Figure 11: (Discharging phase)Capacitor and Source Voltage vs time for 5.6 kΩ resistor and 100Hz frequency

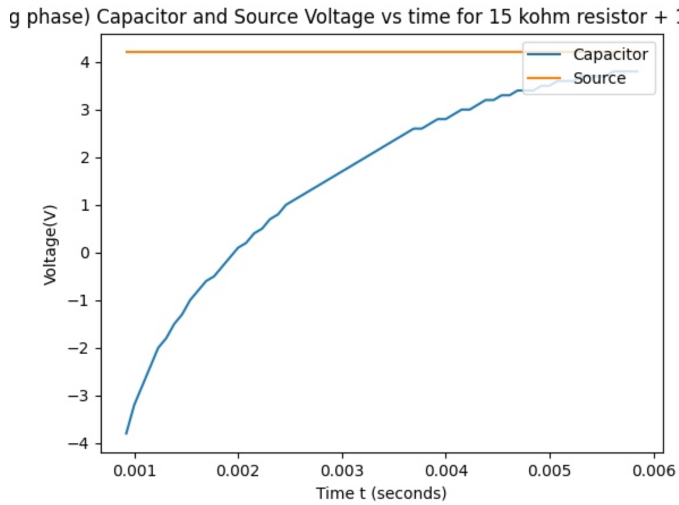

Figure 12: (Charging phase)Capacitor and Source Voltage vs time for 15 kΩ resistor and 100Hz frequency

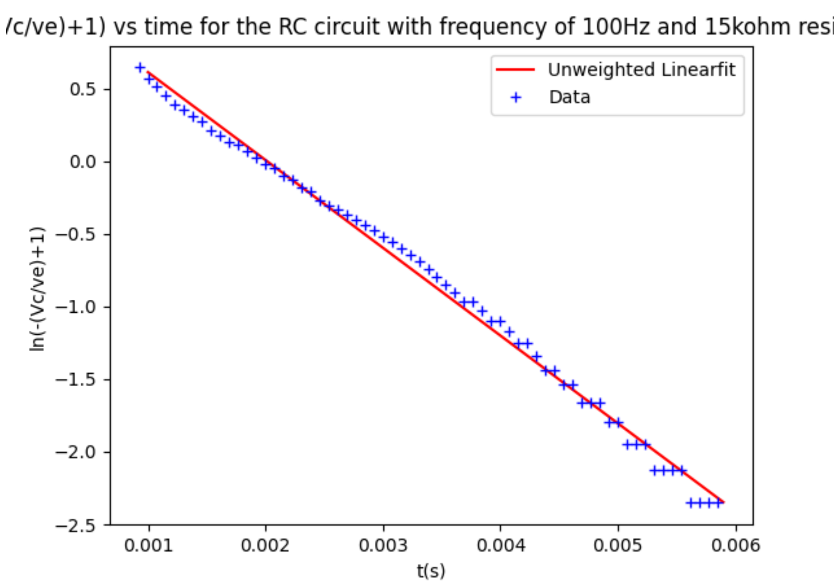

Figure 13: $ln(\frac{-V_C}{V_\epsilon} + 1)$versus time for the circuit with 15kΩ resistance and $100Hz$ frequency during the charging phase

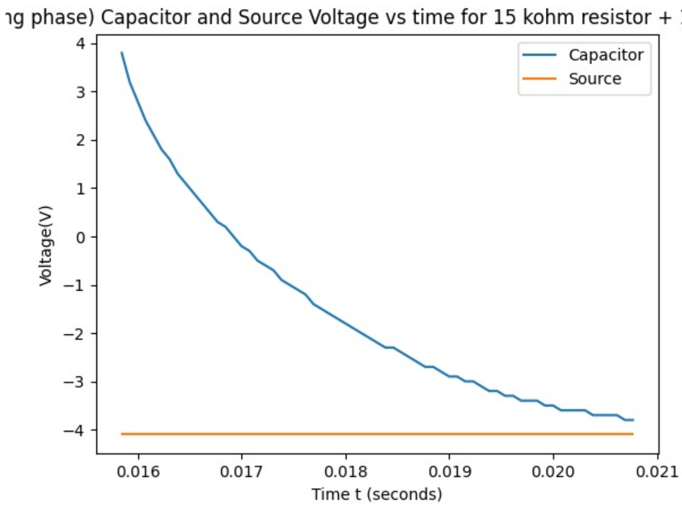

Figure 14: (Discharging phase)Capacitor and Source Voltage vs time for 15 kΩ resistor and 100Hz frequency

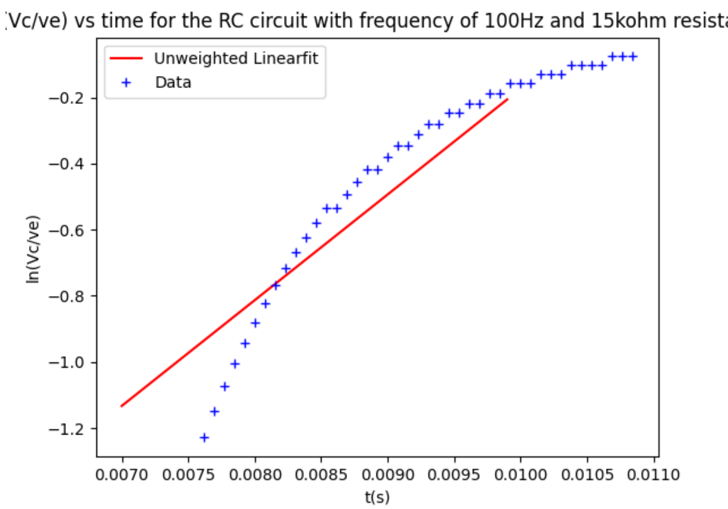

Figure 15: (Discharging phase)$ln(\frac{V_c}{V_\epsilon})$ vs time for 15 kΩ resistor and 100Hz frequency

2.3 Phase difference between Current I and Source Voltage V_ϵ in RC Circuit

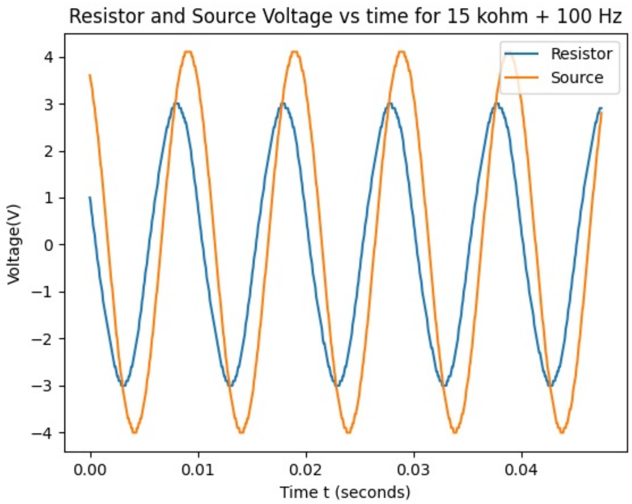

Figure 16: Resistor and Source Voltage vs time for 15kΩ and 100 Hz

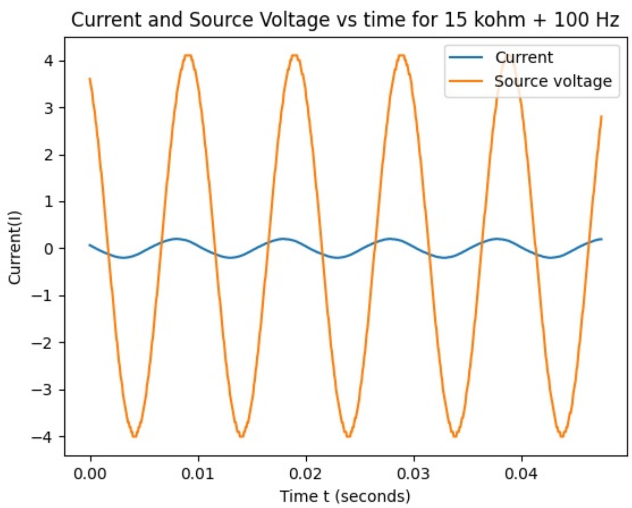

Figure 17: Current and Source Voltage vs time for 15 kΩ and 100 Hz

2.4 Verifying Capacitance

Graphs can be found on section 2.1.

3 Analysis

3.1 Frequency and Resistor Response

The increasing and decreasing of the resistance of a circuit has an effect on the output signal across the capacitor that is shown on Figure 8, the increase in the resistance results in a reduction in the time constant since the Voltage vs time graph on figure 8 has a sharper ascent and descent for the low resistance and a lower gradient for the higher resistances.

3.2 Time Constant $\tau = RC$

To calculate the time constant(τ), The Equationa5 was linearised to $ln(\frac{-V_C}{V_\epsilon}+1) = -\frac{t}{RC}$, The graph of $tvsln(\frac{-V_C}{V_\epsilon}+1)$ was plotted and the gradient and its uncertainty was noted.

From the charging phase of the capacitor with a resistance of 5.6kΩ and a frequency of $100Hz$ in figure 9 and figure 10: gradient of the fit (m) = -1653 ± 5.69

$$m = -\frac{1}{RC} \tag{14}$$

$$m = -\frac{1}{\tau} \tag{15}$$

$$\tau = 6.0496 \times 10^{-4} s \tag{16}$$

Using propagation of uncertainties:

$$\tau = -m^{-1} \tag{17}$$

$$u(\tau) = \tau\sqrt{(-\frac{u(m)}{m})^2} \tag{18}$$

$$u(\tau) = 30.95 \tag{19}$$

$$\tau = 6.0 \times 10^{-4} \pm 30.9 \tag{20}$$

From the charging phase of the capacitor with a resistance of 15kΩ and a frequency of $100Hz$ in figure 12 and figure 13: gradient of the fit (m) = -603.7839 ± 31

$$m = -\frac{1}{RC} \tag{21}$$

$$m = -\frac{1}{\tau} \tag{22}$$

$$\tau = 1.65622 \times 10^{-3} s \tag{23}$$

Using propagation of uncertainties:

$$\tau = -m^{-1} \tag{24}$$

$$u(\tau) = \tau\sqrt{(-\frac{u(m)}{m})^2} \tag{25}$$

$$u(\tau) = 0.0569 \tag{26}$$

$$\tau = 1.656 \times 10^{-3} \pm 0.057 \tag{27}$$

Using the values of the $5.6k\Omega$ resistor and the Capacitance of the capacitor which is $100nF$:

$$(5.6^3)(100 \times 10^{-9}F) \tag{28}$$

$$5.6 \times 10^{-4} \tag{29}$$

The time constant from the 5.6kΩ resistor is equal to the time constant calculated using the resistance and the capacitance of the circuit but the uncertainty is rather large compared to the norm. The time constant calculated using the circuit with the 15kΩ resistor is way off in terms of accuracy from the time constant calculated using the resistance and the capacitance.

4 Interpretation, discussion and conclusion

The Results for the time constant were not close in proximity to the value calculated for the time constant, this is due to the techniques that were used to retrieve the value of the time constant and its uncertainties. There was also a negligence on my part when it came to finding the ratings of the apparatus. The time constant according to this experiment is the following: $\tau = 6.0496 \times 10^{-4} \pm 30.9$ and the calculated value is: $\tau = 1.656 \pm 0.056$

5 References

-Figure1: commons.wikimedia.org/wiki/File:Parallel_plate_capacitor.svg
-WJ Sarjeant, Capacitors,IEEE Transactions on Electrical Insulation Vol. 25 No. 5, October 1990