

#### About Me

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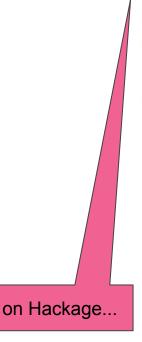


# Agenda

- Artificial Neural Networks
- Gradient Descent
- Backpropagation
- The *neurals* library
  - Automatic Differentiation
  - Components
  - Layers
  - o <u>Pipes</u>
  - Other Features
  - o <u>Examples</u>
- Questions & Comments

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>> neural: Neural Networks in native Haskell

#### The neural package

[Tags:benchmark, library, mit, program, test]

The goal of neural is to provide a modular and flexible neural network library written in native Haskell.

Features include

- composability via arrow-like instances and pipes,
- automatic differentiation for automatic gradient descent/ backpropagation training (using Edward Kmett's fabulous ad library).

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The idea is to be able to easily define new components and wire them up in flexible, possibly complicated ways (convolutional deep networks etc.).

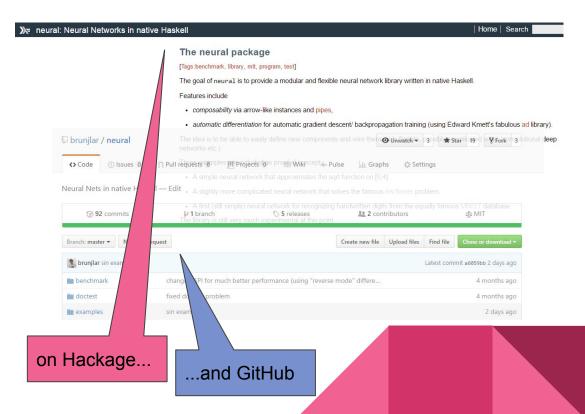
Three examples are included as proof of concept:

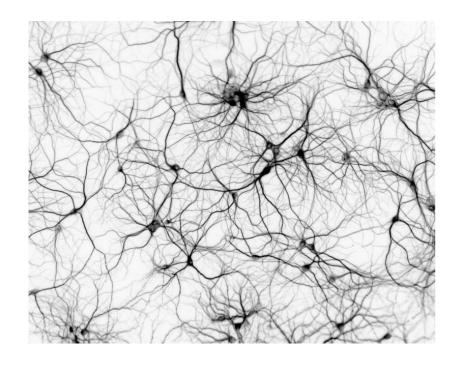
- · A simple neural network that approximates the sqrt function on [0,4].
- A slightly more complicated neural network that solves the famous Iris flower problem.
- · A first (still simple) neural network for recognizing handwritten digits from the equally famous MNIST database.

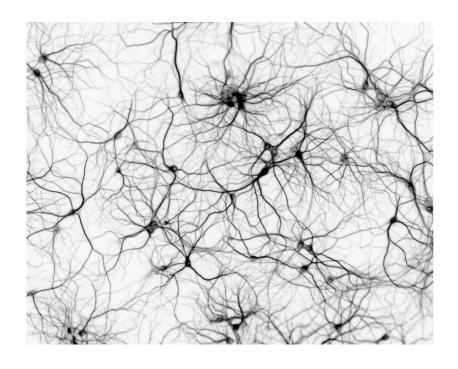
The library is still very much experimental at this point.

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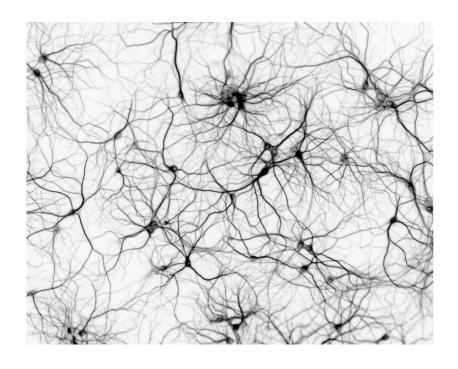




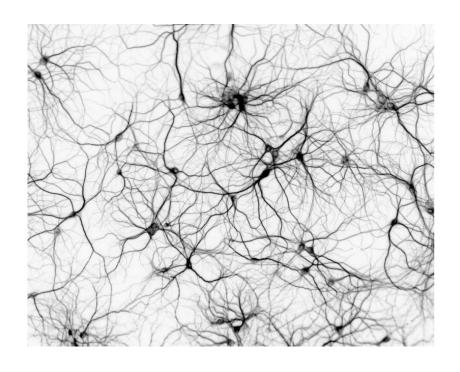


Taking inspiration from biological brains to approach Artificial Intelligence.

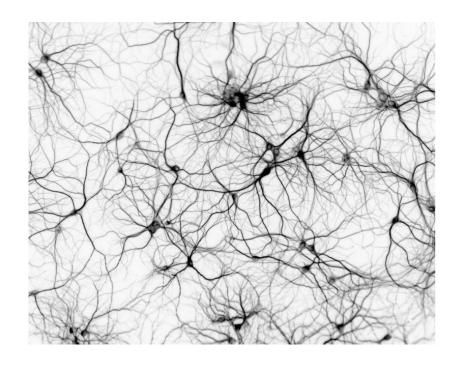
1943 McCulloch-Pitts Neuron



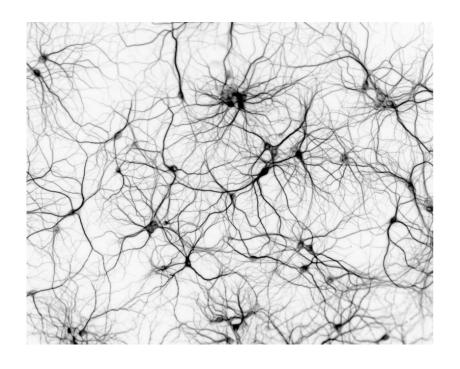
- 1943 McCulloch-Pitts Neuron
- 1958 Perceptron



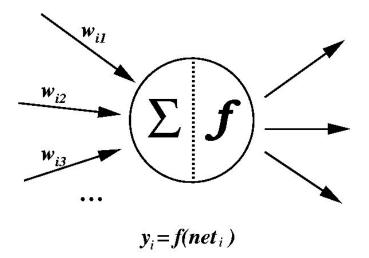
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- 1958 Perceptron
- 1975 Backpropagation

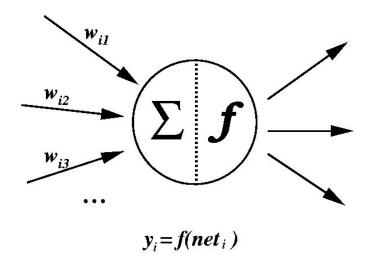


- 1943 McCulloch-Pitts Neuron
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- 1975 Backpropagation
- 2006 Deep Learning

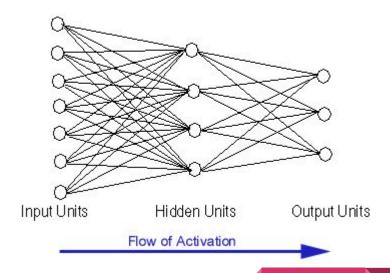


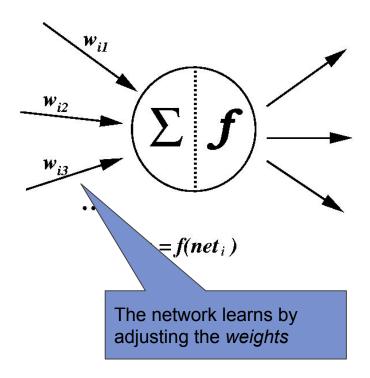
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- 2015 AlphaGo



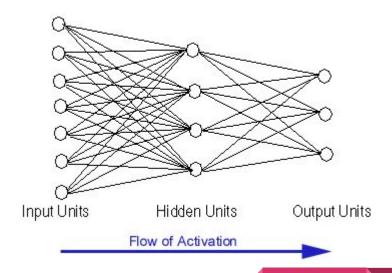


#### Schematic Diagram of a Neural Network





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Extremely generic method for finding minima (or maxima) of differentiable functions:

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We want to **minimize** the **error** of our artificial neural networks!

Extremely generic method for finding minima (or maxima) of differentiable functions:

• The "gradient" points into the direction of steepest incline.  $\nabla f = \left(\frac{\partial f}{\partial x_i}\right)_i$ 

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- At each step, move into the opposite direction.
- Eventually, you'll end up at a (local) minimum.

All we need is to compute the gradient!

$$\frac{\partial E_{p}}{\partial Y_{ji}} = \frac{\partial E_{p}}{\partial net} \frac{\partial Y_{Mi}}{\partial D_{pi}} \frac{\partial P_{Mi}}{\partial net} \frac{\partial P_{pi}}{\partial net} \frac{\partial$$

## ...is just Gradient Descent

Mathematically simple, a rather tedious application of the Chain Rule.

But details change when...

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- A different activation function is used for neurons.
- A different error function is used.
- A different layout is used.

Furthermore, the classical formulas are **index-based**, i.e. complicated when network topologies become complex.

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#### **Automatic Differentiation**

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Less well known is the third way, **Automatic Differentiation** (see Conal M. Elliott's fantastic paper "Beautiful Differentiation"), an **exact numeric** method.

**Basic Idea:** Manipulate numbers and their derivatives simultaneously (can be done conveniently in Haskell by overloading the numeric type classes).

### Edward Kmett's ad Library

```
grad :: (Traversable f, Num a) => (forall s. Reifies s Tape => f (Reverse s a) -> Reverse s a) -> f a -> f a
  The grad function calculates the gradient of a non-scalar-to-scalar function with reverse-mode AD in a single pass.
   >>> grad (\[x,y,z] -> x*y+z) [1,2,3]
   [2,1,1]
grad' :: (Traversable f, Num a) => (forall s. Reifies s Tape => f (Reverse s a) -> Reverse s a) -> f a -> (a,
fa)
  The grad' function calculates the result and gradient of a non-scalar-to-scalar function with reverse-mode AD in a single pass.
   >>> grad' (\[x,y,z] -> x*y+z) [1,2,3]
   (5,[2,1,1])
gradWith :: (Traversable f, Num a) => (a -> a -> b) -> (forall s. Reifies s Tape => f (Reverse s a) -> Reverse
s a) -> f a -> f b
  grad g f function calculates the gradient of a non-scalar-to-scalar function f with reverse-mode AD in a single pass. The gradient is
  combined element-wise with the argument using the function g.
   grad == gradWith ( dx -> dx)
   id == gradWith const
gradWith' :: (Traversable f, Num a) => (a -> a -> b) -> (forall s. Reifies s Tape => f (Reverse s a) ->
Reverse s a) -> f a -> (a, f b)
                                                                                                                # Source
```

grad' g f calculates the result and gradient of a non-scalar-to-scalar function f with reverse-mode AD in a single pass the gradient is combined element-wise with the argument using the function g.

```
grad' == gradWith' (_ dx -> dx)
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### Edward Kmett's ad Library

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                                                                                                           # Source
  grad' g f calculates the result and gradient of a non-scalar-to-scalar function f with reverse-mode AD in a single pass the gradient
```

Arguments of "traversable" shape - no indices needed!

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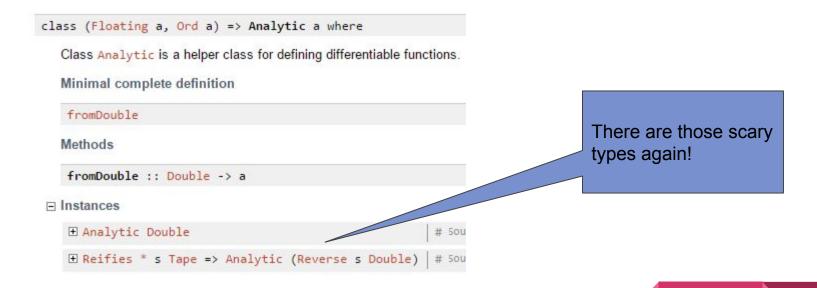
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```

Arguments of "traversable" shape - no indices needed!

These types look intimidating...



```
newtype Diff f g

Type Diff f g can be thought of as the type of "differentiable" functions f Double -> g Double.

Constructors

Diff

runDiff :: forall a. Analytic a => f a -> g a

□ Instances

□ Category (* -> *) Diff | # Source

type Diff' = forall a. Analytic a => a -> a
```

Type Diff' can be thought of as the type of differentiable functions Double -> Double.

#### gradWith'

```
:: Traversable t

=> (Double -> Double -> a) how to combine argument and gradient

-> Diff t Identity differentiable function

-> t Double function argument

-> (Double, t a) function value and combination of argument and gradient
```

Computes the gradient of an analytic function and combines it with the argument.

```
>>> gradWith' (\_ d -> d) (Diff \ (x, y) -> \ (x + 3 + y + 7) \ (2, 1) \ (14.0, [4.0, 3.0])
```

```
newtype ParamFun s t a b # Sou
```

The type ParamFun s t a b describes parameterized functions from a to b, where the parameters are of type t s. When such components are composed, they all share the same parameters.

#### Constructors

#### ParamFun runPF :: a -> t s -> b

#### ■ Instances

newtype ParamFun s t a b

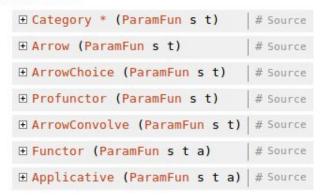
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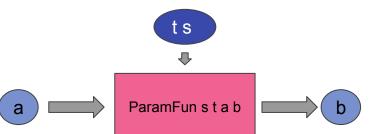
#### Constructors

#### **ParamFun**

runPF :: a -> t s -> b

#### ■ Instances





# Sou

```
data Component f g # Source
```

A Component f g is a parameterized differentiable function f Double -> g Double. In contrast to ParamFun, when components are composed, parameters are not shared. Each component carries its own collection of parameters instead.

#### Constructors

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#### Constructors

#### ■ Instances

```
    ⊕ NFData (Component f g)  # Source
    ⊕ Category (* -> *) Component  # Source
```

The traversable type t is "hidden" by existential quantification

automatically!

```
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  composed, parameters are not shared. Each component carries its own collection of parameters instead.
  Constructors
   (Traversable t, Applicative t, NFData (t Double)) => Component
      weights :: t Double
                                                                   the specific parameter values
      compute :: forall s. An vtic s => ParamFun s t (f s) (g s)
                                                                   the encapsulated parameterized function
      initR :: forall m. MonadRando >> m (t Double)
                                                                   randomly sets the parameters
∃ Instances

    ■ NFData (Component f q)

                                  # Source
   The traversable type t is
                                                   "hidden" by existential
                                                  quantification
          Traversables are composed
```

# Source

```
data Model :: (* -> *) -> (* -> *) -> * -> * -> * -> * where

A Model f g a b c wraps a Component f g and models functions b -> c with "samples" (for model error determination) of type a.

Constructors

Model :: (Functor f, Functor g) => Component f g -> (a -> (f Double, Diff g Identity)) -> (b -> f Double) -> (g Double -> c) -> Model f g a b c

□ Instances

□ Profunctor (Model f g a) | # Source

□ NFData (Model f g a b c) | # Source
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```

```
type StdModel f g b c = Model f g (b, c) b c
```

A type abbreviation for the most common type of models, where samples are just input-output tuples.

```
mkStdModel :: (Functor f, Functor g) => Component f g -> (c -> Diff g Identity) -> (b -> f Double) -> (g Double -> c) -> StdModel f g b c
```

Creates a StdModel, using the simplifying assumtion that the error can be computed from the expected output allone.

#### descent

:: Foldable h	
=> Model f g a b c	the model whose error should be decreased
-> Double	the learning rate
-> h a	a mini-batch of samples
-> (Double, Model f g a b c)	returns the average sample error and the improved model

Performs one step of gradient descent/ backpropagation on the model,

# Layers

#### Layers

```
type Layer i o = Component (Vector i) (Vector o)
```

A Layer i o is a component that maps a Vector of length i to a Vector of length o.

```
linearLayer :: forall i o. (KnownNat i, KnownNat o) => Layer i o
```

Creates a linear Layer, i.e. a layer that multiplies the input with a weight Matrix and adds a bias to get the output.

Random initialization follows the recommendation from chapter 3 of the online book Neural Networks and Deep Learning by Michael Nielsen.

```
layer :: (KnownNat i, KnownNat o) => Diff' -> Layer i o
```

Creates a Layer as a combination of a linear layer and a non-linear activation function.

#### Layers

```
tanhLayer :: (KnownNat i, KnownNat o) => Layer i o

This is a simple Layer, specialized to tanh-activation. Output values are all in the interval [-1,1].

tanhLayer' :: (KnownNat i, KnownNat o) => Layer i o

This is a simple Layer, specialized to a modified tanh-activation, following the suggestion from Efficient BackProp by LeCun et al., where output values are all in the interval [-1.7159,1.7159].

logisticLayer :: (KnownNat i, KnownNat o) => Layer i o

This is a simple Layer, specialized to the logistic function as activation. Output values are all in the interval [0,1].

reLULayer :: (KnownNat i, KnownNat o) => Layer i o

This is a simple Layer, specialized to the rectified linear unit activation function. Output values are all non-negative.
```



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but not enough: ...

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 Training "batches" must be provided - for realistic problems, not all training data will fit into memory.

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#### but not enough:

- Training "batches" must be provided for realistic problems, not all training data will fit into memory.
- We want some progress reporting to see how training is going.
- We want to stop training when some target is reached or no further improvement seems likely.

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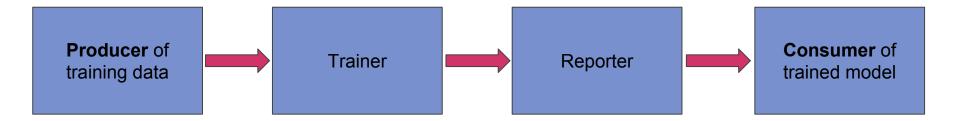
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All this tends to produce imperative and non modular code...

#### Pipes to the rescue!

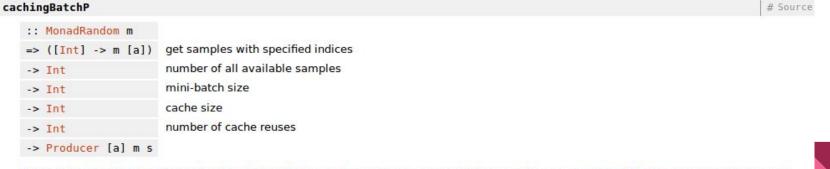


```
Extensions
                                                                                              BangPatterns
data TS f g a b c
                                                                     # Source
  The training state of a model.
  Constructors
   TS
      tsModel :: Model f g a b c updated model
      tsGeneration :: Int
                                     generation
      tsEta :: Double
                                     learning rate
      tsBatchError :: Double
                                     last training error
descentP
                                                                                                        # Source
    :: (Foldable h, Monad m)
                                         initial model
   => Model f g a b c
                                         first generation
    -> Int
                                         computes the learning rate from the generation
   -> (Int -> Double)
    -> Pipe (h a) (TS f g a b c) m r
```

A Pipe for training a model: It consumes mini-batches of samples from upstream and pushes the updated training state downstream.



A simple Producer of mini-batches.



Function simpleBatchP only works when all available samples fit into memory. If this is not the case, cachingBatchP can be used instead. It takes an effectful way to get specific samples and then caches some of those samples in memory for a couple of rounds, drawing mini-batches from the cached values.

#### reportTSP

```
:: Monad m

=> Int report interval

-> (TS f g a b c -> m ()) report action

-> Pipe (TS f g a b c) (TS f g a b c) m r
```

A Pipe for progress reporting of model training.

#### consumeTSP

```
:: Monad m
=> (TS f g a b c -> m (Maybe x)) check whether training is finished and what to return in that case
-> Consumer (TS f g a b c) m x
```

A Consumer of training states that decides when training is finished and then returns a value.

# Other Features

#### Other Features

- already implemented
  - fixed-length vectors (used for layers)
  - support for classification problems (cross entropy,...)
  - normalization (input whitening)
- work in progress
  - convolutional networks
  - regularization
  - deep learning

#### Modules

#### □ Data

□ Data FixedSize fixed-size containers

Data FixedSize Class class definitions

Data FixedSize Matrix fixed-size matrices

Data.FixedSize.Vector fixed-length vectors

Data FixedSize Volume fixed-size volumes

Data.MyPrelude commonly used standard types and functions

□ Data Utils various utilities

Data. Utils. Analytic "analytic" values

Data. Utils. Arrow arrow utilities

Data. Utils. Cache caching

Data. Utils. List list utilities

Data Utils Pipes pipe utilities

Data. Utils. Random random number utilities

Data. Utils. Stack a simple stack monad

Data Utils Statistics statistical utilities

Data. Utils. Traversable utilities for traversables

#### □ Numeric

□ Numeric. Neural neural networks

Numeric Neural Convolution convolutional layers

Numeric. Neural. Layer layer components

Numeric.Neural.Model "neural" components and models

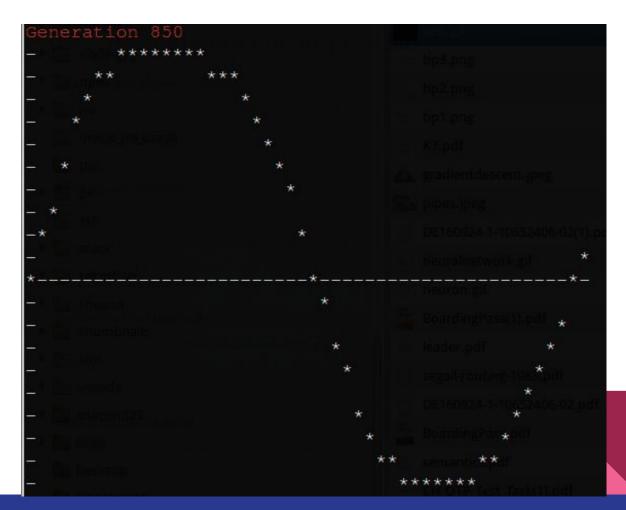
Numeric.Neural.Normalization normalizing data

Numeric. Neural. Pipes a pipes API for models

# Examples

### Sine Example

- approximate sin(x) on [0, 2π]
- 1 input neuron
- 4 hidden neurons
- 1 output neuron
- accurate to 0.1 after 850 generations



#### Sine Example

```
main :: IO ()
main = flip evalRandT (mkStdGen 739570) $ do
   let xs = [0, 0.01 ... 2 * pi]
   m <- modelR $ whiten sinModel xs
   runEffect $
            simpleBatchP [(x, \sin x) | x <- xs] 10
       >-> descentP m 1 (const 0.5)
       >-> reportTSP 50 report
       >-> consumeTSP check
  where
    sinModel :: StdModel (Vector 1) (Vector 1) Double Double
    sinModel = mkStdModel
        (tanhLayer . (tanhLayer :: Layer 1 4))
        (\x -> Diff $ Identity . sqDiff (pure $ fromDouble x))
        pure
        vhead
```

```
getError ts =
   let m = tsModel ts
   in maximum [abs (sin x - model m x) | x <- [0, 0.1 .. 2 * pi]]
report ts = liftI0 $ do
   ANSI.clearScreen
   ANSI.setSGR [ANSI.SetColor ANSI.Foreground ANSI.Vivid ANSI.Red]
   ANSI.setCursorPosition 0 0
   printf "Generation %d\n" (tsGeneration ts)
   ANSI.setSGR [ANSI.Reset]
   graph (model (tsModel ts)) 0 (2 * pi) 20 50

check ts = return $ if getError ts < 0.1 then Just () else Nothing</pre>
```

### Other Examples

• sqrt models the similar regression problem of approximating the square root function on the interval [0,4].

iris solves the famous Iris Flower classification problem.

 MNIST tackles the equally famous MNIST problem of recognizing handwritten digits.

