

# Algorithms for Graph-Based Supervised Learning

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Hello

## 1 Modified Adsorption

This routine is described in Talukdar below.

### 1.1 Prerequisites

I'm looking for a good parallelization strategy for this function. I'm going to expand it as far as I can to see if something presents itself. We are aiming to find  $c_v$  and  $d_v$  for each vertex in the graph. First, let's define

$$m(x) = \sum_u W_{u,x} \tag{1}$$

$$l(x) = \sum_u W_{u,x} \log W_{u,x} \tag{2}$$

Okay, check my math here...

$$p(a|b) = \frac{W_{a,b}}{\sum_u W_{u,b}} \quad (3)$$

$$= \frac{W_{a,b}}{m(b)} \quad (4)$$

$$H(x) = - \sum_y p(y|x) \log p(y|x) \quad (5)$$

$$= - \sum_y \left( \frac{W_{y,x}}{\sum_u W_{u,x}} \right) \log \left( \frac{W_{y,x}}{\sum_u W_{u,x}} \right) \quad (6)$$

$$= - \sum_y \left( \frac{W_{y,x}}{m(x)} \right) \log \left( \frac{W_{y,x}}{m(x)} \right) \quad (7)$$

$$= \frac{-1}{m(x)} \sum_y W_{y,x} [\log W_{y,x} - \log m(x)] \quad (8)$$

$$= \frac{-1}{m(x)} \left[ \sum_y W_{y,x} \log W_{y,x} - \sum_y W_{y,x} \log m(x) \right] \quad (9)$$

$$= \frac{-1}{m(x)} \left[ \sum_y W_{y,x} \log W_{y,x} - m(x) \log m(x) \right] \quad (10)$$

$$= \log m(x) - \frac{1}{m(x)} \sum_y W_{y,x} \log W_{y,x} \quad (11)$$

$$= \log m(x) - \frac{l(x)}{m(x)} \quad (12)$$

Next we have the smoothing function for a given  $\beta$

$$f(x) = \frac{\log \beta}{\log(\beta + e^x)} \quad (13)$$

$$c_x = f(H(x)) \quad (14)$$

$$= \log \beta [\log(\beta + e^{H(x)})]^{-1} \quad (15)$$

$$d_x = (1 - c_x) \sqrt{H(x)} \quad (16)$$

$$z_x = \max(c_x + d_x, 1) \quad (17)$$

Given these values, the authors define

$$p_v^{cont} = \frac{c_v}{z_v}, p_v^{inj} = \frac{d_v}{z_v}, p_v^{abnd} = 1 - p_v^{cont} - p_v^{inj}$$

## 1.2 Algorithm 3: Modified Adsorption

Taken from the book reference below. This will optimize

$$C(\hat{Y}) = \sum_l \left[ \mu_1 \left( Y_l - \hat{Y}_l \right)^T S \left( Y_l - \hat{Y}_l \right) + \mu_2 \hat{Y}_l^T L \hat{Y}_l + \mu_3 \|\hat{Y}_l - R_l\|^2 \right]$$

where  $\mu_{1,2,3}$  are "hyperparameters that determine the relative importance of each term" in  $C$ . So Dealer's Choice on  $\mu$ . I've asked Dr. Talukdar for guidance on this selection and I'm awaiting his response.

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**Algorithm 2** Modified Adsorption

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1: procedure INPUT:
2:    $G = (V, E, W), M, R$ 
3:   Labels =  $Y_v \in \mathcal{R}^{m+1}$  for  $v \in V$ 
4:   Probabilities  $p_v^{inj}, p_v^{cont}, p_v^{abnd}$  for  $v \in V$ 
5:   Constants  $\mu_1, \mu_1, \mu_3$ 
6: procedure OUTPUT:
7:    $\hat{Y}_v$  for  $v \in V$ 
8: procedure ITERATE
9:    $\hat{Y}_v \leftarrow Y_v$  for  $v \in V$  [Initialization]
10:   $M_{vv} \leftarrow \mu_1 \times p_v^{inj} + \mu_2 \times p_v^{cont} \times \sum_u W_{vu} + \mu_3$ 
11:  while Not Converged do
12:     $D_v \leftarrow \sum_x (p_v^{cont} W_{vx} + p_x^{cont} W_{xv}) \hat{Y}_x$ 
13:    for all  $v \in V$  do
14:       $\hat{Y} \leftarrow \frac{1}{M_{vv}} (\mu_1 \times p_v^{inj} \times Y_v + \mu_2 \times D_v + \mu_3 \times p_v^{abnd} \times R_v)$ 

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## 2 References

BibTeX is a pain, so for right now I'm going to do

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@article{doi:10.2200/S00590ED1V01Y201408AIM029,
author = { Amarnag
          Subramanya and Partha Pratim
          Talukdar },
title = {Graph-Based Semi-Supervised Learning},
journal = {Synthesis Lectures on Artificial Intelligence and Machine Learning},
volume = {8},
number = {4},
pages = {1-125},
year = {2014},
doi = {10.2200/S00590ED1V01Y201408AIM029},

URL = {
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