Algorithms for Graph-Based Supervised Learning

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Hello

1 Modified Adsorption

This routine is described in Talukdar below.

1.1 Prerequisites

I'm looking for a good parallelization strategy for this function. I'm going to expand it as far as I can to see if something presents itself. We are aiming to find c_v and d_v for each vertex in the graph. First, let's define the column sum $m(x)$. Since W is symmetric, this is also the row sums. In the code, they are rowSum

$$\begin{aligned} m(x) &= \sum_u W_{u,x} && (1)\\ &= \sum_u W_{x,u} && (2)\\ &= \text{rowSum}(x) && (3)\\ l(x) &= \sum_u W_{u,x} \log W_{u,x} && (4)\\ pc(v) &= p_x^{cont} \sum_x p_x^{cont} W_{xv} && (5)\\ &= \text{pcntW}(v) && (6) \end{aligned}$$

Okay, check my math here...

$$
\begin{aligned}
p(a|b) &= \frac{W_{a,b}}{\sum_u W_{u,b}} && (7)\\
&= \frac{W_{a,b}}{m(b)} && (8)\\
H(x) &= -\sum_y p(y|x)\log p(y|x) && (9)\\
&= -\sum_y \left(\frac{W_{y,x}}{\sum_u W_{u,x}}\right)\log\left(\frac{W_{y,x}}{\sum_u W_{u,x}}\right) && (10)\\
&= -\sum_y \left(\frac{W_{y,x}}{m(x)}\right)\log\left(\frac{W_{y,x}}{m(x)}\right) && (11)\\
&= \frac{-1}{m(x)}\sum_y W_{y,x}\left[\log W_{y,x} - \log m(x)\right] && (12)\\
&= \frac{-1}{m(x)}\left[\sum_y W_{y,x}\log W_{y,x} - \sum_y W_{y,x}\log m(x)\right] && (13)\\
&= \frac{-1}{m(x)}\left[\sum_y W_{y,x}\log W_{y,x} - m(x)\log m(x)\right] && (14)\\
&= \log m(x) - \frac{1}{m(x)}\sum_y W_{y,x}\log W_{y,x} && (15)\\
&= \log m(x) - \frac{l(x)}{m(x)} && (16)
\end{aligned}
$$

Next we have the smoothing function for a given β

$$
\begin{aligned}
f(x) &= \frac{\log\beta}{\log(\beta + e^x)} && (17)\\
c_x &= f(H(x)) && (18)\\
&= \log\beta\,[\log(\,\beta + e^{H(x)}\,)]^{-1} && (19)\\
d_x &= (1-c_x)\sqrt{H(x)} && (20)\\
z_x &= \max(c_x + d_x, 1) && (21)
\end{aligned}
$$

Given these values, the authors define

$$p_v^{cont} = \frac{c_v}{z_v}, p_v^{inj} = \frac{d_v}{z_v}, p_v^{abnd} = 1 - p_v^{cont} - p_v^{inj}$$

1.2 Algorithm 3: Modified Adsoprtion

Taken from the book reference below. This will optimize

$$C(\hat{Y}) = \sum_l \left[\mu_1 \left(Y_l - \hat{Y}_l \right)^T S \left(Y_l - \hat{Y}_l \right) + \mu_2 \hat{Y}_l^T L \hat{Y}_l + \mu_3 \| \hat{Y}_l - R_l \|^2 \right]$$

where $\mu_{1,2,3}$ are "hyperparameters that determine the relative importance of each term" in C. So Dealer's Choice on μ. Y_l and R_l are the l^{th} columns of Y and R. The choice of convergence criteria is unclear at the moment.

1.2.1 Regularization

R is used to provide target for unlabeled vertices, essentially a default value. In the slide deck (but not the lecture notes) they suggest you set $R_v = p_v^{abdn}$ if v is unlabeled.

I've asked Dr. Talukdar for guidance on this selection and I'm awaiting his response.

Algorithm 2 Modified Adsoprtion

procedure INPUT:
  $G = (V, E, W), M, R$
  Labels = $Y_v \in \mathcal{R}^{m+1}$ for $v \in V$
  Probabilities $p_v^{inj}, p_v^{cont}, p_v^{abnd}$ for $v \in V$
  Constants μ_1, μ_1, μ_3
procedure OUTPUT:
  $\hat{Y}_v$ for $v \in V$
procedure ITERATE
  $\hat{Y}_v \leftarrow Y_v$ for $v \in V$ [Initialization]
  $M_{vv} \leftarrow \mu_1 \times p_v^{inj} + \mu_2 \times p_v^{cont} \times \sum_u W_{vu} + \mu_3$
  while Not Converged **do**
    $D_v \leftarrow \sum_x \left(p_v^{cont} W_{vx} + p_x^{cont} W_{xv} \right) \hat{Y}_x$
    for all $v \in V$ **do**
      $\hat{Y} \leftarrow \frac{1}{M_{vv}} \left(\mu_1 \times p_v^{inj} \times Y_v + \mu_2 \times D_v + \mu_3 \times p_v^{abnd} \times R_v \right)$

Taking a closer look, remember $W^T = W$ so

$$D_v \leftarrow \sum_x \left(p_v^{cont} W_{vx} + p_x^{cont} W_{xv} \right) \hat{Y}_x \tag{22}$$

$$= \sum_x \left(p_v^{cont} W_{vx} + p_x^{cont} W_{vx} \right) \hat{Y}_x \tag{23}$$

2 References

BibTeX is a pain, so for right now I'm going to do

```
@article{doi:10.2200/S00590ED1V01Y201408AIM029,
author = {  Amarnag
  Subramanya  and   Partha Pratim
  Talukdar },
title = {Graph-Based Semi-Supervised Learning},
journal = {Synthesis Lectures on Artificial Intelligence and Machine Learning},
volume = {8},
number = {4},
pages = {1-125},
year = {2014},
doi = {10.2200/S00590ED1V01Y201408AIM029},

URL = {
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