# Department of Physics: Computational physics honours module Tutorial 4 - Simulating the ADC spectrum of a $^{60}$Co source as recorded by a scintillator-based gamma-ray detector.

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## Introduction

We can simulate particles by using Monte-Carlo techniques which specifically aims at modelling particles with random scatter. The particles emitted from a $^{60}$Co source travel in random directions albeit in mostly straight paths.

## Particle Generation

In order to generate the photons isotropically we first need to check how many $^{60}$Co atoms decayed. If $N$ photons decayed that does not necessarily mean there at $N$ photons. The decay scheme of $^{60}$Co shows 2 decay paths with their relative decay probabilities.

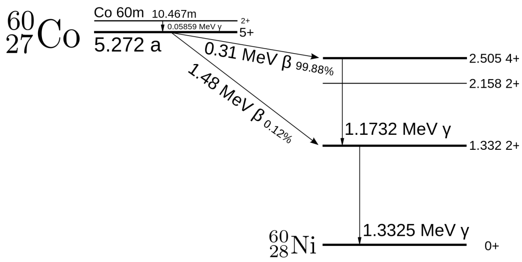


Figure 1: The decay scheme of $^{60}$Co (Preoteasa [2012]).

Using these probabilities we sample from a uniform random number generator $\gamma$. If $\gamma$ is less than the probability of getting the of the $1.1732$ MeV photon then a photon is recorded with the energy $1.1732$ MeV. Another photon is immediately recorded with the energy of $1.3325$ MeV since the $1.1732$ MeV decay path leaves the daughter nucleus in an excited state and it emits another $1.3325$ MeV photon to move into a stable state. If the random number drawn $\gamma$ is more than the probability of going down the $1.1732$ MeV path then we just record a photon of energy $1.3325$ MeV (Preoteasa [2012]).

We now run another Monte Carlo simulation in order to simulate the movement of the photons. The distance between two interaction points will depend on the mean free path which in turn depends on the total cross section of the photon. Since we are dealing with a homogeneous medium this path is exponentially distributed (Vasiliev [2018]):

$$f_l\left(t\right) = \sigma \exp\left(-\sigma t\right) \tag{1}$$

*University of Cape Town - (ref Dietel T, Computational physics, (University of Cape Town), 2020)).

If $l$ is the distance between the interaction points and $t$ is the distance, we can sample the exponential distribution using the inversion method and obtain:

$$l = -\frac{1}{\sigma} \ln \gamma \tag{2}$$

The $1/\sigma$ is equal to the mean free path. We now have a way of of finding the distance the average distance $n$ photons will travel. We now have to find a way of simulating the scattering interaction. We will be doing this in a basic manner just in order to generate the photons before the complex interactions of the photons inside the detector. We now need to find a way of updating the position of the photons after a scattering interaction. This will be done in a basic manner in order to determine if the particle generation is successful. The Cumulative Density Function(CDF) of $\theta$ is given by: (Khungurn [2015])

$$p_\theta = \frac{\sin \theta}{2} \tag{3}$$

The inverse CDF is:

$$p_\theta^{-1} = \arccos(1 - \gamma) \tag{4}$$

The inverse CDF of $\phi$ is:

$$p_\phi^{-1} = 2\phi\gamma \tag{5}$$

The inverse CDF of $\phi$ is simple given that we are sampling uniformly from $[0, 2\pi]$. We can now write down our position update formula as follows Vasiliev [2018]:

$$dx = l \sin(\theta) \cos(\phi) \tag{6}$$
$$dy = l \sin(\theta) \sin(\phi) \tag{7}$$
$$dz = \sin(\theta) \tag{8}$$

The $\theta$ and $\phi$ are sampled from the inverse CDF. We also need a condition for absorption. We will say that absorption occurs when the ratio of the absorption cross section to the total cross section, $\sigma_a/\sigma_t$, is less than a uniformly randomly generated number, $\gamma$, then absorption occurs. We can now check if the algorithm works by plotting the final positions of the photons (Vasiliev [2018]).

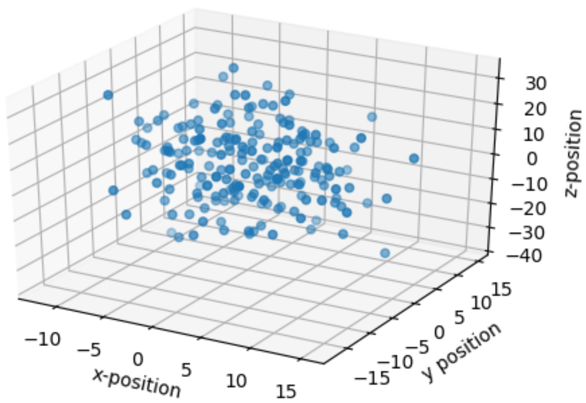


Figure 2: Final positions of photons.

We expect the particles to be distributed isotropically from the origin and rotating the image shows that it is indeed isotropic.

## Efficiency of the photon detection

To simulate the number or percentage of generated photons that reach the detector, an isotropic distribution of photons was randomly generated on a sphere of radius $r$ using the following equations:

$$\theta = \arccos(1 - 2\gamma) \tag{9}$$
$$\phi = 2\pi\gamma \tag{10}$$
$$x = r\sin(\theta)\cos(\phi) \tag{11}$$
$$y = r\sin(\theta)\sin(\phi) \tag{12}$$
$$z = r\cos(\theta) \tag{13}$$

In this case $\gamma$ is a uniformly random generated number as defined above. Since intensity of photon flux on a surface or detector, a distance $r$ decrease via the inverse square law as shown in Figure (3).

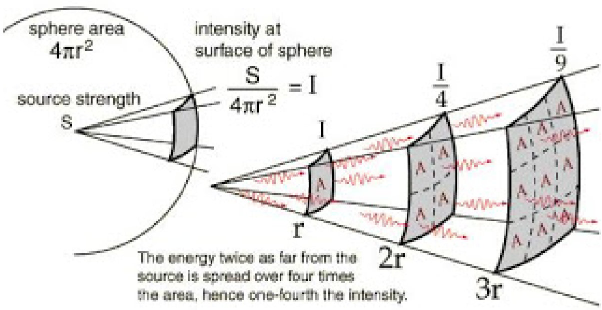


Figure 3: Inverse square law for the intensity of photons on a surface of area $A$ a distance $r$,$2r$ and $3r$ away from the source.

We expect that the as the distance increases the proportion of photons incident on the detector's surface decrease. Two thousand photons where isotropically generated and the number of photons hit the detector with dimension's $40mm \times 40mm$, a distance 25 mm away from the detector was collected.
Ten thousand simulations each with two thousand generated photons were done to check the percentage of generated photons that reach the detector surface given the distance and detector dimensions above. Figure (4) shows the result.

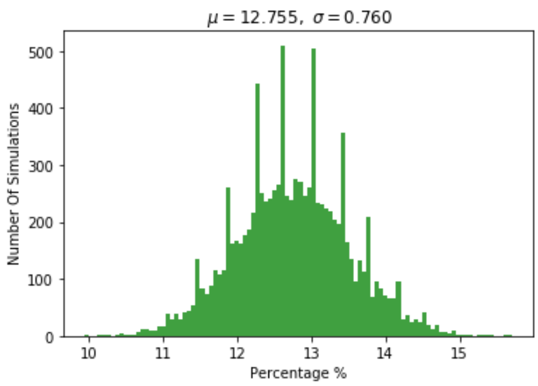


Figure 4: Histogram showing the percentage of generated photons that reach the detector surface for ten thousand simulations. The percentages were binned, and the y- axis gives us the number of simulations that has a given percentage of photons reaching the detector.

From Figure (4) we can see that for 2000 generated photons only about 12.755 ± 2.28 percent reach the detector.

# Photon tracing through the detector geometry

## Detector simulation

We are given the specifications for Sodium Iodide detector to simulate particles interacting with the detector materials.

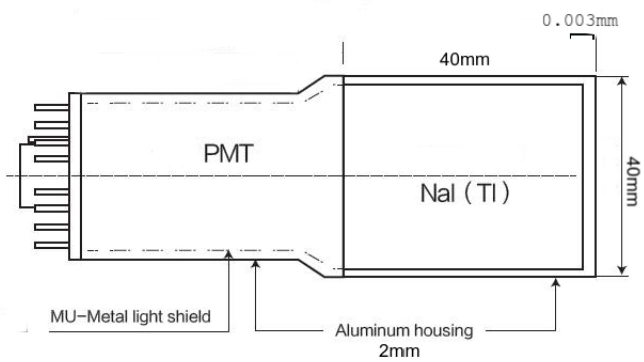


Figure 5: Diagrammatic layout of a the Sodium Iodide detector we want to simulate. The specification for each section is stated in the diagram and is used in the simulation of the detector.

The particles will enter through the window and interact with the Sodium Iodide crystal resulting in Compton scattering and the photoelectric effect. We can detect these interactions using the photomultiplier tube (PMT) which will also be part of the simulation for electrons created during the Compton and photoelectric interactions.

In order to simulate the detector we need to choose a suitable coordinate system specifically in spherical coordinates since our point like source emits particles radially. The detector is orientated such that it is aligned with the $z$-axis assuming the point source is at the origin of our system. According to the details given, the detector is placed centrally 25mm away from the point source.

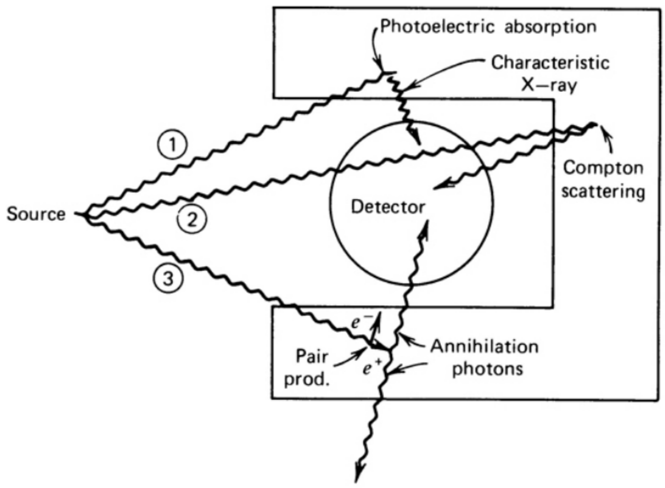


Figure 6: Influence of surrounding materials on detector response from Delone [1993]. When particles enter through the detector window, they interact in different ways through either Compton scattering, the photoelectric effect and pair production. For our investigation we will only be investigating Compton scattering and the photoelectric effect.

Using cylindrical coordinates we specify the coordinate $z_1$ as the aluminium casing where it is placed 25mm away from the source. $z_2$ denotes the Sodium Iodide crystal, where $z_3$ specifies the end of the crystal. From the given information we specify the length of the crystal to be 40mm long. To simplify our simulation we assume that the only interactions occurring is with the Sodium Iodide crystal and the incoming particles - therefore we are only working with one volume. To simplify our coordinate system even further we can set $\rho_1$ as the angle between the axis and the radius of the crystal and $\rho_2$ as the added casing thickness (of 2mm).

### Tracing the photons through different volumes of the detector

To simplify our system we assumed that we are only dealing with one volume, specifically the volume where the particles interact with the crystal. The dimension of this volume is specified to be 40mm $\times$ 40mm

## Tracing the scattered photons through the detector for the Compton scatter

## Energy spectrum measured by the detector - by the photomultiplier

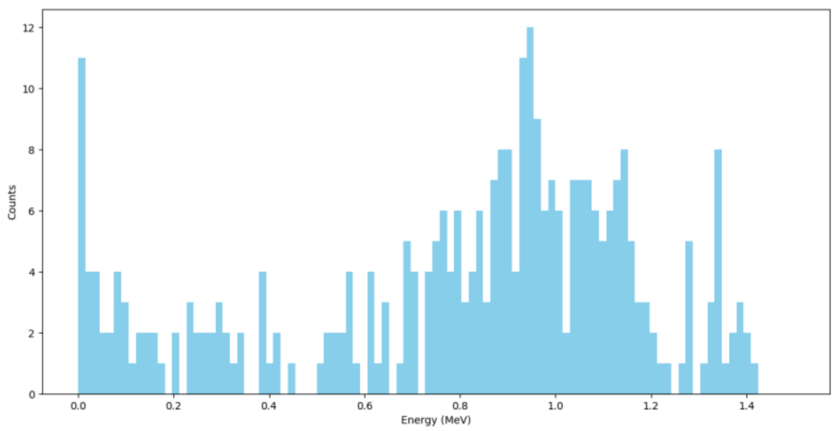


Figure 7: The energy spectrum for $^{60}$Co obtained from the simulated particles passing through the detector.

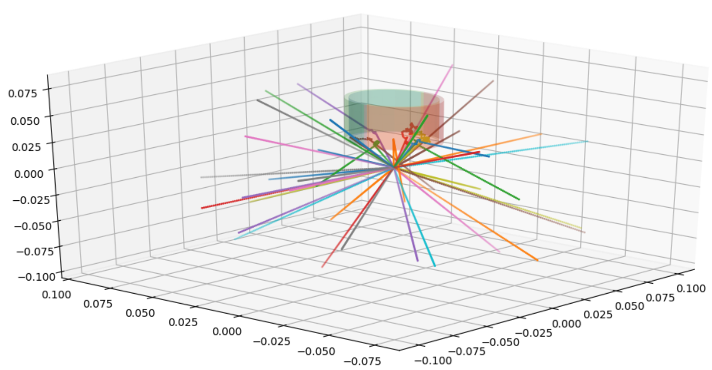

Figure 8: The photons being relased from the source, and entering and scattering within the cyldrincal detector

## Individual contributions

The investigation was divided up into two teams:

1. Particle generation and photon interactions due to the photoelectric effect and Compton scattering - Rayhaan Perin and Boitshoko Moetaesi
2. The detector simulation, tracing through the detector and resulting energy spectrum - Thavish Chetty and Vineshree Pillay
3. The code was edited by everyone therefore not one person did a specific sections of the code but rather everyone had a hand in every section.
4. The report was written up by all members of the group specifically teams Rayhaan and Boitshoko outlining how they did the particle generation and efficiency of photon detection. Sections regarding the detector simulation, tracing and energy spectrum but team Thavish and Vineshree.

## References


N. B. Delone. *Basics of interaction of laser radiation with matter*. Atlantica Séguier Frontières, 1993.

P. Khungurn. Probability Density under Transform. 2015.

E. Preoteasa. *ATOMIC AND NUCLEAR SURFACE ANALYSIS METHODS: A NOVEL PERSPECTIVE FOR THE CHARACTERIZATION OF DENTAL COMPOSITES*. PhD thesis, 2012.

O. N. Vasiliev. *Monte Carlo Methods for Radiation Transport*. Springer, 2018.