University of Cape Town

PHY4000W

Computational Physics Exam

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Abstract

This report focuses Particle accelerators where quadruple magnets are used to focus a particle beam going through the magnetic field,this is called strong focusing.The investigate the trajectories of particles moving through a magnetic quadrupole field and how this can be used to focus a particle beam.

1 Particle Trajectories

Figure 1 given the magnetic quadruple used for focusing.For the magnetic field given[1] the dynamical variable vector y(t) and the generalized velocity g(y; t) for the protons passing through an LHC quadruple magnet are given by

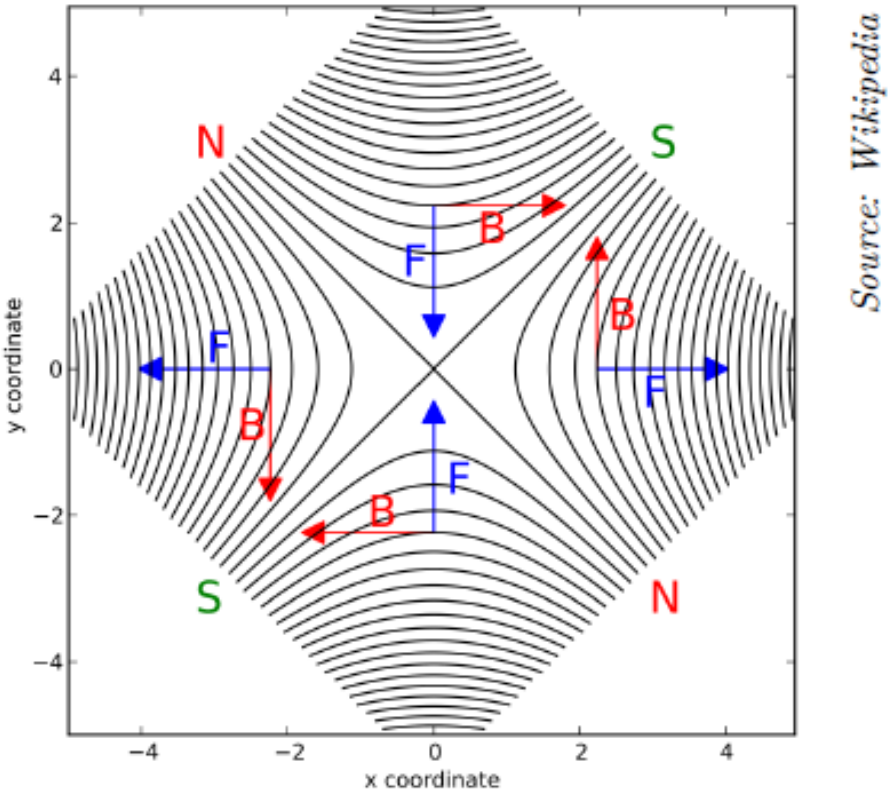

Figure 1: Quadruple magnetic field .

[1] given in the exam question

$$Y(t) = \begin{pmatrix} x0 \\ y0 \\ z0 \\ P_x \\ P_y \\ P_z \end{pmatrix} \tag{1}$$

$$g(Y,t) = \begin{pmatrix} P_x \\ P_y \\ P_z \\ -CP_z x \\ CP_z y \\ C(P_x x - P_y y) \end{pmatrix} \tag{2}$$

Were C is given by

$$C = \frac{eKc^2}{\sqrt{m^2c^4 + (P_x^2 + P_y^2 + P_z^2)c^2}} \tag{3}$$

These equations are obtained from Vojtech Horny[1].For the calculation we will use the nominal values of CERN's Large Hadron Collider (LHC), with proton beams of 7TeV passing through 3:10m-long quadrupole magnets with a field gradient(K) of 223Tm. The nominal beam position is along the z-axis at x = y = 0, where the magnetic field is zero. The protons may however be displaced with respect to the nominal position, or travel on an inclined trajectory. The vectors for a proton with x = 1mm, y = 0 mm and z = 0 mm moving parallel to the z-axis is given by

$$Y(t) = \begin{pmatrix} x0 \\ 0 \\ 0 \\ 0 \\ 0 \\ P_{z0} \end{pmatrix} \tag{4}$$

$$g(Y,t) = \begin{pmatrix} 0 \\ 0 \\ P_{z0} \\ -CP_{z0}x0 \\ 0 \\ 0) \end{pmatrix} \tag{5}$$

Here xo=0.01 m and $P_{z0} = 7Tev$

Integrating the generalized velocity vector using 4th order Runger Kutter method to trace the protons through the magnetic quadrupole field.The position (in metres) and momentum (in GeV=c) of a proton with initial position of x = 1mm and y = 0mm and an initial momentum in z-direction at the end of the quadruple is plotted for protons are moving parallel to the nominal beam line, entering the quadrupole

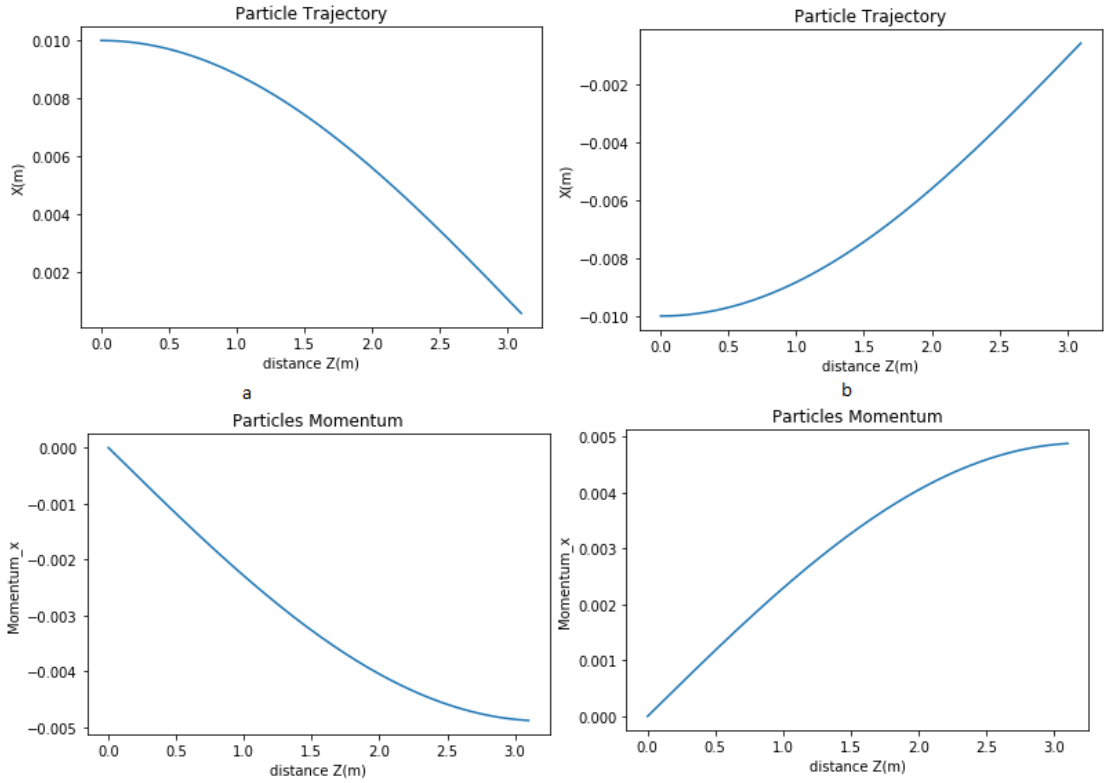

Figure 2: Particle trajectory's along when starting a the x or -x position.This shows that the quadrupole magnetic filed are horizontally focusing the protons.

magnet at x positions up to 1 cm and y = 0.The trajectories are given by.

Trajectories for protons moving parallel the nominal beam at y positions up to 1 cm and x = 0, i.e. in the plane perpendicular to the previous part. When protons start from a point on the z axis and enter the quadrupole magnet at a finite angle with respect to the beam axis, but still within the aperture of the beam pipe,the trajectory looks like this for when angle is 45 degrees with both the x.

The trajectories from part (c) are oscillatory like,Though they are not to scale the do show that quadripole magnetic field focus protons on one axis and defocuses them on another.The optical analogy of focusing a ray of light through a lens converges if

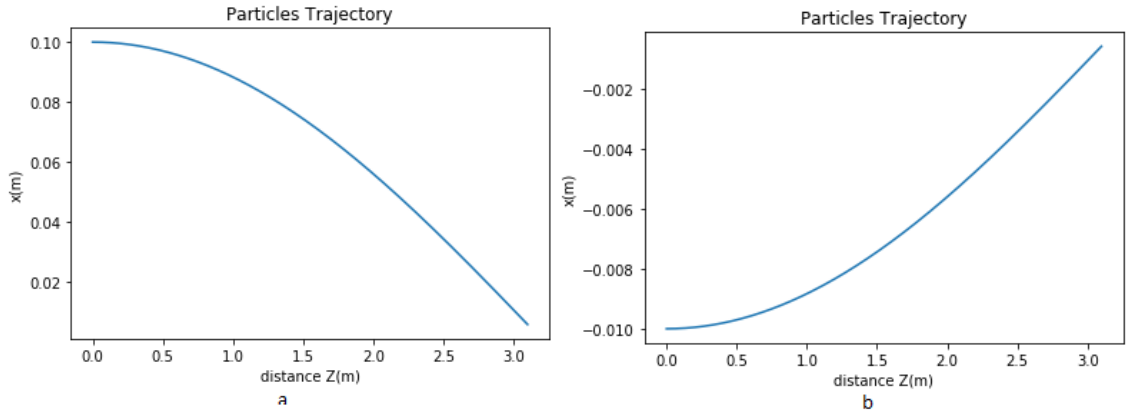

Figure 3: Particle trajectory's along when starting a the x or -x position.This shows that the quadrupole magnetic filed are horizontally focusing the protons.

lens concave or diverge if lens convex.

For the second question i only have an approximation but not a good one

2 References

[1] Vojtech Horny,Relativistic motion equation solver,February 21, 2016 `http://kfe.fjfi.cvut.cz/~horny/NME/NME-motionsolver/pohyboverovnice.pdf` [2]

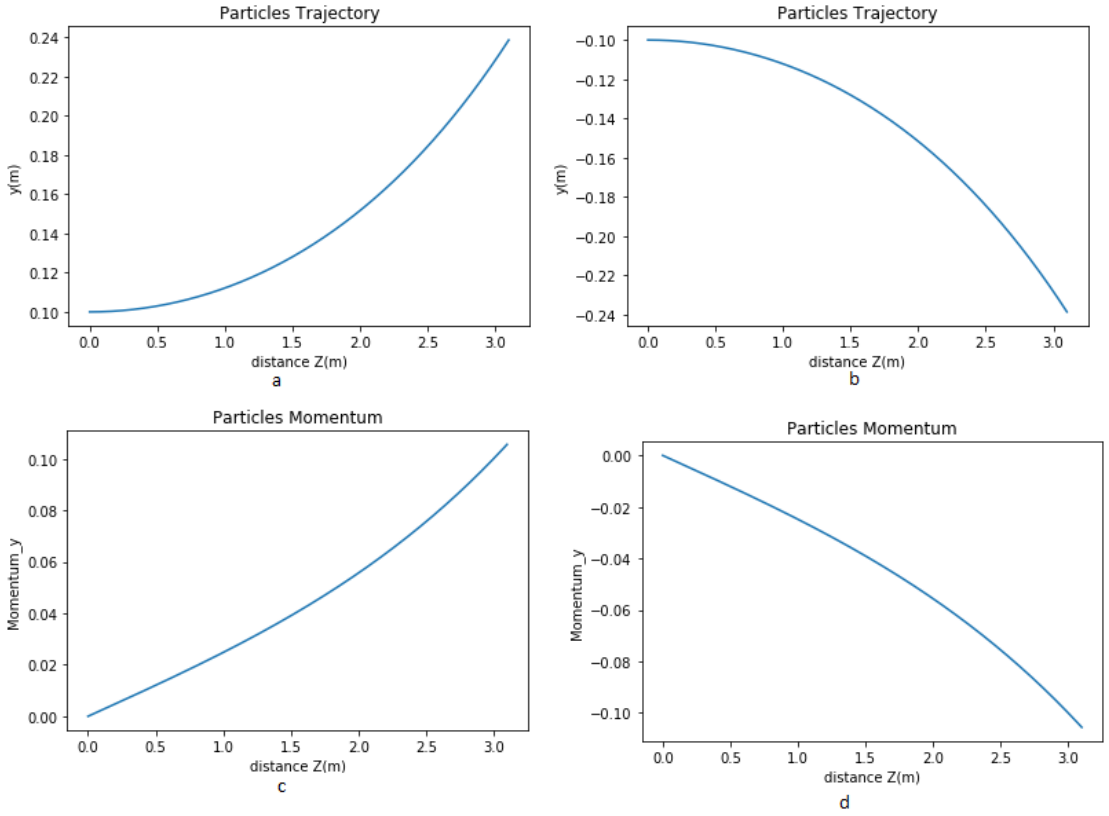

Figure 4: Particle trajectory's along when starting at y or -y position,momentum for the for the particle is also included.From the figure we can see that the quadrupole magnetic field are vertically de-focusing the protons.