University of Cape Town

PHY4000W

Computational Physics Tutorial 5

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Abstract

The aim of this report is to calculate use Monte-Carlo integration to Determine the volume of a 2 to 10 -dimensional sphere with radius r = 1.This is done by drawing random points in a n-dimensional cube and use the probability of them to lie in a sphere to estimate the volume of the sphere.The other objective is to Calculate the motion of a double pendulum, consisting of two mass-less rods with lengths L1 and L2, connecting two masses m1, m2.This is done by determining the behavior of this system for different starting values $\theta1, \theta2, \omega1, \omega2$ using the 4th order Runge-Kutta method.Trajectories of the mass m2 for initial conditions for chaotic and one non-chaotic behaviour were studied and plotted.

1 Monte Carlo Volumes

Volume of a hyper-sphere is give by

$$V = (\frac{\pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}+1\right)})R^n \tag{1}$$

Which we will use to compare to our estimations for n dimensional volume of the sphere. Here we will start by determining the volume of a 3-dimensional sphere with radius r = 1 using Monte-Carlo integration,this is done by drawing random points in a 3-dimensional cube and use the probability of them to lie in a sphere to estimate the volume of the sphere.n the convergence of the integration for a few runs is visualized by plotting the estimated value of the integral as a function of samples drawn as shown in figure 1.

As can be seen from the figure,the spread between the runs[1] is decreasing as we increase the number of trials. The simulation was run again but this time for a 10000 samples.To get this sample size,The simulation was ran for 1000 trails,at each trail 10 runs where done to determine the volume.The mean was calculated and the standard deviation was used to calculate the uncertainty of the mean as follows.

$$\sigma = \frac{\sigma_{N-1}}{\sqrt{N}} \tag{2}$$

The volume and uncertainty of the volume of the 3d sphere was determined to be 4.18 +/- 0.03888 and the results to the literature value is about 4.18879.Thus the

[1] Here the runs where a constant 10.

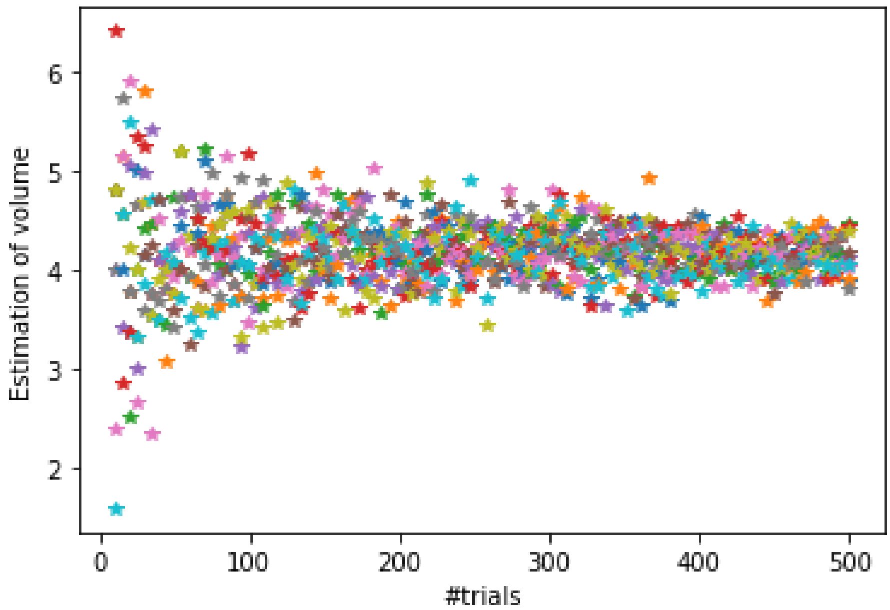


Figure 1: Volume estimators for 10 runs after of 100 trials

value is with the window of the true value.The analysis for hyper-sphere in 2 to 10 dimensions was estimated, the uncertainty ,volume and computation time was tabulated.

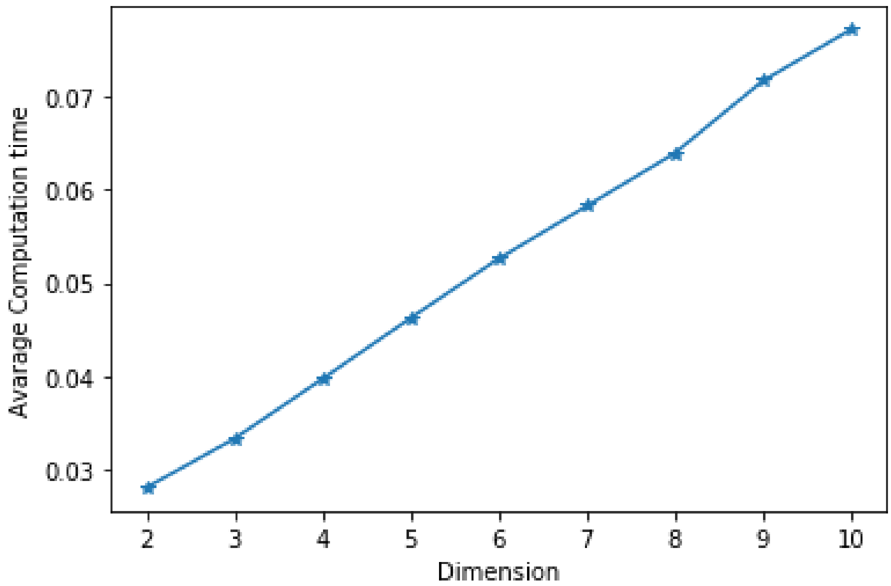


Figure 2: This plot shows how average computational time to calculate volume using using Monte-Carlo integration increases with increasing dimensions of the sphere.

Table 1:Estimated value,true value and uncertainly of an N-dimensional sphere's volume.The comp also computed.

Dimentions	Estimated Volume	Uncertainty	Calculated Volume	Averaged computational tim
3D	4.186	0.037	4.1887	0.0395
4D	4.920	0.073	4.934	0.044
5D	5.12	0.11	5.263	0.0514
6D	5.09	0.18	5.167	0.063
7D	5.07	0.25	4.724	0.074
8D	3.9424	0.32	4.059	0.078
9D	2.7136	0.36	3.299	0.0872
10D	2.46	0.50	2.550	0.0906

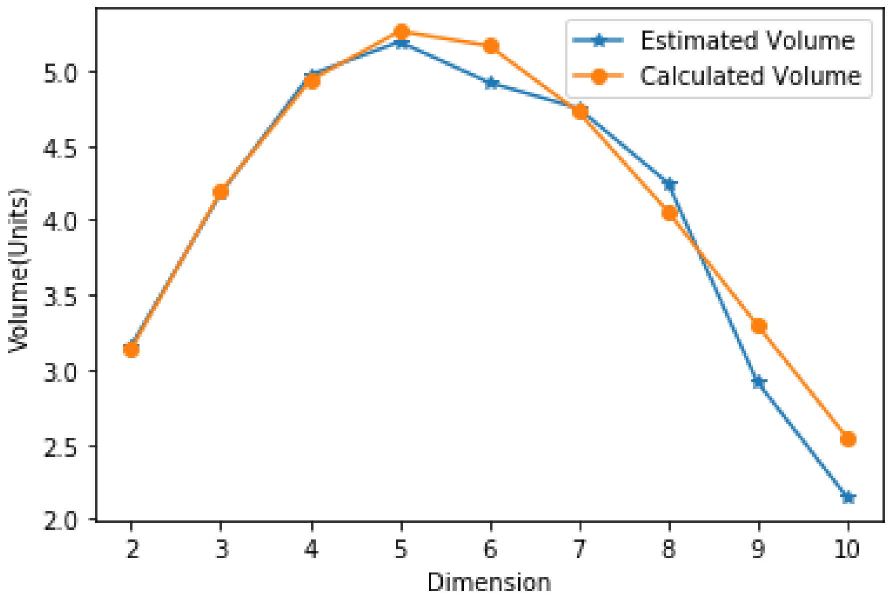


Figure 3: This plot shows volume obtained using using Monte-Carlo integration and volume obtained using equation 1^2 for hyper-sphere's of different dimensions.

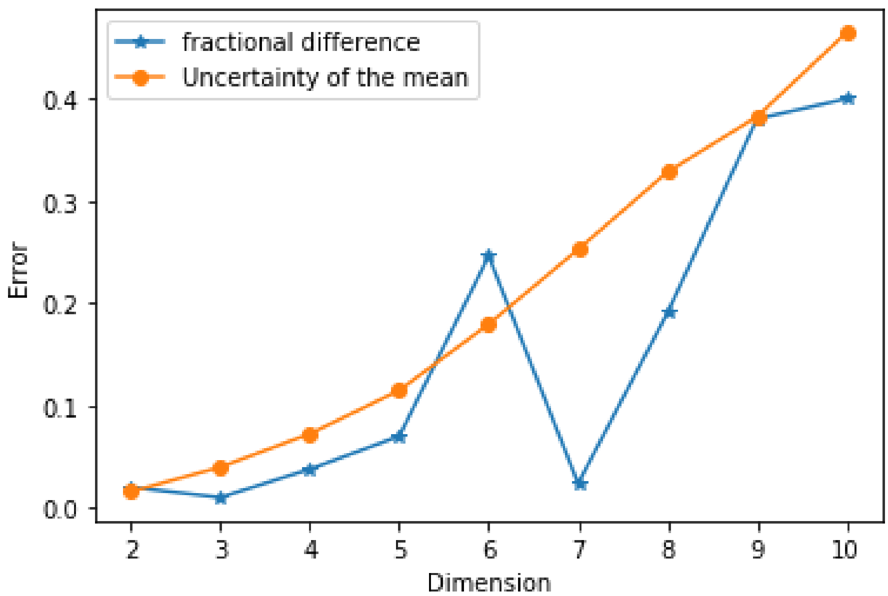


Figure 4: This plot shows uncertainty and difference between true value and estimated value of the volume of hyper-sphere in n dimensional space.

Figure 2 shows that as the dimensions increase the computational time increases linearly.This is expected because

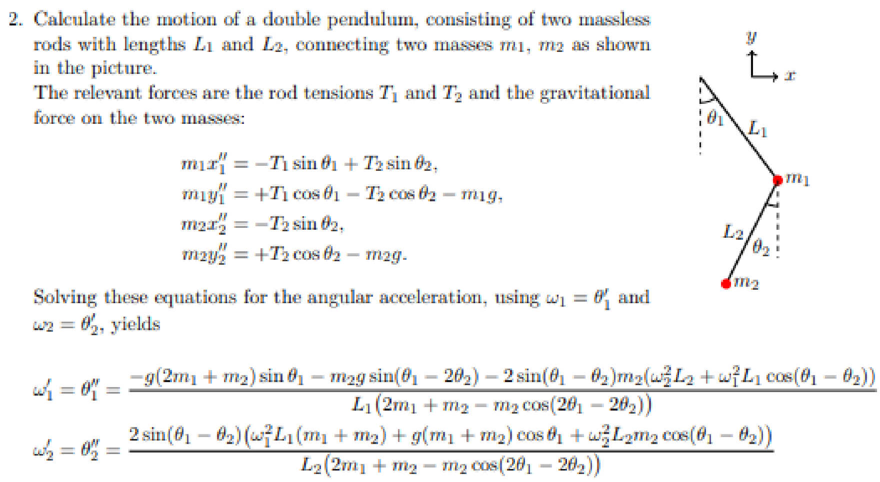
$$R = \sqrt{\sum_i x_i{}^2} \tag{3}$$

where R is the radius and x_i are the coordinates.thus adding a dimension increases the computational complexity by one operation.As the be seen from table 1 and figure 3,Monte Carlo integration does give a good approximation to the volume of the hyper-spheres.But figure 4 shows us that the error[3] grows and the number of dimensions increases,why?.

Because if we wanted to get better precision we would need to increase the number of trials as the number of dimensions increases.

2 Runge-Kutta Methods

Figure 5 shows the double pendulum problem.


2. Calculate the motion of a double pendulum, consisting of two massless rods with lengths L_1 and L_2, connecting two masses m_1, m_2 as shown in the picture.
The relevant forces are the rod tensions T_1 and T_2 and the gravitational force on the two masses:

$$m_1 x_1'' = -T_1 \sin\theta_1 + T_2 \sin\theta_2,$$
$$m_1 y_1'' = +T_1 \cos\theta_1 - T_2 \cos\theta_2 - m_1 g,$$
$$m_2 x_2'' = -T_2 \sin\theta_2,$$
$$m_2 y_2'' = +T_2 \cos\theta_2 - m_2 g.$$

Solving these equations for the angular acceleration, using $\omega_1 = \theta_1'$ and $\omega_2 = \theta_2'$, yields

$$\omega_1' = \theta_1'' = \frac{-g(2m_1+m_2)\sin\theta_1 - m_2 g\sin(\theta_1 - 2\theta_2) - 2\sin(\theta_1-\theta_2)m_2(\omega_2^2 L_2 + \omega_1^2 L_1\cos(\theta_1-\theta_2))}{L_1\big(2m_1 + m_2 - m_2\cos(2\theta_1 - 2\theta_2)\big)}$$
$$\omega_2' = \theta_2'' = \frac{2\sin(\theta_1-\theta_2)\big(\omega_1^2 L_1(m_1+m_2) + g(m_1+m_2)\cos\theta_1 + \omega_2^2 L_2 m_2\cos(\theta_1-\theta_2)\big)}{L_2\big(2m_1 + m_2 - m_2\cos(2\theta_1 - 2\theta_2)\big)}$$

Figure 5: Double pendulum problem

To determine the behavior of this system for different starting values $\theta 1, \theta 2, \omega 1$,$\omega 2$ using the 4^{th} order Runge-Kutta method. For some initial conditions, we will start by observing non-chaotic behaviour.Starting with a simple pendulum approximation.where m1>>m2 and θ1=0,$\theta 2 < \frac{\pi}{2}$.Trajectories are plotted bellow.

[3]this is for both uncertainty of the mean and difference.

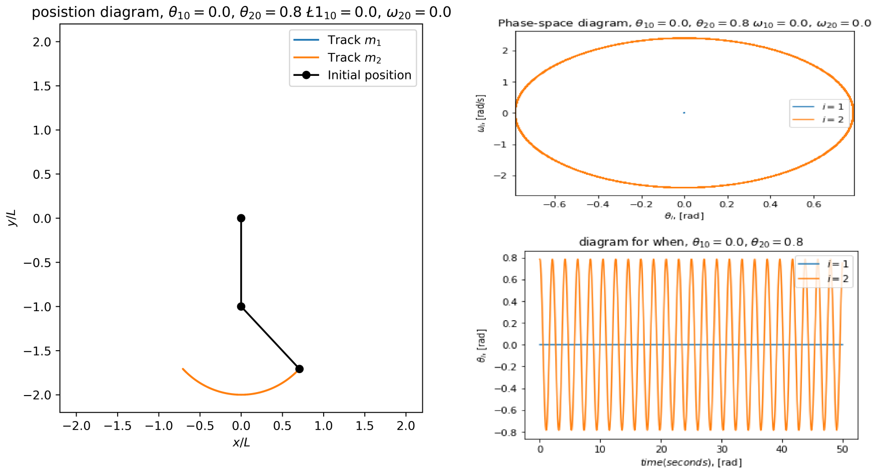


Figure 6: This figure shows the double pendulum, as a simple pendulum approximation.

As we expected.now study m1=m2,and variety of theta1,theta2 ect.

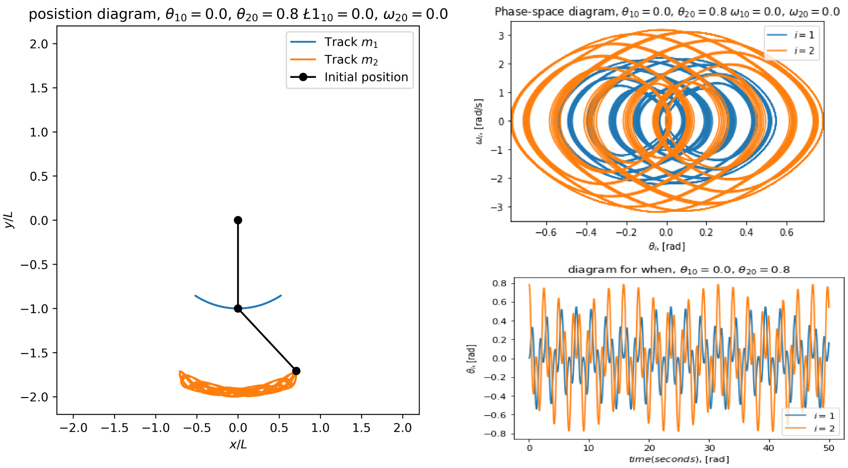


Figure 7: This figure shows chaotic double pendulum,for $\theta1=\pi/4$.

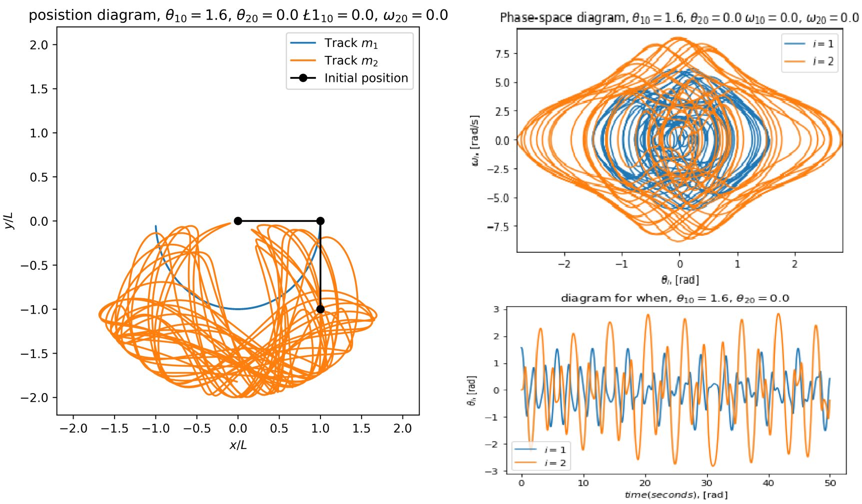


Figure 8: This figure shows the chaotic double pendulum, $\theta2=\pi/4$.

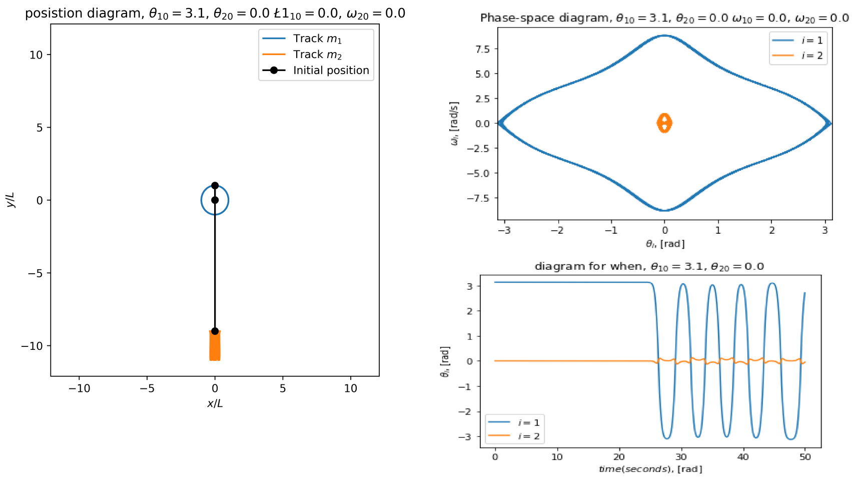


Figure 9: This figure shows a non chaotic double pendulum, $\theta2{=}\pi$.

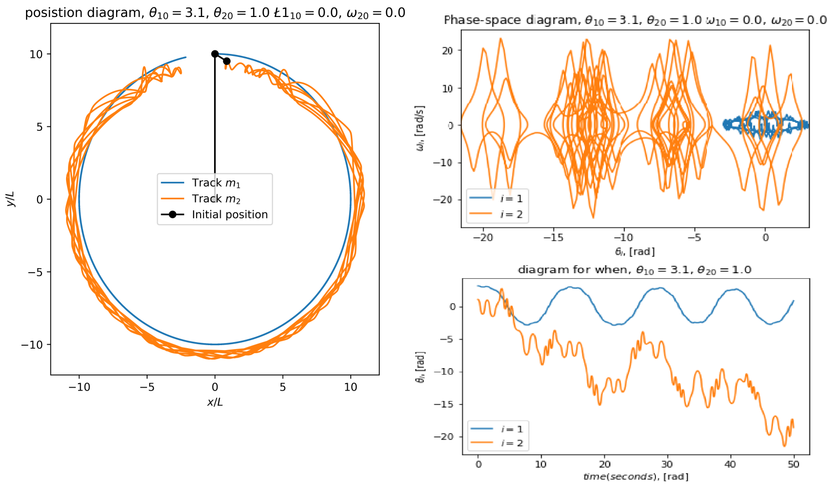


Figure 10: This figure shows the chaotic double pendulum,with initial $\theta2{=}\pi/3$,$\theta1{=}\pi$ and L1>L2