positions and momenta of the system, this will determine in a completely Newtonian way the distribution of the momenta and positions for any future time. It will then be possible to make statements about these distributions, and some of these will have the character of assertions that the future system will have certain characteristics with probability one, or certain other characteristics with probability zero.

Probabilities one and zero are notions which include complete certainty and complete impossibility but include much more as well. If I shoot at a target with a bullet of the dimensions of a point, the chance that I hit any specific point on the target will generally be zero, although it is not impossible that I hit it; and indeed, in each specific case I must actually hit some specific point, which is an event of probability zero. Thus an event of probability one, that of my hitting *some* point, may be made up of an assemblage of instances of probability zero.

Nevertheless, one of the processes which is used in the technique of the Gibbsian statistical mechanics, although it is used implicitly, and Gibbs is nowhere clearly aware of it, is the resolution of a complex contingency into an infinite sequence of more special contingencies—a first, a second, a third, and so on—each of which has a known probability; and the expression of the probability of this larger contingency as the sum of the probabilities of the more special contingencies, which form an infinite sequence. Thus we *cannot* sum probabilities in all conceivable cases, to get a probability of the total event—for the sum of any number of zeros is zero—while we *can* sum them if there is a first, a second, a third member, and so on, forming a sequence of contingencies in which every term has a definite position given by a positive integer.

The distinction between these two cases involves rather subtle considerations concerning the nature of sets of instances, and Gibbs, although a very powerful mathematician, was never a very subtle one. Is it possible for a class to be infinite and yet essentially different in multiplicity from another infinite class, such as that of the positive integers? This problem was solved toward the end of the last century by Georg Cantor, and the answer is "Yes." If we consider all the distinct decimal fractions, terminating or nonterminating, lying between 0 and 1, it is known that they cannot be arranged in 1, 2, 3 order—although, strangely enough, all the *terminating* decimal fractions can be so arranged. Thus the distinction demanded by the Gibbs statistical mechanics is not on the face of it an impossible one. The service of Lebesgue to the Gibbs