1 Selected Topics

1.1 Power Spectrum and Filtering

To investigate the properties of the power spectrum and the effects of filtering, we consider the simple discretely sampled sinusoidal signal:

$$V_j = A\sin(2\pi f t_j - \phi) \tag{1}$$

1.1.1 Power Spectra

The power spectrum of a continuous signal is simply the square of the continuous Fourier transform. However, when computing the power spectrum of a discretely sampled signal, the leakage of power from some frequencies to others is a problematic result of the digitization of the signal. To illustrate this, the power spectra for two slightly different frequencies are considered: $f = 60$Hz and $f = 59.673$Hz.

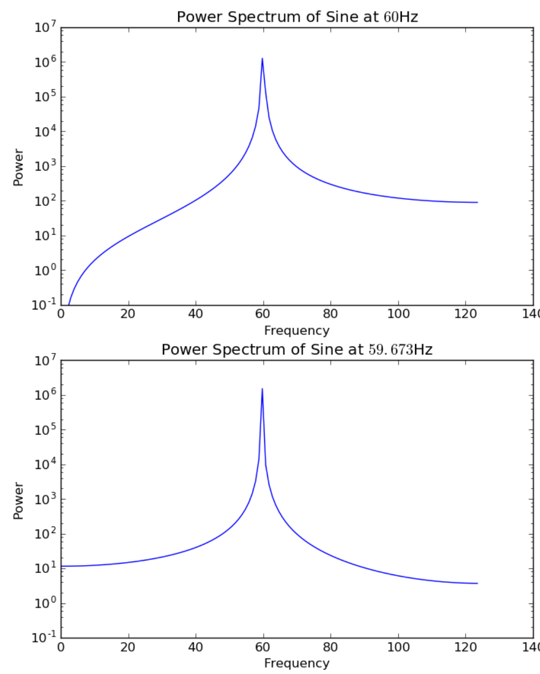

Figure 1: The power spectra of equation 1 with two very close frequencies.

1.1.2 Windows

One of the standard ways to reduce spectral leakage is to multiply the data in time space with a window before performing the Fourier transform. Two of the most common windows we look into are the Hann window and the Blackman-Harris window. The Hann function and its Fourier transform are shown in figure 2.

Now the Hann window can be applied to the two signals that were used previously with frequencies 60Hz and 59.673Hz, shown in figures 3 and 4, respectively.

2 General Applications

2.1 Heart Beats

The data gives a time sequence of heart beats sampled at 125Hz. Since the data is evenly

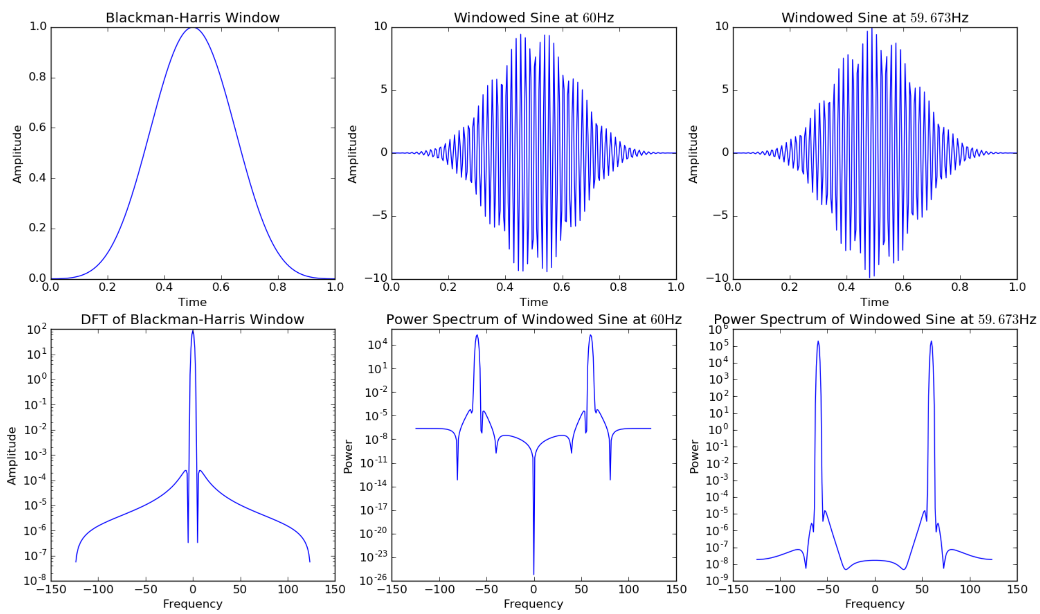

Figure 5: The Blackman-Harris window is also applied to equation **??** with frequencies $f = 60$Hz and $f = 69.673$Hz. The functions in time space are shown in the top row while the bottom row shows the Fourier transform of the Blackman-Harris window and the power spectra of the functions above.

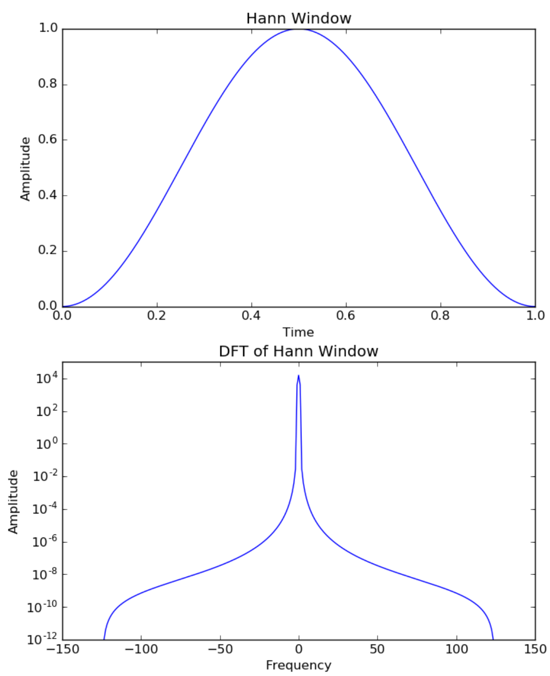

Figure 2: The Hann window (top) and its Fourier transform (bottom). It can be thought of as a portion of a cosine wave.

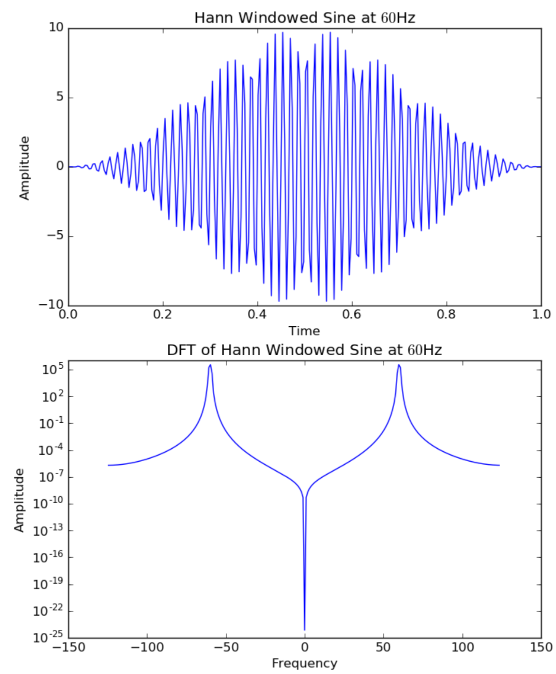

Figure 3: The Hann window applied to equation 1 with $f = 60$Hz in time space (top) and in frequency space (bottom).

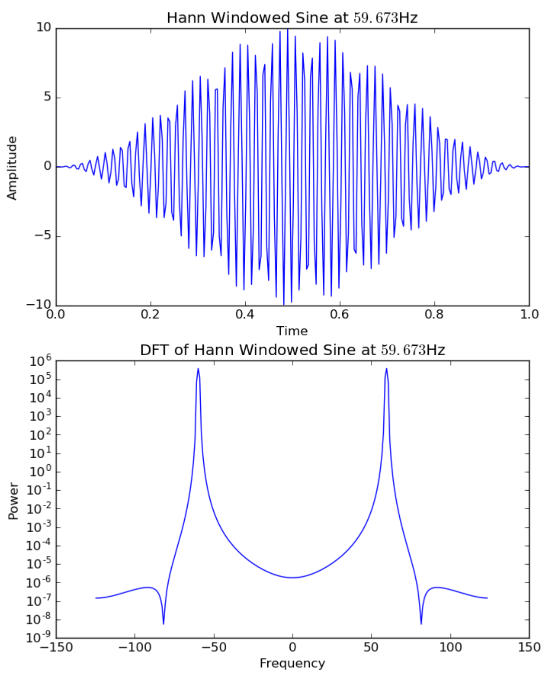

Figure 4: The Hann window applied to equation 1 with $f = 59.673$Hz in time space (top) and in frequency space (bottom).

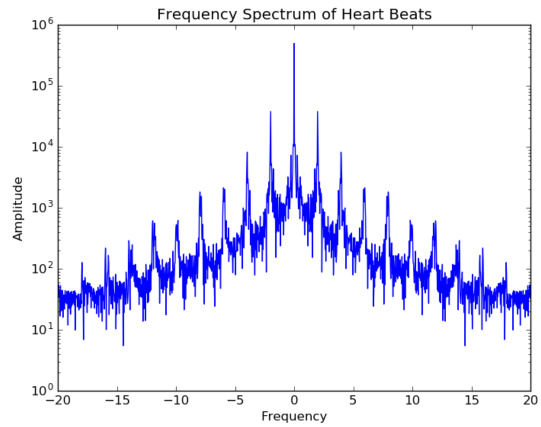

Figure 6: The amplitude spectrum of the patient's heart beats. The data was sampled at 125Hz and has 4096 samples.

sampled and the number of samples is a power of two, padding is not necessary in this case. The amplitude spectrum is shown in figure 6. The two most dominant frequencies are approximately 2Hz and 4kHz, with corresponding periods 0.5s and 0.25s, respectively.

The average resting heart rate is about 60 to 100 beats per minute. However, this translates to periods within a range of 1s to 1.6s, which are significantly faster than that of the patient. This would suggest that the patient was not at rest before the measurement or some other reason for a faster heart rate.

2.2 Financial Series

For this section, we analyze the stock prices for 6 companies, shown in figure 7.

This data is then used to compute the continuously compounded returns R_i, given by the

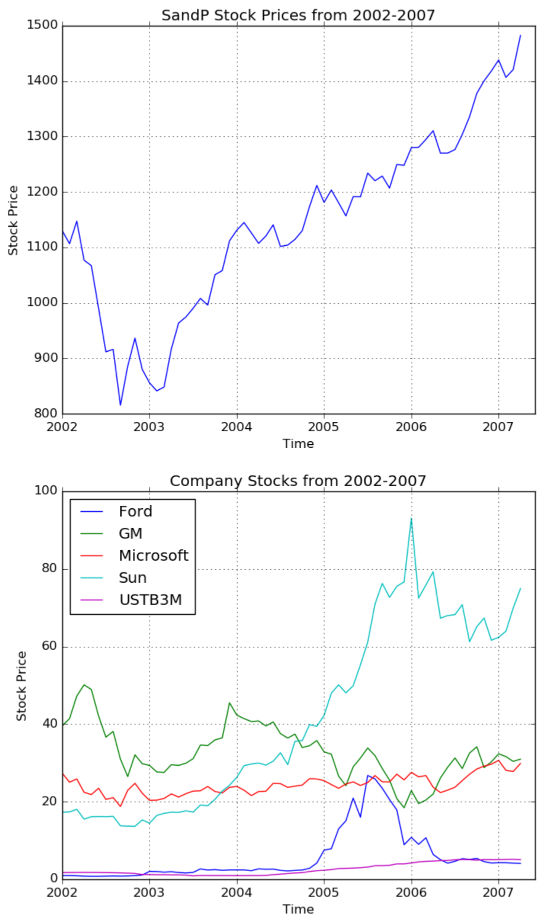

Figure 7: The monthly stock prices of six companies from 2002 through to mid-2007.

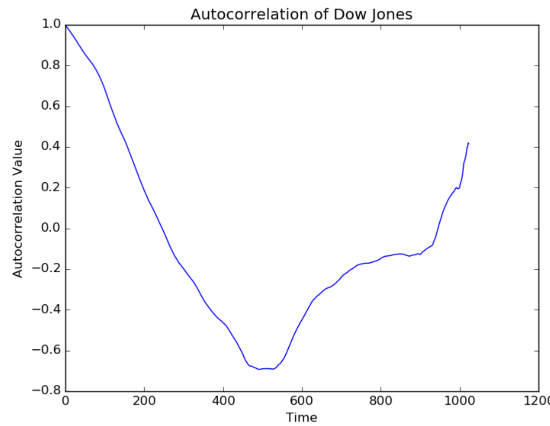

Figure 12: The autocorrelation values of the data shown in figure 11.

relation:

$$R_i = \ln\left(\frac{P_i}{P_{i-1}}\right) \tag{2}$$

which are shown in figure 8. The autocorrelation of the data points are shown in 9.

Next, we take a look at the daily closing value of the Dow Jones Industrial Average, which is a measure of the average stock price in the US market. In figure 11, the original data and its amplitude spectrum are shown.

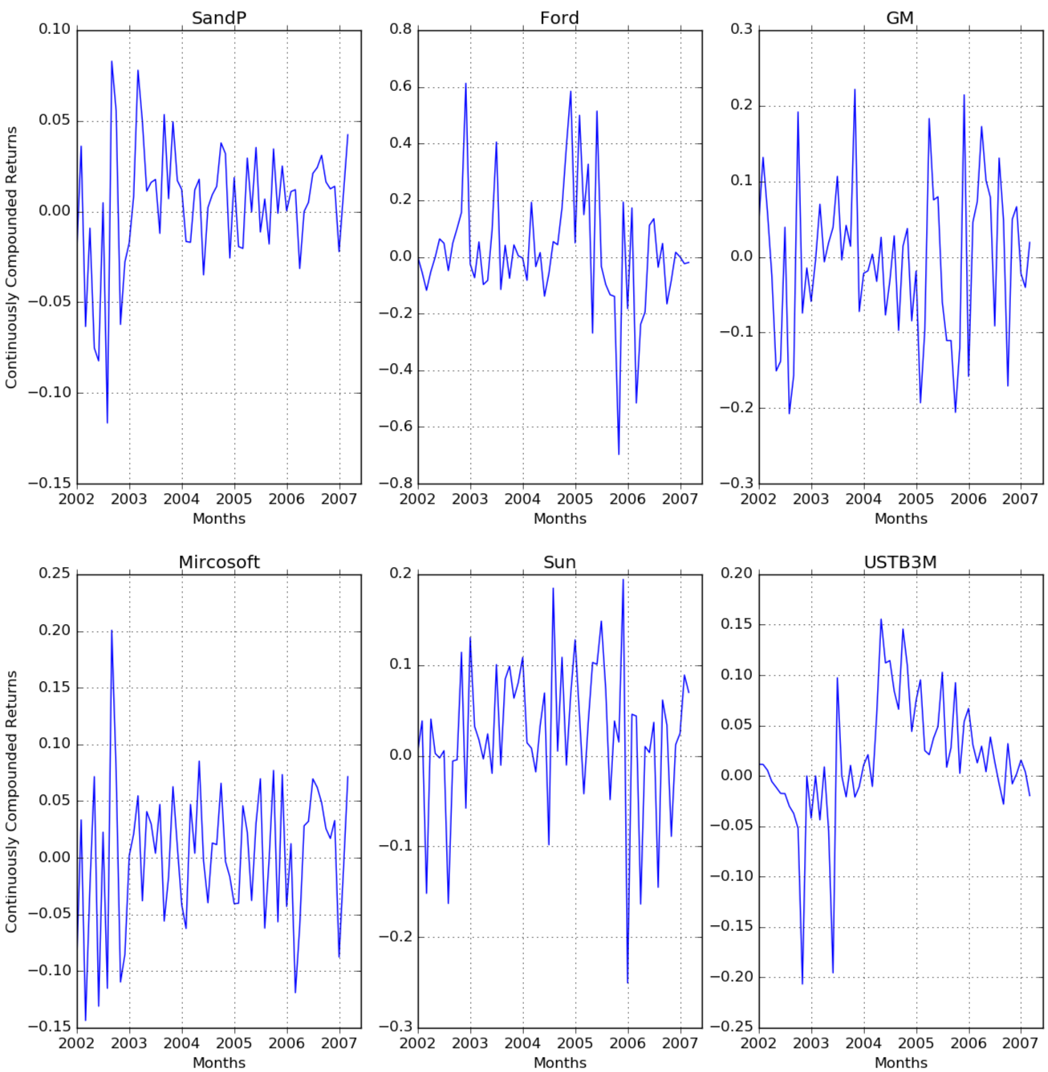

Figure 8: The continuously compunded returns of six companies. The returns were computed from the data in figure 7 using equation 2.

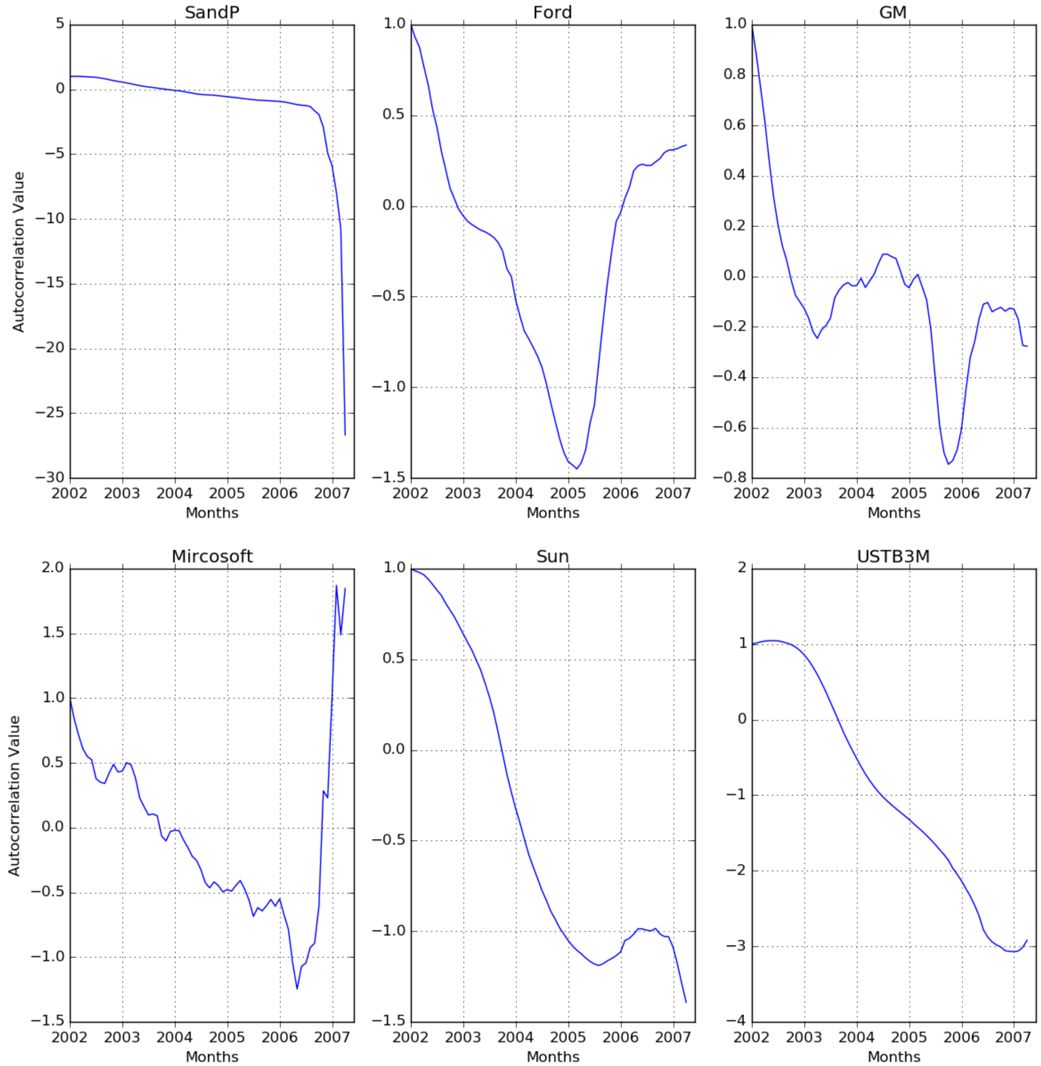

Figure 9: The autocorrelation data of six companies.

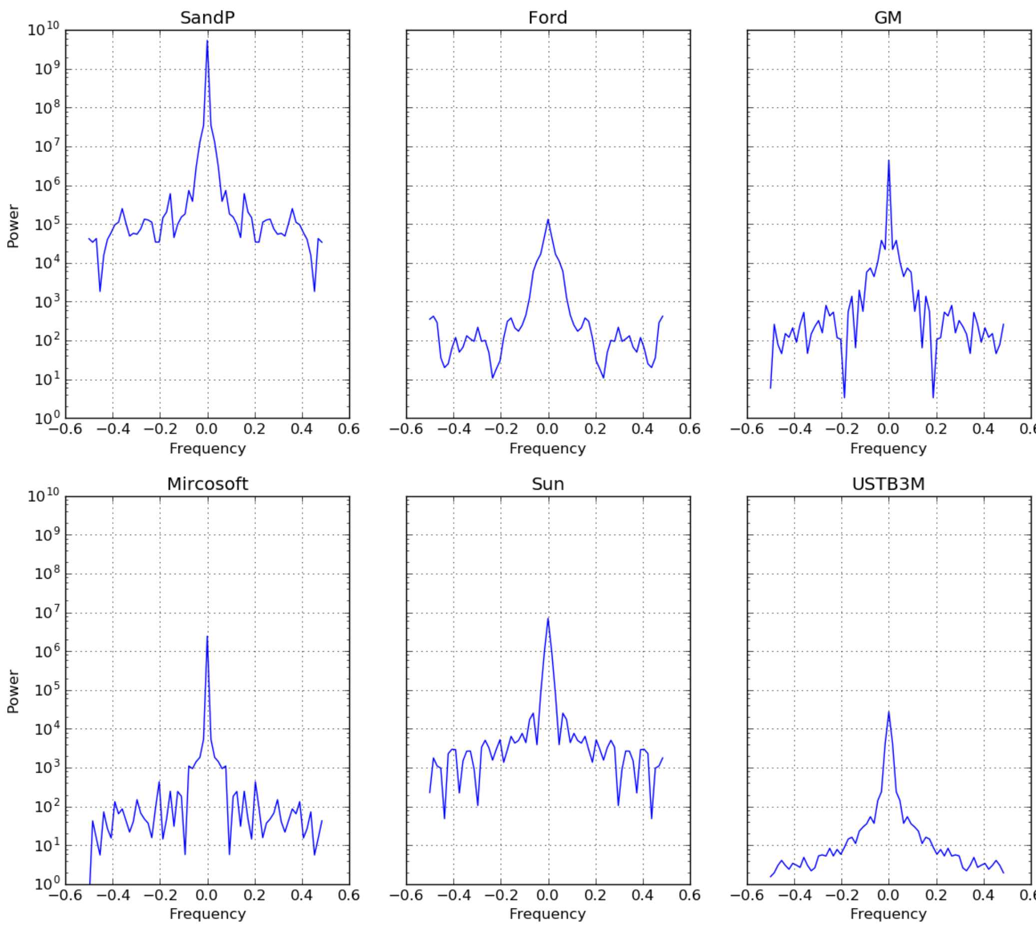

Figure 10: The power spectra of the stock prices of six companies. The frequencies are in units of per month.

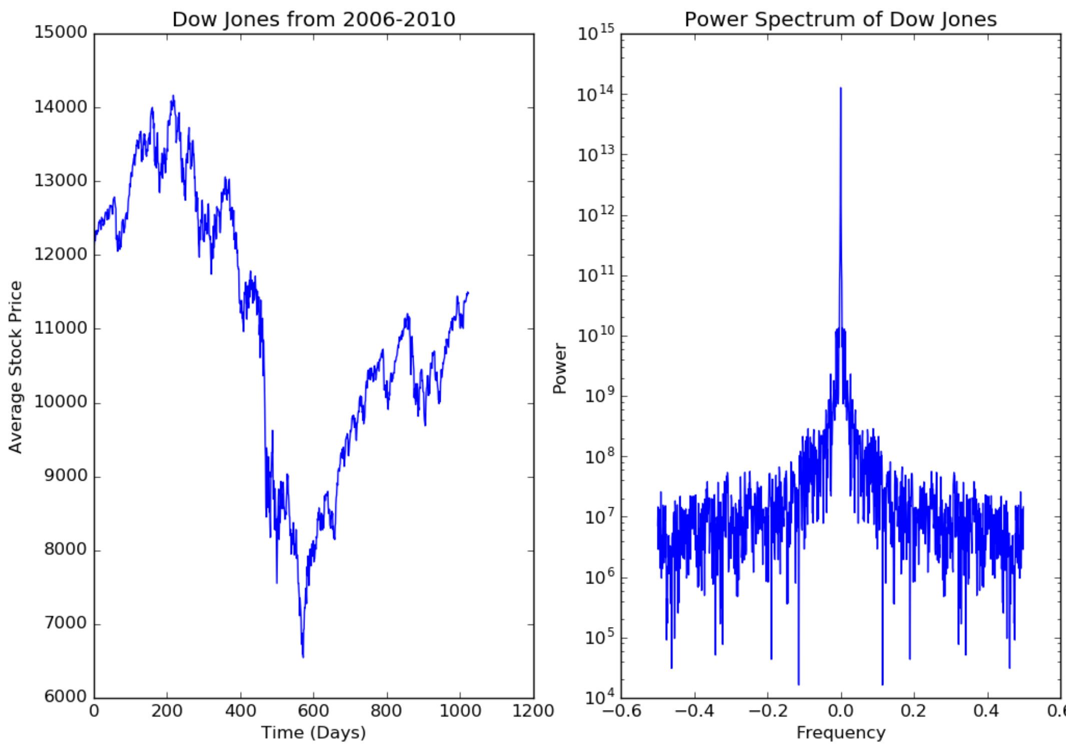

Figure 11: The given data of daily average stock prices from 2006 to 2010 is plotted on the left and its power spectrum is given on the right.