1 Selected Topics

1.1 DFT vs. FFT

This section investigates different aspects of the discrete Fourier transform (DFT) and its inverse (IDFT). First, consider the discrete boxcar frequency spectrum:

$$X(\omega) = \{\, 1\ , for|\omega| \leq \frac{\pi}{3} 0, otherwise \tag{1}$$

We can transform this back into time space using the inverse transform.

Both the forward and inverse Fourier transform can be thought of as a linear transformation on a vector, and thus can be represented through matrix multiplication, with each element corresponding to the equation

$$F_{nm} = e^{-2\pi i \frac{nm}{N}} F_{nm}^{-1} = \frac{1}{N} e^{2\pi i \frac{nm}{N}} \tag{2}$$

where N is the total number of points. To transform a 3-point vector, the forward transformation matrix is then the 3×3 matrix

$$F = e^{-\frac{2\pi}{3}i} e^{-\frac{4\pi}{3}i} e^{-2\pi i} e^{-\frac{4\pi}{3}i} e^{-\frac{8\pi}{3}i} e^{-4\pi i} e^{-2\pi i} e^{-4\pi i} e^{-6\pi i} \tag{3}$$

and its inverse is

$$F^{-1} = \frac{1}{3} ccce^{\frac{2\pi}{3}i} e^{\frac{4\pi}{3}i} e^{2\pi i} e^{\frac{4\pi}{3}i} e^{\frac{8\pi}{3}i} e^{4\pi i} e^{2\pi i} e^{4\pi i} e^{6\pi i} \tag{4}$$

We can easily confirm that these matrices are inverses of one another through matrix multiplication, or by using their definitions as written in equation **??**. The result of matrix multiplication should be the identity matrix, which can be proved by showing the diagonal elements (when $n = m$) are equal to 1:

$$\sum_{j=1}^{N} F_{nj} F_{jn}^{-1} = \frac{1}{N} \sum_{j=1}^{N} e^{-\frac{2\pi}{N} nji} e^{\frac{2\pi}{N} nji} = \frac{1}{N} \sum_{j=1}^{N} 1 = 1 \tag{5}$$

and that the off-diagonal elements (when nm)are equal to zero:

$$\sum_{j=1}^{N} F_{nj} F_{jn}^{-1} = \frac{1}{N} \tag{6}$$