Written Section

1. (a) ${}^{A}T_{B} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & c\theta & -s\theta & 2 \\ 0 & s\theta & c\theta & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(b) ${}^{B}T_{A} = {}^{A}T_{B}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & c\theta & s\theta & (-2c\theta - 3s\theta) \\ 0 & -s\theta & c\theta & (2s\theta - 3c\theta) \\ 0 & 0 & 0 & 1 \end{bmatrix}$

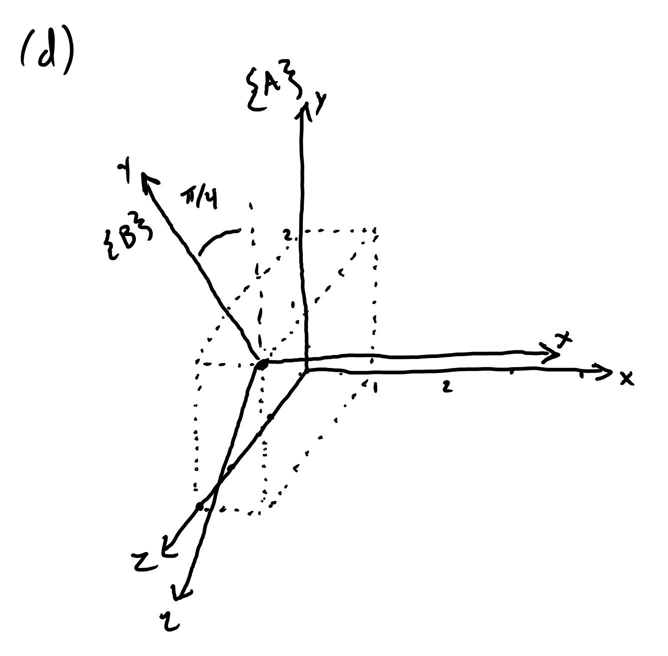
(c) ${}^{B}p = {}^{B}T_{A}\,{}^{A}p$

$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 & (-2\sqrt{2}/2 - 3\sqrt{2}/2) \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 & (2\sqrt{2}/2 - 3\sqrt{2}/2) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 4 - 1 \\ 5\sqrt{2}/2 + 6\sqrt{2}/2 - \sqrt{2} - 3\sqrt{2}/2 \\ -5\sqrt{2}/2 + 6\sqrt{2}/2 + \sqrt{2} - 3\sqrt{2}/2 \\ 1 \end{bmatrix}$

${}^{B}p = \begin{bmatrix} 3 \\ 3\sqrt{2} \\ 0 \end{bmatrix}$

(d)



2. ${}^{A}R_{C} = \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$

$= \begin{bmatrix} c\phi c\theta & -s\phi & c\phi s\theta \\ s\phi c\theta & c\phi & s\phi s\theta \\ -s\theta & 0 & c\theta \end{bmatrix}$

3. (a) ${}^{0}T_{1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

${}^{0}T_{2} = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

${}^{0}T_{3} = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(b) ${}^{3}T_{2} = {}^{3}T_{0}\,{}^{0}T_{2}$

$= \begin{bmatrix} 0 & 1 & 0 & -1.5 \\ 1 & 0 & 0 & 0.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(c) ${}^{0}\text{Cube} = {}^{0}T_{3}\,{}^{3}\text{Cube}$

$= \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2 \\ -0.3 \\ 2 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} -0.3 - 0.5 \\ 0.2 + 1.5 \\ -2 + 3 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} -0.8 \\ 1.7 \\ 1 \end{bmatrix}$

(d) ${}^{3}\text{cube} = {}^{3}T_{0}\,{}^{0}T_{1}\,{}^{1}\text{cube}$

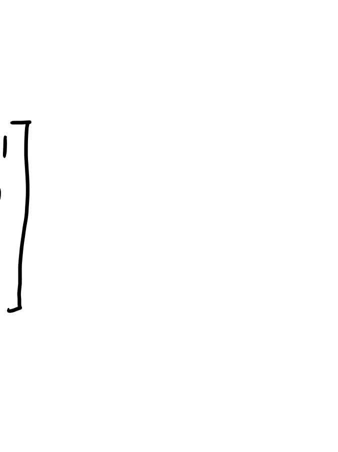
$= \begin{bmatrix} 0 & 1 & 0 & -1.5 \\ 1 & 0 & 0 & 0.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.1 \\ 0.1 \\ 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 0.5 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.1 \\ 0.1 \\ 0 \\ 1 \end{bmatrix}$

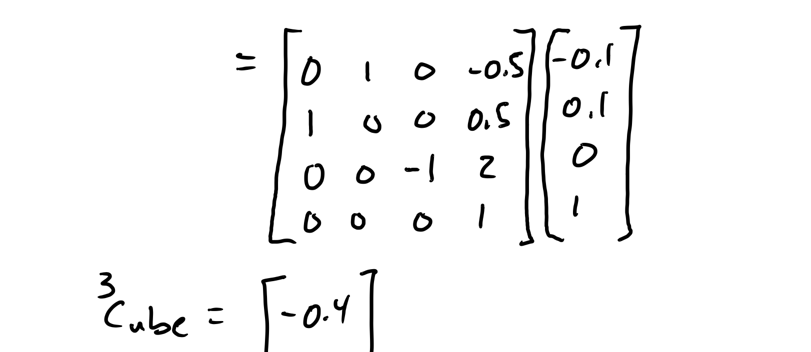
${}^{3}\text{Cube} = \begin{bmatrix} -0.4 \\ 0.4 \\ 2 \end{bmatrix}$

4. modified DH

${}^{i-1}T_{i} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$



(a)



${}^{0}T_{3} = {}^{0}T_{1}\,{}^{1}T_{2}\,{}^{2}T_{3}$

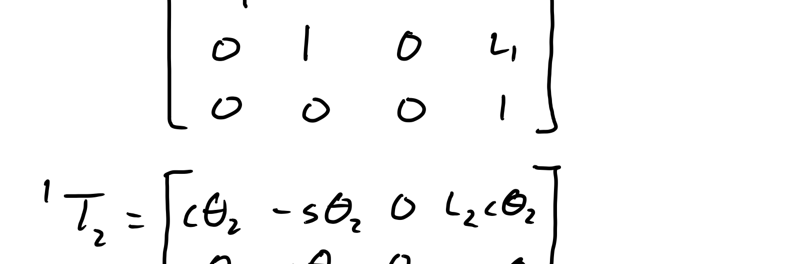
i	α_i	a_i	d_i	θ_i
1	90	0	L_1	θ_1
2	0	L_2	0	θ_2
3	0	L_3	0	θ_3

${}^{0}T_{1} = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

${}^{1}T_{2} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & L_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

${}^{2}T_{3} = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & L_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(b)



${}^{0}T_{3} = {}^{0}T_{1}\,{}^{1}T_{2}\,{}^{2}T_{3}$

i	α_i	a_i	d_i	θ_i
1	90	0	L_1	θ_1
2	0	L_2	0	θ_2
3	90	0	d_3	0

${}^{0}T_{1} = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

${}^{1}T_{2} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & L_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

${}^{2}T_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

5. (a) $r^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos\beta$

$\cos\beta = \dfrac{l_1^2 + l_2^2 - r^2}{2 l_1 l_2}$

(b) $\theta_2 = 180 - \beta$

$\theta_2 = 180 - \cos^{-1}\left(\dfrac{l_1^2 + l_2^2 - r^2}{2 l_1 l_2}\right)$

(c) $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\dfrac{\sin\theta_2}{d} = \dfrac{\sin 90}{l_2}$

$d = l_2 \sin\theta_2$

$\gamma = \sin^{-1}\left(\dfrac{d}{r}\right)$

$\gamma = \sin^{-1}\left(\dfrac{l_2 \sin\theta_2}{r}\right)$

(d) $\alpha = \sin^{-1}\left(\dfrac{y_w}{r}\right)$

$\theta_1 = \alpha - \gamma$

$= \sin^{-1}\left(\dfrac{y_w}{r}\right) - \gamma$

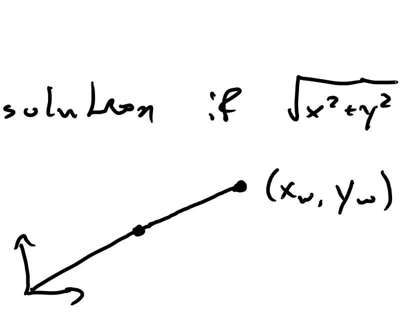
(e) zero solutions if:

$\sqrt{x^2+y^2} > (l_1 + l_2)$

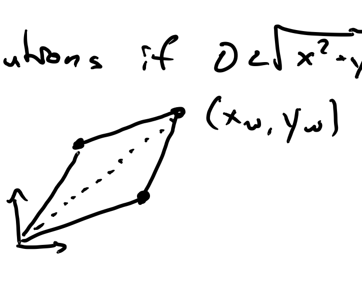
or

$l_2 \neq l_1$ AND $\sqrt{x^2+y^2} < |l_1 - l_2|$

one solution if $\sqrt{x^2+y^2} = l_1 + l_2$



two solutions if $0 < \sqrt{x^2+y^2} < l_1 + l_2$



inf solutions if $\sqrt{x^2+y^2} = 0$ and $l_1 = l_2$

$0 \leq \theta_1 \leq 360°$

$\theta_2 = 180°$