הטכניון—מכון טכנולוגי לישראל, הפקולטה להנדסת מכונות

TECHNION—Israel Institute of Technology, Faculty of Mechanical Engineering

Advanced Control Laboratory (034406)

Spring Semester, 2022/2023 Academic Year

Preparatory Work 1

Question 1 (30%)

Two dynamic systems are defined by the following transfer functions:

$$G_1(s) = \frac{15^2}{s^2 + 30s + 15^2}, \quad G_2(s) = \frac{15s}{s^2 + 30s + 15^2}.$$

What are the static gains of the systems above? What does a static gain mean in the sense of the unit step input response? Plot in MATLAB the unit step responses of these systems (attach the appropriate code).

Question 2 (70%)

A direct current motor (DC motor) is connected to a load. This load is described by the following equation: $J\ddot{\theta} + b\dot{\theta} = T_l$, where T_l is the torque that acts on the load, and θ is the rotation angle of the load. The physical parameters are:

- $K_m = 39.6[\frac{\text{mNm}}{\text{A}}]$ - motor torque constant
- $K_b = 6.46[\frac{\text{mV}}{\text{rpm}}]$ - back electromotive force constant
- $R_a = 6.8[\Omega]$ - electric resistance of the armature
- $L_a = 620[\mu\text{H}]$ - electric inductance of the armature
- $J = 0.06[\text{kg}\,\text{m}^2]$ - moment of inertia of the load
- $f = 0.01[\frac{\text{Nm sec}}{\text{rad}}]$ - viscous friction constant of the load
- moment of inertia of the motor rotor is insignificant
- viscous friction constant in the motor rotor is negligible

Tasks:

1. (15%) Assuming a direct connection between the motor rotor and the load, build the block diagram that presents the DC motor and the load, where the input is the applied voltage V, and the output is the load rotation angle θ.
   <u>Remark</u>: it is recommended to model the DC motor and the load by equations, and then to build the appropriate block diagram.
2. (15%) What is the transfer function $P(s)$ of the process from the applied voltage V to the load rotation angle θ? Why can the value of L_a be negligible? Is the process $P(s)$ stable?
3. (5%) Simulate in Simulink the responses of $P(s)G_1(s)$ and $P(s)G_2(s)$ to the unit step input, where $G_1(s)$ and $G_2(s)$ are defined in the previous problem. Save and display these responses with respect to simulation time by the block *Scope* (see the help for this block), and prepare the code in MATLAB to plot the saved in the block *Scope* responses.
   Are the systems $P(s)G_1(s)$ and $P(s)G_2(s)$ stable?

4. (5%) Repeat the previous question with the simulation in MATLAB (not in Simulink) by function *step* (see the help for this function). Compare the results of the previous question and this one.

5. (20%) Repeat the tasks 1 and 2 assuming the motor rotor and the load are connected by the gear with the gear ratio of $N_r : 1$ (N_r rotations of the motor rotor cause one rotation of the load).

6. (10%) Assume $P(s)$ is the transfer function from the voltage $\widetilde{V}$ to the load rotation angle $\widetilde{\theta}$ relative to some reference frame. Also assume that the positive rotation direction of $\widetilde{\theta}$ is counterclockwise relative to the above mentioned frame. It is known that a positive $\widetilde{\theta}$ is obtained from a positive voltage input $\widetilde{V}$. Obviously a negative $\widetilde{\theta}$ is obtained from a negative voltage input $\widetilde{V}$.
   In the laboratory implementation of the system, the voltage V (the same units as $\widetilde{V}$ and $|V| = |\widetilde{V}|$) rotates the load by the rotation angle θ (the same units as $\widetilde{\theta}$ and $|\theta| = |\widetilde{\theta}|$), but the positive rotation direction of θ is clockwise relative to the reference frame defined for $\widetilde{\theta}$. The laboratory apparatus is built so that a positive θ is obtained from a positive voltage input V, and a negative θ is obtained from a negative voltage input V.
   Prove that the transfer function from V to θ is given by $P(s)$.
   Note: find constants K_i and K_o in Figure 1, and prove that $K_i \cdot K_o = 1$.

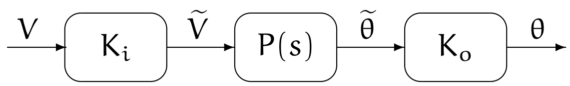

Figure 1: Laboratory implementation of $P(s)$