Advanced Control Laboratory Preparation Work 1

Zhiyuan Li 941190894
Shubhankar Jadhav 941190704

Question 1

For $G_1(s) = \frac{15^2}{s^2+30s+15^2}$, the static gain equals to $G_1(0) = \frac{15^2}{15^2} = 1$

For $G_2(s) = \frac{15s}{s^2+30s+15^2}$, the static gain equals to $G_2(0) = \frac{0}{15^2} = 0$

The static gain tells the ratio of the output and the input under steady state condition. For a unit step input, static gain is the steady state output.

Shown below is the unit step response of $G_1(s)$ and $G_2(s)$, as we can see, the steady state output is consistent with the static gain we calculated before. Related MATLAB code can be found in the 1.1 section of Appendix.

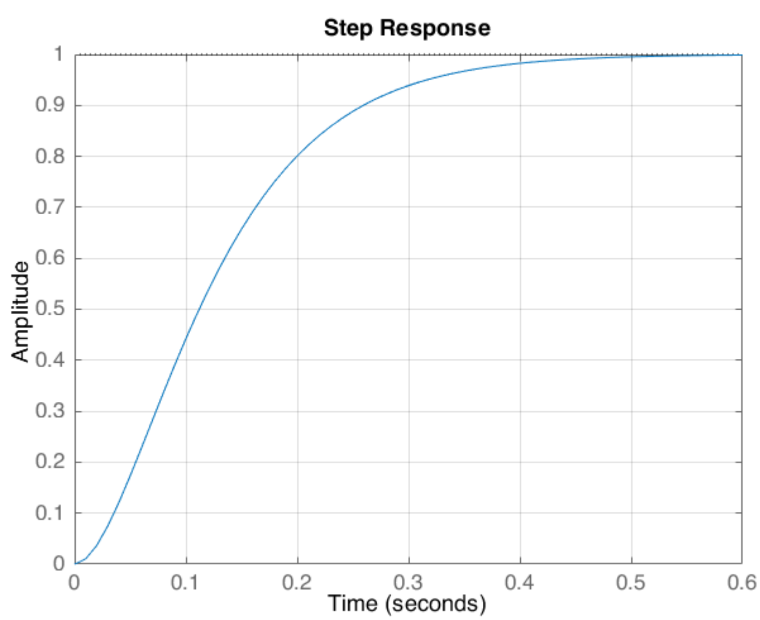


Figure.1 Unit step response of $G_1(s)$

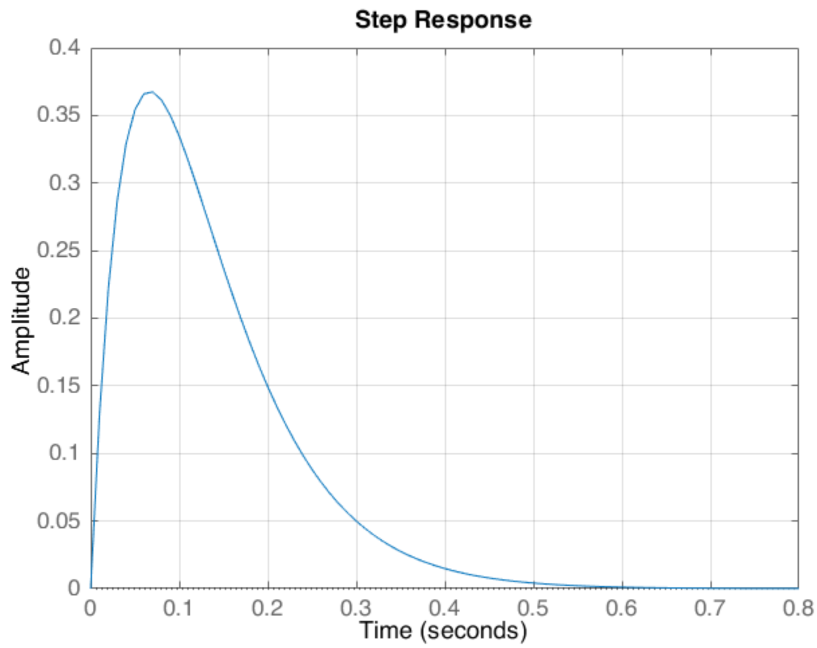


Figure.2 Unit step response of $G_2(s)$

Question 2

1.

For a classic DC motor, we have

$$\begin{cases} R_a I(t) + L_a \dot{I}(t) = V(t) - V_e(t) \\ V_e(t) = K_b \omega(t) = K_b \dot{\theta}(t) \\ T_1 = K_m I(t) \end{cases}$$

The substitute 2^{nd} equation into the 1^{st}, we get

$$\begin{cases} R_a I(t) + L_a \dot{I}(t) = V(t) - K_b \dot{\theta}(t) \\ T_1 = K_m I(t) \end{cases}$$

Apply Laplace transform on both sides of 1^{st} rearranged equation, we get

$$R_a I(s) + sL_a I(s) = V(s) - sK_b \theta(s)$$

Rearranging the equation,

$$I(s) = G_{VI}(s)V(s) - G_{\theta I}(s)\theta(s)$$

Where,

$$G_{VI}(s) = \frac{1}{L_a s + R_a}, G_{\theta I}(s) = \frac{K_b s}{L_a s + R_a}$$

Also, we are given that

$$J\ddot{\theta} + f\dot{\theta} = T_1$$

Apply Laplace transform on both sides, we get

$$s^2 J\theta(s) + sf\theta(s) = T_1(s)$$

Thus, the transform function from T_1 to θ can be written as

$$G_{T\theta} = \frac{1}{Js^2 + fs}$$

Therefore, the block diagram can be built in the following way

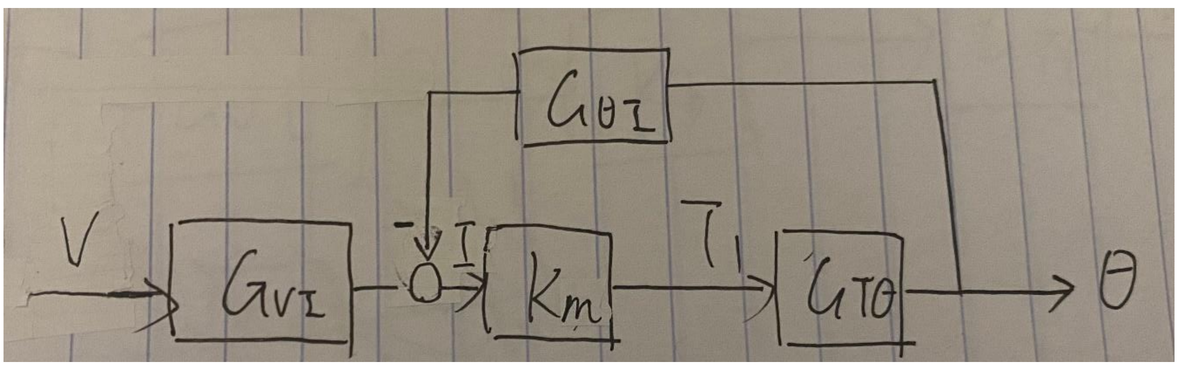


Figure.3 block diagram of the system.

<u>2.</u>

From the block diagram we built, the overall transform can be written as

$$P(s) = \frac{G_{VI} G_{T\theta} K_m}{1 + G_{T\theta} G_{\theta I} K_m} = \frac{\frac{1}{L_a s + R_a} \cdot \frac{1}{Js^2 + fs} \cdot K_m}{1 + \frac{1}{Js^2 + fs} \cdot \frac{K_b s}{L_a s + R_a} \cdot K_m}$$
$$= \frac{K_m}{JL_a s^3 + \ (L_a f \ + JR_a)s^2 + \ (R_a f \ + \ K_b K_m)s}$$

Substituting the numerical values of given parameters, we get

$$P(s) = \frac{1065}{s^3 + 10968\, s^2 + 1894\, s}$$

It can be observed that the 3rd order term on the dominator which include L_a are many orders of magnitude smaller than other terms. For the 2nd order term on the dominator, it can also be calculated that $JR_a = 0.408$, while $L_a f = 6.2e - 6$ with the same unit. Therefore, the value of L_a affects the transfer function in a negligible way and can assume to be zero in the calculation. When assuming $L_a = 0[H]$, $P(s)$ becomes

$$P(s) = \frac{K_m}{JR_a s^2 + (R_a f + K_b K_m)s} = \frac{0.09706}{s^2 + 0.1727\, s}$$

It is obvious that the $P(s)$ is unstable since it contains a pole on the origin, and it will diverge in response to a unit step input.

3.

As is shown below, $P \cdot G_1(s)$ is unstable, and $P \cdot G_2(s)$ is stable.

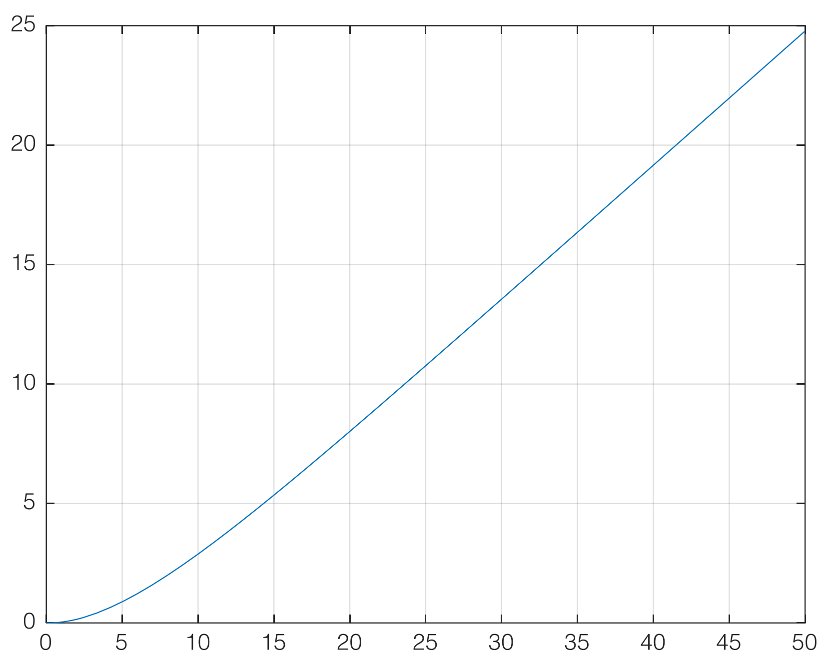


Figure.4 step response of $P \cdot G_1(s)$ in Simulink

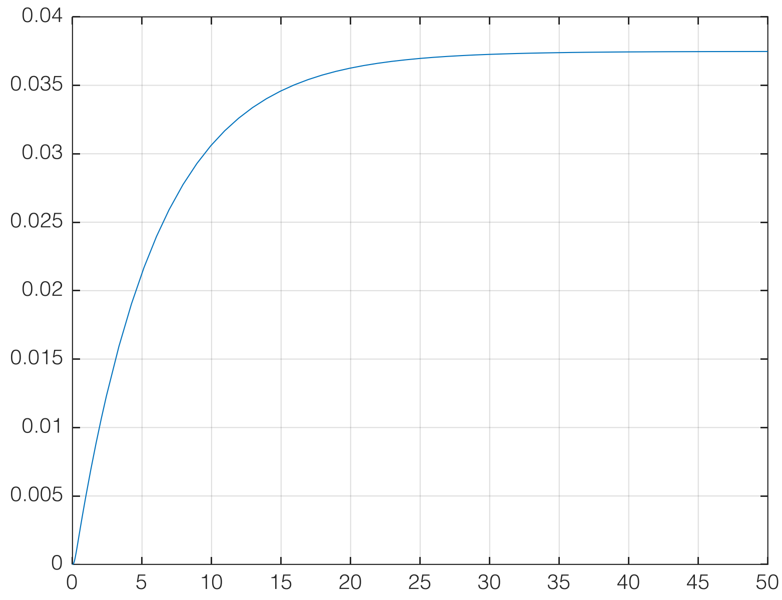


Figure.5 step response of $P \cdot G_2(s)$ in Simulink

4.

We use step function in MATLAB for the process, as we can see we obtain similar results as we did for the previous part on Simulink.

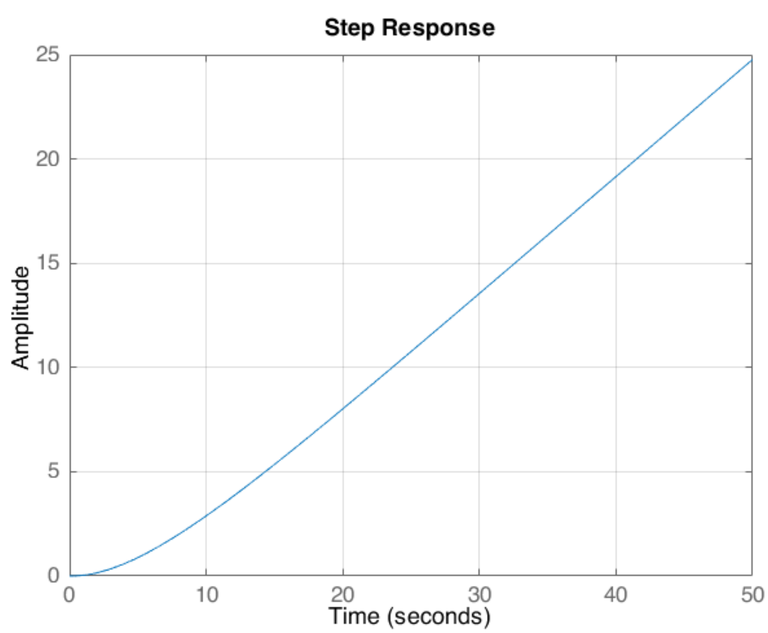


Figure.6 step response of $P \cdot G_1(s)$ in MATLAB

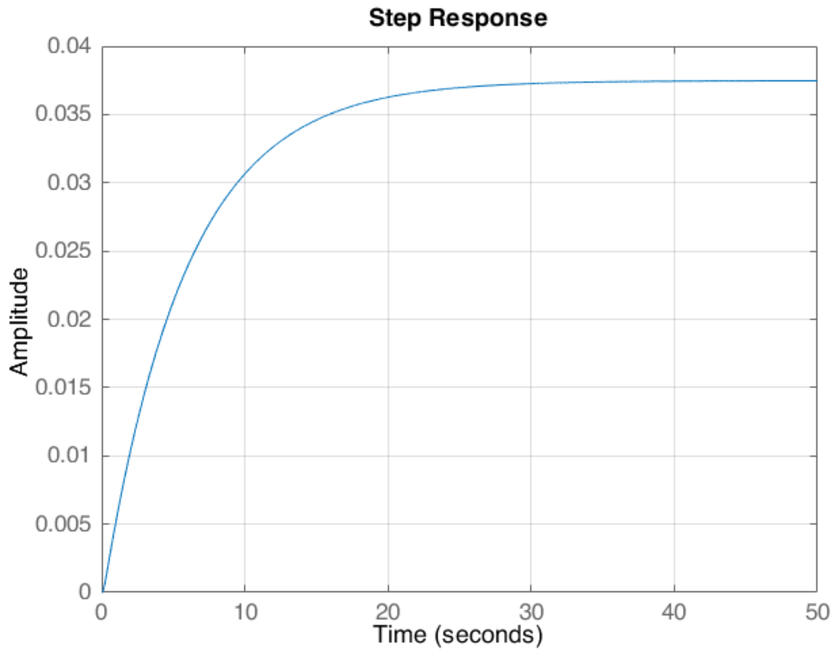


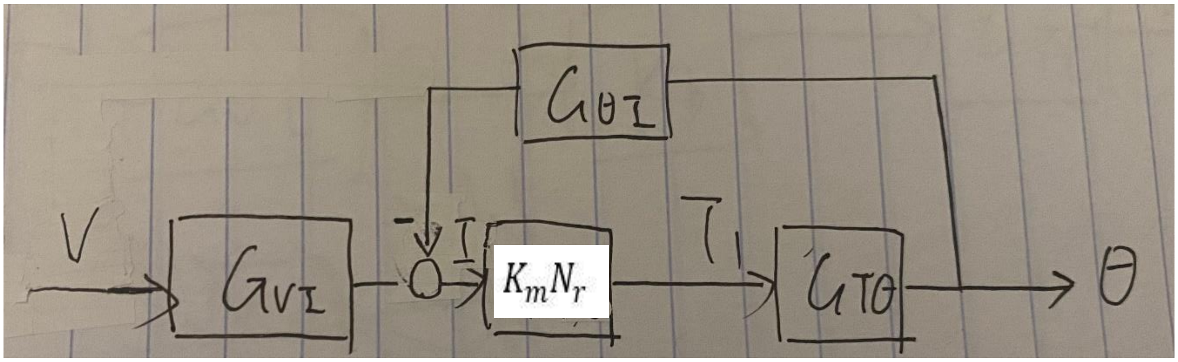
Figure.7 step response of $P \cdot G_2(s)$ in MATLAB

5.

In this question we add the gear ratio to the transfer function. The DC motor equation is modified as follows

$$\begin{cases} R_a I(t) + L_a \dot{I}\,(t) = V(t) - V_e(t) \\ V_e(t) = K_b \omega(t) = \dfrac{K_b \dot{\theta}(t)}{N_r} \\ T_1 = N_r K_m I(t) \end{cases}$$

The form of block diagram remains the same except we modify the K_m block in the CL into $N_r K_m$, and change $G_{\theta I}(s)$ from $\frac{K_b s}{L_a s + R_a}$ to $\frac{K_b s}{(L_a s + R_a) N_r}$



After adding the gear ratio, the transfer function changes to

$$P(s) = \frac{\theta}{V} = \frac{G_{VI}G_{T\theta}K_m}{1 + G_{T\theta}G_{\theta I}K_m} = \frac{\frac{1}{L_a s + R_a} \cdot \frac{1}{Js^2 + fs} \cdot K_m N_r}{1 + \frac{1}{Js^2 + fs} \cdot \frac{K_b s}{(L_a s + R_a)N_r} \cdot K_m N_r}$$
$$= \frac{K_m N_r}{JL_a s^3 + (L_a f + JR_a)s^2 + (R_a f + K_b K_m)s}$$

Again, L_a is negligible for the same reason. And when we neglect the inductor $L_a = 0$

$$P(s) = \frac{\theta}{V} = \frac{K_m N_r}{JR_a s^2 + (R_a f + K_b K_m)s}$$

The consideration of the gear ratio does not affect the system as one of the pole=0, will make the system **unstable**

6.

We have been given $|\tilde{\theta}| = \theta$ which has directions clockwise and counter clockwise respectively. From this we can say that the block diagram K_i need to equal -1 to satisfy the equality condition. The direction of the load voltage is same as the direction of the rotation angle $|\tilde{V}| = V, K_0$ also needs to be equal to -1 in order to satisfy the direction equality constraints. From this we can see that $K_i * K_0 = -1 * -1 = 1$.

Appendix

1.1

```
G1=tf([15^2],[1 30 15^2]);
G2=tf([15 0],[1 30 15^2]);
figure(1);step(G1);grid on;
figure(2);step(G2);grid on;
```