AA203 Final Project Report
Optimal Control for A Knee-Ankle Exoskeleton

Roy Bian, Zhiyuan Li, and Ziqin Zhu

{roybian, zhiyuanl, ziqinzhu}@stanford.edu

June 6, 2024

Abstract

Conventional exoskeleton controls like proportional-integral-derivative (PID) require extensive hyper-parameter tuning and do not guarantee global optimality. This paper introduces a new exoskeleton control system using Model Predictive Control (MPC) to assist walking along prescribed trajectories. MPC calculates motor torques for each degree of freedom by solving a Quadratic Programming (QP) problem. It also compares MPC with Linear Quadratic Regulator (LQR) and its iterative variant (iLQR). Simulation results show that MPC provides better trajectory tracking for knee and ankle motions compared to LQR and iLQR.

1 Introduction

This paper presents a novel knee-ankle exoskeleton control system to assist walking in prescribed trajectories. The system is designed for KATEE, a unilateral knee-ankle exoskeleton developed for stroke patient rehabilitation at Stanford Biomechatronics Lab. Currently, the most effective control scheme is proportional–integral–derivative (PID) control at each joint, which is difficult to implement due to the numerous tunable parameters needed to optimize for different users. Instead, we propose implementing Model Predictive Control (MPC), which iteratively refines its control strategy through a combination of open-loop and closed-loop methods with the known dynamics and constraints. Additionally, we compared the performance of MPC with Linear Quadratic Regulator (LQR) and its iterative variant (iLQR). Simulation results demonstrate that MPC achieves superior trajectory tracking for both knee and ankle motions, outperforming LQR and iLQR.

This is a joint project for AA 222 and AA 203. For AA 222, we focus on the constrained optimization method. For AA 203, we focus on optimal control methods for trajectory tracking.

2 Related Works

This project is closely related to the family of exoskeleton trajectory tracking. Implementations of MPC in the field of upper and lower limb exoskeletons focus on providing enough assistance for trajectory tracking while minimizing the input torques. For lower limb exoskeletons, Jin and Guo investigated utilizing extended state observer (ESO) and MPC to predict and compensate for external and human disturbances and proved it to have 35-39% better tracking accuracy compared to a fuzzy PID mode [2]. Caulcrick et al. used EMG signals to predict human torque and find the motor torque required for on-the-fly transitions between three tracking schemes through MPC: relaxed, assistive, and resistive, thanks to MPC's flexibility in applying different controls for each time step [3].

Researchers of upper limb exoskeletons, Dunkelberger et al. studied the effect of MPC implementation for spinal cord injury and proved that an MPC-based controller showed a reduction sum of squared torques by an average of 48.7 and 57.9% on the elbow flexion/extension and wrist flexion/extension joints respectively with minimal impact on tracking accuracy [4]. On the other hand, Bonilla et al. developed a hand exoskeleton that used accelerating velocity references as inputs to the MPC system and proved that the tracking errors remain insignificant even for input references up to 7mm/s. In brief, MPC has proved to be effective in minimizing control torques while tracking with reasonable accuracy, and is more efficient to implement than a conventional PID method, and therefore is suitable for our problem.

3 Problem Statement

3.1 Dynamics

KATEE is a unilateral knee-ankle exoskeleton designed for stroke patient rehabilitation at Stanford Biomechatronics Lab. It has three degrees of freedom (DOFs): knee extension, knee flexion, and ankle plantar flexion, and each DOF is individually controlled by a motor which applies forces through cables. Since the forces are not directly attached to the joint centers, some offset parameters need to be introduced for the dynamics system.

Finding the internal joint torques and forces in the user's ankle and knee belongs to the realm of biomechanics and demands an inverse dynamics analysis, To simplify the problem for the purpose of our study, we only consider the summation of internal torques τ_1 and τ_2 that could be obtained from Electromyography (EMG). As a result, at each joint, the summation of moments is composed of the internal torque that is known as well as external torques that are commanded through the corresponding cables.

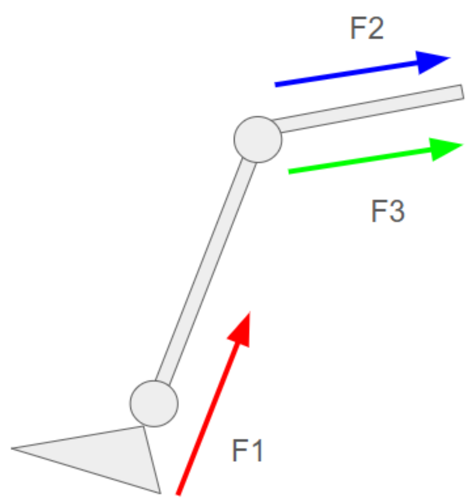


Figure 1: A sketch of the forces applied to the exoskeleton. The first cable is attached to the back of the ankle to pull and causes plantar flexion, the second cable is attached to the front of the knee to extend the knee, and the third is attached to the back of the knee and pulls to induce knee flexion. Knee extension and flexion are controlled separately due to the nature of cables that can only pull, not push. The three forces can be converted to commanded torques using the known motor radius. Forces are applied along the direction of the next segment, namely, along the shank and the thigh. Those two directions are also the references for ankle and knee angular displacements.

The state and control of the system are described by vectors $x \in \mathbb{R}^4$ and $u \in \mathbb{R}^3$:

$$x = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}, \quad u = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix}$$

where θ_1, θ_2 denote the angular displacements at ankle and knee respectively, and T_1, T_2, T_3 denote the motor torques at each DOF. The equations of motion are:

$$\tau_1 + \frac{T_1}{r} \cdot \frac{b_1(\sqrt{a^2 + b_1^2} + l_1 - l_{cm1})}{\sqrt{a^2 + b_1^2}} = I_1 \ddot{\theta}_1$$

$$\tau_2 - \frac{T_2}{r} \cdot b_2 + \frac{T_3}{r} \cdot b_3 = I_2 \ddot{\theta}_2$$

where τ_1, τ_2 are the internal torques predicted from muscle activities, I_1, I_2 are the moments of inertia of the foot and shank sections of the exoskeleton, r is the motor radius, a is the vertical offset from the ankle cable to the ankle joint, b_1, b_2, b_3 are the horizontal offsets between the points of three applied cable forces and joints. A discrete system of dynamics can be written as:

$$x_{t+1} = f(x(t), u(t)) = x_t + \Delta t \dot{x}$$

$$\dot{x} = \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \frac{\tau_1 + \frac{T_1}{r} \cdot \frac{b_1(\sqrt{a^2 + b_1^2} + l_1 - l_{cm1})}{\sqrt{a^2 + b_1^2}}}{I_1} \\ \frac{\tau_2 - \frac{T_2}{r} \cdot b_2 + \frac{T_3}{r} \cdot b_3}{I_2} \end{pmatrix}$$

The system can be transcribed into an affine estimate $f(x, u) \approx Ax + Bu + C$ using Taylor expansion. Around any iterate, A, B and C could be found as:

$$\begin{aligned} A &= \frac{\partial f}{\partial x}(x_k, u_k) \\ B &= \frac{\partial f}{\partial u}(x_k, u_k) \\ C &= f(x_k, u_k) - Ax - Bu \end{aligned}$$

3.2 Model Predictive Control Problem

Model Predictive Control (MPC) is an advanced process control technique that optimizes for the current time slot while considering future time slots by utilizing dynamic models of the process, typically linear empirical models. It can be effective in dealing with uncertainties because of its features of feedback correction and moving horizon optimization. It performs differently than usual PID controllers, which do not have predictive capabilities. MPC can anticipate future events and take proactive actions in control accordingly. At present there is no other technique other than MPC to design controllers for general large linear multivariable systems with input and output constraints with a stability guarantee [6]. By continuously optimizing over a finite time horizon and implementing the current time slot's control action, MPC can iteratively refine its control strategy, which differs from the Linear-Quadratic Regulators (LQR) approach [7]. Also, with its ability to optimize within constraints, MPC is advantageous in various application scenarios including power system balancing models and power electronic applications.

There are two objectives of our algorithm, one is to minimize tracking error and the other is to minimize command efforts, while adhering to various constraints. This multiobjective approach ensures that the system not only follows the desired trajectory closely but also operates efficiently by minimizing the effort required from the actuators. The trajectory tracking system is different from stabilizing to one goal pose since the steady state control does not need to be zero. Rather, the system should minimize control changes.

For iteration k, the MPC problem can be formulated as:

$$J_0^*(x(t)) = \min_{\delta u_0, \dots \delta u_{N-1}} \sum_k ||y_k - r_k||_Q^2 + ||\delta u_k||_R^2$$

$$\begin{aligned}
\text{subject to } & x_{k+1} = Ax_k + Bu_k + C, & k = 0, \dots N-1 \\
& y_k = Dx_k & k = 0, \dots N-1 \\
& x_k \in X, u_k \in U, & k = 0, \dots N-1 \\
& x_N \in X_f & \\
& u_k = u_{k-1} + \delta u_k & k = 0, \dots N-1 \\
& x_0 = x(t), u_{-1} = u(t) &
\end{aligned}$$

where A, B, and C are the estimated dynamics matrices, X and U are the constraint sets, cost matrices $P \succ 0$, $Q \succ 0$, and $R \succ 0$ and the reference state $r_k \in \mathbb{R}^4$. For optimal performance, P is set to be the unique positive definite solution to the discrete algebraic Riccati equation (DARE):

$$A^T PA - P - A^T PB(R + B^T PB)^{-1} B^T PA + Q = 0$$

3.3 Reference Trajectories

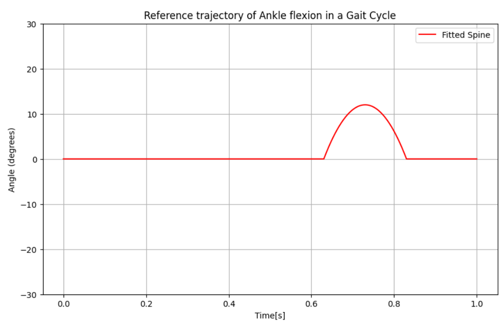
In this project, we aim to perform an informed methodology of the MPC algorithm to track the following desired ankle and knee trajectories which usually apply to a normal person's gait. Using the knowledge of biomechanics of movements, the reference trajectories for knee and ankle are generated (2a, 2b).

3.4 Constraints

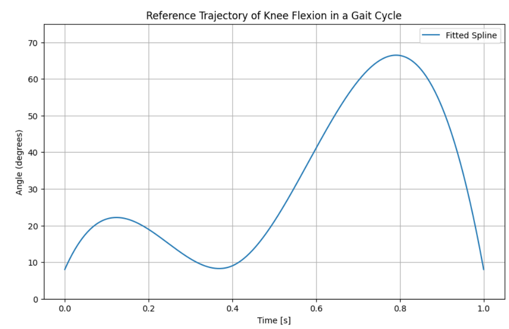
There are a few hard constraints for the system to follow. First, considering the physical limits of the user of the exoskeleton, for the purpose of walking, the ankle and knee angular displacements must be confined for safety.

$$0 \leq \theta_1 \leq 70^o$$

$$0 \leq \theta_2 \leq 75^o$$



(a) Ankle plantar flexion desired trajectory. The neutral pose is defined as standing straight. No negative angular displacement is commanded due to the design limit of the exoskeleton since the cable can only pull, not push.



(b) Knee angular displacement desired trajectory. The neutral pose is defined as standing straight. No negative angular displacement is commanded since hyper extension of knee is not desired for human movements.

Figure 2: Ankle and Knee Desired Trajectories

Second, the motors have torque limits. Since the forces are transmitted through cables, there are no negative torques since cables can only pull.

$$0 < T_1, T_2, T_3 < 10Nm$$

4 Conventional Approach: LQR and iLQR

Before implementing MPC, we will approach the problem using LQR [7] and iLQR [8]. These more conventional and simpler algorithms will provide a foundational comparison for evaluating the performance improvements offered by MPC, which will be discussed in a later section.

4.1 LQR

Linear Quadratic Regulator (LQR) is an optimal control strategy used to regulate the state of a dynamic system to minimize a quadratic cost which balances state performance and control effort. The LQR algorithm computes the optimal gain matrix K, providing control inputs that drive the system towards the desired state while minimizing the cost. This method is widely used by virtue of its simplicity and effectiveness in handling linear systems. However, LQR struggles with non-linear system and does not allow direct inclusion of constraints on states or control inputs, thus limiting its application in practice.

For our dynamcis, We choose state cost matrix Q to be an identity matrix with first two diagonal elements set to 50, and the control input cost matrix to be an indentity matrix. This ensures the controller can focus more on reference trajectory following rather than angular velocity or control input. solving Algebraic Riccati Equation iteratively, the optimal gain matrix turns out to be:

$$K = R^{-1}B^TP = \begin{bmatrix} -1.836 & 0 & -0.289 & 0 \\ 0 & 4.384 & 0 & 0.854 \\ 0 & -4.384 & 0 & -0.854 \end{bmatrix}$$

Based on the gain matrix, we simulate the system and ontain the following result illustrated in Figure 3,

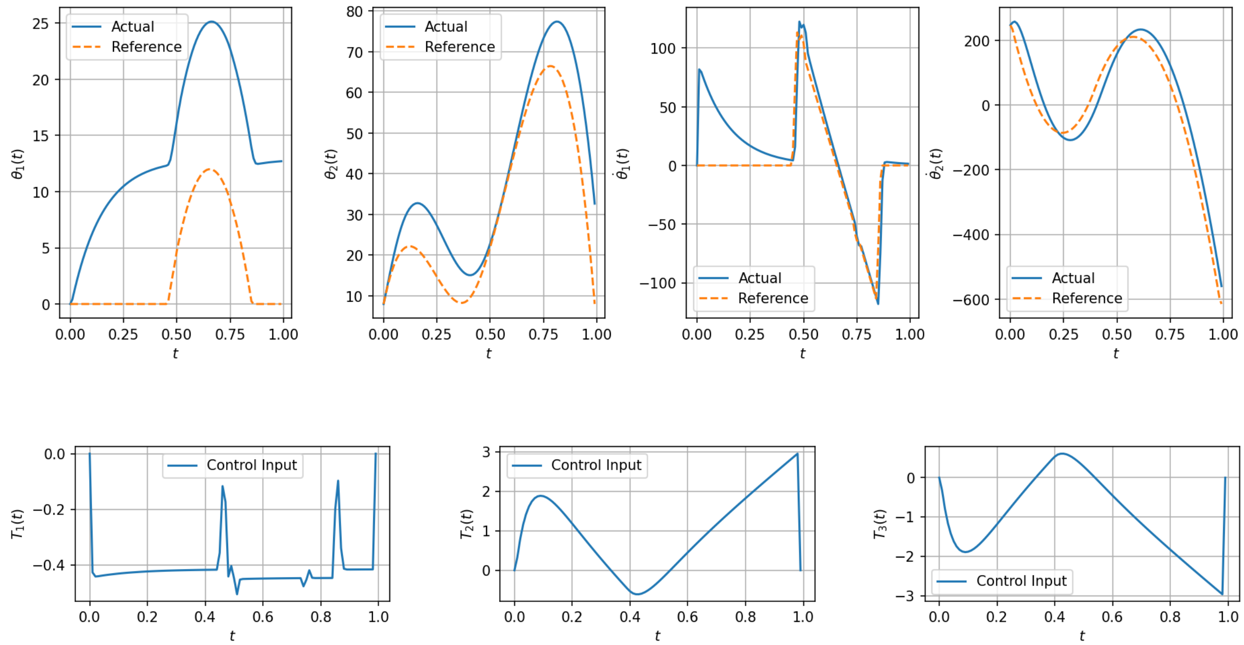


Figure 3: A set of plots showing the time response of the system controlled by LQR. The top row illustrates the actual vs. reference angles (θ) for the ankle (θ_1) and knee (θ_2), along with their respective angular velocities. The bottom row shows the control inputs (T) applied to the system. The blue solid lines represent the actual values, while the orange dashed lines represent the reference values.

The LQR controller shows several performance issues: there are significant deviations between actual and reference trajectories for both the ankle (θ_1) and knee (θ_2). The RMSE for Ankle Angle = 11.38 degrees, and RMSE for Knee Angle = 10.52 degrees. Also, the actual trajectories often exceed the reference values, highlighting difficulties in maintaining the desired range and efficiency. The control inputs exhibit oscillations and peaks, and have inputs in negative regions, which are not desired. These issues suggest that while LQR provides some level of control, it struggle to handle our dynamic systems effectively.

4.2 iLQR

The iterative Linear Quadratic Regulator (iLQR) is an extension of LQR designed for nonlinear systems, refining the control policy iteratively by re-linearizing the system dynamics around the current trajectory. Unlike LQR, which is limited to linear systems, iLQR handles nonlinear dynamics more effectively. The matrices used in iLQR include $R = 1 \times 10^{-1} \times I$ (control cost matrix), $Q_N = 1 \times 10^1 \times I$ (terminal state cost matrix), and Q, which is the same as that in LQR. These matrices balance the trade-off between minimizing control effort and achieving accurate state tracking. Shown in Figure 4 is simulation of system controlled by iLQR.

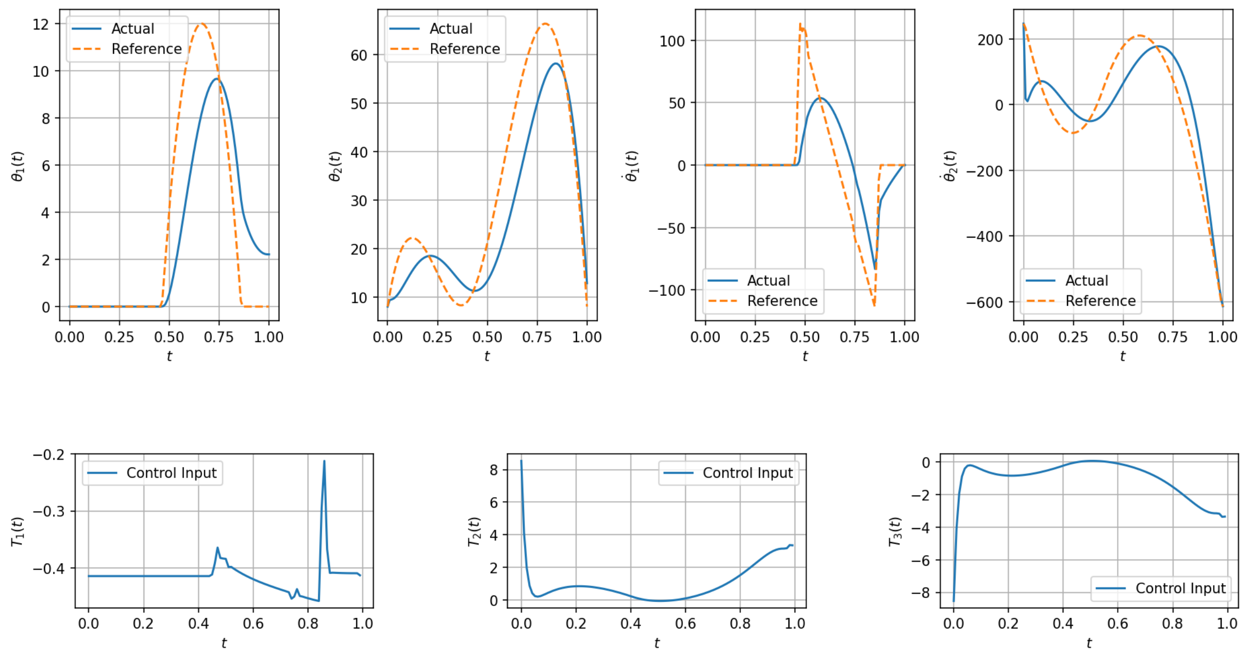


Figure 4: A set of plots showing the time response of the system controlled by iLQR. All other labels are the same as in the previous figure of LQR, including actual vs. reference angles ((θ_1) for the ankle and (θ_2) for the knee), their respective angular velocities, and control inputs (T).

The iLQR controller shows an improvement in trajectory tracking compared to LQR, with the actual angles and velocities closer to the reference values. It has a longer computation time than LQR due to the predictive iteration each step. However, performance issues remain, as there are still notable deviations, especially at peak values. The RMSE for Ankle Angle = 2.82 degrees, and RMSE for Knee Angle = 11.55 degrees. The control inputs are smoother than with LQR but still exhibit oscillations and peaks, indicating inefficiency. Overall, while iLQR improves performance, it is still not sufficient for precise control and may require further optimization or advanced strategies.

5 Simulations & Results

The simulation results are demonstrated with closed-loop control using MPC. In all simulations, CVX, a Matlab-based solver for convex optimization problems (http://cvxr.com), is used for optimization. The proposed approach combines parameter tuning and multi-objective optimization techniques to improve the performance of the MPC algorithm for the exoskeleton system.

The MPC problem is solved iteratively, providing the optimal torque inputs at each step. Results demonstrate that MPC can accurately track the prescribed joint angle trajectories by effectively handling the system dynamics and constraints, outperforming traditional PID control.

Based on the test results, the MPC implementations for ankle and knee trajectory tracking are demonstrated as follows with much more accurate tracking capabilities compared to LQR and iLQR methods. The RMSE for Ankle Angle = 0.26 degrees, and RMSE for Knee Angle = 2.61 degrees. This directly illustrates MPC's ability in achieving superior trajectory tracking for both knee and ankle motions, outperforming LQR and iLQR.

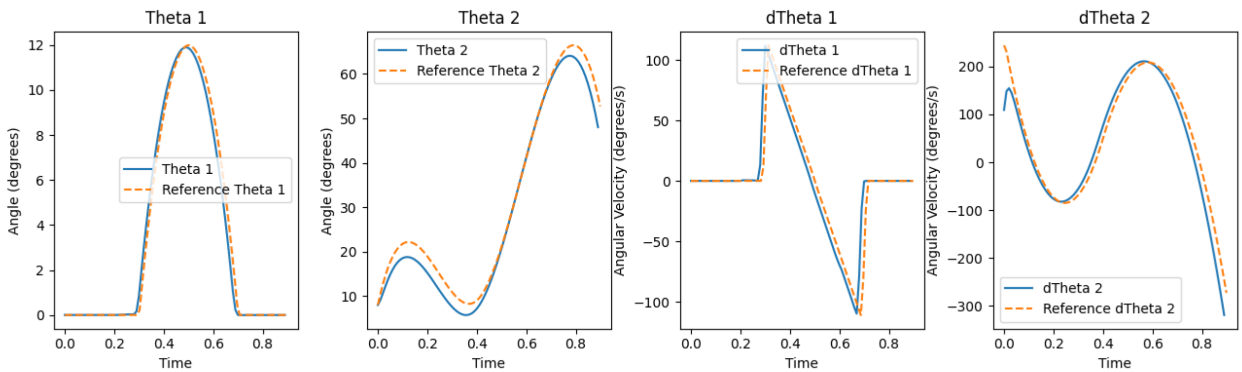


Figure 5: MPC implementations for knee and ankle trajectory tracking.

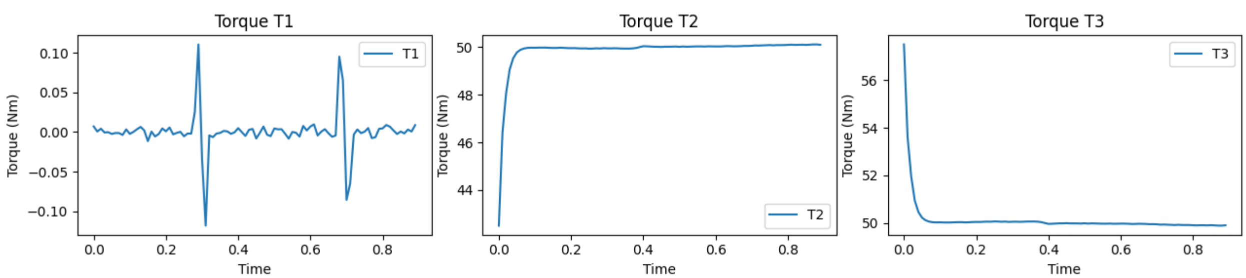


Figure 6: Control Inputs for MPC

6 Conclusion and Future Work

This study compared three control methods—Model Predictive Control (MPC), iterative Linear Quadratic Regulator (iLQR), and Linear Quadratic Regulator (LQR)—for achieving precise trajectory tracking of ankle and knee joint angles.

The results showed that MPC performed significantly better than iLQR and LQR in accurately following the desired joint angle trajectories. MPC had the lowest root mean square errors (RMSEs) of 0.26° for the ankle and 2.61° for the knee. iLQR outperformed LQR, with lower RMSEs of 2.82° for ankle and 11.55° for knee. However, it still exhibited deviations from the reference trajectories, especially at peak values. LQR had the poorest performance, with high RMSEs of 11.38° for the ankle and 10.52° for the knee. The actual trajectories frequently exceeded the reference, and the control inputs showed undesirable oscillations and peaks.

In summary, MPC provided superior trajectory tracking compared to iLQR and LQR methods. Future work should focus on optimizing iLQR and LQR, exploring advanced techniques, and improving numerical stability to enhance control system accuracy and efficiency.

The code for this project is uploaded to: `https://github.com/roybian/aa203project.git`

The presentation for this project is uploaded to: `https://drive.google.com/file/d/12daCD5E1LaTW2JKix-xxm_vVqIkbWvaa/view?usp=sharing`

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