## Advanced Control Lab Lab Report-5

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## **Objective:**

We want to design a controller for the pendulum angle,  $C_{\theta}$  for a dampening of the pendulum oscillations, and it also need to fulfill requirements regarding open/closed loop gain. We will compare the system with and without this controller to verify its performance.

## $C_{\theta}$ upholds the following requirements:

- the gain of the full closed loop should be no more than 3 [dB]
- the controller gain should be less than 0 [dB] for frequencies higher than  $200 \left[ \frac{rad}{s} \right]$
- the controller slope should be at least  $-20 \left[ \frac{db}{decade} \right]$

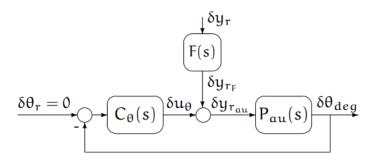


Figure: 1 Dampening loop with Pau

 $P_{au}$  is given by:

$$P_{au} = \frac{C_y P_y P_\theta}{1 + C_y P_y}$$

Substituting values we get previous lab, we get

 $P_{au} = \frac{-214.19 \text{ s}^4 \text{ (s} + 5.78) \text{ (s} + 7.549) \text{ (s} + 23.68) \text{ (s} + 1.17) \text{ (s}^2 + 0.1679s + 35.96) \text{ (s}^2 + 1.586s + 42.34)}{\text{s}^2 \text{ (s} + 23.68) \text{ (s} + 7.549) \text{ (s} + 3.671) \text{ (s} + 1.619) \text{ (s}^2 + 1.295s + 32.49) \text{ (s}^2 + 0.1679s + 35.96) \text{ (s}^2 + 1.586s + 42.34) \text{ (s}^2 + 26.23s + 255.6)}$ 

The lead controller we have

We get the lead controller from the given gain crossover frequency 4.061 rad/s. We then find the phase of the controller  $\phi$  using the formula of the phase margin:  $PM = \pi + phase \ of \ (P) - 5.7^0 - \phi$ 

We use the  $\phi$  to obtain the lead coefficient of the lead controller  $\alpha = \frac{1+\sin\phi}{1-\sin\phi}$  and we get  $\alpha$  as 7.584 and we apply the lead controller formula

$$C_{lead} = \frac{2.754 \, s + \, 4.061}{s \, + \, 11.18}$$

We choose to design the controller in the following form:

$$C_{\theta} = -\frac{1}{C_{lead}} \cdot C_{LPF},$$

Where  $C_{LPF} = \frac{200}{s+200}$  so that we could largely attenuate the signal when the frequency is higher than  $200 \left[ \frac{rad}{s} \right]$ .

In the end, we get:

$$C_{\theta} = \frac{-200 \,s - 2237}{2.754 \,s^2 + 554.8 \,s + 812.3}$$

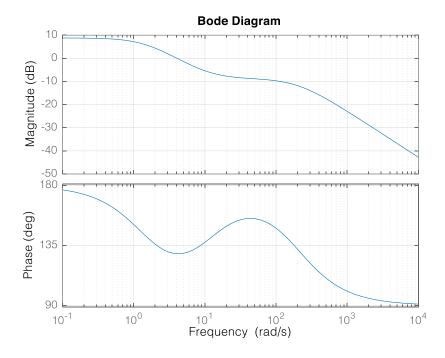


Figure 2: bode diagram of  $C_{\theta}$ 

It is clear that that the controller gain is less than O[dB] for the frequencies from 200[rad/sec] to infinity, the controller **slope** is around  $\frac{-43.2dB+23.1dB}{dec} = -20.1\frac{db}{dec}$ , which also satisfies the related requirements.

Add the designed  $\mathcal{C}_{\theta}(s)$  to Figure 4 in lab 4, we get the new Simulink:

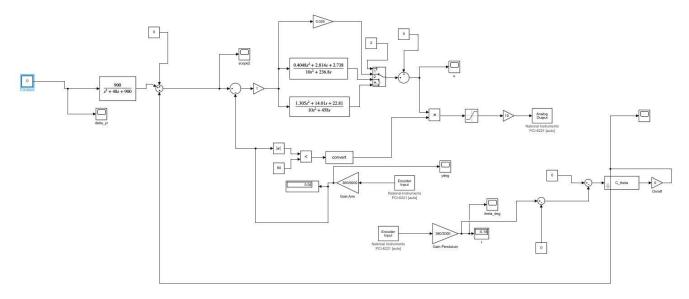


Figure:3 Simulink built in lab 5

Now, let's start to plot the related graphs.

1.Bode plot, Nyquist plot (or Nichols plot) of the open loop transfer function  $L(s) = P_{au}(s)C_{\theta}(s)$ ,

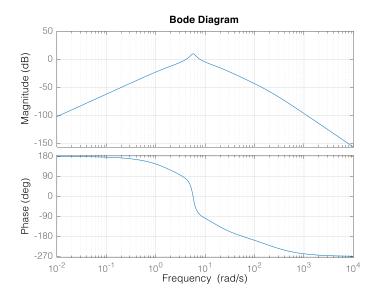


Figure 4: Bode plot of the open loop transfer function  $L(s) = P_{au}(s)C_{\theta}(s)$ 

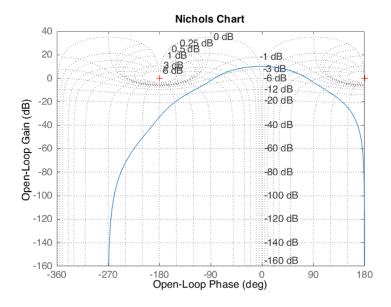


Figure: 5 Nichols plot of the open loop transfer function  $L(s) = P_{au}(s)C_{\theta}(s)$ 

2. Bode plots of the transfer functions from  $\delta y_r$  to  $\delta \theta_{deg}$  in open loop (with  $C_{\theta}(s)=0$ ) and in closed loop (with designed  $C_{\theta}(s)$ )),

In open loop:

$$\frac{\delta\theta_{deg}}{\delta y_r} = F(s)P_{au}(s)$$

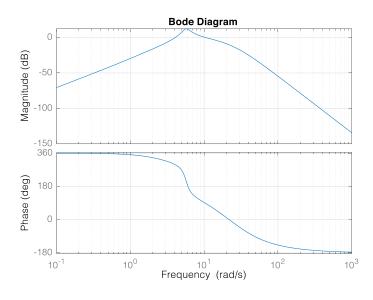


Figure:8 Bode plots of the transfer functions from  $\delta y_r$  to  $\delta \theta_{deg}$  in open loop (with  $C_{\theta}(s) = 0$ )

In closed loop:

$$\frac{\delta\theta_{deg}}{\delta y_r} = \frac{F(s)P_{au}(s)}{1+C_{\theta}(s)P_{au}(s)}$$

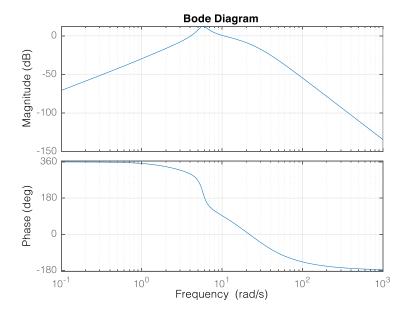


Figure 9: Bode plots of the transfer functions from  $\delta y_r$  to  $\delta \theta_{deg}$  in closed loop (with designed  $C_{\theta}(s)$ )

3. Bode plots of the transfer functions from  $\delta y_r$  to  $\delta y_{deg}$  in open loop (with  $C_{\theta}(s)=0$ ) and in closed loop (with designed  $C_{\theta}(s)$ )

In open loop:

$$\frac{\delta y_{deg}}{\delta y_r} = \frac{F(s)C_y(s)P_y(s)}{1+C_y(s)P_y(s)}$$

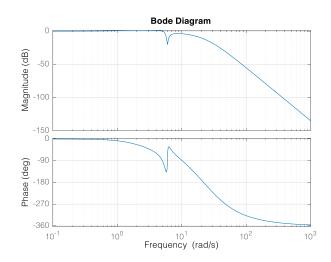


Figure: 6 Bode plots of the transfer functions from  $\delta y_r$  to  $\delta y_{deg}$  in open loop (with  $C_{\theta}(s) = 0$ )

In closed loop:

$$\frac{\delta y_{deg}}{\delta y_r} = \frac{F(s)P_{au}(s)}{P_{\theta}(1+C_{\theta}(s)P_{au}(s))}$$

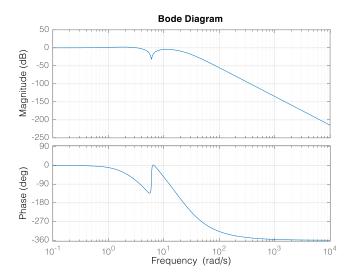


Figure 7: Bode plots of the transfer functions from  $\delta y_r$  to  $\delta y_{deg}$  in closed loop (with designed  $C_{\theta}(s)$ )

4. compare the signals obtained by the experiment to the corresponding signals in the simulations with the linear and nonlinear models of the experimental setup.

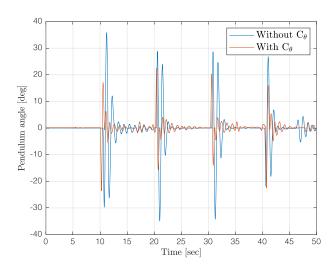


Figure 10: comparison of pendulum degree vs time with and without  $C_{\theta}$ 

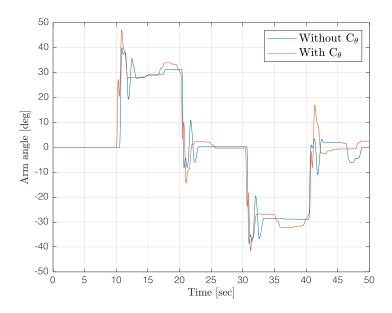


Figure 11: comparison of arm degree vs time with and without  $C_{\theta}$ 

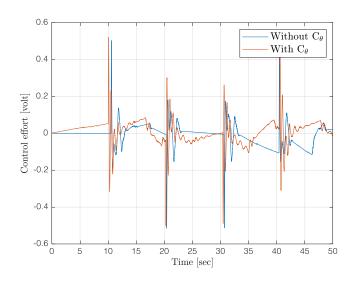


Figure 12: comparison of control effort vs time with and without  $C_{\theta}$ 

The controller  $C_{\theta}$  not only reduced the oscillations of the system, but also yielded faster convergence to the steady state of the system. The improved performance is evident in the systems abilities to achieve desired results more accurately and efficiently.

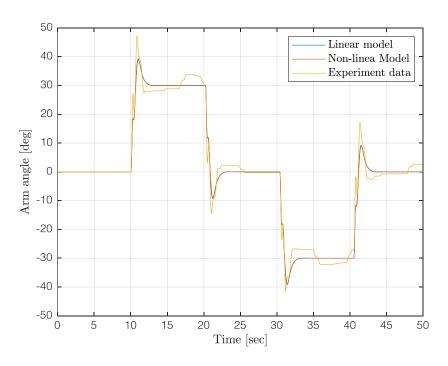


Figure 13: comparison of arm angle vs time between linear model, non-linear model, and experimental data

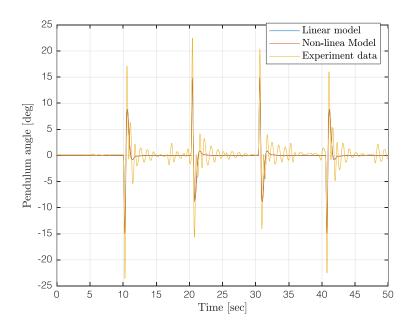


Figure 14: comparison of pendulum angle vs time between linear model, non-linear model, and experimental data

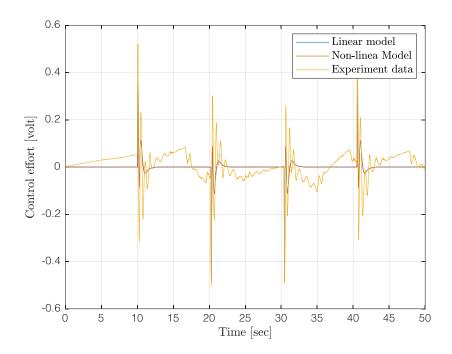


Figure 15: comparison of control effort vs time between linear model, non-linear model, and experimental data

Here we can see that the experimental results specifically for the ones with the pendulum angles oscillate a lot, whereas we see that the Linear and Non-Linear model reaches steady state very quickly.

## **Conclusion:**

We also see that due to the implementation of the  $C_{\theta}$  controller in our system yielded very good results that are in very good agreement with our theoretical results, and exhibited better control and performance compared to other controllers implemented. The controller  $C_{\theta}$  not only reduced the oscillations of the system, but also yielded faster convergence to the steady state of the system. The improved performance is evident in the systems abilities to achieve desired results more accurately and efficiently. We can clearly see the difference in the outputs where the controller  $C_{\theta}$  was and wasn't implemented, the systems in which the controller wasn't implemented showed increased oscillations and slower convergence rates. These results serve as a testament to the impact of the  $C_{\theta}$  controller for the enhancement of the control of our system.