Control Lab Report-3

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• Experiment objective

The objective of this experiment was to design servo controllers for position control of DC motor with load. For this we were given requirements that needed to be met for the proper function of the DC motor. We designed two servo controllers for different requirements of our DC motor. We then checked the stability margin for the closed loop feedback for internal stability. We analyzed the bode diagrams of the different members of the 'gang of 4' for their internal stability and the smooth functioning of the DC motor. Then using the designed controllers, we initiated the system with step and sine wave input without the pendulum, and also step input with the pendulum, and compare them to the simulation results.

Q1) Compute the stability margins (phase and gain margins) for the controller $\boldsymbol{C(s) = k_c = 0.02}$ and the process model obtained in lab

For $C(s) = k_c = 0.02$, the bode diagram of the open loop is shown below,

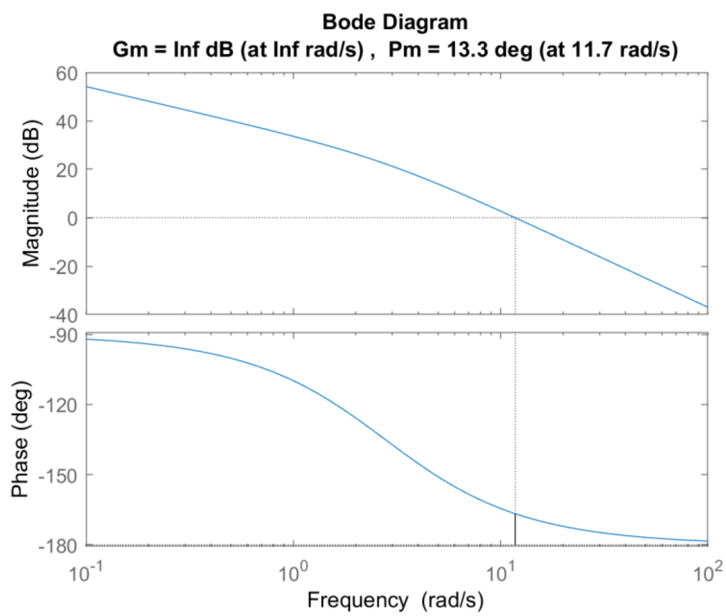


Figure:1 Bode diagram of the open loop when $C(s) = k_c = 0.02$

As is indicated in the figure 1:

The Phase Margin is 13.3 degrees.

And the gain crossover frequency is 11.7 secs.

Illustrated below is the bode plot of the following transfer function: $T = \frac{PC}{1+PC}, S = \frac{1}{1+PC}, T_d = \frac{P}{1+PC}, T_c = \frac{C}{1+PC}$

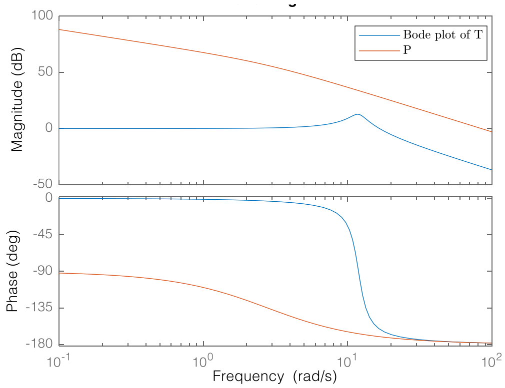


Figure:2 Bode diagram of T *and* P *when* $C(s) = k_c = 0.02$

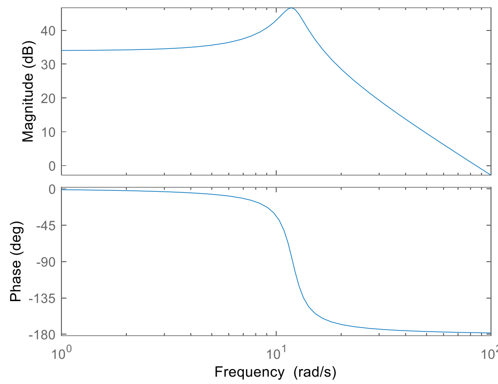


Figure:3 Bode diagram of T_d *when* $C(s) = k_c = 0.02$

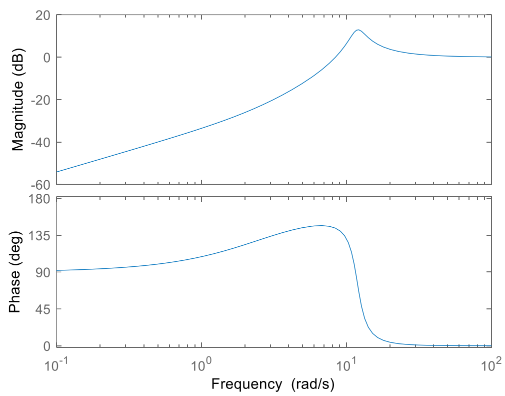


Figure:4 Bode diagram of S *when* $C(s) = k_c = 0.02$

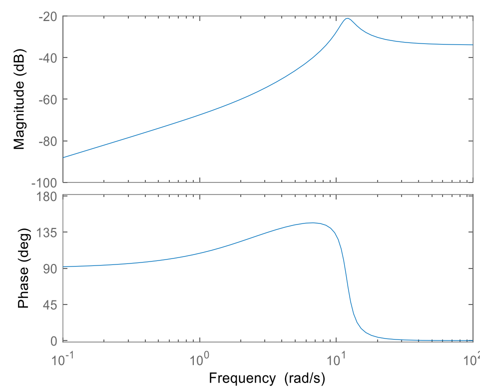


Figure:5 Bode diagram of T_c *when* $C(s) = k_c = 0.02$

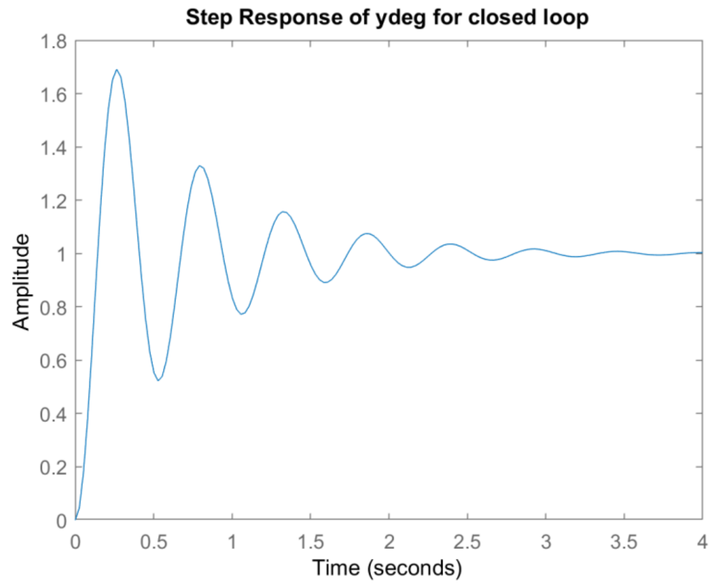


Figure:6 Step response y_{deg} *when* $C(s) = k_c = 0.02$

2. What is a relationship between the open loop frequency response (Bode plot) and the closed loop response y_{deg} for the step input in y_r?

We can see from the bode plot that when $\omega \to 0$, the magnitude of $L = PC$ goes to infinity. Meaning that $T_{(\omega \to 0)} = \frac{L}{1+L} \approx 1$. As we can see in the following step response plot, after the transient stage, and reaching steady state, the system output reaches the reference 1. Also, it can be found that the bandwidth of closed loop is around 1.2-1.5 times of the crossover frequency 11.7rad/sec, consistent with the rule of thumb we know.

Q2) Design the first servo system for the process model so that the closed loop system satisfies the following specifications(without prefilter, that is $F(s) = 1$):

1. zero steady state error for unit step in y_r,

2. zero steady state error for unit step in d,

3. the gain crossover frequency is the same as the one of the closed loop with the above mentioned proportional controller,

4. PM = 45° .

What are overshoot, settling time, and rise time of the designed servo system ?

We choose the proportional controller to be 0.02, so that the gain crossover frequency is the same as before.

Since we need zero steady state error for the step input for both $r(t), d(t)$, we need an integrator in the controller. For that purpose, we will use the lag controller where $\beta = \infty$. So the Lag controller will become

We choose the parameter $\omega_m = \omega_c$.

$$C_{lag} = \frac{10s + 11.7}{10s}$$

For the given phase margin of 45 degree, we need to use a lead controller.

We will use the Phase Margin, the Phase of the Plant, and the phase of the lag controller.

$$C_{lead} = \frac{\sqrt{\alpha} \cdot s + \omega_c}{s + \sqrt{\alpha} \cdot \omega_c}$$

From the Phase, we will find

$$\alpha = \frac{1 + \sin(\phi)}{1 - \sin(\phi)} \approx 5.2433$$

$$C_{lead} = \frac{2.024s + 11.7}{s + 23.8}$$

$$C = k_c * C_{lag} * C_{lead} = \frac{0.4048s^2 + 2.814s + 2.738}{10s^2 + 236.8}$$

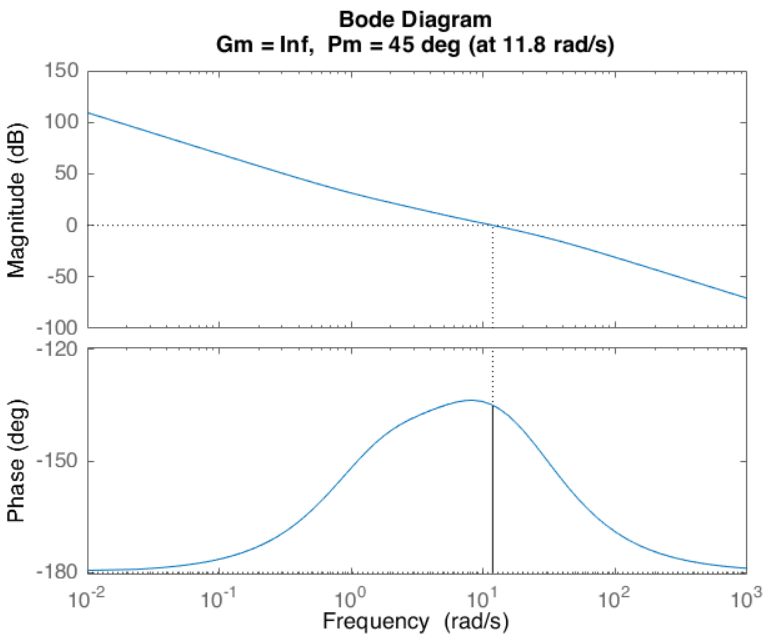


Figure:7 Bode diagram of the open loop using servo controller 1.

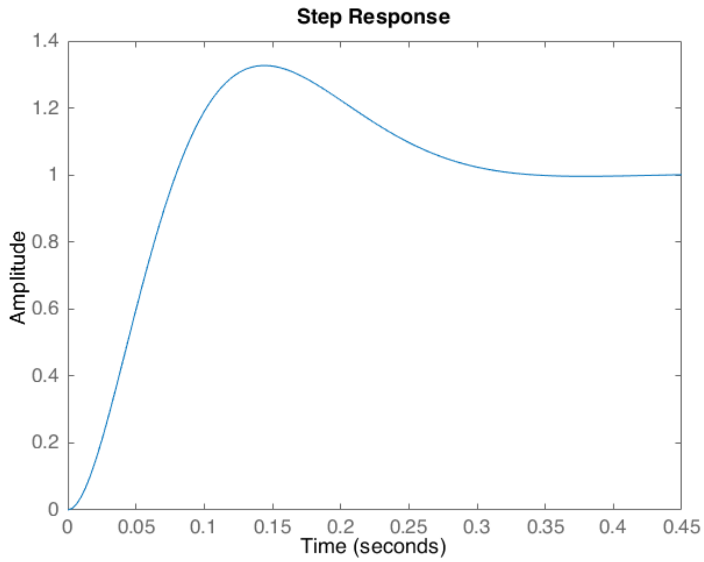


Figure:8 Step response of the closed loop system using servo controller 1

From the step response shown above, we can find the property of the designed servo system.

Rise Time: 0.0953 seconds

Settling Time: 0.5017 seconds

Settling Min: 0.9136

Settling Max: 1.3095

Overshoot: 30.9502%

Undershoot: 0

Peak: 1.3095

Peak Time: 0.2467 seconds

Illustrated below is the bode plot of the following transfer function: $T = \frac{PC}{1+PC}, S = \frac{1}{1+PC}, T_d = \frac{P}{1+PC}, T_c = \frac{C}{1+PC}$

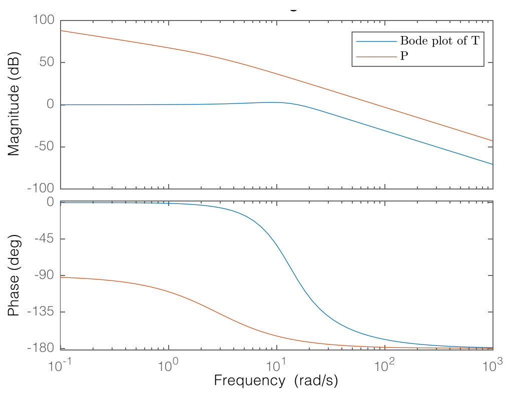


Figure:9 Bode diagram of P and T using servo controller 1.

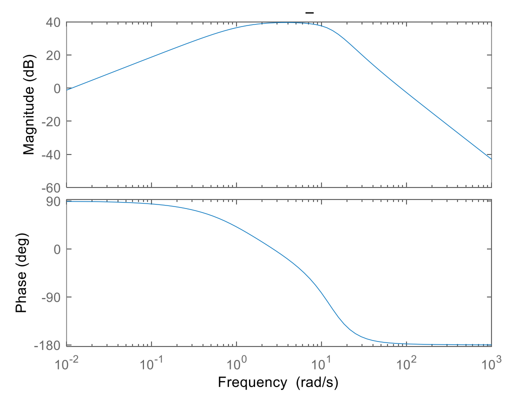


Figure:10 Bode diagram of T_d using servo controller 1.

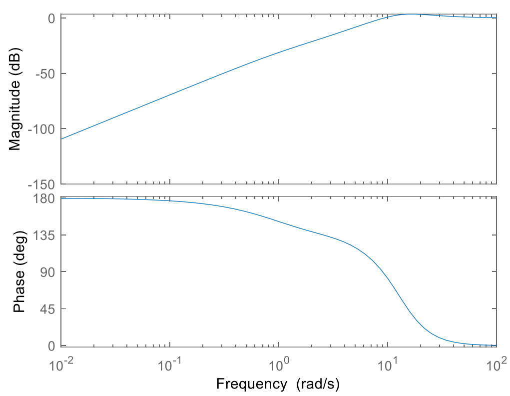


Figure:11 Bode diagram of S using servo controller 1.

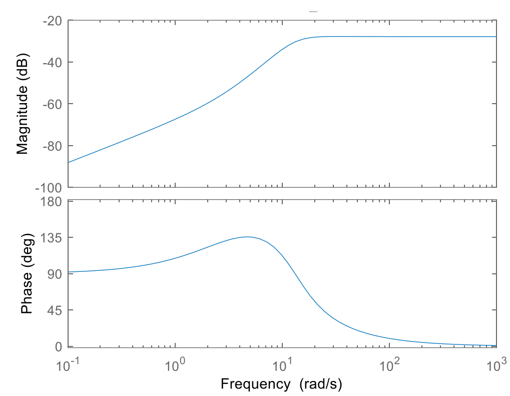
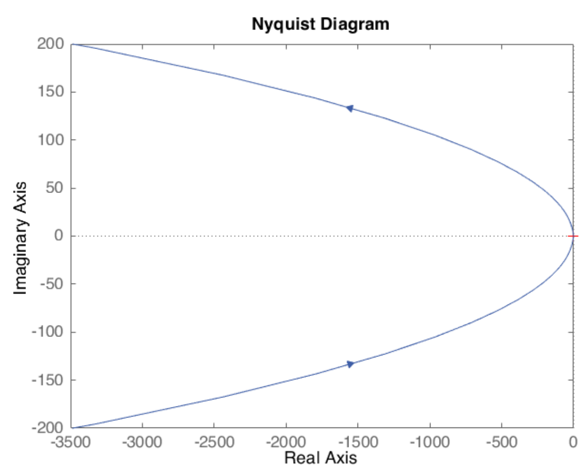
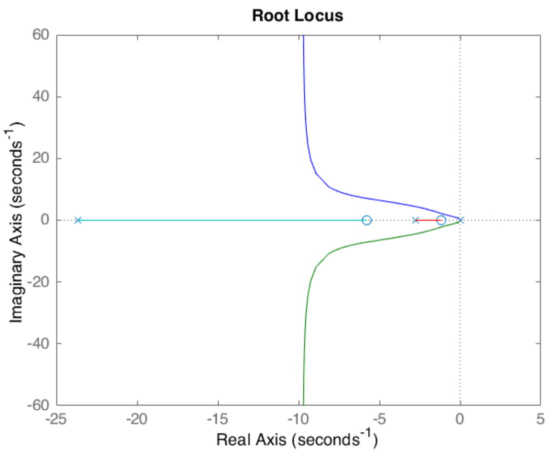


Figure:12 Bode diagram of T_c using servo controller 1.

It can find that T, complementary sensitivity transfer function, is a low pass filter, meaning only signal with low frequency such as a step input is passed. T_d and T_c are high pass filters, S is a band-pass filter.



. *Figure:13 Nyquist of L using servo controller 1*



. *Figure:14 Root locus when using servo controller1*

Q3) Compute the proportional controller and stability margins (phase and gain margins) for the process model obtained in lab 2, if it is known that gain crossover frequency is given by 20[rad/s].

For this system, we have been given the plant and gain crossover frequency. So we will first find the proportional controller

$$C = k_c = \frac{1}{|P(20j)|} = 0.057$$

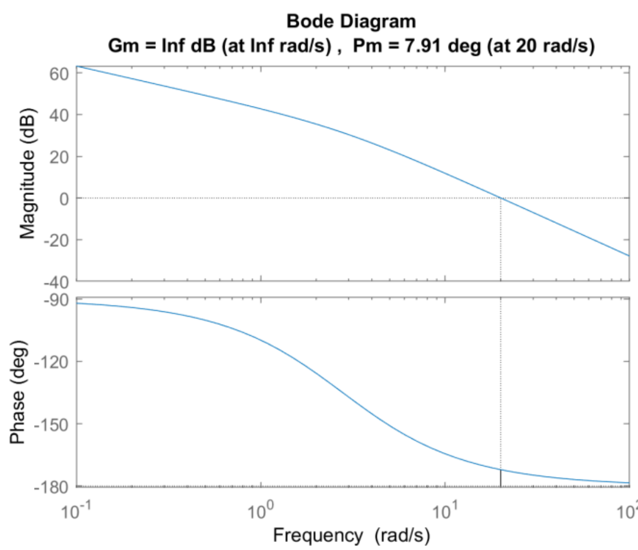


Figure:15 Bode diagram of the open loop when $C = 0.057$

The Phase margin is 7.91 degrees, the gain margin is infinity since it is always larger than $-180°$

Q4) Design the second servo system for the process model so that the closed loop system satisfies the following specifications(without prefilter, that is $F(s) = 1$):

1. zero steady state error for unit step in y_r,

2. zero steady state error for unit step in d,

3. the gain crossover frequency equals 20 $[rad/s]$,

4. $PM = 45°$.

What are overshoot, settling time, and rise time of the designed servo system ?

$$k = \frac{1}{|P(\omega_c)|} = 0.0570$$

Since we need zero steady state error for the step input for both $r(t), d(t)$, we need an integrator in the controller. For that purpose we will use the lag controller where $\beta = \infty$. So the Lag controller will become

We choose the parameter $\omega_m = \omega_c$

$$C_{lag} = \frac{10s + 20}{10s}$$

For the given phase margin of 45 degree, we need to use a lead controller.

We will use the Phase Margin, the Phase of the Plant, and the phase of the lag controller.

$$C_{lead} = \frac{\sqrt{\alpha} \cdot s + \omega_c}{s + \sqrt{\alpha} \cdot \omega_c}$$

From the Phase, we will find

$$\alpha = \frac{1 + \sin(\phi)}{1 - \sin(\phi)} \approx 5.2433$$

$$C_{lead} = \frac{2.29s + 20}{s + 45.8}$$

$$C = k * C_{lag} * C_{lead} = \frac{1.305s^2 + 14.01s + 22.81}{10s^2 + 458s}$$

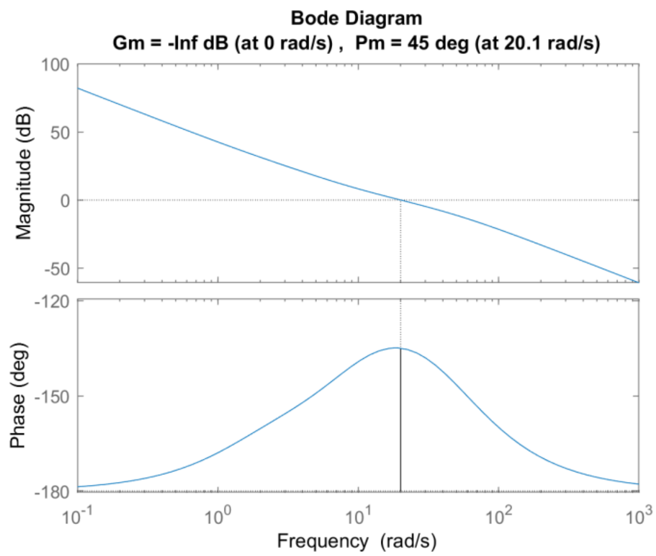


Figure:16 Bode diagram of the open loop when using servo controller2

Rise Time: 0.0545 seconds

Settling Time: 0.3040 seconds

Settling Min: 0.9017

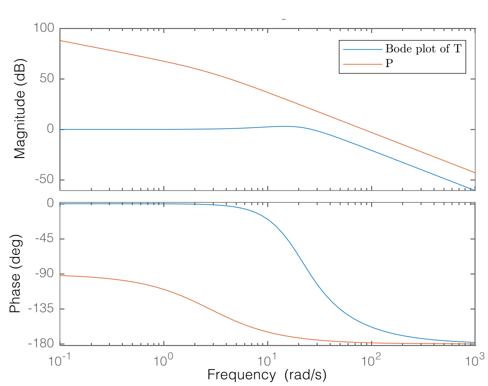
Settling Max: 1.3268

Overshoot: 32.6838%

Undershoot: 0

Peak: 1.3268

Peak Time: 0.1452 seconds



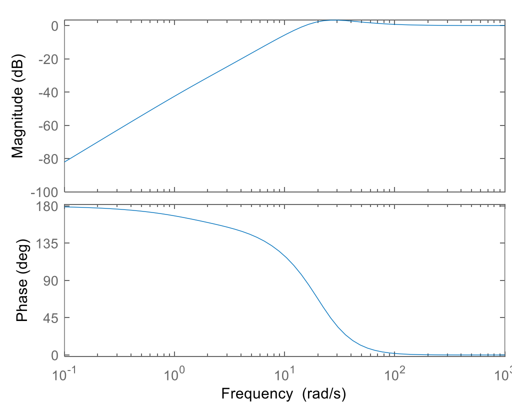


Figure:17 Bode diagram of P and T using servo controller 2. *Figure:18 Bode diagram of T_d using servo controller 2.*

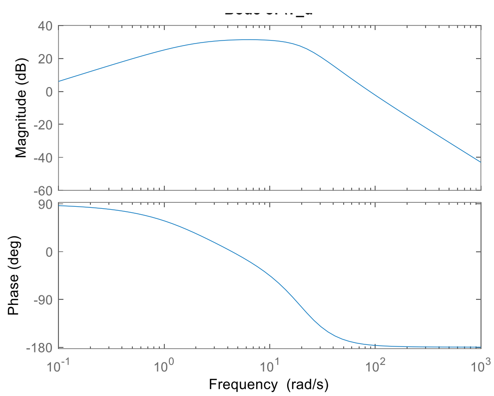


Figure:19 Bode diagram of S using servo controller 2.

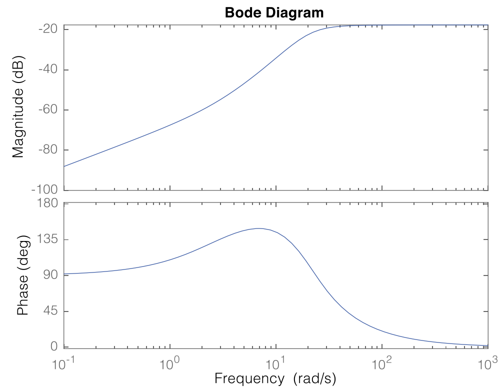


Figure:20 Bode diagram of T_c using servo controller 2.

It can find that T, complementary sensitivity transfer function, is a low pass filter, meaning only signal with low frequency such as a step input is passed. T_d and T_c are high pass filters, S is a band-pass filter.

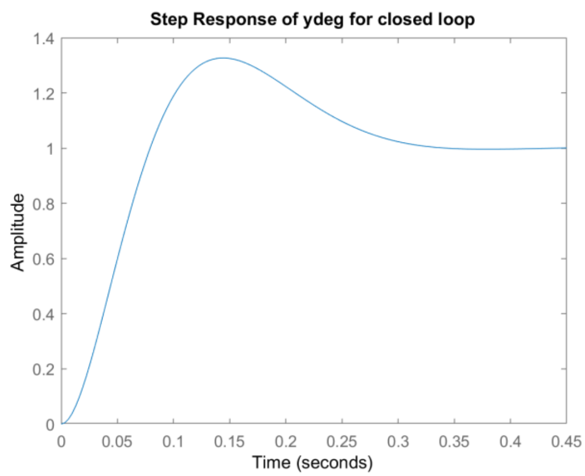
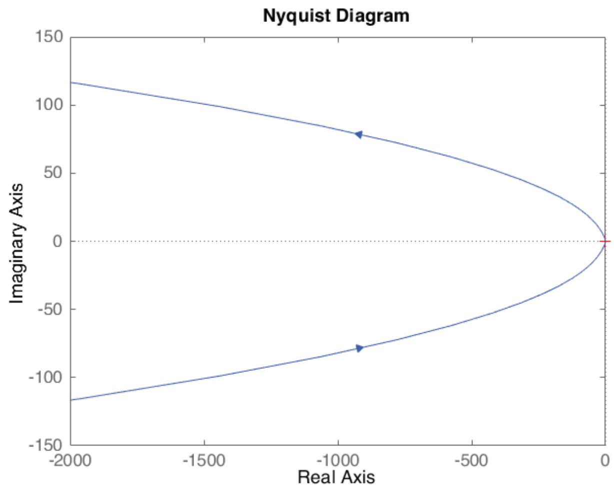


Figure:21 Step response of the closed loop system *when* $C = 0.057$



. *Figure:22 Nyquist of L using servo controller 2*

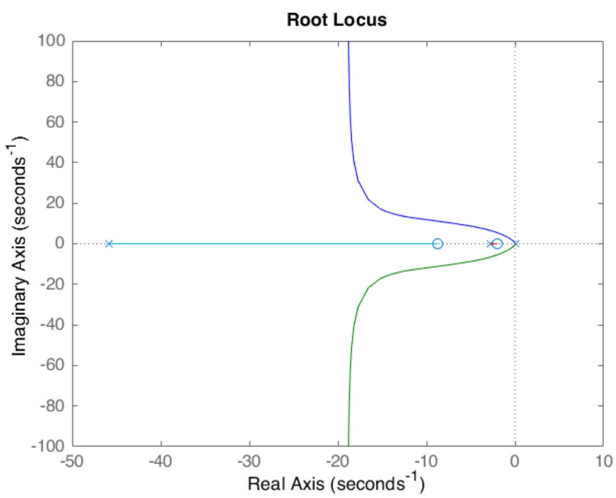


Figure:23 Root locus when using servo controller2

Q5) Show by the corresponding simulations in SIMULINK and Bode plots that the two servo systems satisfy the above specifications.

• Simulate the two servo systems with prefilter

$$F = \frac{30^2}{s^2 + 1.6 \cdot 30s + 30^2}$$

and the step input of 30o in yr. Verify that the control signal u belongs to the range of [−1, 1].

4. The responses (u and ydeg) of the two designed servo systems for the step of 30o in yr without pendulum. The reference signal yr should be filtered by F(s). Compare these

responses to the corresponding simulations in SIMULINK. What are reasons for the differences between the corresponding signals?

It should be noted that the **control efforts** from the simulation and from the experiment are within the range of $[-\mathbf{1}, \mathbf{1}]$ for both servo controllers.

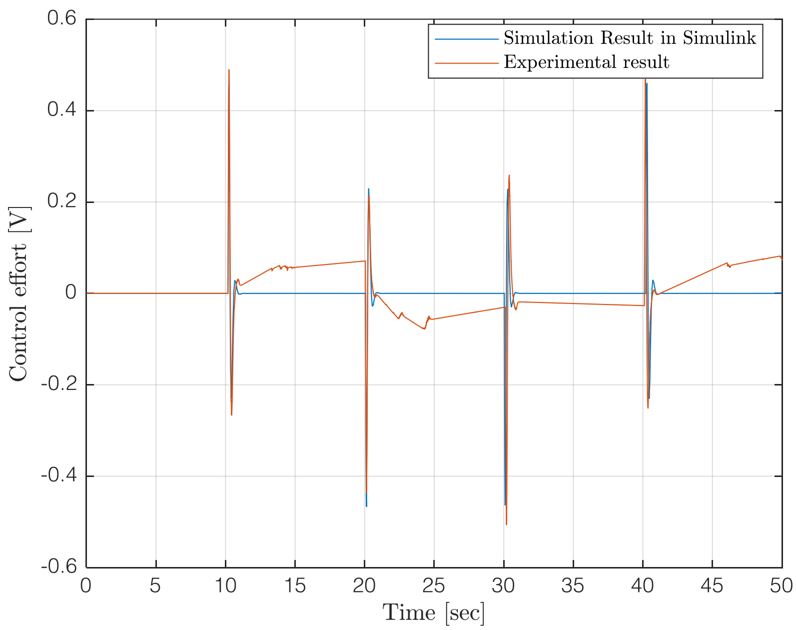


Figure 24: Comparison between simulation and experimental results of the control effort when using servo1 controller.

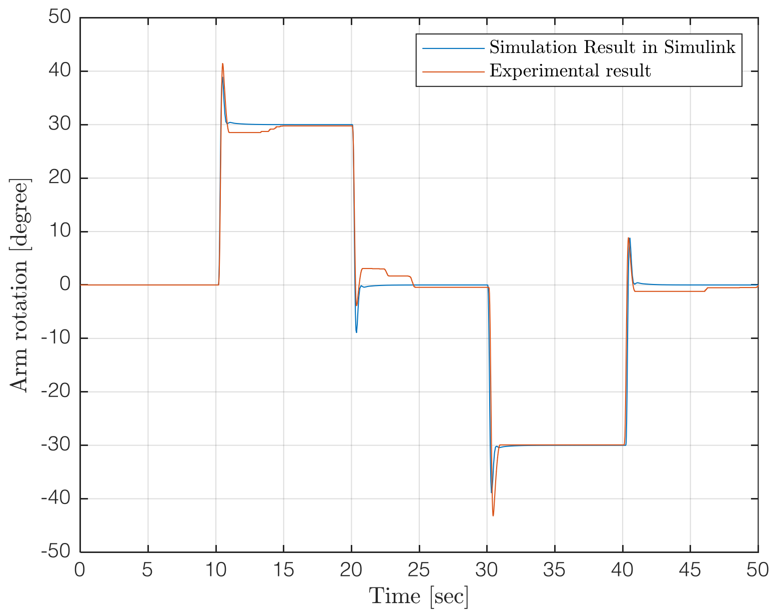


Figure 25: Comparison between simulation and experimental results of the control effort when using servo2 controller.

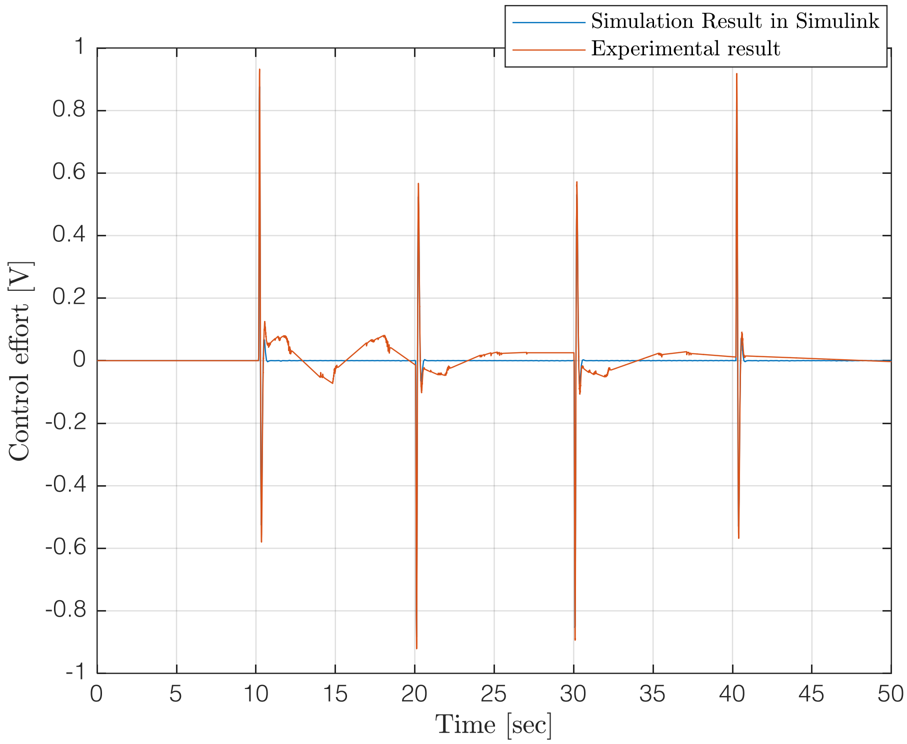


Figure 26:Comparison between simulation and experimental results of the control effort when using servo1 controller.

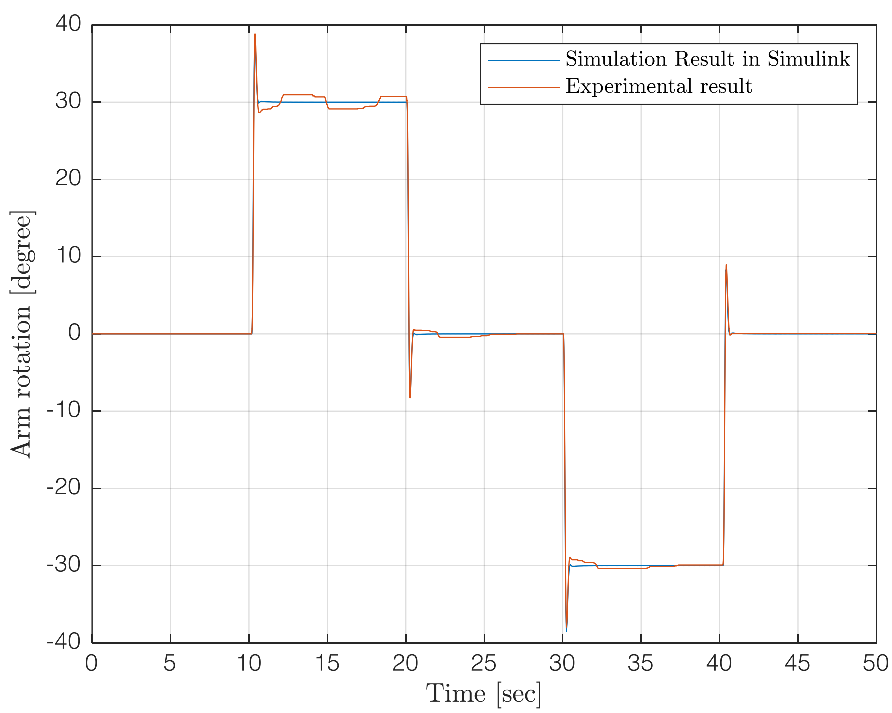


Figure 27:Comparison between simulation and experimental results of the control effort when using servo2 controller.

As is observed from the figures above, the simulation gives a good prediction during the transient stage, however the simulation results go back to steady state quickly after each step, while for experimental results, it takes much longer for the signal to decay to steady state and stop oscillating during each step.

Probable reasons may derive from: external disturbance and noise which are not considered in this model; the sensor we used in the experiment is not sensitive enough, which may cause a delay of the whole system; systematic measurement error of the sensor; frictions that exist in the experimental setup.

5. The responses (u and $ydeg$) of the two designed servo systems for the sinusoidal input of the 2[Hz] and 3[Hz] frequencies, and the amplitude of 15o in yr without pendulum. The reference signal yr should be filtered by F(s). Compare the steady state response of arm angle ydeg to the corresponding points of Bode plot (magnitude and phase of the corresponding transfer function).

<u>For controller servo1</u>

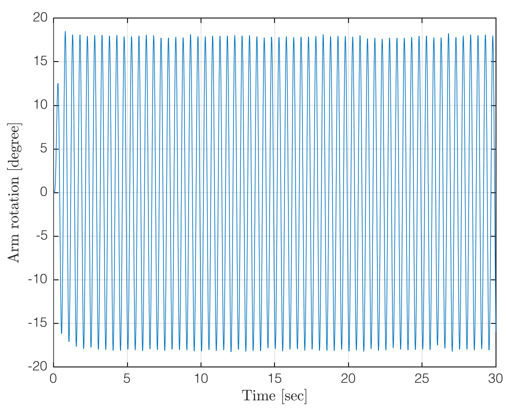


Figure 28: Response when the input sine wave is 2Hz

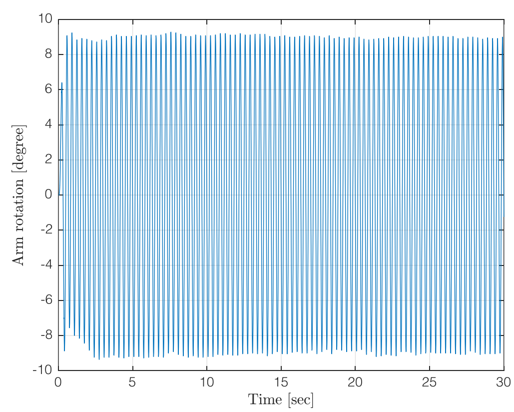


Figure 29: Response when the input sine wave is 3Hz

When the input sine wave is 2Hz, the steady state amplitude of the response signal is 18.08, and when the input sine wave is 2Hz, the steady state amplitude of the response signal is 9.04.

It is easy to know that $2[Hz] = 2 * 2\pi \left[\frac{rad}{s}\right] \approx 12.566 \left[\frac{rad}{s}\right]$, and $3[Hz] = 3 * 2\pi \left[\frac{rad}{s}\right] \approx 18.850 \left[\frac{rad}{s}\right]$. We can find magnitude and phase for corresponding frequencies on the bode diagram of the transfer function.

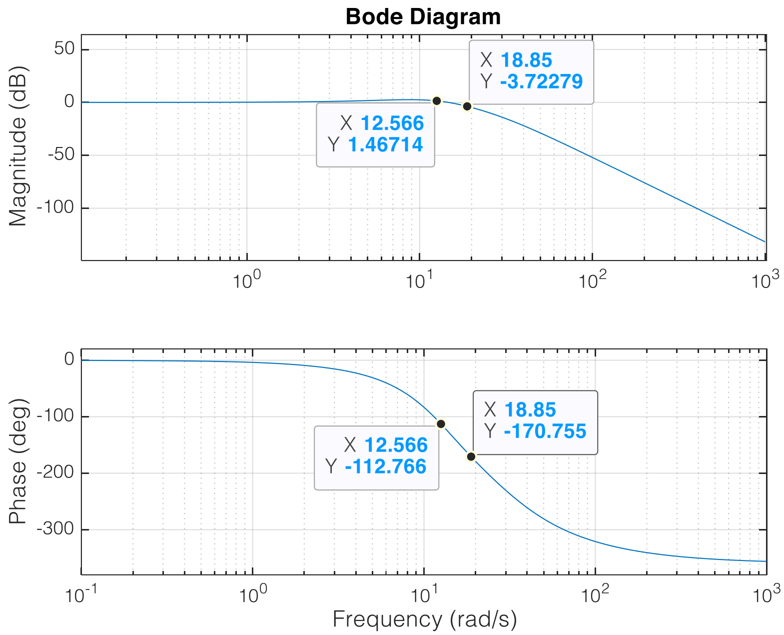


Figure 30: bode plot of the transfer function of the system

Frequency response theorem states that given an asymptotically stable LTI system $G(s)$. The steady state response when input $u(t) = Asin(\omega t)$ is $y_{ss}(t) = ARsin(\omega t + \phi)$, where $R(\omega) = |G(jw)|,\ \phi(\omega) = \angle G(jw)$.

In our case, when the input $u(t) = 15sin(4\pi t)$, We can see from the figure above that when $\omega = 12.566\ [rad/sec]$, $|G| = 1.46714dB = 1.1840, \phi(\omega) = -112.766° = -1.9681\ [rad]$, meaning the theoretical response of the steady state output is

$$\boldsymbol{y_{ss}(t)} = ARsin(\omega t + \phi) = \mathbf{17.760}\boldsymbol{sin}(\mathbf{4}\boldsymbol{\pi t} - \mathbf{1.9681})$$

Plot the theoretical response and experimental result on the same graph, we can see that they are well matched with each other after the transient stage.

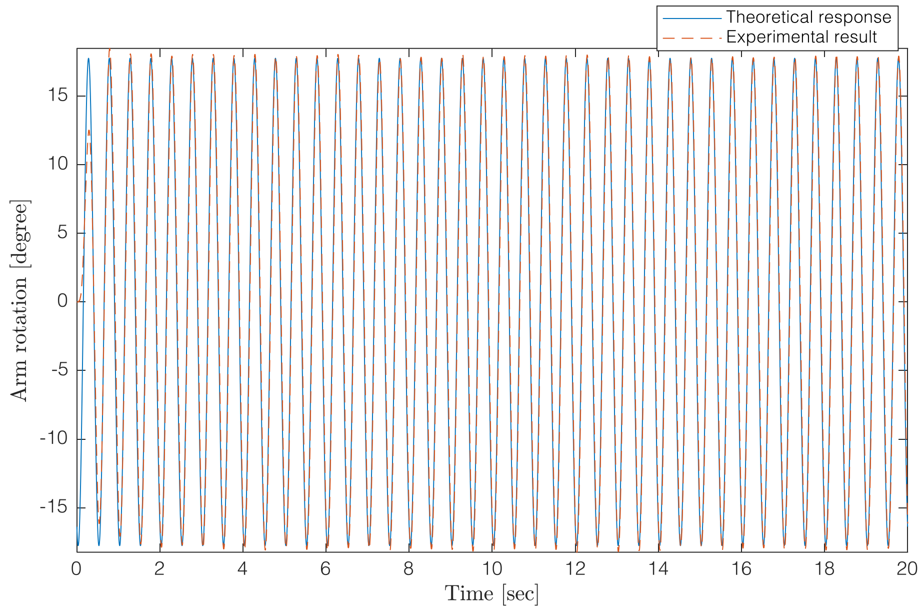


Figure 31: experimental result vs theoretical response when the input signal is $y_r(t) = 15sin(4\pi t)$

When the input $u(t) = 15sin(6\pi t)$, We can see from the figure above that when $\omega = 18.850\ [rad/sec]$, $|G| = -3.7228dB = 0.6514, \phi(\omega) = -170.755°$, meaning the theoretical response of the steady state output is

$$\boldsymbol{y_{ss}(t)} = ARsin(\omega t + \phi) = \boldsymbol{9.771sin(6\pi t - 2.9802)}$$

Plot the theoretical response and experimental result on the same graph, we can see that the phases are well matched with each other after the transient stage, as for the amplitude, the theoretical response is larger, possible reason for the error includes unconsidered friction in the motor, not accurate enough approximated model.

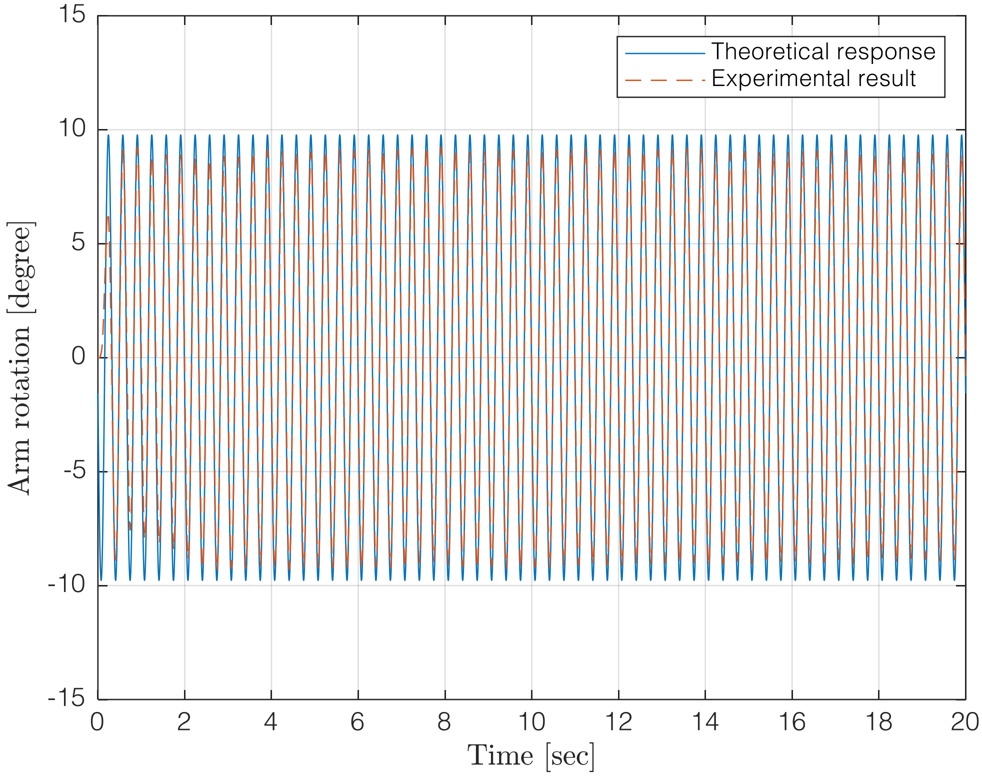


Figure 32: experimental result vs theoretical response when the input signal is $y_r(t) = 15sin(6\pi t)$

<u>For controller servo2</u>

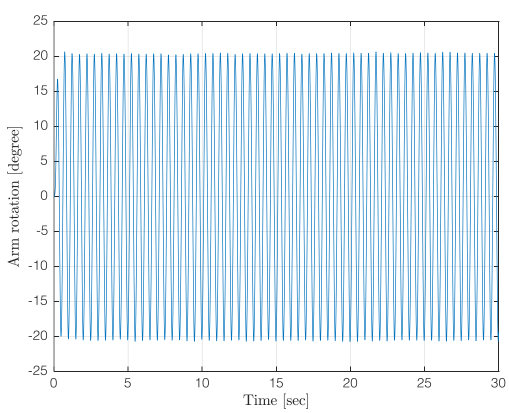


Figure 33: Response when the input sine wave is 2Hz

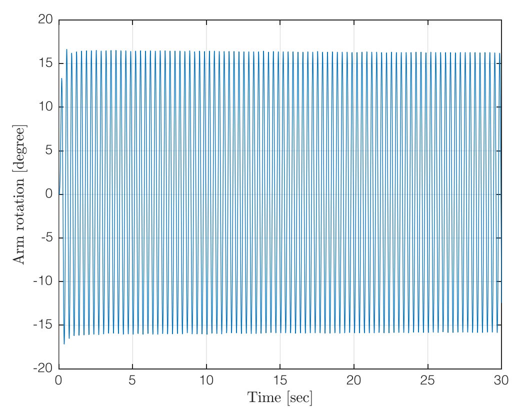


Figure 34: Response when the input sine wave is 3Hz

When the input sine wave is 2Hz, the steady state amplitude of the response signal is 20.06, and when the input sine wave is 2Hz, the steady state amplitude of the response signal is 16.32.

We can find magnitude and phase for corresponding frequencies on the bode diagram of the transfer function.

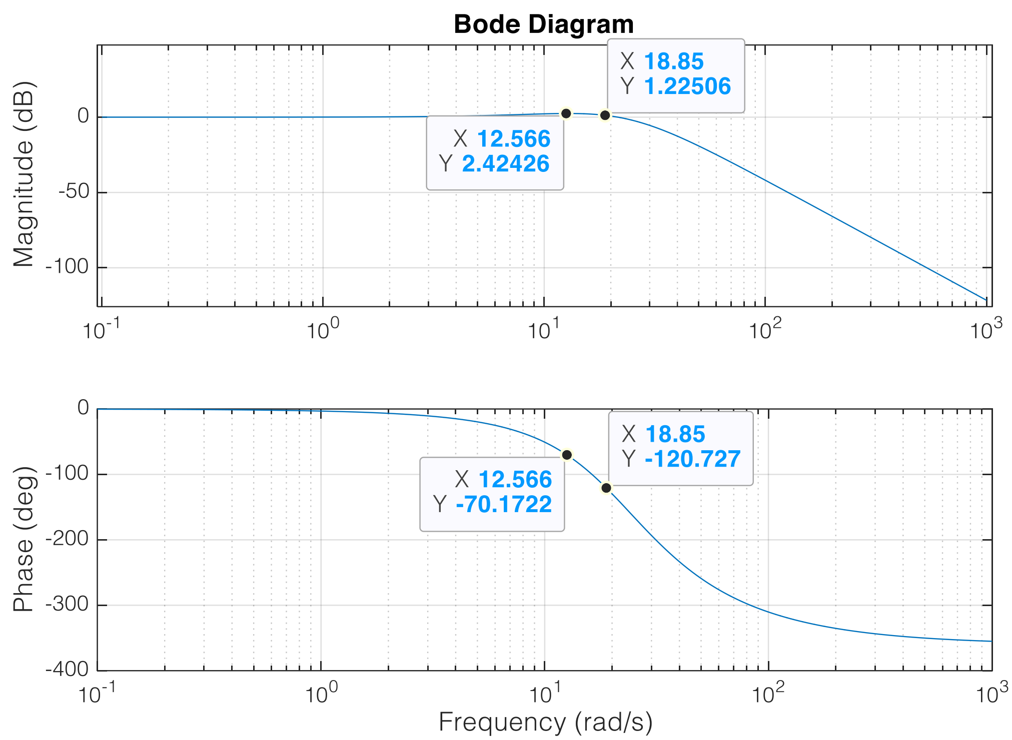


Figure 35 bode plot of the transfer function of the system.

Similarly, we will find theoretical response using frequency response theorem. In our case, when the input $u(t) = 15sin(6\pi t)$, We can see from the figure above that when $\omega = 12.566\ [rad/sec]$, $|G| = 2.4243dB = 1.3219, \phi(\omega) = -70.1722° = -1.2247\ [rad]$, meaning the theoretical response of the steady state output is

$$\boldsymbol{y_{ss}(t)} = ARsin(\omega t + \phi) = \mathbf{19.8292}\boldsymbol{sin}(\boldsymbol{4\pi t} - \mathbf{1.2247})$$

Plot the theoretical response and experimental result on the same graph, we can see that they are well matched with each other after the transient stage except minor changes in amplitude.

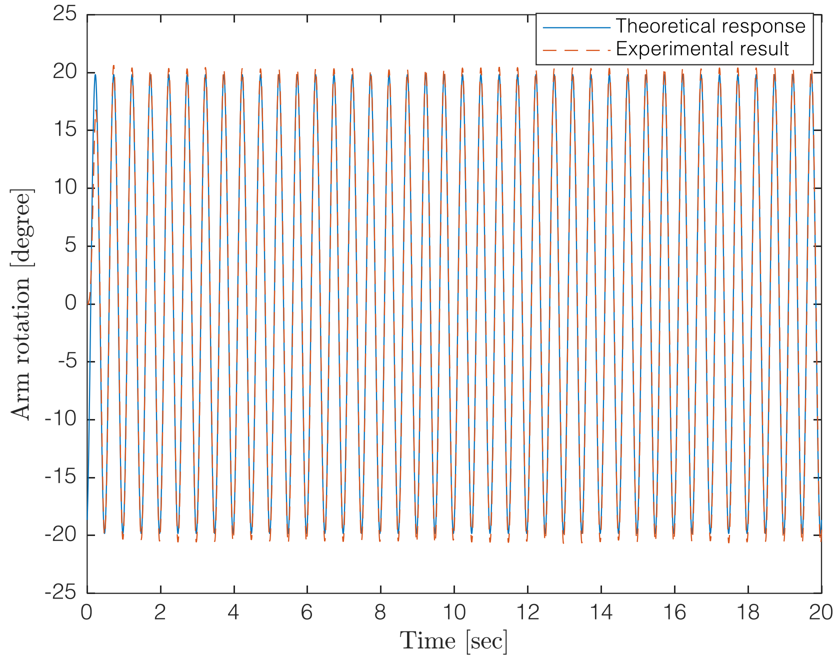


Figure 36: experimental result vs theoretical response when the input signal is $y_r(t) = 15sin(4\pi t)$

When the input $u(t) = 15sin(6\pi t)$, We can see from the figure above that when $\omega = 18.850\ [rad/sec]$, $|G| = 1.2251dB = 1.1515, \phi(\omega) = -120.727° = -2.0966\ [rad]$, meaning the theoretical response of the steady state output is

$$\boldsymbol{y_{ss}(t)} = ARsin(\omega t + \phi) = \mathbf{17.2721}\boldsymbol{sin}(\mathbf{6}\boldsymbol{\pi t} - \mathbf{2.0966})$$

Plot the theoretical response and experimental result on the same graph, we can see that the phases are well matched with each other after the transient stage, as for the amplitude, the theoretical response is larger, possible reason for the error includes unconsidered friction in the motor, not accurate enough approximated model.

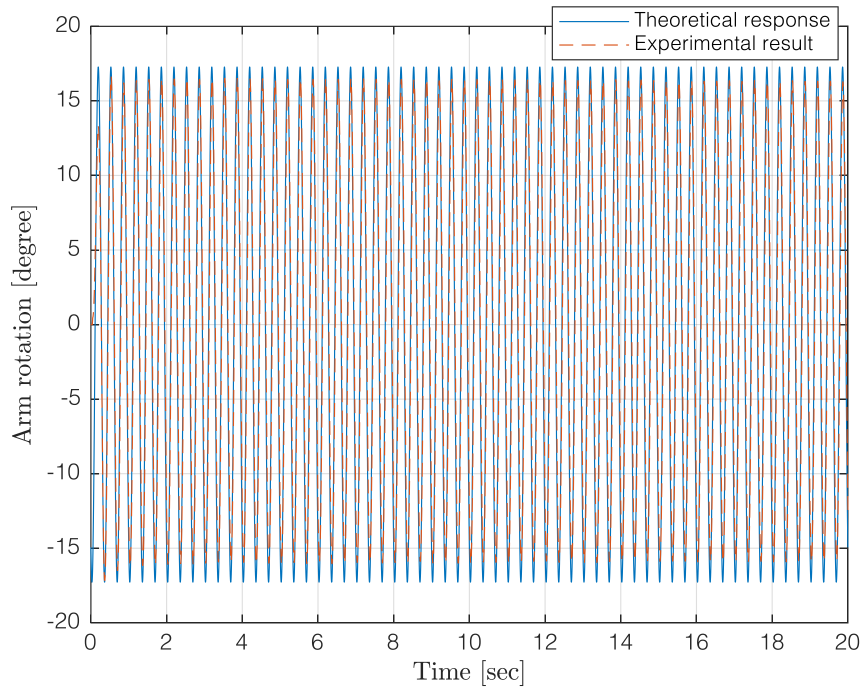


Figure 37: experimental result vs theoretical response when the input signal is $y_r(t) = 15sin(6\pi t)$

Similar to servo controller 1 when the input frequency becomes larger, we can see that the phases are well matched with each other after the transient stage, as for the amplitude, the theoretical response is larger, possible reason for the error includes unconsidered friction in the motor, not accurate enough approximated model.

6. The responses (u and y_{deg}) of the second servo system for the step of $30°$ in y_r with pendulum. What can you deduce from these responses about the closed loop robustness in the presence of plant perturbations?

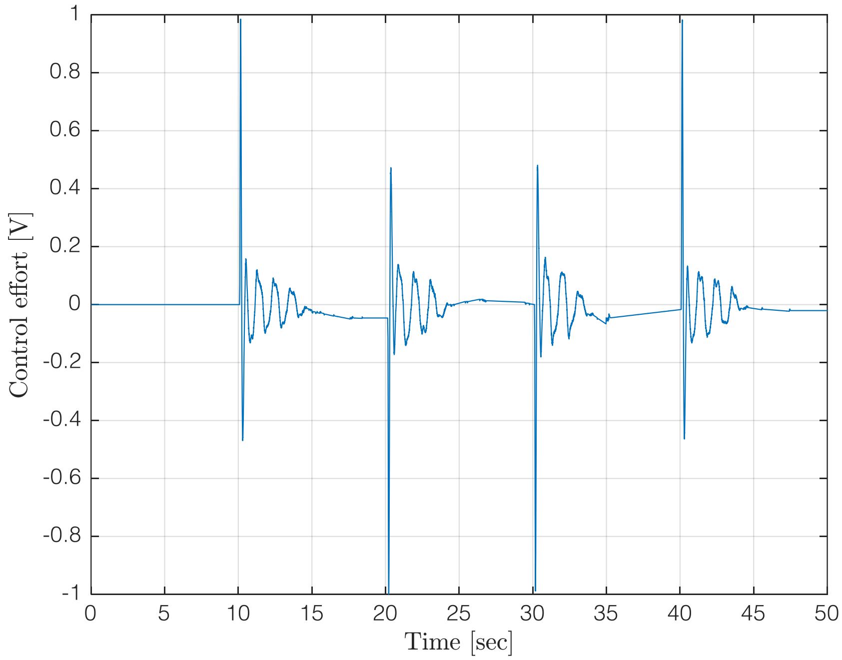


Figure 38: control effort vs time when pendulum in response to a step input of 30°

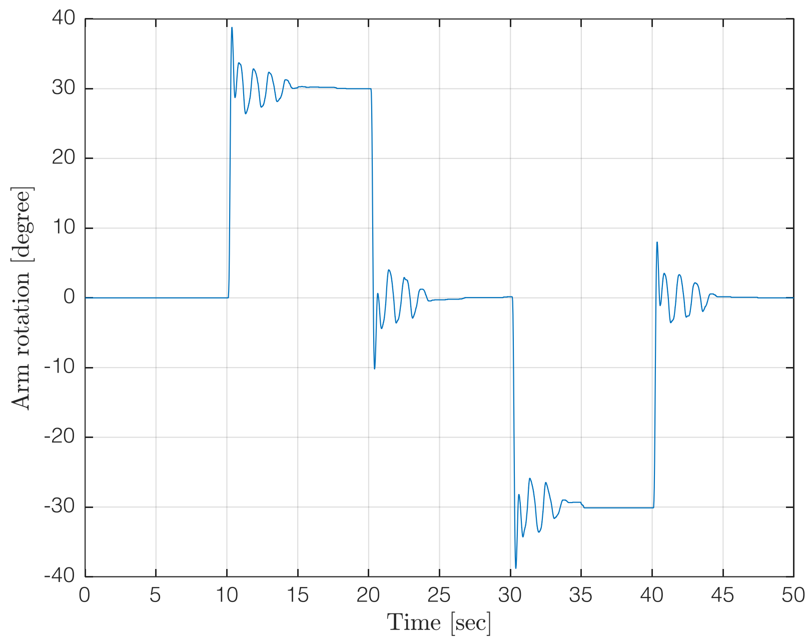


Figure 39: Arm degree vs time when pendulum in response to a step input of 30°

From the figure above, we can see that the arm still has a good tracking of the step input, though larger oscillations are present in both arm rotation and control efforts due to the passive pendulum added to the system.

• Conclusions

In this lab, we designed two servo controllers which satisfy different specifications separately. We further investigated the property of the designed controller by plotting related transfer function of the system.

In the next stage, we start to test our controllers with the motor in the lab. We first actuated the arm with step input and found that the simulation and experimental result are well matched with each other. Then, we input sine waves with different frequencies, and compare the result with the theoretical response derived from frequency response theorem. The phases of the output signals almost perfectly match with the theoretical prediction, while the amplitudes deviate a bit possibly due to motor friction or external disturbance or noise. Last, we performed the step input again with the pendulum added to the arm end. The response turned out to be more oscillatory and took longer time to return to the steady state, and more control effort was also required with the pendulum assembled.

• Suggestions for improvement (if there are).

In the section of sine wave input, the amplitude of the output signal from the experimental result doesn't match really well with the theorem, we can probably use sine inputs with more frequencies and amplitudes so that we could better understand what factors might contribute to the errors.

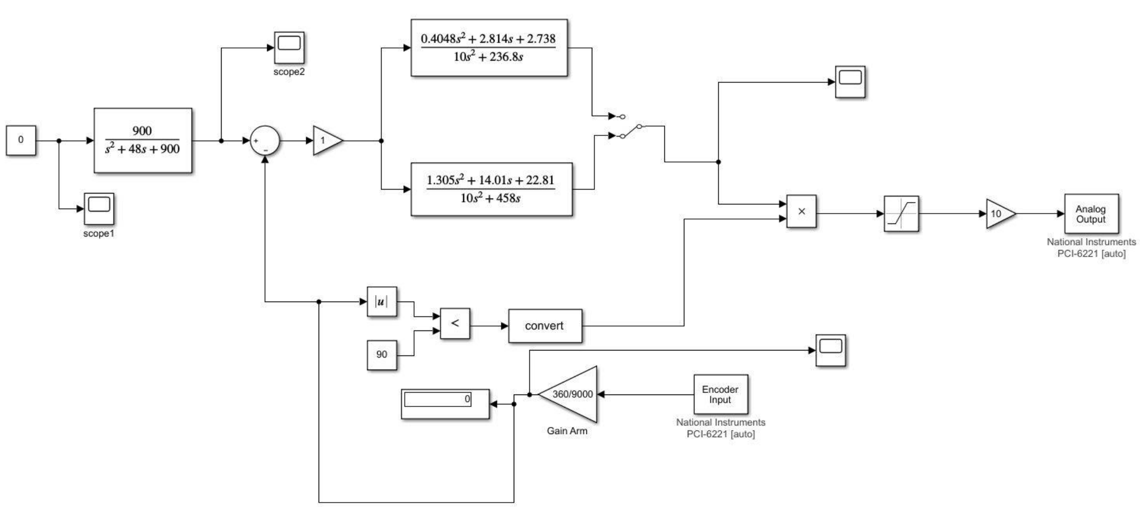


Figure 40: Simulink built in the lab (constant input)

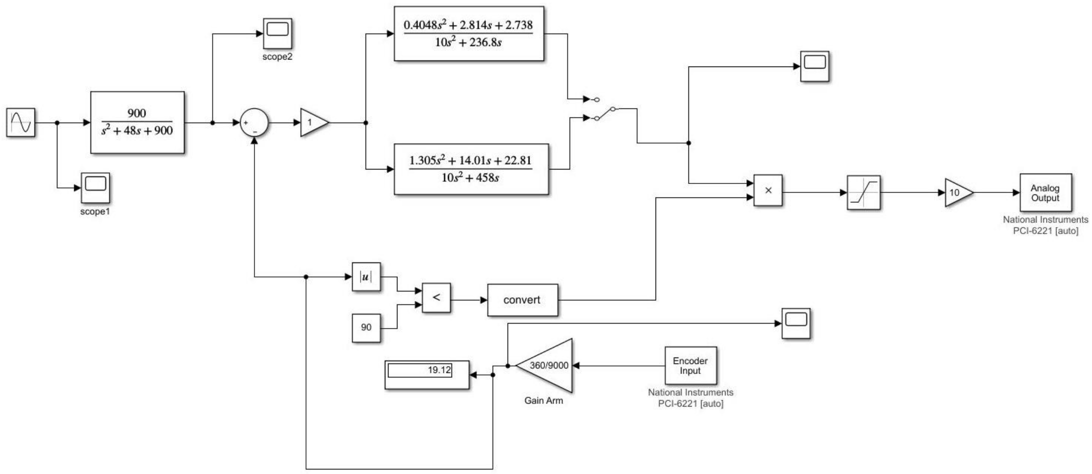


Figure 41: Simulink built in the lab (sine wave input)