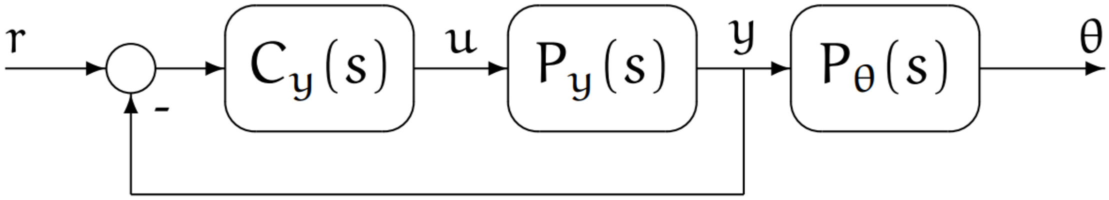
Advanced Control Lab

Pre-lab 5

Student Name	Student Name
Zhiyuan Li	941190894
Shubhankar Jadhav	941190704

Question 1



Control System given to us

Where we have $P_y(s) = \frac{100}{s(s+100)}$

$$P_\theta(s) = -\frac{180}{\pi}\frac{50s^2}{s^2 + 0.49s + 50}$$

The Servo Controller given to us is:

$$C_y(s) = \frac{40s + 179}{s}$$

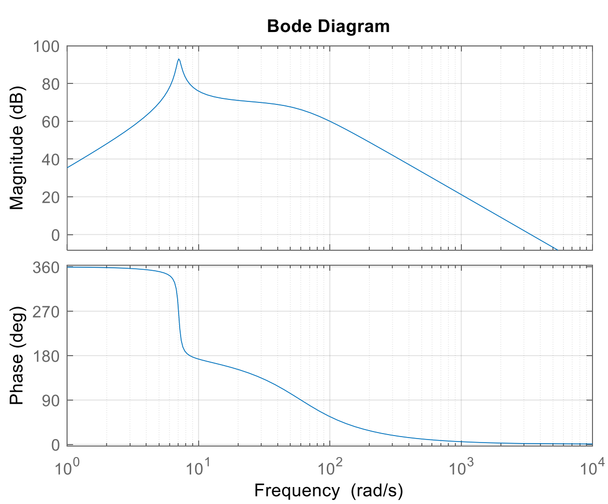
1. (15%) Show Bode plot of the transfer function from r to θ.

• What are conclusions?

• What are the resonant peak and frequency?

Solution:

$$P_{r\theta} = \frac{C_y P_y}{1 + C_y P_y} P_\theta$$



We used the Matlab command:

```
% Get the magnitude and frequency data from the Bode plot
```

```
[mag, phase, wout] = bode(L);

% Find the index of the resonant peak (maximum magnitude)
[peak_mag, peak_index] = max(mag);
resonant_frequency = wout(peak_index);

% Display the resonant peak and frequency
fprintf('Resonant Peak: %.2f dB\n', 20*log10(peak_mag));
fprintf('Resonant Frequency: %.2f rad/s\n', resonant_frequency);
```

We get the Resonant Peak and Frequency as:

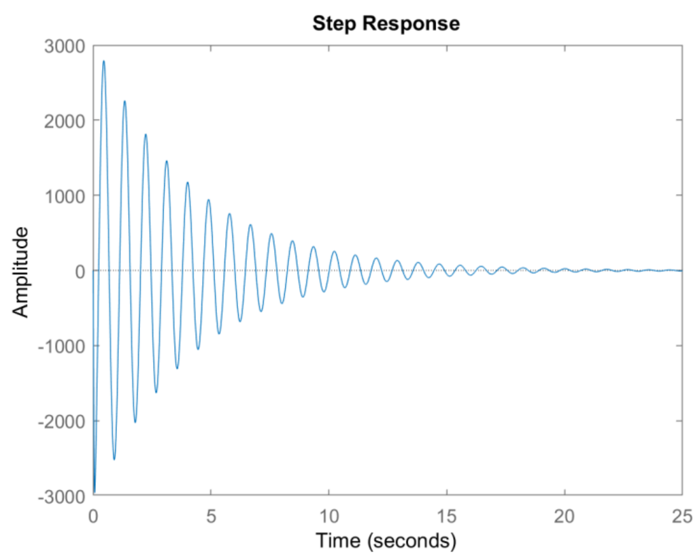
Resonant Peak: 93.04 dB

Resonant Frequency: 7.06 rad/s

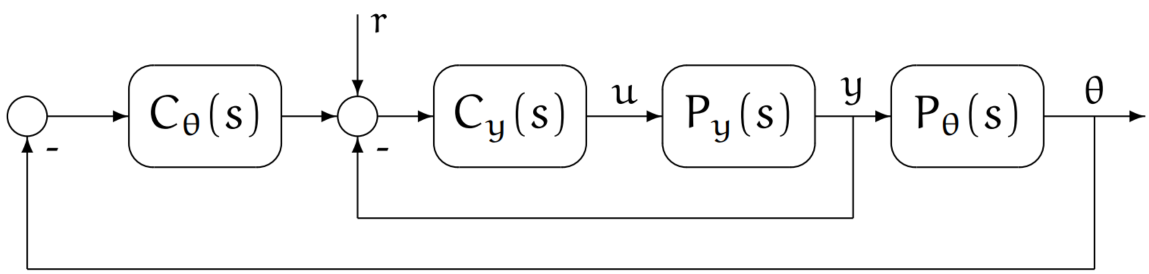
We can clearly see that the system has a very large resonant peak and 2 crossover frequencies at 0.14[rad/s] and 3395[rad/s]. This maybe due to the fact that the controller is non causal and unstable. Hence we see the system is highly oscillatory and unstable.

2. (10%) Show step response of the transfer function from r to θ.

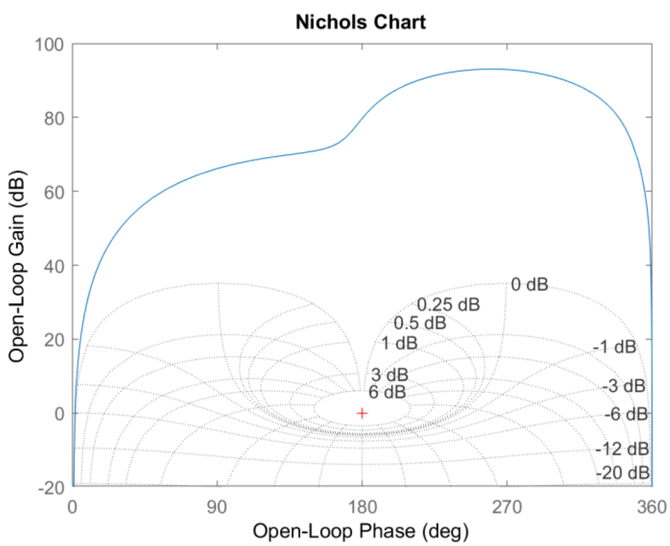
Solution:



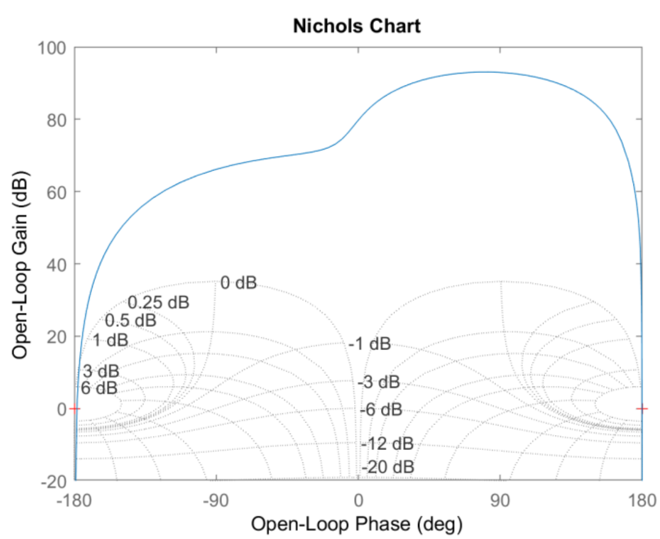
3. (35%) In order to decrease an amplitude of the pendulum oscillations the outer closed loop is added as shown in Figure 2. Design a controller Cθ that reduces the resonance peak of the outer closed loop transfer function between r and θ to a value which is no more than 75[dB]. Note: the minimal order of Cθ that satisfies the above specification is the first order.



We will use the Nichols chart to get a better insight of the system to design a better controller



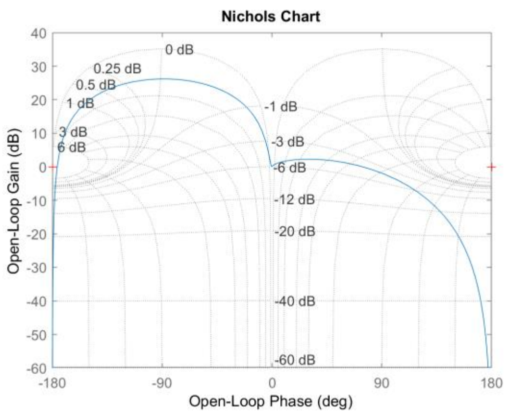
We can clearly see that the system is unstable, so we will try with the negative gain



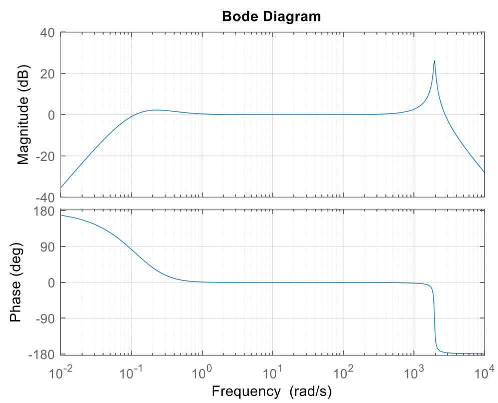
The negative gain is directly between two ciritcal points

So, we will add an inverse lead controller for the first crossover frequency of 0.14[rad/s]

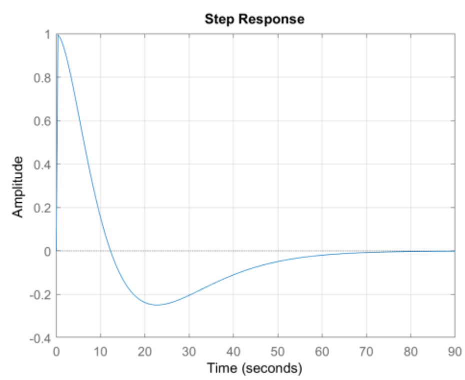
$$C_\theta = \frac{1}{C_{lead}} = \frac{s + 0.28}{2s + 0.14}$$



Nichols diagram of the new loop



Bode of new outer closed loop TF between r and θ



The step response of the transfer function

4. (25%) Compare the following characteristics of the systems described in Figure 1 and Figure 2, respectively:

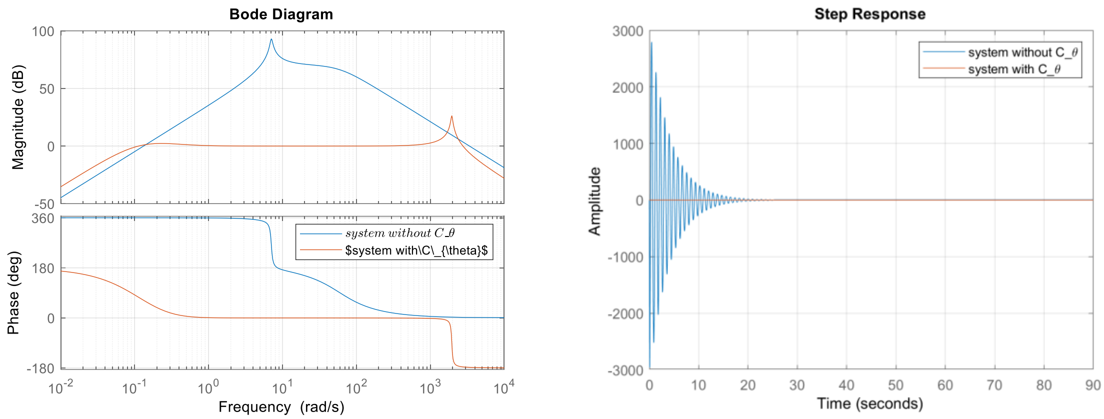
• Bode plots of the transfer functions from r to θ,

• step responses of the transfer functions from r to θ,

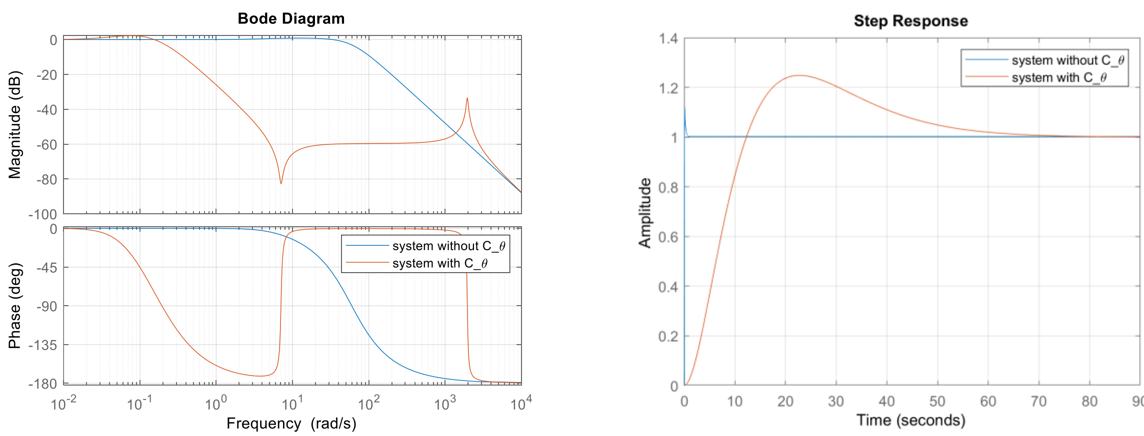
• Bode plots of the transfer functions from r to y,

• step responses of the transfer functions from r to y. What is the contribution of Cθ? Utilize the above mentioned plots to answer the question.

Solution:



Bode and Step response Comparison for TF between r to θ



As we can see that the new controller C_θ enhances the performance of the $T_{r\theta}$ by accelerating response and attenuating the oscillations. But the performance of T_{ry} suffers because of the improvement to $T_{r\theta}$ where T_{ry} suffers from greater overshoot and much longer settling time. In this way the improvement of performance of $T_{r\theta}$ reduces the performance of T_{ry}.

Question 2 (15%)

1. (10%) Find the values of $a, y(0^-), \dot{y}(0^-)$ that guarantee y(t) = 0 · 1(t).

In order to guarantee y(t) = 0 · 1(t), we must ensure that $a = -\infty,\ y(0^-), \dot{y}(0^-) = 0$. This is a theoretical answer, and if we want to simulate it in the MATLAB, we can try to make a negative, and its absolute value large enough.

2. (5%) Use the functions lsim (the response of the system to the given input in the zero initial conditions), and impulse (the response of the system to the zero input in the given initial conditions) of MATLAB in order to show the correctness of the obtained in the previous section values of $\boldsymbol{a}, \boldsymbol{y}(\mathbf{0}^-), \dot{\boldsymbol{y}}(\mathbf{0}^-)$.

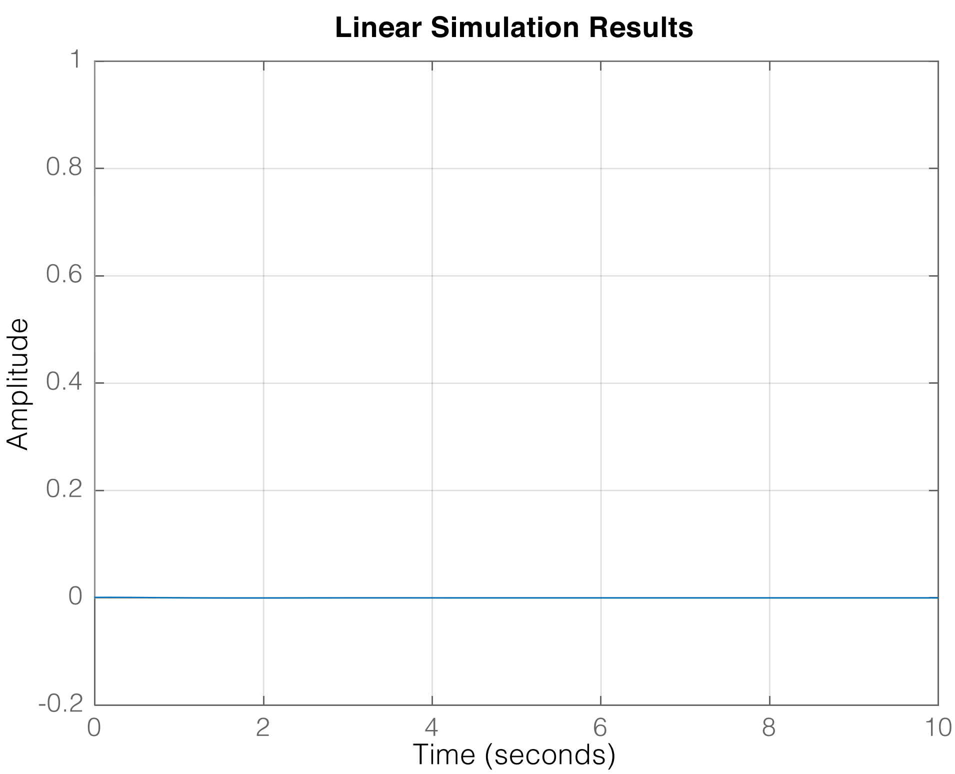


Figure: the response of the system to the given input in the zero initial conditions

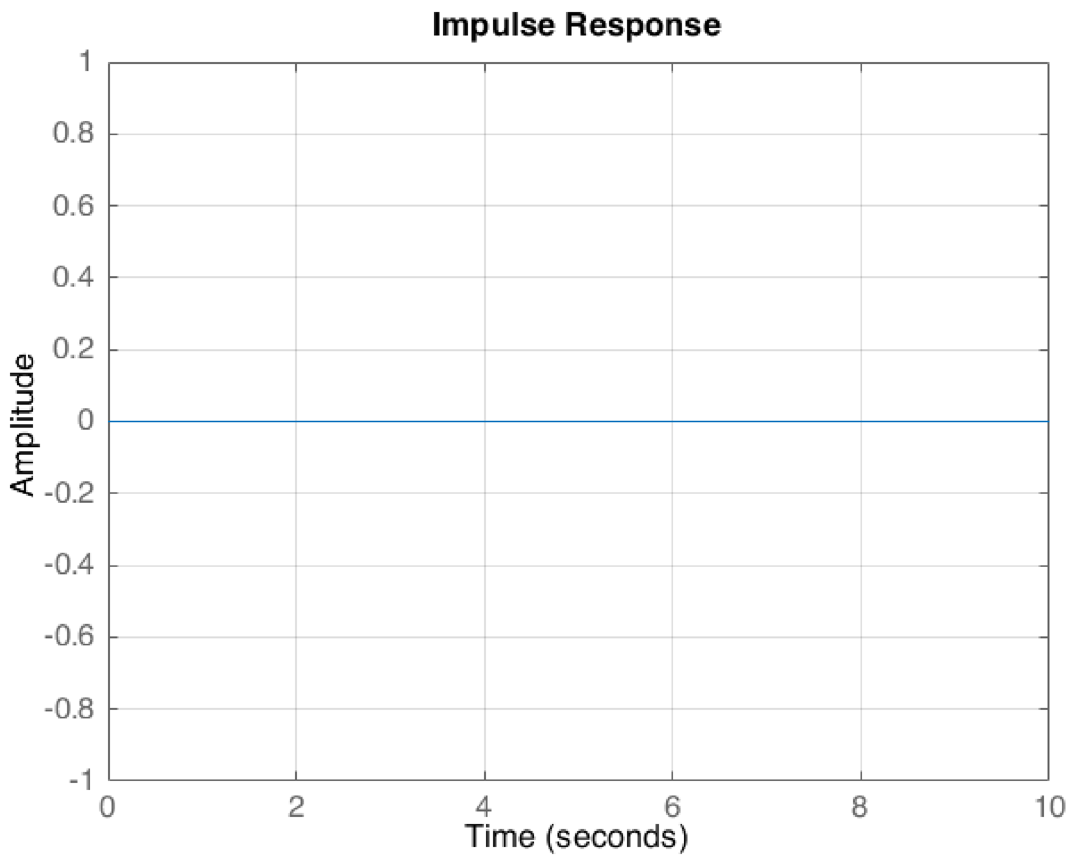


Figure: the response of the system to the zero input in the given initial conditions

As we can see, we get exactly the same graph using two methods.