**Supplementary Material — Appendices**

Appendix B – Explanatory Notes on Core Definitions and Results

B.1 Luck Measure

Applied Interpretation: The luck measure acts as a synthesizer of four quantities: probability, utility, entropy, and surprisal.

Theoretical Note: Defined on (Ω,Σ,ℙ) with information σ-algebra, measurability and monotone dependence on U,H,S ensure axiomatic consistency.

**Table B.1: Mini-examples***Input Output*P=0.2,U=1.5,H=1.1,S≈2.3 L≈1.32

B.2 Luck Bias

Applied Interpretation: Positive LB means experienced favorability exceeds bare probability. Useful for backtesting.

Theoretical Note: E[P(E|ℱ)] = P(E). Thus calibration requires unbiasedness relative to available info.

**Table B.2: Mini-examples***Input Output*L=0.40,P=0.28 LB=0.12

B.3 Normalization

Applied Interpretation: Normalization allows comparison across domains (lottery vs stocks).

Theoretical Note: Order-isomorphism preserved under any monotone transform with essential bounds.

**Table B.3: Mini-examples***Input Output*L∈{3.52,1.68,0.86} Normalized={1.00,0.477,0.244}

B.4 Multiplicative Model

Applied Interpretation: Parameters tune sensitivity to gain (α), uncertainty (β), rarity (γ).

Theoretical Note: Positive parameters ensure monotonicity; log form ensures linearity in log-space.

**Table B.4: Mini-examples***Input Output*(α,β,γ)=(1,1,1),P=0.2,U=1.5,H=1.1 L≈1.32

B.5 Logistic Model

Applied Interpretation: Maps inputs to [0,1] with interpretable coefficients.

Theoretical Note: Bounded, differentiable; GC/ULLN yields ERM consistency.

**Table B.5: Mini-examples***Input Output*(a,b,c,d)=(1,0.5,0.2,0),U=0.8,H=1.39,S=1.386 L≈0.864

B.6 Bayesian-adjusted Model

Applied Interpretation: Combines surprisal and uncertainty, weighting unexpected events.

Theoretical Note: As P→1, both H,S→0 so L→0, satisfying axioms.

**Table B.6: Mini-examples***Input Output*(λ=0.7,P=0.2,U=1.5,H=1.1,S=1.609) L≈1.68

B.7 Information Collapse

Applied Interpretation: After an event resolves, residual luck vanishes.

Theoretical Note: Doob martingale convergence ensures collapse of uncertainty as H\_t,S\_t→0.

**Table B.7: Mini-examples***Input Output*Event realized (1\_E=1) L\_t→0

B.8 ERM/M-Estimation Consistency

Applied Interpretation: Train on rolling windows, expect stabilization.

Theoretical Note: Under GC/ULLN, argmin-consistency holds with identifiability.

**Table B.8: Mini-examples***Input Output*Fit (α,β,γ) on 10k games Stable estimates

B.9 Lipschitz Stability

Applied Interpretation: Small probability changes shouldn’t cause big L jumps.

Theoretical Note: Ensures error bounds; required for GC/ULLN families.

**Table B.9: Mini-examples***Input Output*ΔP=0.02 ΔL ≤0.02·const

B.10 Chaotic Extension

Applied Interpretation: Captures sensitivity in chaotic domains (markets, weather).

Theoretical Note: Linked with Lyapunov exponents; sensitive dependence ⇒ derivative growth.

**Table B.10: Mini-examples***Input Output*Δx0 small ||∇E[U]|| increases ⇒ larger L\_chaos

**Appendix C — Lottery (OPAP Joker): Mechanics, Expected Jackpot Frequency, and Calibration Notes**

**4.X Lottery (Joker) Case Study – Calibration and Luck Features**

We integrate OPAP Joker draws (1997–2024). For 2015–2024 we extract sales per draw (columns\_sold) and winners per tier directly from OPAP-format spreadsheets, enabling a ground-truth calibration of the probability that at least one jackpot (5+1) winner occurs.

Using the per-column jackpot probability p⋆=1/(C(45,5)·20) and total sales N, the predicted probability for ≥1 jackpot winner in a draw is p̂(N)=1−(1−p⋆)^N. We compare p̂ to the observed jackpot frequency across sales deciles. Calibration is strong in the middle deciles; expected deviations at extremes reflect sparse data. These calibrated probabilities define P for the Luck measure; we derive H (Bernoulli entropy) and S (surprisal) per draw. Utility U can be set from tier payouts (e.g., jackpot amount).

|  |  |
| --- | --- |
| Metric | Value |
| Sample (2015–2024) with sales+winners, N draws | 1144 |
| Brier score (jackpot occurrence) | 0.1038 |
| Log loss (jackpot occurrence) | 0.3540 |

*Figure C.1 – Joker (5/45) main-number frequency (1997–2014).*

*Figure C.2 – Joker (1/20) bonus-number frequency (1997–2014).*

*Figure C.3 – Jackpot calibration vs sales deciles (2015–2024).*

*Figure C.4 – Distribution of rollover streaks (2015–2024).*

These results certify a calibrated P for ≥1 jackpot winner. In Section 2 we proved that, under GC/ULLN and Lipschitz conditions, the ERM estimator for Lθ is consistent; here we instantiate P,H,S from lottery physics and set U from observed payouts, enabling out-of-sample evaluation of multiplicative and logistic Luck models with proper scores (Brier/log loss) and decision utility.

**4.X.1 ERM with Tier Payouts (Utility U)**

We enrich the feature set with per-tier dividents from OPAP spreadsheets (2015–2024). Utility is defined as the total euro payout across tiers per draw, U\_total = Σ\_t winners\_t × divident\_t. We transform U with log1p(·/1e6) for numerical stability. A logistic Lθ model using [U\_tot\_m, H, S, 1−P] is trained on 2015–2019 and evaluated on 2020–2024.

|  |  |  |
| --- | --- | --- |
| Model | Brier (↓) | Log loss (↓) |
| Baseline p̂(N) | 0.0859 | 0.3074 |
| No-U (H,S,1−P) | 0.0866 | 0.3088 |
| Lθ (U,H,S,1−P) | 0.0609 | 0.2136 |

Estimated coefficients on the test configuration (signs match intuition): U\_tot\_m=6.55 (+), H=-0.80 (−), S=3.21 (+), 1−P=0.74 (+), intercept=-12.23.

*Figure C.5 – Reliability: baseline vs No-U vs Lθ (2020–2024).*

*Figure C.6 – Brier scores (test 2020–2024).*

*Figure C.7 – Log loss (test 2020–2024).*

Result: Lθ with U markedly improves proper scores versus probabilistic baseline p̂(N), and ablation confirms U’s contribution; without U, metrics revert to baseline. This supports the claim that utility-aware luck captures decision relevance beyond rarity/surprisal alone.

**4.X.2 Methods – OPAP Data Enrichment & ERM Setup**

Data sources: We parse OPAP Joker spreadsheets (2015–2024) using a robust two-row header layout that pairs winners and dividents (ΚΕΡΔΗ) for each tier. We extract per-draw sales (columns\_sold), winners per tier, and per-winner dividents. For 1997–2014, we ingest draw IDs, dates, and numbers, and join them into a unified dataset.

Probability model: With Ω = C(45,5) × 20 outcomes per column and p⋆ = 1/Ω for the 5+1 hit, the predicted probability of ≥1 jackpot winner given N sold columns is p̂(N) = 1 − (1 − p⋆)^N. This provides the calibrated P per draw. We derive H(P) = −P log P − (1−P) log(1−P) and surprisal S = −log P.

Utility: We compute U\_total = Σ\_t winners\_t × divident\_t across tiers (per draw), and use U\_tot\_m = log1p(U\_total / 10^6) as a stabilized feature. ERM: We fit a logistic Lθ on training years (2015–2019) over features [U\_tot\_m, H, S, 1−P] and evaluate on 2020–2024 with Brier/log-loss and reliability plots; ablations remove U to test contribution.

**Data & Reproducibility**

We supply an enriched CSV (1997–2024) combining draws, sales, winners, and dividents, along with calibration and ERM evaluation plots. Results are reproducible by re-running the exact parsing and ERM steps.

|  |  |
| --- | --- |
| Artifact | Path |
| Enriched dataset (1997–2024) | /mnt/data/joker\_1997\_2024\_rich.csv |
| Reliability comparison (2020–2024) | /mnt/data/joker\_reliability\_comparison\_2020\_2024.png |
| Brier bars | /mnt/data/joker\_brier\_bars.png |
| Log-loss bars | /mnt/data/joker\_logloss\_bars.png |

**4.X.3 Limitations & Future Work**

Limitations — Our calibration relies on OPAP-reported sales per draw; missing values or reporting errors can bias the estimator p̂(N). Seasonal demand and jackpot-advertising shocks shift the distribution of columns\_sold (covariate shift); decile-wise calibration mitigates but does not eliminate the effect. We assume a constant single-column jackpot probability p⋆ under fixed game rules (5/45, 1/20); rule changes would require updating. Utility U was instantiated as aggregate euro payout per draw; alternative U definitions (e.g., player-level expected utility with risk aversion, per-euro return, tier-weighted utilities, or inclusion of costs) may better reflect specific decision contexts. The logistic Lθ head is parametric and may underfit local nonlinearities; more flexible yet regularized classes (e.g., isotonic regression, monotone GBMs) are a natural extension. Finally, uncertainty on Brier/log-loss deltas was assessed out of sample; bootstrap or block-bootstrap over time could tighten confidence intervals and quantify drift.

Future Work — Extend to 2025+ via the live OPAP API; add covariates (advertising intensity, rollover size, day-of-week/seasonality) to explain sales-driven shifts; incorporate ROI-aware decision analysis with explicit costs; test alternative proper scores; and develop a Bayesian hierarchical formulation that links U,H,S and demand dynamics while enforcing the axioms (normalization, information collapse, Lipschitz stability).

**Appendix D — Online Slots: Starburst (Low-Vol) vs Book of Dead (High-Vol)**

**Online Slots – Low vs High Volatility: Starburst vs Book of Dead**

We construct RTP- and hit-frequency–calibrated surrogates for two iconic titles: Starburst (NetEnt, low volatility) and Book of Dead (Play’n GO, high volatility). We then simulate ROI distributions, risk-of-ruin curves, and show payout mass functions.

|  |  |
| --- | --- |
| Metric | Value |
| Starburst-like RTP | 0.9609 |
| Starburst-like hit frequency | 0.226 |
| ROI median (20k spins) | -0.041 |
| Ruin @10k spins (B=1000) | 0.048 |
| Risk of ruin: B=1000, b=1, T=10k | 0.031 |
| Risk of ruin: B=2000, b=1, T=10k | 0.000 |
| Risk of ruin: B=1000, b=1, T=50k | 0.940 |

*Figure D.1 – Starburst-like payout mass function (semi-log).*

*Figure D.2 – Starburst-like ROI distribution after 20,000 spins.*

*Figure D.3 – Starburst-like wealth snapshots (2k/5k/10k spins).*

**High Volatility Foil – Book of Dead-like**

|  |  |
| --- | --- |
| Metric | Value |
| Book of Dead-like RTP | 0.9621 |
| Book of Dead-like hit frequency | 0.291 |
| ROI median (20k spins) | -0.096 |
| Ruin @10k spins (B=1000) | 0.766 |

*Figure D.4 – Payout mass functions: low vs high volatility.*

*Figure D.5 – ROI distributions after 20,000 spins: low vs high volatility.*

*Figure D.6 – Risk of ruin vs horizon (B=1000, b=1).*

**Appendix E — Roulette: Theoretical Benchmarks, La Partage / En Prison, and Betting Systems**

**Roulette – Theoretical Benchmarks, La Partage / En Prison, and Betting Systems**

Single-zero European wheel: 37 pockets. Double-zero American: 38 pockets. Payouts: 35:1 (straight), 17:1 (split), 11:1 (street), 8:1 (corner), 5:1 (six-line), 2:1 (dozen/column), 1:1 (even-money). House edge ≈2.70% (EU) and ≈5.26% (US); American five-number (0,00,1,2,3) ≈7.89%. Even-money with La Partage halves loss on zero → ≈1.35% edge.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Bet | P(win) EU | House edge EU | P(win) US | House edge US |
| Straight-up (1 number) | 0.02703 | 0.0270 | 0.02632 | 0.0526 |
| Split (2 numbers) | 0.05405 | 0.0270 | 0.05263 | 0.0526 |
| Street (3 numbers) | 0.08108 | 0.0270 | 0.07895 | 0.0526 |
| Corner (4 numbers) | 0.10811 | 0.0270 | 0.10526 | 0.0526 |
| Six line (6 numbers) | 0.16216 | 0.0270 | 0.15789 | 0.0526 |
| Dozen / Column (12 numbers) | 0.32432 | 0.0270 | 0.31579 | 0.0526 |
| Even-money (Red/Black, Even/Odd, Low/High) | 0.48649 | 0.0270 | 0.47368 | 0.0526 |
| Five-number (0,00,1,2,3) – American only |  |  | 0.13158 | 0.0789 |
| Even-money with La Partage (EU) | 0.48649 | 0.0135 |  |  |

*Figure E.1 – House edge by bet type (European vs American).*

*Figure E.2 – Even-money house edge: EU vs EU+La Partage vs US.*

**En Prison vs La Partage (Even-Money)**

*Figure E.3 – Even-money: La Partage and En Prison are EV-equivalent (simulation confirmation).*

**Monte Carlo: EV Convergence & Table-Limit Pressure**

*Figure E.4 – EV convergence (mean±sd) across bet types (EU vs US).*

*Figure E.5 – Martingale: ending-wealth distribution under table limits.*

*Figure E.6 – Anti-Martingale (Paroli): ending-wealth distribution (streak cap=3).*

*Figure E.7 – Martingale: stake trajectory (example session).*

*Figure E.8 – Anti-Martingale: stake trajectory (example session).*

Appendix F — Football (Super League Greece): Market-Implied Probabilities, Utility & Luck

**Super League (Greece) – Market-Implied P, Utility and Luck**

Data source: Football-Data.co.uk (league code G1). We use closing/average odds to derive market-implied probabilities for {Home, Draw, Away}, corrected for overround. The realized outcome’s probability defines surprisal S; entropy H is computed over the 3-way distribution. Utility U is modeled as unit-stake net return on the argmax-probability selection (illustrative baseline).

|  |  |
| --- | --- |
| Metric | Value |
| Season (latest) | 2024/25 |
| Matches | 233 |
| Brier score (3-way) | 0.5812 |
| Log loss (3-way) | 0.9794 |
| Mean unit utility (per match) | -0.1176 |

*Figure F.1 – Reliability of argmax market probabilities (latest season).*

*Figure F.2 – Distribution of per-match unit utility (latest season).*

*Figure F.3 – Cumulative unit-stake profit (latest season).*

*Figure F.4 – Kelly log-wealth (λ\_K=0.25, cap 5%/match).*

**SPI Features & Kelly Staking (Optional Enhancements)**

If FiveThirtyEight SPI probabilities are available, blend or recalibrate market probabilities with SPI to minimize 3-way log-loss. Kelly staking with fractional λ\_K and per-match caps evaluates bankroll growth; block-bootstrap over matchdays yields uncertainty bands.

**Comparative Season Analysis – 2023/24 vs 2024/25**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Season | Matches | Brier (3-way) | Log-loss (3-way) | Mean unit utility |
| 2023/24 | 240 | 0.5306 | 0.9004 | -0.0640 |
| 2024/25 | 233 | 0.5812 | 0.9794 | -0.1176 |

*Figure F.5 – Reliability comparison (2023/24 vs 2024/25).*

*Figure F.6 – Unit-stake cumulative profit comparison (2023/24 vs 2024/25).*

**Appendix G — Equity Tail-Event Calibration: SPX (R<−2%) & AAPL (R<−3%), Normal vs t vs EWMA**

**Stocks Mini-Case – SPX (R<-2%) vs AAPL (R<-3%), Normal vs t(ν=6) vs EWMA**

We add SPX heavy-tail and time-varying volatility estimators for symmetry. Models: 60-day Normal; 60-day Studentt (ν=6, variance-matched); EWMA Normal (λ=0.94, mean≈0). Thresholds: SPX R<−2%, AAPL R<−3%.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ticker | Threshold | Model | Obs | Event rate | Brier | Log-loss |
| SPX | -2% | Normal(60d) | 39,451 | 0.0311 | 0.02815 | 0.11617 |
| SPX | -2% | t(ν=6, 60d) | 39,451 | 0.0311 | 0.02796 | 0.11298 |
| SPX | -2% | EWMA(λ=0.94) | 39,260 | 0.0305 | 0.02785 | 0.12115 |
| AAPL | -3% | Normal(60d) | 10,256 | 0.0824 | 0.07286 | 0.26441 |
| AAPL | -3% | t(ν=6, 60d) | 10,256 | 0.0824 | 0.07223 | 0.26153 |
| AAPL | -3% | EWMA(λ=0.94) | 10,065 | 0.0814 | 0.07264 | 0.26669 |

*Figure G.1 – AAPL (R<−3%): reliability across Normal, t(ν=6), and EWMA.*

*Figure G.2 – SPX (R<−2%): reliability across Normal, t(ν=6), and EWMA.*

*Figure G.3 – AAPL (R<−3%): 20-day tail-event incidence over time.*

### *Conclusions*

We introduced a unified “Luck” functional that fuses probability, utility, surprisal, and information collapse, and proved normalization and convergence properties (GC/ULLN) enabling consistent ERM. Applications across lotteries, football markets, roulette, and slots demonstrate that the framework is domain-agnostic yet empirically testable: calibration curves align with theory, and variance-driven phenomena (e.g., table limits or high-volatility slots) appear exactly where the axioms predict.

### Practical Implications

For applied modelers, the framework yields: (i) transparent calibration targets, (ii) robustness via Lipschitz constraints, and (iii) decision metrics that remain comparable across domains with different base rates and paytables.

### Limitations & Future Work

Our results depend on sales reporting quality (lotteries), seasonal demand (sports), operator RTP settings (slots), and simple return dynamics (equities). Future work: richer utility U (risk-averse/CRRA, prospect-theory), heavy-tail & regime-switching for returns, multi-market information flows (cross-entropy regularizers), and non-parametric calibration under partial observability.

### Data & Code Availability

All code and data to reproduce figures and tables (lottery tiers, football odds, roulette/slots simulations, SPX/AAPL analyses) will be released at [repository/DOI placeholder]. A reproducibility README includes versions, seeds, and run scripts.

### CRediT Author Statement

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### Ethics Statement

This study does not involve human participants, personal data, or animal subjects. All analyses use public or synthetic datasets.

### Disclaimer (Research-Only)

This article is for research and education. It is not gambling or investment advice. Casino games have negative expected value; sports and markets involve financial risk; past performance does not guarantee future results.

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Starburst (NetEnt) and Book of Dead (Play’n GO) are trademarks of their respective owners. Slot models herein are surrogate, RTP/hit-rate–calibrated approximations, not reverse-engineered paytables.

### Supplementary/Appendices

* Appendix B – Explanatory Notes on Core Definitions and Results (with mini-examples).
* Appendix C – Lottery (OPAP Joker): Mechanics & Jackpot Frequency.
* Appendix D – Online Slots: Starburst (Low-Vol) vs Book of Dead (High-Vol).
* Appendix E – Roulette: Theory, La Partage / En Prison, Systems & Variance.
* Appendix F – Football (Super League): Market‑Implied Probabilities (Home/Draw/Away), Utility & Luck.
* Appendix G – Equities: SPX (R<−2%) & AAPL (R<−3%), Normal vs t vs EWMA.

**Appendix H — Unluck: Axioms, Models, and Convergence**

We define an Unluck functional as the adverse counterpart to Luck. Let U be decision utility and set D = max{−U, 0}. Unluck 𝒰\_θ(D,H,S,P) is non‑negative, increases with disutility D, amplifies rare unfavorable events via surprisal S and (1−P), and collapses to 0 as information is fully revealed (H,S→0).

Axioms (U1–U8):

• U1 Non‑negativity & Nullity: 𝒰=0 if D=0 or P=1 (S=0).

• U2 Monotonicity in D: higher loss ⇒ higher 𝒰.

• U3 Rarity amplification: for fixed D,H, 𝒰 increases with S and as (1−P) rises.

• U4 Information collapse: along a filtration, H\_t,S\_t→0 ⇒ 𝒰\_t→0.

• U5 Order‑preserving reparametrization in D (strictly increasing transforms preserve ranking).

• U6 Calibration compatibility with E[1\_E|ℱ]=P.

• U7 Regularity: continuity and local Lipschitzness on compacts.

• U8 ERM identifiability: distinct θ induce distinct risks.

(H.1)

A normalized Unluck (H.1) enables cross‑domain comparability and clean thresholds for alerts.

(H.2)

Sketch: with P\_t(E)=E[1\_E|ℱ\_t] a Doob martingale and (H\_t,S\_t)→0, continuity and boundedness yield a.s. and L¹ convergence to 0.

**Appendix I — Unified Luck–Unluck**

We merge upside Luck and downside Unluck into a single signed composite. Let U⁺=max{U,0}, U⁻=max{−U,0} and information terms (H,S) with base probability P.

(I.1)

In the symmetric case ϕ=ψ=id, we obtain a centered 𝓛 = Luck⁺ − Unluck⁻. A normalized [−1,1] mapping follows.

(I.2)

Mini‑examples (Input → Output):

• P=0.10, H≈0.802, S≈2.303, U=+1.2 → Luck⁺>0, Unluck⁻=0.

• P=0.05, H≈0.898, S≈2.996, U=−1.8 → Luck⁺=0, Unluck⁻>0.

• P=0.20, H≈1.029, S≈1.609, U=+0.7 → modest Luck⁺.

• P=0.01, H≈0.424, S≈4.605, U=−2.5 → large Unluck⁻.

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