A Formal Measure of Luck: Theoretical Foundations, Convergence, and Empirical Validation

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We propose a rigorous mathematical framework for the concept of luck, treated as distinct from probability. Our measure integrates probability, utility, entropy, and surprisal, and satisfies a set of axioms ensuring consistency with intuition. We develop normalization and convergence theorems, establish empirical risk minimization (ERM) consistency, and demonstrate applications in games of chance, sports betting, and financial markets with numerical examples and calibration plots.

# Theoretical Framework: Formalizing Luck

## Foundational Setting

Let be a probability space. Let denote an information set available to an observer, and let denote a filtration representing refining information over time.

Define:

* A utility function , with . We assume normalization unless stated otherwise.
* Conditional probability (a bounded martingale in ).
* Conditional Shannon entropy of partitions measurable with respect to .
* Surprisal .

**Definition 1** (Luck Measure). *A luck measure is a functional where is parameterized and admissible.*

## Axioms of Luck

**Axiom 1** (Non-negativity). *.*

**Axiom 2** (Triviality under certainty). *If , then .*

**Axiom 3** (Monotonicity in utility). *If , then .*

**Axiom 4** (Sensitivity to surprisal). *.*

**Axiom 5** (Information-collapse). *If and , then .*

## Luck Bias and Equivalence

**Definition 2** (Luck Bias).

**Theorem 1** (Affine Equivalence). *If with , then the two measures induce the same event ranking.*

# Normalization and Stability

**Definition 3** (Normalization). *Define*

**Theorem 2** (Preservation of Axioms). *If satisfies the axioms, then any strictly monotone transform (with ) also satisfies them.*

**Theorem 3** (Uniqueness up to Monotone Transform). *Any two admissible measures with the same ranking are related by a strictly increasing transformation.*

**Proposition 1** (Lipschitz Stability). *If is -Lipschitz, then .*

# Convergence Under Refining Information

**Theorem 4** (a.s. Convergence). *If , , , and continuous, then realized luck a.s.*

**Theorem 5** ( Convergence). *If , then .*

**Theorem 6** (Consistency of ERM). *With compact, continuous loss , and Glivenko–Cantelli conditions, the ERM estimator converges in probability to the population minimizer.*

# Experimental Protocols

## Datasets

(a) Simulated roulette ( spins). (b) Historical football odds ( matches). (c) Quarterly earnings surprises (Compustat, CRSP).

## ERM Procedure

Compute empirical risk with losses: squared error, logistic, Brier.

## GC/ULLN Proof Sketch

For squared error , bounded by 1 on , the class is GC by equicontinuity and compactness. Thus uniformly, yielding ERM consistency.

# Enhanced Applications

## Roulette Simulation

Event , , . Multiplicative model : . As , .

## Sports Betting

Event: away underdog wins, , . Bayesian model : .

## Financial Earnings

Event: EPS surprise , , . Logistic model : .

# Conclusion

Luck can be mathematically formalized as a functional of probability, utility, entropy, and surprisal. We proved normalization, convergence, and consistency results, and validated with simulations and empirical data.