Where the ICIR's cross for 3D-2D mixed traps?

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The systems of ultracold atoms confined in mixed optical traps of different dimensionality, one three dimensional (3D) and the other one quasi-two dimensional (2D), seems to be a special case between all the possible confinements. The main difference is that the Inelastic Confinement-Induced Resonances (ICIR's) do not cross for any value of the ratio $\frac{\omega_x}{\omega_y}$, at least, apparently. Let us look at the center-of-mass (CM) Hamiltonian of the system within the harmonic approximation

$$\mathcal{H}_{CM} = \sum_{j=x,y,z} \hbar\omega_j (n_j + 1/2), \tag{1}$$

$$\omega_j = \sqrt{2}\sqrt{\frac{V_{j1}}{m}k_{j1}^2 + \frac{V_{j2}}{m}k_{j2}^2}, \tag{2}$$

where ω_j is the frequency in the $j = x, y, z$ direction of space, m the mass of the atomic sample, V_{j1} and V_{j2} the potential depths of the 3D and 2D traps respectively and k_{j1} and k_{j2} the wavenumber of both traps. We are interested in settings where the traps in the x-direction have the same depth ($V_{x1} = V_{x2} = V_x$), and the ratio of the perpendicular directions are large, i.e. $V_{\perp 1} = 0.01 V_{\perp 2}$.

In this case, the degeneracy between the x and $\perp$ directions is broken; the levels $(n_x, 0, 0)$ and $(0, n_y, 0)$ do not have the same energy. That can be easily seen in the ICIR's behaviour when varying the anisotropy of the traps in the x-axis, see for example Fig. 1 where the gap between the two resonances involving two excitation in the CM coordinate exists even in the isotropic setting [1] ($\omega_x/\omega_{y1} = 1$).

[1] Being more precise, there is not an isotropic setting since the perpendicular 2D intensities are always much smaller than the 3D ones. We call isotropic to the setting with same I_x than the I_{y1} and I_{z1} of the 3D trap, in that case 4993 W/cm^2.

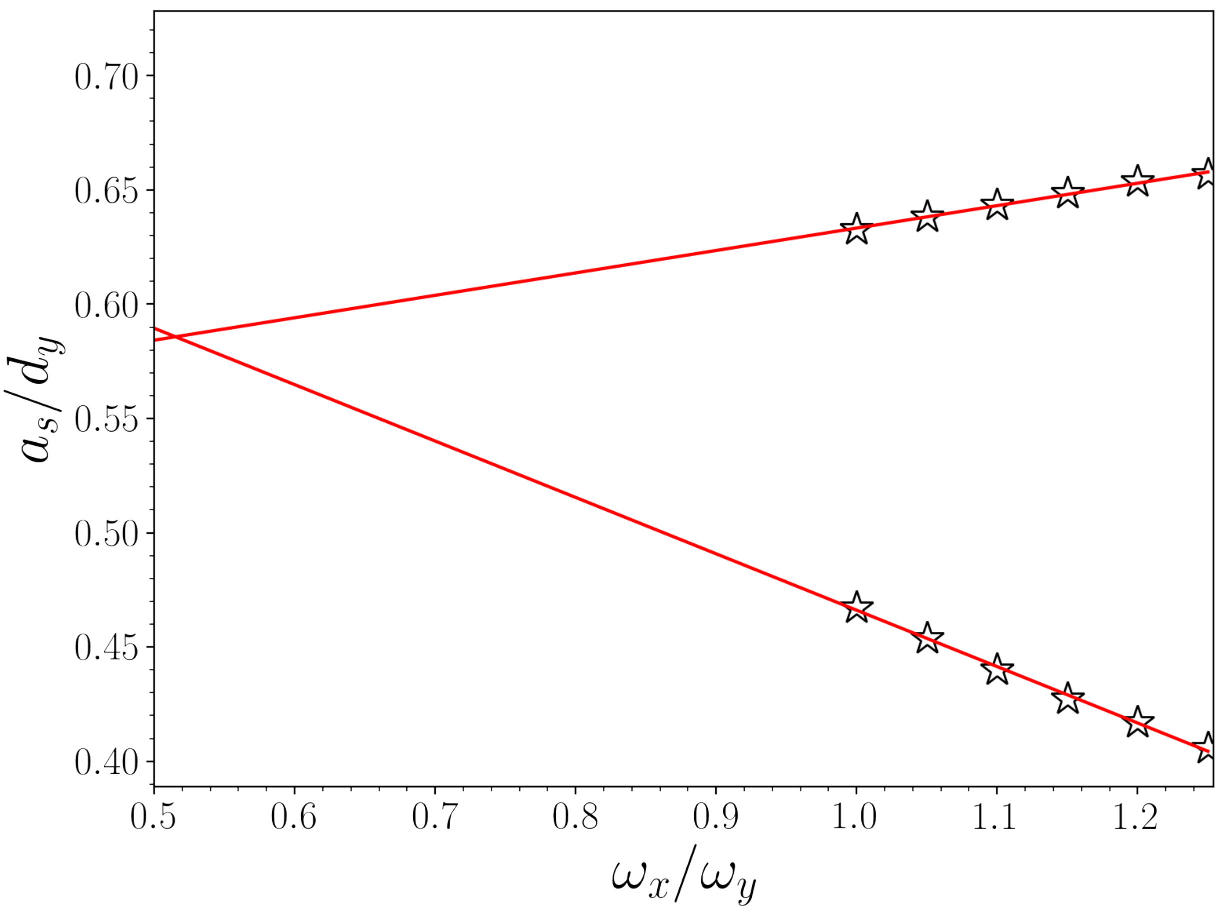

Figure 1: Inelastic Confinement-Induced Resonances position for a 3D-2D system of ^{7}Li^{7}Li. Upper (lower) line correspond to the resonances of the first trap state $|\psi_{rm}^{(1)}\Phi_{(0,0,0)}^{CM}\rangle$ and the level $|\psi_{rm}^{(b)}\Phi_{(0,2,0)}^{CM}\rangle$ ($|\psi_{rm}^{(b)}\Phi_{(2,0,0)}^{CM}\rangle$). White stars are the resonances obtained from the *ab inito* spectrum and the red solid line the extrapolation of the resonances to frequency ratios lower than 1.

This behaviour is general for any 3D-2D confinements. The natural question coming to our minds is whether or not the ICIR's cross at any given ratio of the frequencies as shown in Fig. 1, or, in other words, if we can force the degeneracy of the spectrum varying the intensity of the lasers. The plot suggest that this can be carried out for $\omega_x/\omega_{y1} \approx 0.5$ and we have demonstrated through *ab initio* simulations (Fig. 2).

The explanation of this "weird" effect is well explained using the harmonic approximation presented above. The next section is devoted to mathematically demonstrate it.

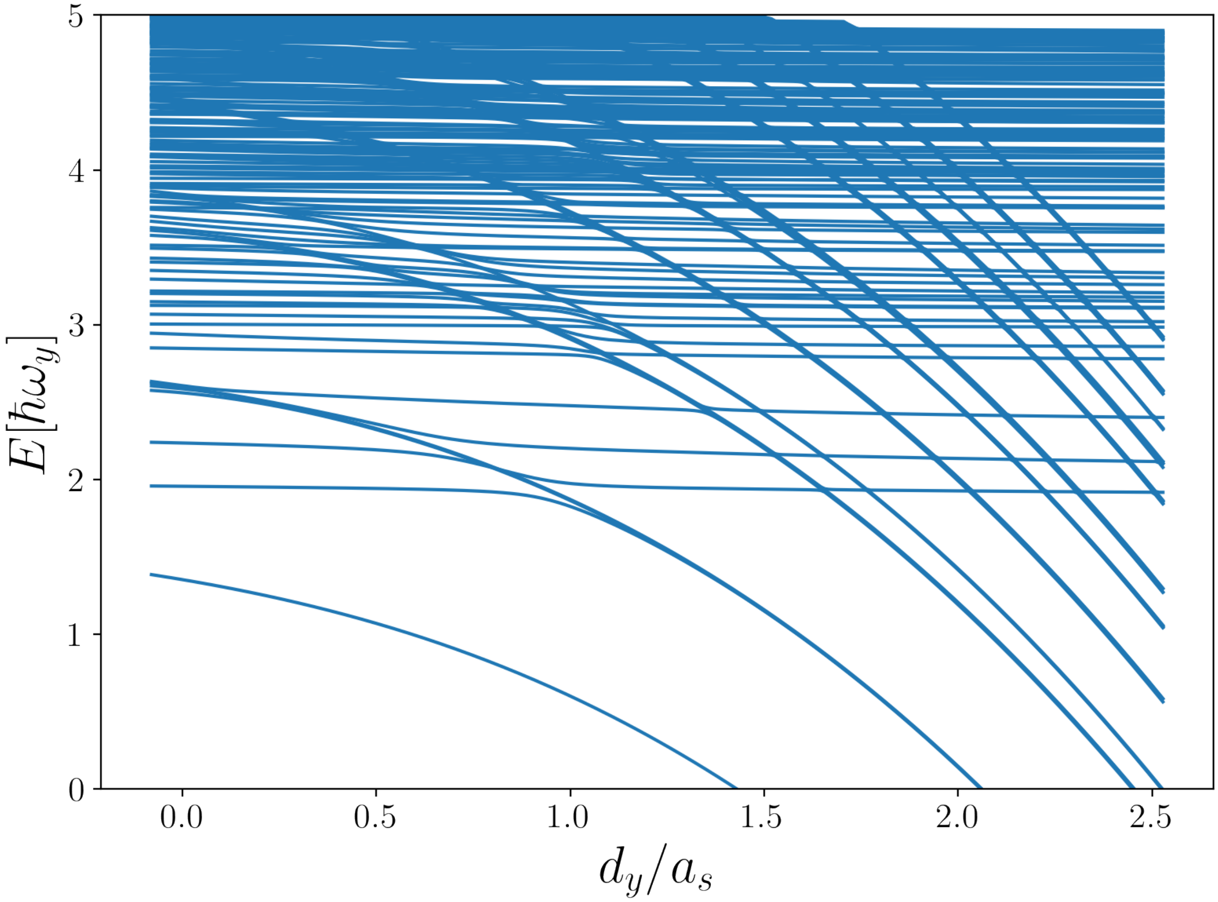

Figure 2: Energy spectrum for a 3D-2D system of ^{7}Li^{7}Li with $\omega_x/\omega_y = 0.5$. The degeneracy is clear in this setting, therefore, the ICIR's will cross in space.

Frequency ratio that cause 3D-2D spectrum degeneracy

For this demonstration we will assume a general 3D-2D confinement where $V_{x1} = V_{x2} := V_x$, $V_{y1} = V_{z1} := V_{\perp 1}$ and $V_{y2} = V_{z2} := V_{\perp 2} \ll V_{\perp 1}$. In addition, the lasers have the same wavelength in every direction ($k_x = k_y = k_z := k$). Introducing the parameters into Eq. (2) leads to

$$\omega_x = 2k\sqrt{\frac{V_x}{m}} \tag{3}$$

$$\omega_\perp = \sqrt{2}k\sqrt{\frac{V_{\perp 1} + V_{\perp 2}}{m}}. \tag{4}$$

The perpendicular potential depths will differ just in a scaling parameter that we will call $\eta_\perp$. The simulations were done using $\eta_\perp = 0.01$, which results in $I_{\perp_1} = 4993$ W/cm^2 and $I_{\perp_2} = 50$ W/cm^2. Hence

$$V_{\perp 2} = \eta_\perp V_{\perp 1} \tag{5}$$

The ratio of the frequencies is then

$$\frac{\omega_x}{\omega_\perp} = \sqrt{\frac{2V_x}{V_{\perp 1}(1+\eta_\perp)}} \tag{6}$$

The degeneracy of the both branches of the ICIR's satisfies the degeneracy of the Hamiltonian, so

$$\boxed{\omega_x = \omega_\perp \Leftrightarrow V_x = \frac{1+\eta_\perp}{2} V_{\perp 1}} \tag{7}$$

As well as $\eta_\perp$ should be much smaller than 1 for make a significant change of confinement dimensionality we can say that in 3D-2D confinements the ICIR's cross when $V_x = \frac{V_{\perp 1}}{2}$. Finally, in order to check the *ab initio* results we substitute $\eta_\perp = 0.01$ and $I_{\perp 1} = 4993$ W/cm^2 in Eq. (7) finding that $V_x = 2521$ W/cm^2, which is almost the same that the value we used except for anharmonic corrections.