CS201: Discrete Math for Computer Science
2025 Spring Semester Written Assignment #1
Due: 23:55 on Mar. 17th, 2025, please submit through Blackboard

Please answer questions in English. Using any other language will lead to a zero point.

Q. 1. Let p, q be the propositions
p: You get 100 marks on the final.
q: You get an A in this course.
Write these propositions using p and q and logical connectives (including negations).

(a) You do not get 100 marks on the final.

(b) You get 100 marks on the final, but you do not get an A in this course.

(c) You will get an A in this course if you get 100 marks on the final.

(d) If you do not get 100 marks on the final, then you will not get an A in this course.

(e) Getting 100 marks on the final is sufficient for getting an A in this course.

(f) You get an A in this course, but you do not get 100 marks on the final.

(g) Whenever you get an A in this course, you got 100 marks on the final.

Q. 2. Construct a truth table for each of these compound propositions.

(a) $(p \oplus q) \rightarrow (p \wedge q)$

(b) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

Q. 3. "Logic is difficult or not many students like logic."
"If mathematics is easy, then logic is not difficult."
Which of the following are valid conclusions?

(a) That mathematics is not easy, if many students like logic.

(b) That not many students like logic, if mathematics is not easy.

(c) That mathematics is not easy or logic is difficult.

(d) That logic is not difficult or mathematics is not easy.

(e) That if not many students like logic, then either mathematics is not easy or logic is not difficult.

Q. 4. Determine whether the following statements are correct or incorrect. Explain your answer. Assume that p, q and r are logical propositions, x and y are real numbers, and m and n are integers.

(1) $(\neg p \wedge (p \to q)) \to \neg q$ is a tautology.

(2) $(p \vee q) \to r$ and $(p \to r) \wedge (q \to r)$ are equivalent.

(3) Under the domain of all real numbers, the truth value of $\exists x \forall y (y \neq 0 \to xy = 1)$ is T.

(4) Under the domain of all integers, the truth value of $\exists n \exists m (n^2 + m^2 = 5)$ is T.

Q. 5. For each of the following argument, determine whether it is valid or invalid. Explain using the validity of its argument form.

(1) Premise 1: If you did not finish your homework, then you cannot answer this question.

Premise 2: You finished your homework.

Conclusion: You can answer this question.

(2) Premise 1: If all students in this class has submitted their homework, then all students can get 100 in the final exam.

Premise 2: There is a student who did not submit his or her homework.

Conclusion: It is not the case that all student can get 100 in the final exam.

Q. 6. Suppose that p, q, r, s are all logical propositions. You are given the following statement

$$(\neg r \vee (p \wedge \neg q)) \to (r \wedge p \wedge \neg q)$$

Prove that this implies $r \vee s$ using logical equivalences and rules of inference.

Q. 7. Use logical equivalences to prove the following statements.

(a) $\neg(p \oplus q)$ and $p \leftrightarrow q$ are equivalent.

(b) $\neg(p \to q) \to \neg q$ is a tautology.

(c) $(p \to q) \to ((r \to p) \to (r \to q))$ is a tautology.

Q. 8. Let $C(x)$ be the statement "x has a cat", let $D(x)$ be the statement "x has a dog" and let $F(x)$ be the statement "x has a ferret." Express each of these sentences in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

(a) A student in your class has a cat, a dog, and a ferret.

(b) All students in your class have a cat, a dog, or a ferret.

(c) Some student in your class has a cat and a ferret, but not a dog.

(d) No student in your class has a cat, a dog, and a ferret.

(e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

Q. 9. Prove that if $p \wedge q$, $p \to \neg(q \wedge r)$, $s \to r$, then $\neg s$.

Q. 10. (a) Give the negation of the statement

$$\forall n \in \mathbb{N}\ (n^3 + 6n + 5 \text{ is odd} \Rightarrow n \text{ is even}).$$

(b) Either the original statement in (a) or its negation is true. Which one is it and explain why?

Q. 11. Give a direct proof that: Let a and b be integers. If $a^2 + b^2$ is even, then $a + b$ is even.

Q. 12. Prove that $\sqrt[3]{2}$ is irrational.