Problem 1:

The definition of the Euler number is:

$$e = \lim_{n \to \infty} (1 + \frac{1}{n})^n$$

where e = 2.718281828459

a) Using single precision arithmetic:
 create a table of e versus n for $n = (10, 10^2, 10^3, 10^5, 10^5, 10^6, 10^7, 10^8)$
What do you observe?

b) Improve your estimation of e. Use SINGLE PRECISION
Hint:

$$\log (1 + \frac{1}{n})^n = n \log (1 + \frac{1}{n})$$

$$\lim_{n \to \infty} \log (1 + \frac{1}{n}) = \lim_{x \to 0} \log (1 + x)$$

How can $\log(1+x)$ be approximated as $x \to 0$?

Problem 2:

Consider x=100. Apply $\sqrt{x}$ recursively, that is

$$do\, i = 1 \ldots n : x = \sqrt{x}$$

Then, reconstruct the original value of x:

$$do\, 1 = 1 \ldots n : x = x * x$$

for $n = 2, 5, 10, 20, 30, 40$
What happens? Explain.
Perform all computations in SINGLE PRECISION.

Problem 3:

Compute: $z = \frac{e^x - 1}{x}\, as\; x \to 0$
that is, for: $x = 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8}, 10^{-9}, 10^{-10}, 10^{-11}, 10^{-12}, 10^{-13}, 10^{-14}, 10^{-15}$

Note that $\lim_{x \to 0} z = 1$ (L'Hôpital's rule)

Alternative approach

Compute: $y = e^x$
if y==1 then $z = 1$
else $z = \frac{(y-1)}{\log y}$

Compare to the original approach.

Perform all computations in DOUBLE PRECISION

Problem 4:

Estimate the exponent of a DOUBLE PRECISION floating point number.

Expensive way: $\log_{10}|(x)|$

Alternative way: You learned about bit manipulation. Recall $\log_{10} x = \log_{10} 2 \times \log_2 x$
IEEE double precision format:
- mantissa: 52 bits
- exponent: 11 bits
- sign: 1bit

Remember that the exponent bits store the ***biased*** value of the base 2 exponent