

Task 04:

We will divide a for-loop over the array into $n/(n/\log(n)) = \log(n)$ parts for the $n/\log(n)$ processors.

Define an array B to store the minimum index of first nonzero entry in the k -th part.

1. iterate over the array:

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for i from 0 to n dopar:  
  if A[i] != 0:  
    B[k] = i  
    terminate for-loop for k-th processor  
  if no nonzero entry in k-th part:  
    B[k] = undefined
```

n operations using $\frac{n}{\log(n)}$ processors for a runtime of $O(\log(n))$.

2. iterate over B to find the first non-undefined entry, which gives the answer. ($O(\log(n))$)

Task 05:

See next page.

Task 05

(Index of arrays starts from 1 as is given by the problem.)

We need an intermediate array C for this task, where

$$C[k] := \sum_{i=1}^k A[i].$$

As $A[]$ is a boolean array consisting of 0s and 1s, it is easy to notice, if $A[k] == 1$, $B[C[k]]$ should be k .

Note that the LHS of the formula above is in a form of prefix sum, the main task becomes computing the prefix sum in $O(\log n)$.

Prefix Sum Steps:

1. Fill B with undefined;
2. Split A into $\log n$ parts. $A_k := A[k \log n + 1, \dots, (k+1) \log n]$; Denote B_k, C_k in the same way;
3. Do in parallel $C_k \leftarrow \text{sequential_prefix_sum}(A_k)$; ($O(\log n)$)
4. Let $\text{last}(C_k) :=$ the last element of C_k , $\text{offset}_k := \sum_{i=1}^k \text{last}(C_i)$;
5. Do in parallel $C_k \leftarrow C_k + \text{offset}_{k-1}$; ($O(\log n)$)

The total time complexity of Prefix Sum Steps is $O(\log n)$.

Compute Final Solution:

Do in parallel $B[C[j]] \leftarrow j$, if $A[j] == 1$. ($O(\log n)$)

All in all, the overall time complexity is $O(\log n)$. ■

An Example of How It Works:

$$A = \{0, 1, 1, 1, 0, 1\}, n = 6, p = n/(\log n) \approx 3.$$

1. $A_1 = \{0, 1\}, A_2 = \{1, 1\}, A_3 = \{0, 1\}$;
2. $C_1 = \{0, 1\}, C_2 = \{1, 2\}, C_3 = \{0, 1\}$;
3. $\text{offset}_1 = 1, \text{offset}_2 = 1 + 2 = 3$;
4. $C_2 \leftarrow C_2 + \text{offset}_1 = \{2, 3\}$;
5. $C_3 \leftarrow C_3 + \text{offset}_2 = \{3, 4\}$;
6. The complete C is $\{0, 1, 2, 3, 3, 4\}$;
7. Compute the final solution $B \leftarrow \{2, 3, 4, 6, \text{undefined}, \dots\}$.