Exercises 1

Problem 1

The nonparallel version of computing pi by Monte Carlo.

```
void compute_pi_monte_carlo(int n_steps)
{
    int m = 0;
    unsigned int seed = time(NULL) ^ omp_get_thread_num();
    srand(seed);

    for (int i = 0; i < n_steps; i++)
    {
        double x = (double) rand() / RAND_MAX;
        double y = (double) rand() / RAND_MAX;
        if (x * x + y * y < 1.)
        {
            m += 1;
        }
    }

    double pi = 4.0 * m / n_steps;
    printf("Estimated value of pi = %f\n", pi);
}
```

Problem 2

Here, I implemented a OpenMP version of Monte Carlo. I used `reduction` to avoid **data race** for `m`.

```
void compute_pi_monte_carlo_parallel(int n_steps)
{
    int m = 0;
#pragma omp parallel
    {
        unsigned int seed = time(NULL) ^ omp_get_thread_num();

#pragma omp for reduction(+:m)
        for (int i = 0; i < n_steps; i++)
        {
            double x = (double) rand_r(&seed) / RAND_MAX;
            double y = (double) rand_r(&seed) / RAND_MAX;
            if (x * x + y * y < 1.)
            {
                m++;
            }
        }
    }

    double pi = 4.0 * m / n_steps;
    printf("Estimated value of pi = %f\n", pi);
}
```

Test Wall Time Elapsed for Our Programs

To test how good our parallel program performs, we need to measure the **Wall Time** (Time consumed in the real world) of our program instead of CPU Time. For the measurement of this, the recommended way is the Linux/MacOS command `time` or the function `omp_get_wtime` provided by `OpenMP` other than `clock`.

I use the following code in my code for the measurement purpose (use `clock` if you wish to compile without `OpenMP`):

```
int n_steps = 100000000;
double start = omp_get_wtime();
...Your Code/Function For Parallel Computation
double end = omp_get_wtime();
double time_spent = end - start;
printf("Wall time taken: %f seconds\n", time_spent);
```

The testing result when we set `n_steps=100000000` :

```
# Parallel version
Estimated value of pi = 3.141505
Wall time taken: 0.229149 seconds
-----------------------------------
# Nonparallel version
Estimated value of pi = 3.141704
Wall time taken: 1.495120 seconds
```

Problem 3

In this part we implement a MPI version of the Monte Carlo example.

First make sure we have installed OpenMPI properly:

```
mpicc --version # For compiling MPI program (We can also edit CMakeLists.txt to enable gcc to compile it)
mpiexec --version # For executing MPI program
```

Include `mpi.h` to make MPI functions visible to our code. In MPI program, `rank` is the id of the current thread that is doing the parallel tasks; `size` indicates how many threads are running in parallel. To boot up a MPI program, we use `MPI_Init(argc, argv)` at first.

Initialize MPI and get the parameters of our parallel program:

```
MPI_Init(&argc, &argv);
MPI_Comm_rank(MPI_COMM_WORLD, &rank);
MPI_Comm_size(MPI_COMM_WORLD, &size);
```

Similarly, we assign a random seed for each thread and use `rand_r()` for random number generation. We collect statistics respectively for each thread,

```
    local_points = num_points / size;

    unsigned int seed = time(NULL) + rank;

    for (long long int i = 0; i < local_points; i++) {
        double x = (double)rand_r(&seed) / RAND_MAX;
        double y = (double)rand_r(&seed) / RAND_MAX;
        if (x*x + y*y <= 1.0) {
            local_in_circle++;
        }
    }
```

and reduce after all the jobs are done.

```
MPI_Reduce(&local_in_circle, &total_in_circle, 1, MPI_LONG_LONG, MPI_SUM, 0, MPI_COMM_WORLD);

if (rank == 0) {
        double pi_estimate = 4.0 * (double)total_in_circle / (double)num_points;
    printf("Estimated Pi = %.10f\n", pi_estimate);
}
```

Put `MPI_finalize()` in the end of the MPI program to free the threads.

To execute MPI program with 4 threads, we use:

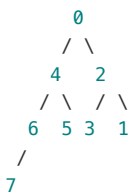
```
mpiexec -np 4 your_program_fullname
```

The result of our program:

```
Process 3 out of 4
Process 0 out of 4
Process 2 out of 4
Process 1 out of 4
Estimated Pi = 3.1414960000
```

Problem 4

A broadcast binomial tree with 8 nodes looks like:



Once received a message, the node will send the message to the node in the layer below. The smaller children are those lie in the last layers (i.e. 1, 3, 5, 6, 7). Once they recive the message, their inability to inform more nodes hinders the efficiency of broadcast. Moreover, the cost of sending messages between nodes increases as the distance gets bigger. Sending message from node 0 to nodes on the last layer is $\log p$ (given we have p processors). Thus, the total cost of serving small children first is $\Omega(\sum_{i=1}^{\log p} i)$, which is $\Omega(\log^2 p)$.

Problem 5

My implementation of parallel scalar product between vector and vector:

```
    double a[5] = {1.0, 2.0, 3.0, 4.0, 5.0},
           b[5] = {1.0, 2.0, 3.0, 4.0, 5.0};
    double res = .0;

#pragma omp parallel for reduction(+:res)
    for (int i = 0; i < 5; i++) {
        res += a[i] * b[i];
    }

    printf("Dot product = %f\n", res);

    return 0;
```

Suppose we use n processors for vector of order n, the time complexity of doing multiplication is $O(1)$ while the complexity of reduction is $O(\log n)$, when we *divide and conquer*. As a result,

$$T(n) = O(\log n).$$

Problem 6

My implementation of parallel dot product between matrix and vector:

```
    // dot product mat and vec
    double mat[2][2] = {{1.0, 2.0}, {3.0, 4.0}},
           vec[2] = {1.0, 2.0};
    double res2[2] = {0.0, 0.0};
    double start = omp_get_wtime();

    // collaspe(2) is used to collapse the two loops into one
#pragma omp parallel for collapse(2) reduction(+:res2[:2])
    for (int i = 0; i < 2; i++) {
        for (int j = 0; j < 2; j++) {
            res2[i] += mat[i][j] * vec[j];
        }
```

```
    }
    double end = omp_get_wtime();
    double time_spent = end - start;
    printf("Wall time taken: %f seconds\n", time_spent);
    printf("Matrix-vector product = [%f, %f]\n", res2[0], res2[1]);
```

Each row of the resulting vector is calculated in the same way with vector dot product, thus having a time complexity of $O(\log n)$. In the reduction phase, we still can reduce the results in parallel, which results in a $O(\log n)$ time complexity. What is different is that we need $n \cdot n$ processors for the reduction phase. Hence,

$$T(n) = O(\log n).$$