

Ex 2

$$1. \quad (a) \quad \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\text{let } u(x) = e^x - e^{-x}, \quad v(x) = e^x + e^{-x}$$

$$u'(x) = e^x + e^{-x} = v(x) \quad v'(x) = e^x - e^{-x} = u(x)$$

$$\frac{d}{dx} \tanh(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v(x)^2}$$

$$= \frac{v(x)^2 - u(x)^2}{v(x)^2}$$

$$= 1 - \frac{u(x)^2}{v(x)^2}$$

$$= 1 - \tanh^2(x)$$

(b)

$$\frac{\partial \mathcal{L}}{\partial \tilde{z}^{(2)}} = \frac{\partial \mathcal{L}}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial \tilde{z}^{(2)}}$$

$$= \left(\frac{\hat{y} - y}{\hat{y}(1-\hat{y})} \right) \cdot (\hat{y}(1-\hat{y}))$$

$$= \hat{y} - y$$