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Worked with Cole TerBush and Daniel Aronov

1 • Graphs

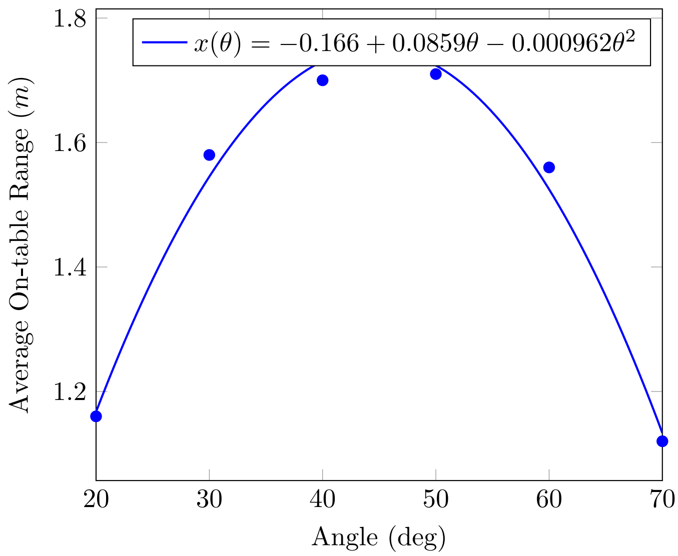

Figure 1: The on-table ranges for each launch degree, along with an approximating polynomial function.

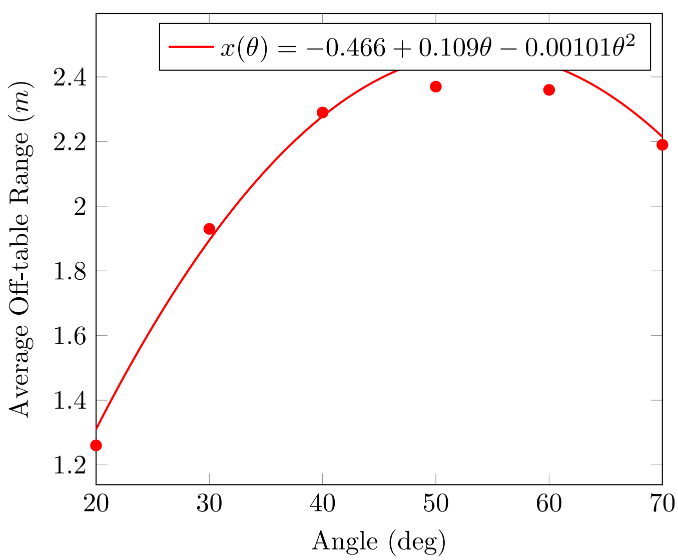

Figure 2: The off-table ranges for each launch degree, along with an approximating polynomial function.

2 • First Derivative Test

Style II Launch

The derivative of the on-table range equation shown in Figure 1 is as follows:

$$\frac{dx}{d\theta} = 0.0859 - 0.001924\theta$$

To find the maximum of the function, and thus the greatest range, we set the derivative equal to zero and solve for the angle.

$$0 = 0.0859 - 0.001924\theta$$
$$0.001924\theta = 0.0859$$
$$\theta = \frac{0.0859}{0.001924}$$
$$\theta = 44.65^\circ$$

This angle agrees with the angle found visually by looking at Figure 1.

Style III Launch

The derivative of the off-table range equation shown in Figure 2 is as follows:

$$\frac{dx}{d\theta} = 0.109 - 0.00202\theta$$

To find the maximum of the function, and thus the greatest range, we set the derivative equal to zero and solve for the angle.

$$0 = 0.109 - 0.00202\theta$$
$$0.00202\theta = 0.109$$
$$\theta = \frac{0.109}{0.00202}$$
$$\theta = 53.96^\circ$$

This angle agrees with the angle found visually by looking at Figure 2.

3 • Style II Range Complements

In any kinematic launch, the distance traveled by an object in the y-axis direction is as follows:

$$\Delta y = v_{0_y} t + \frac{1}{2} a_y t^2$$

In a style II launch, Δy is 0 and a_y is $-g$. v_{0_y} can also be rewritten as $v_0 \sin(\theta)$, where θ is the angle of launch, giving:

$$0 = v_0 \sin(\theta) t - \frac{1}{2} g t^2$$

Factoring out t, we obtain:

$$0 = t(v_0 \sin(\theta) - \frac{1}{2} g t)$$

There are two possible time solutions for this. One is the trivial solution of 0, but the other is the solution of interest. Solving for that solution, we obtain:

$$t = \frac{2v_0 \sin(\theta)}{g}$$

We may now use this time value in the distance equation for the x-axis direction:

$$\Delta x = v_{0_x} t + \frac{1}{2} a_x t^2$$

a_x is 0, and v_{0_x} can be rewritten as $v_0 \cos(\theta)$, so the equation can be rewritten as:

$$\Delta x = v_0 \cos(\theta) t$$

Plugging in the t we obtained earlier, we now obtain:

$$\Delta x = \frac{2v_0^2 \sin(\theta) \cos(\theta)}{g}$$

Applying the trigonometric identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$, we obtain the range equation:

$$\Delta x = \frac{v_0^2 \sin(2\theta)}{g}$$

If an object in a style II launch is launched at a complementary angle, then the angle will be $90° - \theta$, which results in:

$$\Delta x = \frac{v_0^2 \sin(2(90° - \theta))}{g}$$

Which simplifies to:

$$\Delta x = \frac{v_0^2 \sin(180° - 2\theta)}{g}$$

Since the values of sin are equivalent for all supplementary angles, $\sin(180° - 2\theta)$ is equivalent to $\sin(2\theta)$. Plugging that in, we obtain:

$$\Delta x = \frac{v_0^2 \sin(2\theta)}{g}$$

Which is the original range equation. Therefore, any object launched at an angle's complement will have the same range as the same object launched from the original angle.