Baye's Net 1

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1.Independence

We say X, Y are independent if and only if $\forall x, yP(x, y) = P(x)P(y)$, we write $x \perp\!\!\!\perp y$. But it's so rare in reality. 可以发现某些变量其实之间的关系很微弱，但是独立性的条件太强了，所以提出了条件独立(conditional independence)的概念。

2.Conditional Independence

If $P(x|y, z) = P(x|z)$. We can say that x i conditional independence of y given z. Or we can write $P(x, y|z) = P(x|z)P(y|z)$, which means if I know the information of z, then y will not change my belief of x.

2.1 n example

- Given Traffic, Umbrella, Raining. What's the relationship between the them?

If it's raining, then I should carry umbrella, which has nothing to do with traffic is heavy or not. So the answer is $T \perp\!\!\!\perp U \mid R$.

- Given Fire, Smoke, Alarm, what is in this scenario?

If there is smoke, then the alarm will go off, which has nothing to with fire. So the answer is $F \perp\!\!\!\perp A \mid S$.

- The key thing is to find a thing like Raining will directly affect the other thing like Umbrella.

2.2 Chain

While $P(x, y|z) = P(x|z)P(y|z)$ is not always right, the following formula is right all the time.

$$P(x_1, x_2, x_3, x_4, ..., x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1, x_2)...P(x_n|x_1, x_2, ..., x_{n-1})$$

You can use this to solve some problems very quickly.

3. Baye's Net

3.1 What is baye's net?

It's a graph, which has nodes ,arcs and is acyclic. The arc between A and B means the direct influence between A and B. So it can split big, complicated things into tiny pieces.

- $A \to B$, we can could find what's B now given A? Or calculated P(B|A).

3.2 Example

$$\begin{aligned} P(i, h, !l, p, !e) &= P(i)P(h|i)P(!l|h, i)P(p|i, h, !l)P(!e|i, h, !l, p) \\ &= P(i)P(h)P(!l|h, i)P(p|i, h, !l)P(!e|p) \end{aligned}$$

So when you encounter a big probability formula, just split it using the chain rule. Then you can find some information from the Baye's net. For example, there doesn't exist an arc between the node H and the node I, which means whether you know this or not doesn't affect your belief on H.