Computer Science 384 | Version W24.1
St. George Campus | University of Toronto

Take-Home Quiz: Uncertainty

---

**Instructions**:

Submissions must be a single PDF file named `csc384probquiz.pdf`, submitted to MarkUs. Submissions must be typed/legible. Please refer to the course information sheet for the late submission policy.

1. Bayes' Burrito Bowls is a food truck that serves burrito bowls (obviously). You can choose:

   - a type of rice, which can be; white, brown
   - a type of bean, which can be; black, pinto, lima, or kidney
   - any of the following toppings; guacamole, corn, salsa, sour cream, onion, lettuce, or cheese

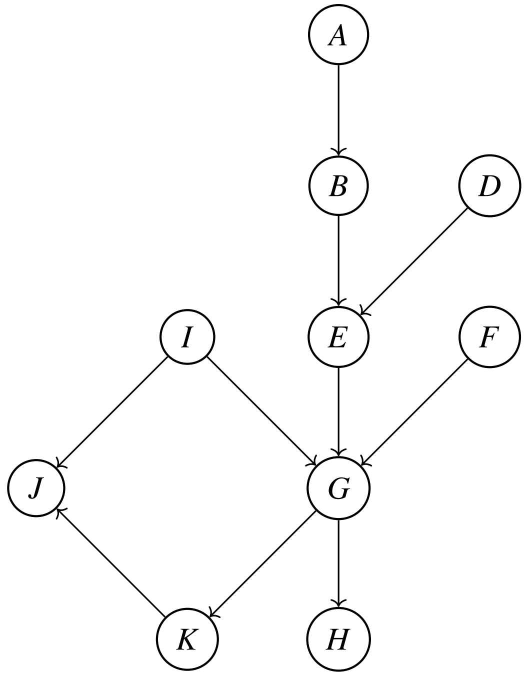
   We model each type of burrito as a triple $(R,B,T)$, where $R$ and $B$ are the types of rice and bean, and $T$ is any subset of the toppings. The type of burrito a customer picks depends on his/her preferences. We can model such preferences using a probability distribution over $(R,B,T)$.

   (a) How many different types of burrito bowls are there?

   (b) Eshan builds a burrito bowl as follows: He first chooses the type of rice and bean independently of each other. Based on both these choices, he chooses the toppings (the toppings are not chosen independently of each other). Provide an appropriate factorization of $P_{\text{Eshan}}(R,B,T)$ and the number of values that must be known to compute it for any $R,B,T$.

   (c) Zafeer builds a book as follows: He first chooses type of rice. Based on this choice, he chooses a type of bean. He chooses the toppings independently of the other choices (toppings are not chosen independently of each other). Provide an appropriate factorization of $P_{\text{Zafeer}}(R,B,T)$ and the number of values that must be known to compute it for any $R,B,T$.

2. Consider the Bayesian network shown below:




   (a) Which variables are independent of $I$?

   (b) Which variables are conditionally independent of $A$ given $K$?

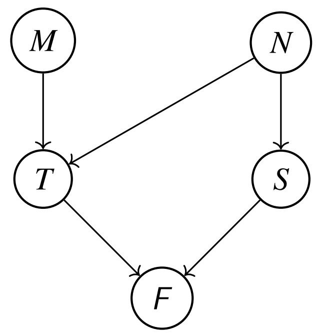
   (c) Which variables are conditionally independent of $B$ given $E$?

3. Draw a Bayesian network over the variables $W, X, Y, Z$, such that:
   - $X$ and $Y$ are independent given $Z$
   - $Z$ and $W$ are independent given $X$ and $Y$

4. You and your friend are on the Amazing Race (see Wikipedia article for details) and the next leg is in a few hours. Once the leg begins, you will need to make your way to the airport. You want to use this time to decide how you are going to get there, and how many bags you need to pack.

| $P(M = \text{train})$ |
|---|
| 0.6 |

| $M$ | $N$ | $P(T = \text{early}\|M,N)$ |
|---|---|---|
| train | 0 | 0.9 |
| train | 1 | 0.85 |
| train | 2 | 0.60 |
| car | 0 | 0.75 |
| car | 1 | 0.75 |
| car | 2 | 0.75 |




| $P(N = 0)$ | $P(N = 1)$ |
|---|---|
| 0.4 | 0.5 |

| $N$ | $P(S = \text{fast}\|N)$ |
|---|---|
| 0 | 0.9 |
| 1 | 0.8 |
| 2 | 0.7 |

| $T$ | $S$ | $P(F = \text{yes}\|T,S)$ |
|---|---|---|
| early | fast | 0.95 |
| early | slow | 0.7 |
| late | fast | 0.7 |
| late | slow | 0.35 |

Both of these decisions influence whether you will make your flight or miss it by influencing when you get to the airport and how long it takes you to get through security. This scenario is modelled using the Bayesian network above:

- $M \in \{\text{train}, \text{car}\}$ is how you will get to the airport
- $N \in \{0, 1, 2\}$ is the number of bags you pack
- $T \in \{\text{early}, \text{late}\}$ is when you get to the airport
- $S \in \{\text{fast}, \text{slow}\}$ is how long it takes to get through security
- $F \in \{\text{yes}, \text{no}\}$ is whether you make the flight or not

(a) Compute $P(F = \text{yes}|N, M)$ for all $M$ and $N$. Show your work.

(b) How should you get to the airport, and how many bags should you bring to maximize your chances of making the flight?