CSC384: Take-Home Quiz
Uncertainty

1) Bayes' Burrito Bowls

(a)

We are given:

1. 2 types of rice
2. 4 types of beans
3. Any subset of the 7 toppings

The total # of different burrito bowls is the product of the # of choices for each category:
Total burrito bowls = types of rice * types of beans * combinations of toppings

$\therefore Total\ different\ types\ of\ burrito\ bowls\ =\ 2\ \times\ 4\ \times\ 2^{7}\ =\ 1024$

(b)

Denote:

$P(R)$:
the probability of choosing a type of rice R (independent)

$P(B)$:
the probability of choosing a types of beans B (independent)

$P(T)$:
the conditional probability of choosing a subset of toppings T (dependent on R & B);
but the toppings are not chosen independently of each other

An appropriate factorization is:
$\therefore P_{Eshan}(R, B, T)\ =\ P(R)\ \times\ P(B)\ \times\ P(T|R, B)$

The # of values needed for $P(T|R, B)$ is $8\ \times\ 2^{7}$, because we have 2^{7} possible combinations of toppings for each of the 8 rice and bean combinations.

Therefore, the total number of values (specific probabilities associated with the choices of rice, beans, and toppings for the burrito bowls) required is:
2 (rice) + 4 (beans) + $8\ \times\ 2^{7}$ (all topping combinations given rice & bean combinations)

$\therefore$ Total number of values needed for any (R,B,T) = 1030

(c)
Denote:
$P(R)$:
the probability of choosing a type of rice R (independent)

$P(B)$:
the probability of choosing a types of beans B (dependent on R)

$P(T)$:
the conditional probability of choosing a subset of toppings T (independent);
but the toppings are not chosen independently of each other

An appropriate factorization is:
$\therefore P_{Zafeer}(R, B, T) = P(R) \times P(B|R) \times P(T)$

The # of values needed for $P(B|R)$ is 2×4, because there are 4 types of beans and we need 2 sets of 4 probabilities, one for each type of rice.

Therefore, the total number of values needed is:
2 (rice) + 2×4 (type of beans given type of rices) + 2^7 (topping combinations)

∴ Total number of values needed for any (R,B,T) = 138

2) Bayesian Network & Independence

(a)

$variables \perp I$:

A, B, D, E, F

(b)

$variables \perp A|K$:

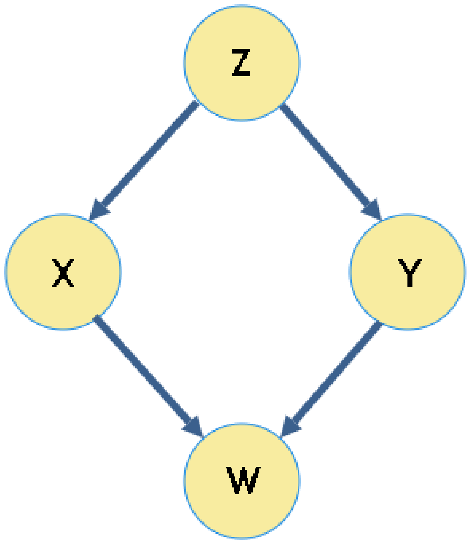
Nothing

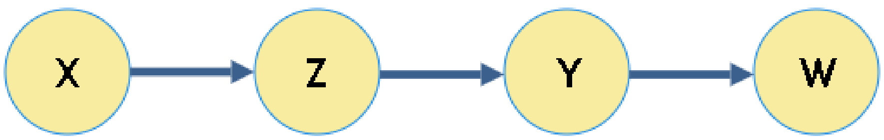
(c)

$variables \perp D|E$:

F, G, I, J, K, H

3) Bayesian Network & Graph





4) Bayesian Network & Probability

(a)

Find: $P(F = yes|N, M), \forall N, M$

$\Rightarrow P(F = yes|N, M) = \Sigma_S P(F = yes|T, S) \cdot P(T|M, N) \cdot P(S|N)$

For all possible M and N, we have $M = \{train, car\}$ and $N = \{0, 1, 2\}$.
So we have a total of 6 scenarios to consider:

1. **M = train and N = 0:**

P(F=yes | N=0, M=train) =
P(F=yes | T=early ,S=fast)*P(T=early | N=0, M=train)*P(S=fast | N=0)
+P(F=yes | T=early, S=slow)*P(T=early | N=0, M=train)*P(S=slow | N=0)
+P(F=yes | T=late, S=fast)*P(T=late | N=0, M=train)*P(S=fast | N=0)
+P(F=yes | T=late, S=slow)*P(T=late | N=0, M=train)*P(S=slow | N=0)

= 0.95*0.9*0.9 + 0.7*0.9*0.1
+ 0.7*0.1*0.9 + 0.35*0.1*0.1

∴ P(F=yes|N=0, M=train) = 0.899

2. **M = train and N = 1:**

P(F=yes | N=1, M=train) =
P(F=yes | T=early, S=fast)*P(T=early | N=1, M=train)*P(S=fast | N=1)
+P(F=yes | T=early, S=slow)*P(T=early | N=1, M=train)*P(S=slow | N=1)
+P(F=yes | T=late, S=fast)*P(T=late | N=1, M=train)*P(S=fast | N=1)
+P(F=yes | T=late, S=slow)*P(T=late | N=1, M=train)*P(S=slow | N=1)

= 0.95*0.85*0.8 + 0.7*0.85*0.2
+ 0.7*0.15*0.8 + 0.35*0.15*0.2

∴ P(F=yes | N=1, M=train) = 0.8595

3. **M = train and N = 2:**

P(F=yes | N=2, M=train) =
P(F=yes | T=early, S=fast)*P(T=early | N=2, M=train)*P(S=fast | N=2)
+P(F=yes | T=early, S=slow)*P(T=early | N=2, M=train)*P(S=slow | N=2)
+P(F=yes | T=late, S=fast)*P(T=late | N=2, M=train)*P(S=fast | N=2)
+P(F=yes | T=late, S=slow)*P(T=late | N=2, M=train)*P(S=slow | N=2)

= 0.95*0.6*0.7 + 0.7*0.6*0.3
+ 0.7*0.4*0.7 + 0.35*0.4*0.3

∴ P(F=yes | N=2, M=train) = 0.763

4. **M = car and N = 0:**

P(F=yes | N=0, M=car) =
P(F=yes | T=early ,S=fast)*P(T=early | N=0, M=car)*P(S=fast | N=0)
+P(F=yes | T=early, S=slow)*P(T=early | N=0, M=car)*P(S=slow | N=0)
+P(F=yes | T=late, S=fast)*P(T=late | N=0, M=car)*P(S=fast | N=0)
+P(F=yes | T=late, S=slow)*P(T=late | N=0, M=car)*P(S=slow | N=0)

= 0.95*0.75*0.9 + 0.7*0.75*0.1
+ 0.7*0.25*0.9 + 0.35*0.25*0.1

∴ P(F=yes | N=0, M=car) = 0.86

5. **M = car and N = 1:**

P(F=yes | N=1, M=car) =
P(F=yes | T=early, S=fast)*P(T=early | N=1, M=car)*P(S=fast | N=1)
+P(F=yes | T=early, S=slow)*P(T=early | N=1, M=car)*P(S=slow | N=1)
+P(F=yes | T=late, S=fast)*P(T=late | N=1, M=car)*P(S=fast | N=1)
+P(F=yes | T=late, S=slow)*P(T=late | N=1, M=car)*P(S=slow | N=1)

= 0.95*0.75*0.8 + 0.7*0.75*0.2
+ 0.7*0.25*0.8 + 0.35*0.25*0.2

∴ P(F=yes | N=1, M=car) = 0.8325

6. **M = car and N = 2:**

P(F=yes | N=1, M=car) =
P(F=yes | T=early, S=fast)*P(T=early | N=2, M=car)*P(S=fast | N=2)
+P(F=yes | T=early, S=slow)*P(T=early | N=2, M=car)*P(S=slow | N=2)
+P(F=yes | T=late, S=fast)*P(T=late | N=2, M=car)*P(S=fast | N=2)
+P(F=yes | T=late, S=slow)*P(T=late | N=2, M=car)*P(S=slow | N=2)

= 0.95*0.75*0.7 + 0.7*0.75*0.3
+ 0.7*0.25*0.7 + 0.35*0.25*0.3

∴ P(F=yes | N=1, M=car) = 0.805

(b)
To maximize our chances of making the flight:
∵ The highest probability of making the flight is P(F=yes|N=0, M=train).
∴ I should get to the airport **by train** and **bring no bags**.