Rendering: equation with a sphere

1 Introduction

The equation of a sphere is

$$(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 = r^2 \tag{1}$$

where x, y, z are the coordinates of the point where the ray **r(t)** intersects the sphere, with equation

$$\begin{aligned} x &= o_x + t * d_x \\ y &= o_y + t * d_y \\ z &= o_z + t * d_z. \end{aligned} \tag{2}$$

Introducting Eq. 2 in Eq. 1 yields

$$(o_x + t * d_x - x_c)^2 + (o_y + t * d_y - y_c)^2 + (o_z + t * d_z - z_c)^2 = r^2. \tag{3}$$

We want to find t that solves Eq. 3.

This is a second order equation in t, that can be rewritten as

$$at^2 + bt + c = 0 \tag{4}$$

with

$$\begin{aligned} a &= d_x^2 + d_y^2 + d_z^2 \\ b &= 2 * ((d_x * (o_x - x_c)) + (d_y * (o_y - y_c)) + (d_z * (o_z - x_z))) \\ c &= (o_x - x_c)^2 + (o_y - y_c)^2 + (o_z - x_z)^2 - r^2 \end{aligned} \tag{5}$$

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