The Breakdown of Protest-Based Information Aggregation under Strategic Pricing and Precise Individual Signals*

Tomoya Tajika†

July 3, 2025

Click here for the latest version

Abstract

Consumer protests can aggregate dispersed information about product externalities. This paper demonstrates a paradox: when consumers' private signals become more precise, protest turnout can become uninformative. We model a monopolistic firm that sets prices strategically. We show that higher signal precision induces the firm to set a high price, which effectively screens out consumers receiving unfavorable signals. While profitable, this screening homogenizes the protest incentives of consumers, causing turnout to lose its informational value. This breakdown of information aggregation holds under both committed and uncommitted pricing, thereby limiting a protest's effectiveness as a disciplinary device.

Keywords: Protest, information aggregation, ethical agents, monopoly pricing

JEL Classification: D42, D72, D81, D82

1. Introduction

Consumers frequently engage in protest campaigns against firms over products associated with negative externalities, such as labor abuses, environmental damage, or controversial political stances.[1] Despite their prevalence, the effectiveness of these

*The author thanks the seminar participants at Kyoto University, Meiji University, the Game Theory Workshop at Kanazawa, Tokyo Social Choice Theory Workshop, and CoED at the University of Essex.

†College of Economics, Nihon University, Kanda-Misaki cho 1-3-2, Chiyoda Tokyo, 101-8360. Email: tajika.tomoya@nihon-u.ac.jp

[1]In the real world, there are many protest activities aiming for product discontinuation, e.g., protests against Nike for labor violations (e.g., "Protesters call on NYU Bookstore to cut ties with Nike," *Washington Square News*, October 13, 2023), a threat against the publication of a book ("Japan firm nixes translation of U.S. book questioning trans surgery," *Japan Times*, December 6, 2023), and so on.

campaigns in influencing firm behavior remains uncertain. While firms are not obliged to discontinue products based on protest participation, protests can still impact firm decisions by conveying information about consumer preferences.

This paper develops a theoretical model of product protests and highlights a surprising result: when consumers receive highly precise private signals about a product's value, the informativeness of protests—as a mechanism for aggregating dispersed information—declines. As a result, more informed consumers can undermine the effectiveness of protest campaigns. This stands in contrast to the conventional wisdom in the information aggregation literature, where more precise individual signals are typically associated with more informative collective outcomes.

Our model features a monopolistic firm selling a product that may impose negative externalities. Consumers privately observe binary signals about the product's intrinsic value, which includes both its consumption utility and a moral disutility. Before purchasing, they may participate in a protest campaign to influence the firm's decision to discontinue the product. Participation incurs a cost, and both the firm and consumers observe a noisy public signal of the protest turnout. The firm then decides whether to continue selling the product. Crucially, the firm sets the product price either before or after observing the signal of the protest turnout, and this decision critically shapes the protest's informativeness.

We show that when prices are high, consumers with both positive and negative signals derive similar marginal utility from discontinuing the product, leading to convergence in their protest incentives. This convergence of the incentives undermines the informativeness of protest turnout. A key insight from this result is that the price mechanism itself possesses the power to undermine information aggregation.

The firm's incentive to set a high price when consumer signals are precise is also central to the mechanism. With more precise private signals, consumers with a favorable signal are more confident that the product is valuable and are thus willing to pay more. Anticipating this, the firm can profitably set a higher price that effectively screens out consumers with unfavorable signals. However, this pricing strategy fundamentally alters protest incentives. For consumers with unfavorable signals, who are now priced out of the market, the motivation to protest is driven solely by the negative externality. For consumers with favorable signals, the high price diminishes the potential surplus they would lose from product discontinuation. This narrows the gap in protest incentives between the two types of consumers, causing their motivations to converge. As a result, protest turnout fails to reflect meaningful variation in underlying beliefs, rendering it uninformative. Nevertheless, an informative equilibrium exists when consumers have less precise signals.

We obtain this result under both committed and uncommitted pricing regimes. In the committed case, the firm sets the price before observing protest turnout, while in the uncommitted case, the price is chosen ex post. In both scenarios, increased signal precision reduces the informativeness of protests, although the underlying mechanisms differ.

Our main result is that signal precision and protest informativeness are negatively correlated, under both committed and uncommitted pricing regimes. This result differs sharply from Ekmekci and Lauermann (2022), who find that protests become more informative as signals become more precise in the context of political regimes without strategic price setting. In our setting, price serves as a selection mechanism that reduces heterogeneity in protest participation, thereby dampening information aggregation.

Our work adds to the growing literature on protest and information design. Direct evidence for our mechanism is still absent, yet the 2020 boycott of Goya Foods' offers suggests supportive evidence for our mechanism: prices remained unchanged or even rose because the firm could profit from a smaller pool of highly committed buyers, thereby limiting the campaign's impact (see Liaukonytė et al., 2023). The parallel is clear: when the customer base consists mainly of high-WTP consumers, a firm can "price-screen" the market and neutralize the informational power of protest, exactly the channel our model formalizes.

From a policy perspective, our results suggest that unregulated pricing may undermine protests as an information-transmitting device. Regulating prices in ethically sensitive markets could help preserve the social value of consumer protest, especially in contexts where firm behavior depends on perceived consumer sentiment.

The rest of the paper is organized as follows: Section 2 presents the model. Section 3 analyzes equilibrium under fixed prices. Section 4 endogenizes the price-setting behavior. Section 5 considers an uncommitted pricing regime. Section 6 discusses the model assumptions and concludes.

1.1. Related Literature

This paper contributes to three strands of literature: the theory of protest and information aggregation, economic models of consumer protests, and the modeling of ethical behavior in collective decision-making.

First, we relate to the literature on protest turnout as a signal of latent preferences or information. Lohmann (1993, 1994), Battaglini (2017), and Ekmekci and Lauermann (2022) analyze how collective action can aggregate dispersed information. However, these studies typically consider settings in which individual signals are independent of the market structure, as they consider situations in political contexts. In contrast, we show that in a private market with endogenous pricing, increased signal precision can reduce the informativeness of protest turnout. This arises because the firm uses pricing to screen consumers, which homogenizes participation incentives.

Second, our work contributes to the literature on consumer boycotts and protests (e.g., Baron, 2001, Feddersen and Gilligan, 2001, Baron, 2003, Innes, 2006, Miyagiwa, 2009, Glazer et al., 2010, Egorov and Harstad, 2017, Peck, 2017, and Correa, 2024). For example, Egorov and Harstad (2017) model a protest as a dynamic war of attrition, while Diermeier and Van Mieghem (2008) and Delacote (2009) study protests as public goods games with fixed thresholds. Our approach differs in that it emphasizes protests as information transmission mechanisms and considers how pricing behavior endogenously interacts with the informativeness of protests.

Finally, we build on studies that incorporate ethical concerns and social norms into decision-making. Feddersen and Sandroni (2006) and Ali and Lin (2013) model ethical voting based on group utilitarianism and reputational concerns. Our model incorporates a similar structure by assuming that consumers behave based on group utilitarian, but with signal-dependent motivations that lead to strategic protest participation.

2. Model

A firm sells a product to a continuum of consumers with unit demand. The population is normalized to 1. The utility of purchasing the product is $w \cdot v$, where $v \in \{0, 1\}$. We refer to v as *product value* and w as the scale parameter, whose role is discussed later. The value v can be interpreted as incorporating a psychological factor, such as feeling guilty about purchasing a product having a negative externality. We assume that v is unknown to all players. For instance, this uncertainty about v captures situations where a negative externality of the product has just been revealed, and people are uncertain about how to respond collectively. The prior probability of $v = 1$ is assumed to be $\frac{1}{2}$ for notational simplicity.

When the product is sold, it imposes a negative externality on consumers, leading to disutility $\varsigma > 0$. Consumers can engage in a protest campaign to stop the sale of the product. Participating in the campaign costs c_i for consumer i. c_i is drawn identically and independently distributed according to function F over $\mathbb{R}_+$ among consumers. We assume that f is the density of F, and $f(0) > 0$. In summary, when the product is sold at price $w \cdot p$, the consumer's utility is as follows.[2]

$$u_i = [w \cdot (v - p) \cdot \chi(\text{purchase}) - \varsigma] \cdot \chi(\text{on sale}) - c_i \cdot \chi(\text{participate in protest}).$$

Before participating in the protest campaign, consumers receive a private signal about the product value v. $\theta \in \{\theta_H, \theta_L\}$ denotes the signal. θ_H suggests that $v = 1$ is more likely. We assume that $\Pr(\theta = \theta_H \,|\, v = 1) = \Pr(\theta = \theta_L \,|\, v = 0) = \mu \in \left(\frac{1}{2}, 1\right)$. μ is referred to as *signal precision*. The parameter μ captures the extent to which individuals tend to receive signals that reinforce a shared perception of the product's value.

In particular, μ reflects the *endogenous nature of social norms*: if μ is close to 1, individuals are likely to receive signals that align with the prevailing belief about v, reinforcing a strong consensus. Conversely, if μ is closer to $\frac{1}{2}$, there is greater ambiguity, and social norms surrounding the product's value are weaker.[3]

Given consumers' private signals, they decide whether to participate in the protest campaign. We assume that consumers act as **ethical agents**. Specifically, they behave according to a **rule-utilitarian** (or **group-utilitarian**) principle, choosing to protest if their participation increases the aggregate welfare of consumers who received the same signal. This framework, which follows Feddersen and Sandroni (2006) and Ekmekci and Lauermann (2022), allows us to directly model the pivotal incentives central to collective action. The precise objective function is given in Section 3.2. A further discussion about the ethical agent assumption is given in Section 6.

One justification for this assumption is that a consumer's participation decision is observed by their social network, leading them to prefer being perceived as ethical. This, in turn, motivates them to act in a way that maximizes consumers' welfare.[4]

---

[2] For each event E, $\chi(E)$ takes the value 1 if E occurs, and 0 otherwise.

[3] An alternative interpretation is that v is *not merely an intrinsic product characteristic* but rather reflects a prevailing social norm shaped by majority opinion. If a fraction $\mu > \frac{1}{2}$ of individuals receive θ_L, the prevailing norm dictates that the product has no value ($v = 0$). Conversely, if a majority receives θ_H, the product is perceived as valuable ($v = 1$). This feature captures the endogenous nature of product value as determined by collective sentiment rather than objective characteristics.

[4] Ali and Lin (2013) explores this rationale in the context of voting decisions.

However, the adequacy of this justification depends on what participation in the protest entails. On the one hand, if participation involves attending an event organized by activists, the cost may be too high for this motivation to hold. On the other hand, if participation means sharing or liking a post on social media that expresses regret or protest against a firm's product, this motivation seems more plausible.[5]

After the protest campaign, the firm and consumers observe a public signal, indicating the protest campaign's scale. Let the actual participation ratio be τ. The firm and consumers observe $t = \tau + \varepsilon$, where ε is a normal noise with 0 mean and σ^2 variance. Let Φ be the standard normal distribution function and φ be its density. ε can be interpreted as noise from media coverage or ambiguity in measuring the protest's true size. For instance, it could reflect noise intentionally created by activists.[6]

Observing the public signal t, the firm decides whether to discontinue the product. We assume that the firm incurs a cost of $wK > 0$ when continuing product sales. This includes the distribution cost and the reputation cost of selling a product that causes negative externalities. We also assume that the marginal cost of production is $w\kappa \geq 0$. Therefore, when the product is on sale, the firm's profit is

$$w \cdot [(\text{demand at } p) \cdot (p - \kappa) - K].$$

Finally, the decision flow is as follows.

1. The firm set a price.
2. Each consumer observes a private signal θ_i and decides whether to participate in the protest.
3. Turnout τ realizes, and the firm and consumers observe public signal $t = \tau + \varepsilon$.
4. The firm decides whether to continue the product sale.
5. If the product is on sale, each consumer decides whether to buy it.

[5]Sharing and liking posts on social media can be enough to compel a firm to discontinue a product. For instance, Adidas withdrew a line of shoes resembling shackles in response to criticism on Facebook (Hoffberger, 2012, "Adidas cancels shackle shoe after Facebook backlash," Daily Dot (https://www.dailydot.com/unclick/adidas-shackle-shoe-controversy/, accessed August 1, 2024)).

[6]This assumption is the same as Ekmekci and Lauermann (2022).

The solution concept is the perfect Bayesian Nash equilibrium.

3. Equilibrium with a given price

3.1. After public signal observation

3.1.1. Consumer's choice

This section considers consumers' purchase decisions after public signal observation. Consider the consumer receiving signal θ. The consumer purchases if the expected product value exceeds the price, that is, $\Pr(v = 1 \mid \theta, t) \geq p$.[7] For notational simplicity, we write $P_\theta(t) = \Pr(v = 1 \mid \theta, t)$. The expected value of the product value is

$$P_\theta(t) = \frac{\Pr(t \mid v = 1)\theta}{\Pr(t \mid v = 1)\theta + \Pr(t \mid v = 0)(1 - \theta)}.$$

Let λ_v be the true turnout when the realized value is $v \in \{0, 1\}$. Then, as $t = \lambda_v + \varepsilon$ at state v, $P_\theta(t)$ is rewritten as follows (recall that φ is the pdf of the standard normal distribution).

$$P_\theta(t) = \frac{\varphi\left(\frac{t-\lambda_1}{\sigma}\right)\theta}{\varphi\left(\frac{t-\lambda_1}{\sigma}\right)\theta + \varphi\left(\frac{t-\lambda_0}{\sigma}\right)(1-\theta)}.$$

By using this formula, when we assume $\lambda_0 > \lambda_1$, inequality $P_\theta(t) > p$ is rewritten as

$$t < \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2}\frac{\ln\left(\frac{\theta}{1-\theta}\right) - \ln\left(\frac{p}{1-p}\right)}{\lambda_0 - \lambda_1} =: T_\theta(p).$$

In other words, $T_\theta(p)$ is the supremum value of public signal t with which the type-θ consumers purchase the product. The following lemma shows that the type-H consumers are more likely to purchase the product than the type-L consumers.

Lemma 1. *If $\lambda_0 > \lambda_1$, $T_L(p) < T_H(p)$ for any p.*

[7] As this decision is in the last stage of the game, the purchase decision does not affect others' utilities. Thus, each consumer's purchase decision is based only on their utility, although the consumers have welfare concerns.

3.1.2. The firm's choice

Given the consumer's choice observing public signal t, this section considers the condition under which the firm discontinues the product sale. We focus on a cutoff equilibrium; there is a cutoff $T_f(p)$ such that the firm discontinues the product sale if and only if $t > T_f(p)$. The following lemma formally characterizes this cutoff, which depends on how the price p compares to the firm's costs K and κ (the proof is in the appendix).

Lemma 2. *Suppose that $T_L(p) < T_H(p)$, and let*

$$\tilde{T} := \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)} \right)}{\lambda_0 - \lambda_1}.$$

Then, the cutoffs are classified as follows.

(*i*) *If* $p - \kappa \geq \frac{K}{1-\mu}$, $T_f(p) = T_H(p)$.

(*ii*) *If* $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$, $T_f(p) = \max\{\min\{T_H(p), \tilde{T}\}, T_L(p)\}$.

(*iii*) *If* $\frac{K}{\mu} \geq p - \kappa \geq K$, $T_f(p) = T_L(p)$.

(*iv*) *If* $K > p - \kappa$, $T_f(p) = -\infty$.

The intuition of this result is as follows. When p is higher, the firm is more likely to continue the sale as long as H-type consumers buy the product. However, when p or t is high so that H-type does not buy, the firm never continues the sale, and therefore, the cutoff of the firm is capped by H-type's cutoff as shown in (i) and (ii). In contrast, if $p < \kappa + K$, the firm's profit is negative even if all consumers buy the product. Then, regardless of the realized public signal, the firm discontinues the sale (i.e., $T_f(p) = -\infty$).

3.2. Before public signal observation

Before the public signal observation, the consumer chooses whether to participate in the protest. This section examines the condition and its informativeness regarding product value.

We focus on the cutoff strategy: each type-θ consumer participates in protest if and only if the marginal benefit of participation exceeds its cost. As mentioned in the model section, we consider *ethical* agents: they maximize the aggregate utility of the same type of consumers written in the following formula.[8,9]

$$\int^{T_f(p)-\mathrm{d}\tau} \Big(w \cdot [P_\theta(t) - p]_+ - \varsigma\Big) \cdot \xi_\theta(t)\,\mathrm{d}t - c_i\,\mathrm{d}\tau - \sum_{j \text{ is a participant}} c_j,$$

where we define $[\cdot]_+ = \max\{\cdot, 0\}$ and $\xi_\theta(t) = \frac{1}{\sigma} \cdot \Big(\theta \cdot \varphi\Big(\frac{t-\lambda_1}{\sigma}\Big) + (1-\theta) \cdot \varphi\Big(\frac{t-\lambda_0}{\sigma}\Big)\Big)$. $\mathrm{d}\tau$ represents a marginal increase in the participation rate. In this formula, the firm applies a cutoff criterion: discontinuing the product if and only if $t > T_f(p)$. A consumer's participation increases turnout by $\mathrm{d}\tau$, which also increases their costs by $c_i\,\mathrm{d}\tau$. Then, consumer i participates in the protest if and only if

$$\underbrace{\Big(\varsigma - w \cdot \big[P_\theta\big(T_f(p)\big) - p\big]_+\Big) \cdot \xi_\theta\big(T_f(p)\big)}_{\text{marginal benefit of participation}} \geq c_i.$$

Now let $c_\theta = \Big(\varsigma - w\big[P_\theta\big(T_f(p)\big) - p\big]_+\Big) \cdot \xi_\theta\big(T_f(p)\big)$ be the cutoff of type-θ consumer. Then, we have that

$$\lambda_1 = \mu \cdot F(c_H) + (1-\mu) \cdot F(c_L),$$

$$\lambda_0 = (1-\mu) \cdot F(c_H) + \mu \cdot F(c_L).$$

Therefore, $\lambda := \lambda_0 - \lambda_1 = (2\mu - 1) \cdot (F(c_L) - F(c_H))$. This value plays a critical role in determining the existence of an informative equilibrium.

3.3. Equilibrium of the continuation game after the price is given

This section defines the equilibrium of the continuation game after the price is given and provides a characterization.

Definition 1. (T, c_H, c_L) is a tuple of *equilibrium cutoffs* with given price p if

[8]Note that assuming this utility does not affect the purchasing decision, as it does not affect the other consumers' utility and the firm's decision.

[9]Our results continue to hold even when each consumer maximizes overall consumer welfare. See Section 6.

(i) $T = T_f(p)$

(ii) $c_\theta = \left(\varsigma - w \cdot [P_\theta(T) - p]_+\right) \cdot \xi_\theta(T)$.

Note that if $c_H = c_L$, which implies $\lambda_0 = \lambda_1$. Then, the public signal t is independent of v. We call this case *uninformative*. In other words, in an uninformative equilibrium, both types of consumers participate in the same cutoff, and therefore, the turnout does not reflect the true state of the world v.

Definition 2. An equilibrium is *informative* if $c_L > c_H$, and is *uninformative* if $c_L = c_H$.

It's important to note that the uninformative equilibrium exists for any parameters. If the firm sets $T = \infty$ or $T = -\infty$, the protest does not affect whether the product sale continues. This leads to both types of consumers having indifferent incentives. Conversely, if the protest is uninformative, the firm's decision remains unchanged regardless of the turnout. This implies that $T = \infty$ or $T = -\infty$.

Therefore, it is crucial to reiterate that our focus is on when an informative equilibrium exists. Our discussion is guided by the following proposition, which characterizes the condition concerning p for the existence of an informative equilibrium: An informative equilibrium exists only when the price is an intermediate value. This proposition is a key tool in our analysis (the proof is relegated to the Appendix).

Proposition 1. *Let* $\overline{P} := \mu \frac{K+(1-\mu)\kappa}{\mu(1-\mu)+(2\mu-1)K}$. *(a) If* $p \geq \max\{\mu, \overline{P}\}$ *or* $p \leq K + \kappa$, *no informative equilibrium exists. (b) Suppose that* $K + \kappa < p < \overline{P}$, $f(0) > 0$ *and* $F(0) > 0$. *Then, if* w *is sufficiently large, an informative equilibrium exists.*

This result makes a crucial point: the price instrument itself can neutralize the informational value of protest, a mechanism that operates irrespective of the noise level σ.

Below, we explain the intuition of the proposition. Here, $\overline{P}$ is a threshold of p: $T_H(p) > T_f(p)$ if and only if $p < \overline{P}$. Note that the consumers' participation decision is determined by the event that they are pivotal, that is, at the firm's cutoff point $T_f(p)$.

If p is large enough, as Lemma 2 shows, $T_f(p) = T_H(p)$. This implies that the H-type's surplus from the product purchase is 0 when the consumer is pivotal on the decision for the sale discontinuation. Their incentive to protest, therefore, becomes driven solely by the desire to avoid the negative externality, just like an L-type consumer who would not have purchased anyway. This equalizes their marginal benefits from protesting, rendering the turnout uninformative.

For part (b), an informative equilibrium exists when an intermediate price creates a divergence in incentives. At such a price, an H-type consumer, when pivotal, still expects to gain a positive surplus from purchasing the product, making them reluctant to protest. An L-type consumer expects no such surplus. This difference in expected utility sustains different protest cutoffs ($c_L > c_H$), allowing for information aggregation. When the scaling parameter w is sufficiently large, this utility difference becomes significant enough to guarantee the existence of an informative equilibrium.

An important consequence of Proposition 1 is that it reveals a fundamental commitment problem faced by the firm: for the equilibrium to be informative, the price should be below $\max\{\mu, \overline{P}\}$. This price ceiling reveals a crucial tension. Even if a protest could perfectly reveal the state (e.g., $v = 1$), the firm cannot fully exploit this information by setting a price close to 1. The reason is that the protest's informativeness is endogenous: the very act of setting a high price would, by the logic of Proposition 1, render the protest uninformative in the first place. This dynamic has a perverse consequence for consumers. Since the price ceiling for an informative protest, $\overline{P}$, increases with signal precision μ, more precise private information does not necessarily benefit consumers; it may simply empower the firm to maintain a higher price. A formal analysis is relegated to Section B in the appendix.

4. Firm's optimal pricing

Assuming that for any given price, the most informative equilibrium prevails, this section endogenizes the firm's price choice to determine the overall equilibrium outcome. All proofs of this section are relegated to the Appendix.

4.1. Benchmark

This section considers the model without protest as a benchmark. In the benchmark case, the H-type's expected product value is $E[v] = \mu$, and that of the L-type consumers is $E[v] = 1 - \mu$. If the firm keeps the product sale, the optimal price is $p = \mu$ or $p = 1 - \mu$. The expected profit when $p = \mu$ is $\frac{\mu - \kappa}{2} - K$, and that when $p = 1 - \mu$ is $1 - \mu - \kappa - K$.

4.2. Optimal price

Now, we consider the firm's optimal pricing with the protest. First, we define the firm's profit. As the firm keeps product sale unless $t < T_f(p)$, the firm's expected profit is

$$\Pi(p) = \sum_{v' \in \{0,1\}} \Pr(v = v') \Big[\Big(\Phi\Big(\tfrac{T_f(p) - \lambda_{v'}}{\sigma} \Big) - \Phi\Big(\tfrac{T_L(p) - \lambda_{v'}}{\sigma} \Big) \Big) \cdot (\mu(p - \kappa) - K)$$

$$+ \Phi\Big(\tfrac{T_L(p) - \lambda_{v'}}{\sigma} \Big) \cdot (p - \kappa - K) \Big].$$

The following proposition shows that the optimal price is $p = \mu$, in which case the protest is uninformative.

Proposition 2. *Suppose that* $\frac{\mu - \kappa}{2} > \max\{K, 1 - \mu - \kappa\}$. *Then, for sufficiently large* $\sigma > 0$, $p = \mu$ *is the optimal price and the protest is uninformative for any* λ_0, λ_1, *and* w.

The intuition for this result hinges on the firm's trade-off between information acquisition and surplus extraction. On one hand, inducing an informative protest is beneficial: by revealing that the state is likely $v = 0$ (when t is high), it allows the firm to discontinue the product and save the fixed cost K. On the other hand, an informative protest is costly: to sustain it, the firm must set a price $p < \mu$ (Proposition 1), thereby forgoing potential profits from its most valuable, high-signal consumers.

While high noise (σ) diminishes the magnitude of both this benefit and cost, their relative importance remains crucial for the firm's pricing decision. We show that when signal precision μ is sufficiently high (specifically, if $\mu > \kappa + 2K$), the profit margin from high-signal consumers becomes so significant that the cost of lowering the price outweighs the potential gain from saving K. Therefore, the firm optimally forgoes the informational benefit of the protest, sets the high price $p = \mu$, and consequently, the protest becomes uninformative.

The above observation depends on whether an uninformative equilibrium with $p = \mu$ yields a sufficient profit. If not, an informative equilibrium leads to a higher profit, as the following proposition shows.

Proposition 3. *Suppose that $1 - \mu - \kappa > K > \frac{\mu - \kappa}{2}$. Then, for each σ, there is $\overline{w}$ such that for each $w > \overline{w}$, the protest is informative at the firm's optimal price.*

With the condition of Proposition 3, while setting the price to $p = \mu$ yields a deficit, setting a low price inducing an informative equilibrium yields a positive profit. Therefore, the firm prefers to set a low price.

By Propositions 2 and 3, we observe that the resulting equilibrium is uninformative if μ is high, while it is informative when μ is small. Theorem 1 summarizes the result, and Figure 1 illustrates it.

Theorem 1. *Suppose that $\kappa < \frac{1}{2}$ and $K \in \left(\frac{1-2\kappa}{3}, \frac{1}{2} - \kappa\right)$. Take sufficiently large σ and w. Then, there exists $\mu^*, \overline{\mu}, \underline{\mu}$ with $\mu^* > \overline{\mu} > \underline{\mu}$ such that the equilibrium is uninformative if $\mu \in (\overline{\mu}, \mu^*)$,[10] and the equilibrium is informative if $\mu < \underline{\mu}$.*

This result implies that the informativeness of individual consumers' signals may negatively correlate with the protest's informativeness. This contrasts sharply with Ekmekci and Lauermann (2022), who shows that each person's signal informativeness positively correlates with the protest campaign's informativeness.

[10]The restriction $\mu < \mu^*$ is needed because Proposition 2 is the statement when μ is fixed.

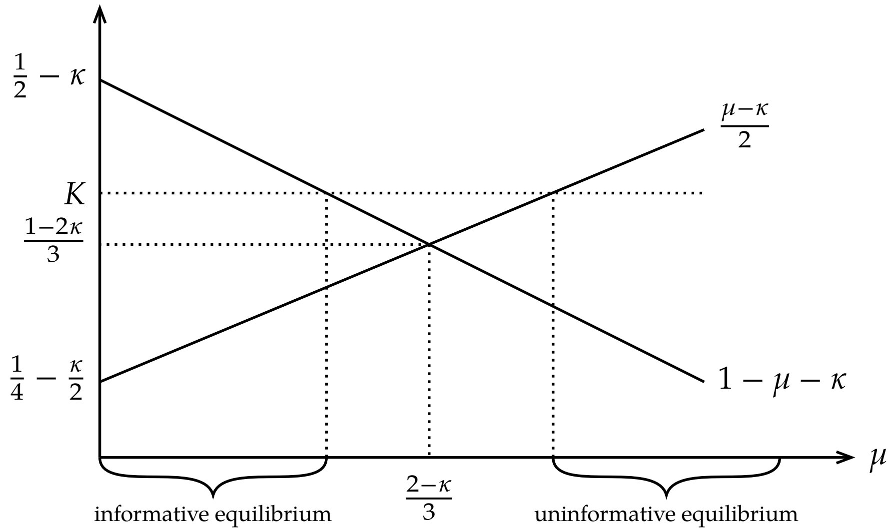

Figure 1: Classification of equilibria

The main difference between their and our models is the existence of a price-setting firm. In the models of Ekmekci and Lauermann (2022) and many protest studies, citizens campaign against the policy-setting government, which affects only the policy decision. However, the firm makes two-dimensional decisions in our model: whether to discontinue the product sale and price. With a higher informativeness of each individual's signal, consumers are more willing to pay, which leads to a greater profit for the firm by setting a higher price. This makes the protest uninformative by reducing the purchase incentive of H-type.

4.3. Welfare Analysis

In this discussion, we examine social welfare, which is the sum of the firm's profit and consumer surplus. Then, social welfare is $((v - \kappa) \cdot \text{purchase} - K - \varsigma)\chi(\text{on sale})$. When $v = 1$, social welfare is increasing in purchases. When $v = 0$, it is decreasing. Specifically, if $v = 1$, the product should be available for sale, and all consumers purchase it at the given price. On the other hand, if $v = 0$, since no consumer buys the product, the firm discontinues the sale.

We refer to $\lambda_0 - \lambda_1$ as the **informativeness** of the protest.[11] As this value increases, the probability of product purchase decreases when $v = 0$, and increases when $v =$

[11] This definition is the same as Ekmekci and Lauermann (2022).

1. Thus, the greater informativeness of the protest led to an improvement in social welfare. Note that the informativeness is 0 in uninformative equilibria.

Our findings suggest that as the consumers' private signals become precise, social welfare can decrease. The following corollary is a summary.

Corollary 1. *Suppose that the same assumption as Theorem 1 holds. Then, there are $\underline{\mu}, \overline{\mu}, \mu^*$ with $\underline{\mu} < \overline{\mu} < \mu^*$ such that social welfare is higher when $\mu < \underline{\mu}$ than when $\mu \in (\overline{\mu}, \mu^*)$ for sufficiently large w and σ.*

5. Uncommitted price

In the previous section, the firm is assumed to commit to a price p before the public signal is observed. This section considers the case in which the firm decides the price after observing the public signal. Therefore, the new decision flow is as follows:

1. Each consumer observes a private signal θ_i and decides whether to participate in the protest.
2. Turnout τ realizes, and the firm and consumers observe public signal $t = \tau + \varepsilon$.
3. The firm decides whether to continue selling the product, and if so, sets a price.
4. If the product is on sale, each consumer decides whether to buy it.

A crucial difference from the committed price case is that the turnout affects the price in the uncommitted case. If the protest is informative, a greater turnout signals a lower value of the product. Then, the firm sets a lower price.[12] Then, protest participation has two effects on type-H consumers' utility. On one hand, a protest threatens to eliminate their potential informational rents, discouraging participation. On the other hand, a larger protest can signal a lower product value to the firm, inducing it to set a lower price. This "price manipulation" motive encourages H-type participation. The

[12]Indeed, protests tend to decrease prices. Hendel et al. (2017) empirically demonstrated that price declines after a consumer boycott, organized in Israel on Facebook.

overall effect on equilibrium informativeness depends on the relative strength of these competing incentives.

Now, we investigate how our conclusion changes. The previous section shows that with higher μ, the equilibrium is less likely to be informative. We show that even in the uncommitted price case, the same conclusion is drawn. The following is a formal analysis. All proofs of the following section are relegated to the Appendix.

5.1. Formal analysis of the uncommitted price case

To focus on the discussion on whether an informative equilibrium exists, we assume that $\lambda_0 > \lambda_1$. Then, we verify the consumers' incentives to participate in the protests. Note that even in the uncommitted price case, after the price is set, the consumers' decision criterion for purchasing the product is unchanged. Thus, type-θ consumer purchases the product if and only if $t < T_\theta(p)$.

Given this strategy, the firm's profit is

$$\Pi = \begin{cases} -K & \text{if } t > T_H(p) \\ [\Pr(v=1\,|\,t)\cdot\mu + \Pr(v=0\,|\,t)\cdot(1-\mu)]\cdot(p-\kappa) - K & \text{if } t \in (T_L(p), T_H(p)] \\ p-\kappa-K & \text{if } t \le T_L(p). \end{cases}$$

This implies that the optimal price is either $p = P_H(t)$ or $P_L(t)$. Therefore, we need to compare the following equations.

$$\Pi^*_H(t) := [\Pr(v=1\,|\,t)\cdot(2\mu-1)+1-\mu)]\cdot(P_H(t)-\kappa) - K$$

$$\Pi^*_L(t) := P_L(t) - \kappa - K.$$

$\Pi^*_H(t)$ represents the profit when the firm focuses solely on H-type consumers, while $\Pi^*_L(t)$ indicates the profit when targeting both consumer types. Also, the firm discontinues the product sales rather than selling it with $p = (T_L)^{-1}(t)$ if $p - \kappa < K$. In other words, $(T_L)^{-1}(t) < K + \kappa$. Equivalently, $t > T_L(K+\kappa)$.

To simplify the discussion, we assume $\kappa = 0$ hereafter. The following lemma identifies the condition under which one profit exceeds the other.

Lemma 3. *Assume that $\kappa = 0$. Then, there exist values μ^* and μ^{**} with $\mu^* < \mu^{**}$ such that: (a) If $\mu > \mu^{**}$, there is $\hat{t} < T_L(K)$ such that $\Pi_H^*(t) < \Pi_L^*(t)$ if and only if $t < \hat{t}$. (b) If $\mu < \mu^{**}$, $\Pi_H^*(t) < \Pi_L^*(t)$ for each $t < T_L(K)$. (c) $T_L(K) \to \infty$ if $\mu < \mu^*$, and $T_L(K) \to -\infty$ if $\mu > \mu^*$.*

Given this firm's behavior, we consider the consumers' incentive to participate in the protest. We first consider the case (a): there is $\hat{t} < T_L(K)$ such that $\Pi_L^*(t) > \Pi_H^*(t)$ if and only if $t < \hat{t}$. Let T^* be the solution to $\Pi_H^*(t) = 0$ with respect to t. This implies that at the optimal strategy, the firm sets price $P_L(t)$ when $t \in (-\infty, \hat{t})$, sets price $P_H(t)$ when $t \in (\hat{t}, T^*)$, and when $t > T^*$, discontinues the product sale.

Under this firm's behavior, the expected utility of the H-type is

$$-\varsigma \cdot \int^{T^*-\,\mathrm{d}\tau} \xi_H(t)\,\mathrm{d}t + w\int_{\hat{t}-\,\mathrm{d}\tau}^{T^*-\,\mathrm{d}\tau} [P_H(t) - P_H(t+\mathrm{d}\tau)]\xi_H(t)\,\mathrm{d}t$$

$$+w \cdot \int^{\hat{t}-\,\mathrm{d}\tau} [P_H(t) - P_L(t+\mathrm{d}\tau)] \cdot \xi_H(t)\,\mathrm{d}t.$$

Here, $P_H(t)$ is the expected value of the product while $P_L(t)$ is the price. A marginal increase of turnout $\mathrm{d}\tau$ affects the price. As the firm set price $p = P_H(t)$ if $t > \hat{t}$, type-H consumers' the informational rent $P_H(t) - P_L(t)$ vanishes. Therefore, the decision to discontinue the product only affects the externality term ς.

The differentiation concerning $\mathrm{d}\tau$ is the marginal gain by participation. Therefore, consumer i participates in the protest if and only if $c_H \geq c_i$, where

$$c_H := \varsigma \cdot \xi_H(T^*)$$

$$+w \cdot \left[-\underbrace{[P_H(\hat{t}) - P_L(\hat{t})]\xi_H(\hat{t})}_{\text{information rent}} + \underbrace{\int_{\hat{t}}^{T^*} [-P_H'(t)]\xi_H(t)\,\mathrm{d}t + \int^{\hat{t}} [-P_L'(t)]\xi_H(t)\,\mathrm{d}t}_{\text{benefit from price reduction by participation}} \right].$$

Focus on the second line in the above equation. Note that $P_L'(t) < 0$ and $P_H'(t) < 0$. Then, the second and third terms are the gain from price manipulation by increasing turnout. In contrast, with a high turnout, the firm gives up on making all types of consumers purchase the product, and then, type-H's informational rent vanishes. This makes type H consumers reluctant to participate in the protest, captured by the first term.

Now we compare with L-type consumers' utility. The utility of the L-type is $-\varsigma \cdot \int^{T^* - \mathrm{d}\tau} \xi_L(t)\,\mathrm{d}t + w \cdot \int^{\hat{t} - \mathrm{d}\tau} [P_L(t) - P_L(t + \mathrm{d}\tau)] \cdot \xi_L(t)\,\mathrm{d}t$ as the firm extracts type-L consumers' full surplus. Then, the type-L consumers participate in the protest if and only if

$$c_L := \varsigma \cdot \xi_L(T^*) + w \cdot \int^{\hat{t}} \left[-P'_L(t)\right] \xi_L(t)\,\mathrm{d}t \geq c_i.$$

We compare the relation between c_H and c_L with high μ cases. As we focus on a large w case, we consider the coefficients of w in $c_L - c_H$, that is

$$D := \int^{\hat{t}} \left[-P'_L(t)\right] (\xi_L(t) - \xi_H(t))\,\mathrm{d}t - \int_{\hat{t}}^{T^*} \left[-P'_H(t)\right] \xi_H(t)\,\mathrm{d}t + \left[P_H(\hat{t}) - P_L(\hat{t})\right] \xi_H(\hat{t}).$$

As H-type has more price-manipulation incentive than the L-type ($\int_{\hat{t}}^{T^*} \left[-P'_H(t)\right] \xi_H(t)\,\mathrm{d}t$), this exceeds the difference in the information rent. As a result, H-type is more incentivised to participate. The following proposition provides a formal condition.

Proposition 4. *Suppose that $\mu > \max\{\mu^{**}, 2K\}$. Then, for sufficiently large σ, $D < 0$.*

Proposition 4 implies that the protest becomes uninformative with high μ. This is because this inequality $c_H > c_L$ is gained by the supposition that $\lambda_0 > \lambda_1$, which means that $c_L > c_H$. This is a contradiction. So, the equilibrium should be uninformative. This is a similar feature to the committed price case. The intuition is the following. When μ is high, the firm is willing to continue selling even when the public signal t is high. In this scenario, L-type consumers are priced out, but H-type consumers still purchase. Consequently, H-type consumers retain a strong price-manipulation incentive to protest, while L-type consumers do not. This motivates the H-type to participate in the protest more than the L-type.

Next, we consider the case (b): for each $t < T_L(\kappa + K)$, $\Pi^*_L(t) > \Pi^*_H(t)$. This implies that for any $t < T_L(\kappa + K)$, the firm set price $P_L(t)$ and discontinues the product sale if $t > T_L(\kappa + K)$. In this case, the firm never exploits the full surplus of H-type consumers, and therefore, the price manipulation incentive gets closer to that of L -types.

Below, we discuss this point formally. To simplify the notation, we denote $T_* = T_L(\kappa + K)$. The utility of the H-type from the product sales is

$$-\varsigma \int^{T_*-\mathrm{d}\tau} \xi_H(t)\,\mathrm{d}t + w \int^{T_*-\mathrm{d}\tau} [P_H(t) - P_L(t+\mathrm{d}\tau)]\xi_H(t)\,\mathrm{d}t.$$

The differentiation concerning $\mathrm{d}\tau$ is the marginal gain by participation. Therefore, they participate in the protest if and only if

$$c_H := \varsigma\xi_H(T_*) - w[P_H(T_*) - P_L(T_*)]\xi_H(T_*) + w\int^{T_*}[-P'_L(t)]\xi_H(t)\,\mathrm{d}t \geq c_i.$$

Similarly, the utility of the L-type is $-\varsigma \int^{T_*-\mathrm{d}\tau} \xi_L(t)\,\mathrm{d}t + w\int^{T_*-\mathrm{d}\tau}[P_L(t) - P_L(t+\mathrm{d}\tau)]\xi_L(t)\,\mathrm{d}t$. Then, they participate in the protest if and only if

$$c_L := \varsigma\xi_L(T_*) + w\int^{T_*}[-P'_L(t)]\xi_L(t)\,\mathrm{d}t \geq c_i.$$

We also compare c_H with c_L and thus, focus on the coefficient of w in $c_L - c_H$,

$$\tilde{D} := \int^{T_*}[-P'_L(t)](\xi_L(t) - \xi_H(t))\,\mathrm{d}t + [P_H(T_*) - P_L(T_*)]\xi_H(T_*).$$

Compared with case (a), the difference in the price-manipulation incentive decreases, in which case, the difference in information rent makes L-type more likely to participate in the protest. The following proposition provides the formal condition.

Proposition 5. *Suppose that $\mu \in (\mu^*, \mu^{**})$. Then, for sufficiently large σ, $\tilde{D} > 0$.*

$c_L > c_H$ implies that the H-type is likelier to participate in the protest. Then, the equilibrium becomes informative with a large w. Intuition is the following. With a lower μ, the firm gives up selling even with a small t. Then, the incentive for price manipulation becomes smaller, and the effect of losing informational rent dominates that incentive.

In conclusion, the informative equilibrium is more likely with intermediate μ cases, although not in high μ cases. Our conclusion gained in the committed price case remains: precise information that consumers gain does not contribute to the success of the protest.

6. Conclusion and Discussions

This study examines when a protest campaign successfully aggregates information about consumers' values for a product, which determines the firm's decision. We show that consumers' individual signal precision does not necessarily contribute to the protest's informativeness. One implication of our result is that consumers' ignorance may contribute to the success of protest campaigns.

In the remainder of this study, we discuss the assumptions in our model.

Objective of participation in protest: We assumed that consumers consider their type's aggregate utility when deciding whether to participate in the protest. However, as in Ali and Lin (2013), if this motivation stems from a reputation concern, it is more plausible to assume that consumers consider the expected overall consumer welfare.

Consider a scenario where each consumer's decision to participate in the protest is based on maximizing the expected consumer welfare. The consumer's belief, represented by $\theta \in \{\theta_H, \theta_L\}$, influences their expected consumer welfare as follows.

$$\int^{T_f(p)-\mathrm{d}\tau}\Big[\theta\Big(w \cdot [P_\theta(t) - p]_+ - \varsigma\Big)\xi_\theta(t) + (1-\theta)\Big(w \cdot [P_{\theta'}(t) - p]_+ - \varsigma\Big)\xi_{\theta'}(t)\Big]\,\mathrm{d}t,$$

where $\theta' \in \{\theta_H, \theta_L\}$ with $\theta' \neq \theta$. In this case, the H-type participates in the protest if and only if $c_i \leq c_H^* := \mu c_H + (1-\mu)c_L$ while the L-type does if and only if $c_i \leq c_L^* := (1-\mu)c_H + \mu c_L$. As long as $c_L > c_H$, the L-type consumers are more likely to participate. We can apply a similar discussion to obtain the same conclusions.

The ethical agent assumption: A potential concern is our assumption that consumers act as 'ethical agents' when deciding whether to protest. We argue that this assumption, while a simplification, allows for a clear analysis of our main contribution without loss of generality for the core economic trade-off.

First, this framework is a standard and convenient method for directly modeling pivotal incentives, which are central to any model of collective action. The main driver of our results is not the ethical motivation itself, but how the firm's pricing strategy alters the consumers' expected utility of being pivotal. By homogenizing the incentives of high-signal and low-signal consumers at the firm's decision threshold, the firm's pricing, not the specific form of consumer motivation, renders the protest uninformative.

Second, we conjecture that our results are robust and would hold in a setting with purely self-interested consumers. Following the approach in Battaglini (2017) and Ekmekci and Lauermann (2022), one could model a finite n-player game where each agent strategically calculates their probability of being pivotal. However, the key distinction and contribution of our paper is the introduction of a strategic price-setting firm. In such a model, consumers' calculations would have to incorporate the firm's pricing rule, which depends on the informativeness of the protest. The central mechanism we identify—that high signal precision incentivizes the firm to set a high price, which in turn screens the market and dampens information aggregation—would therefore remain. Consequently, the fundamental paradox highlighted in this paper is expected to persist.

Concerns regarding negative externalities: In our model, utility from the product purchase plays an important role, but concern regarding the negative externality (ς) itself does not. If the concern regarding the negative externalities is sufficiently large, the participation rates of H- and L-type consumers converge, making the protest uninformative. This observation holds even when concerns about the externality vary among consumers since the firm does not care about the externality itself. However, if the product value includes guilty feelings about consuming a product causing negative externalities, it would be correlated with the level of concern for the externality. In this case, our conclusion remains true even if concerns about negative externality are high.

Modeling of a Protest: Our model focuses on protest as an expressive act, distinct from the purchase decision itself (a boycott). This focus is motivated by the many contemporary consumer campaigns where public expression on platforms like social media is a central feature. Our contribution is to show how information aggregation can fail at this expressive stage. A richer model could integrate both channels—expression and purchasing—to study their dynamic interplay.

Mechanism Design by the Firm: In our model, the firm's pricing strategy is limited to setting a price. While a firm can indeed use its price-posting mechanism to elicit information, our analysis reveals that the two mechanisms of information —protest and pricing— are in direct conflict. The firm's profit-maximizing use of its pricing power to screen consumers is precisely what destroys the informational value of the protest. This tension between the private incentive to extract surplus and the collective goal of information aggregation lies at the heart of our findings and remains a fruitful area for further exploration.

Furthermore, there is a fundamental limitation to using a price-posting mechanism for information elicitation related to the timing of information. In our model, the firm must incur a cost K to continue selling the product. Therefore, any strategy to learn from realized demand requires the firm to take a risk for this cost. If the product value turns out to be low ($v = 0$), the information arrives 'too late,' only after the cost K has been sunk.

In contrast, the protest mechanism provides a signal before this irreversible cost is paid. An informative protest, therefore, serves as a valuable, ex-ante signal that allows the firm to avoid incurring a loss, a benefit that a post-entry 'price experiment' cannot offer. This highlights the unique economic importance of protests as a pre-commitment signaling device, justifying our focus on their information aggregation properties.

Binary private signal: One could argue that our primary finding is an artifact of the binary-signal structure, which permits a stark form of market screening. We acknowledge that this assumption is a simplification, but we contend that the underlying economic force—price screening undermining information aggregation—is a more general phenomenon.

To see why, consider a setting with a continuum of signals. A firm setting a price p effectively establishes a cutoff type, θ^*, such that only consumers with signals $\theta > \theta^*$ purchase the product. The core mechanism we identify depends on the firm's incentive to set this cutoff. When consumers' signals become more precise, those with high signals become highly confident in the product's value and thus have a higher willingness to pay. This incentivizes the firm to raise its price p, which in turn raises the purchase cutoff θ^*.

The consequence of this strategic pricing is twofold. First, the protest incentive for the screened-out population (those with $\theta < \theta^*$) is driven solely by the externality, as they gain no surplus from consumption. Second, for the remaining pool of buyers ($\theta > \theta^*$), the incentive to protest also converges towards this baseline as their potential consumer surplus shrinks at the margin. A protest, therefore, would primarily reflect the preferences of this skewed, high-type subgroup, rather than aggregating information from the entire population.

Thus, while a protest may not become perfectly uninformative as in our binary model, its effectiveness as a broad information-aggregation device would be fundamentally compromised. The qualitative result—that a firm's endogenous pricing strategy can perversely weaken the informativeness of collective action—should therefore persist. A formal proof of this conjecture presents a challenging but important avenue for future work.

Noisy Campaigns: Our analysis primarily focuses on a high-noise environment. In a low-noise setting, however, the firm faces a different trade-off that could alter the equilibrium outcomes. Although a simple analysis is given in Section B, a full characterization of this case is left for future research.

Appendix

A. Proofs

Proof of Lemma 2. To simplify the notation, in this proof, we use T as the firm's cutoff. Note that the cutoff depends on the consumers' behavior. Therefore, we consider the cases (i) $t \geq T_H(p)$, (ii) $t \in (T_L(p), T_H(p))$, and (iii) $t \leq T_L(p)$.

(i) Suppose that $t \geq T_H(p)$. Note that if $t > T_H(p)$, the firm discontinues the sale as no consumer purchases the product, which leads to a deficit. Consider the case $t = T_H(p)$. Then, as only type H consumers purchase the product, the firm's expected profit when the public signal t is observed is $\Pi_H(t)$, which is defined by

$$\Pi_H(t) := w \cdot ([\Pr(v = 1 \,|\, t) \cdot \mu + \Pr(v = 0 \,|\, t) \cdot (1 - \mu)] \cdot (p - \kappa) - K).$$

We can show the following claim:

Claim 1. *(i) If* $p - \kappa > \frac{K}{1-\mu}$*,* $\Pi_H(t) > 0$*. (ii) If* $\frac{K}{\mu} > p - \kappa$*,* $\Pi_H(t) < 0$*. (iii) If* $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$*,* $\Pi_H(t) > 0$ *if and only if* $t < \tilde{T}$*, where*

$$\tilde{T} := \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)} \right)}{\lambda_0 - \lambda_1}.$$

Therefore, $T = T_H(p)$ if either (a) $p - \kappa > \frac{K}{1-\mu}$, or (b) $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$ and $\tilde{T} \geq T_H(p)$. Otherwise, at $t = T_H(p)$, the profit is negative, and therefore, the cutoff is below $T_H(p)$.

(ii) Suppose that $T_L(p) < t < T_H(p)$. Then, at $t = T$, only the H-type consumer purchases the product. In this case, the firm's profit is $\Pi_H(t)$. Therefore, if $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$ and $\tilde{T} \in (T_L(p), T_H(p))$, $T = \tilde{T}$. Otherwise, there is no t such that $\Pi_H(t) = 0$.

(iii) Suppose that $t \leq T_L(p)$. Then, the firm's profit is

$$P_L := w[(p - \kappa) - K].$$

Therefore, $P_L \geq 0$ if and only if $p - \kappa \geq K$.

The remaining cases to consider the cutoff point are the following.

(a) $p - \kappa \geq \frac{K}{\mu}$ and $T_L(p) \geq \tilde{T}$. Then, the profit is positive if and only if $t < T_L(p)$, the cutoff is $T_L(p)$.

(b) $\frac{K}{\mu} > p - \kappa \geq K$. Then, the profit is positive if and only if $t < T_L(p)$, the cutoff is also $T_L(p)$.

(c) $K > p - \kappa$. Then, the profit is negative for any t. Therefore, the firm never continues the sale; in other words, the cutoff is $-\infty$.

In summary, we have the following:

(i) If $p - \kappa > \frac{K}{1-\mu}$, $T = T_H(p)$.

(ii) If $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$, $T = \max\{\min\{T_H(p), \tilde{T}\}, T_L(p)\}$.

(iii) If $\frac{K}{\mu} > p - \kappa \geq K$, $T = T_L(p)$.

(iv) If $K > p - \kappa$, $T = -\infty$. ■

A.1. Proof of Proposition 1

Before the proof of Proposition 1, we prepare the following lemma.

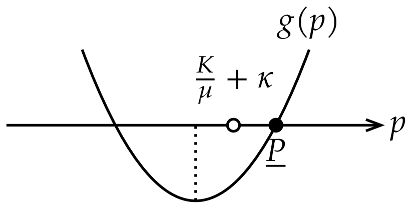

Figure 2: Illustration of g

Lemma 4. *Define $\underline{P}$ be the largest solution to $\frac{1-\mu}{\mu}\frac{1-p}{p} = \frac{\mu-\frac{K}{p-\kappa}}{\frac{K}{p-\kappa}-(1-\mu)}$ with respect to p. Then, $T_f(p) = T_H(p)$ if and only if $p > \overline{P}$ and $T_f(p) > T_L(p)$ if and only if $p > \underline{P}$. Further, $\frac{K}{1-\mu} + \kappa > \overline{P} > \underline{P} > \frac{K}{\mu} + \kappa$.*

Proof of Lemma 4. (a) If $p \geq \frac{K}{1-\mu} + \kappa$, $T_f(p) = T_H(p)$ by Lemma 2. If $p < \frac{K}{1-\mu} + \kappa$, $\tilde{T} < T_H(p)$ if and only if $\frac{\mu-\frac{K}{p-\kappa}}{\frac{K}{p-\kappa}-(1-\mu)} < \frac{\mu}{1-\mu}\frac{1-p}{p}$, which is equivalently $p > \overline{P}$. Further, a simple calculation shows $\frac{K}{1-\mu} + \kappa > \overline{P}$.

(b) If $p < \frac{K}{\mu} + \kappa$, by Lemma 2, $T_f(p) = -\infty$. Consider the case where $p \geq \frac{K}{\mu} + \kappa$. Then, $T_f(p) = \max\{T_L(p), \tilde{P}\}$. Note that $T_L(p) < \tilde{T}$ if and only if $\frac{\mu-\frac{K}{p-\kappa}}{\frac{K}{p-\kappa}-(1-\mu)} > \frac{1-\mu}{\mu}\frac{1-p}{p}$, equivalently, $g(p) > 0$, where g is a convex quadratic function (illustrated in Figure 2). Note that $g(p) < 0$ at $p = \frac{K}{\mu} + \kappa$. Then, $T_L(p) < T_f(p)$ if and only if $p > \underline{P}$. ■

Proof of Proposition 1. (1) Suppose that $p \geq \max\{\overline{P}, \mu\}$.

If $p \geq \overline{P}$, by Lemma 4, $T_f(p) = T_H(p)$. In this case, the equilibrium condition is

$$\lambda_0 - \lambda_1 = (2\mu - 1) \cdot (F(c_L) - F(c_H)),$$

$$c_\theta = \varsigma \cdot \xi_\theta(T_H(p)),$$

$$T_H(p) = \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{p}{1-p}\right)}{\lambda_0 - \lambda_1}$$

$$\xi_\theta(T_H(p)) = \frac{1}{\sigma} \cdot \left[\theta \cdot \varphi\left(\tfrac{T_H(p)-\lambda_1}{\sigma}\right) + (1-\theta) \cdot \varphi\left(\tfrac{T_H(p)-\lambda_0}{\sigma}\right)\right].$$

By $p \geq \mu$, $\frac{\sigma^2}{2} \cdot \frac{\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{p}{1-p}\right)}{\lambda_0-\lambda_1} \leq 0$ when $\lambda_0 \geq \lambda_1$. As φ is symmetric and single-peaked around 0, we show that $\varphi\left(\frac{T_H(p)-\lambda_0}{\sigma}\right) \leq \varphi\left(\frac{T_H(p)-\lambda_1}{\sigma}\right)$. Note also that

$$c_L - c_H = \varsigma \cdot \frac{2\mu - 1}{\sigma} \cdot \left[\varphi\left(\tfrac{T_H(p)-\lambda_0}{\sigma}\right) - \varphi\left(\tfrac{T_H(p)-\lambda_1}{\sigma}\right)\right] \leq 0.$$

Therefore, $\lambda_0 \geq \lambda_1$ implies $c_L \leq c_H$. In contrast, $c_L \leq c_H$ implies that $\lambda_0 \leq \lambda_1$. This shows that $c_H = c_L$ holds at any equilibrium. In other words, no informative equilibrium exists.

(2) Consider the case $p \in \left(\underline{P}, \overline{P}\right)$. Then, if $\lambda_0 > \lambda_1$, $T_L(p) < T_f(p) < T_H(p)$ for any c_H, c_L.

Recall that the following equations characterize the equilibria:

$$\lambda := \lambda_0 - \lambda_1 = (2\mu - 1) \cdot (F(c_L) - F(c_H)),$$

$$c_\theta = \left(\varsigma - w \cdot \left[P_\theta\left(T_f(p)\right) - p\right]_+\right) \cdot \xi_\theta\left(T_f(p)\right),$$

$$T_f(p) - \lambda_1 = \frac{\lambda}{2} + \frac{\sigma^2}{2} \cdot \frac{1}{\lambda} \cdot \ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right),$$

$$T_f(p) - \lambda_0 = -\frac{\lambda}{2} + \frac{\sigma^2}{2} \cdot \frac{1}{\lambda} \cdot \ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right),$$

$$P_\theta\left(T_f(p)\right) = \frac{\varphi\left(\frac{T_f(p)-\lambda_1}{\sigma}\right) \cdot \theta}{\varphi\left(\frac{T_f(p)-\lambda_1}{\sigma}\right) \cdot \theta + \varphi\left(\frac{T_f(p)-\lambda_0}{\sigma}\right) \cdot (1-\theta)},$$

$$\xi_\theta\left(T_f(p)\right) = \frac{1}{\sigma} \cdot \left[\theta \cdot \varphi\left(\frac{T_f(p)-\lambda_1}{\sigma}\right) + (1-\theta) \cdot \varphi\left(\frac{T_f(p)-\lambda_0}{\sigma}\right)\right].$$

This shows that c_L and c_H depend on λ, but not on λ_0, λ_1 individually. Therefore, the equilibrium is a fixed point of a self-map $\zeta : \lambda \mapsto (2\mu - 1)(F(c_L) - F(c_H))$. We can easily verify that ζ is continuous.

Below, we show that $\zeta(\lambda) > \lambda$ for some $\lambda > 0$ when w is large enough. By the definition of $\overline{P}$ and $\underline{P}$, $P_H\left(T_f(p)\right) > p$, and $p > P_L\left(T_f(p)\right)$. This implies that

$$c_H = \left(\varsigma - w\left[P_H\left(T_f(p)\right) - p\right]\right) \cdot \xi_H\left(T_f(p)\right)$$

$$c_L = \varsigma \cdot \xi_L\left(T_f(p)\right) > 0.$$

Then,

$$c_L - c_H = \varsigma \cdot \left(\xi_L\big(T_f(p)\big) - \xi_H\big(T_f(p)\big)\right) + w \cdot \left[P_H\big(T_f(p)\big) - p\right] \cdot \xi_H\big(T_f(p)\big),$$

which is positive for large w. Note that as $\lambda \to 0$, $T_f(p) \to \infty$, which implies that $c_L \to 0$ and $c_H \to 0$. By the Taylor expansion, we can take sufficiently small $\lambda > 0$ so that $(2\mu - 1)(F(c_L) - F(c_H)) \approx (2\mu - 1) \cdot f(0) \cdot (c_L - c_H)$, and then,

$$(2\mu - 1)(F(c_L) - F(c_H)) > (2\mu - 1) \cdot f(0) \cdot w \cdot \left[P_H\big(T_f(p)\big) - p\right] \cdot \xi_H\big(T_f(p)\big)$$

We can take large w that is proportional to $\frac{\lambda}{\left[P_H(T_f(p)) - p\right] \cdot \xi_H(T_f(p))}$. Then, while both c_H and c_L is close to 0, $\zeta(\lambda) = (2\mu - 1)(F(c_L) - F(c_H)) > \lambda$ for small $\lambda > 0$. As $\zeta(2\mu - 1) < 2\mu - 1$, the intermediate value theorem implies the existence of a fixed point of ζ, which is in $(0, 2\mu - 1)$. This implies that the fixed point is a positive value. This shows the existence of an informative equilibrium.

(3) Consider the case $p < \underline{P}$. Then, by Lemma 2 and Lemma 4, if $p - \kappa > K$, $T_f(p) = T_L(p)$, and otherwise, $T_f(p) = -\infty$.

Suppose that $p - \kappa > K$. Then, by the definition of $T_L(p)$,

$$c_H = (\varsigma - w[P_H(T_L(p)) - p]) \cdot \xi_H\big(T_f(p)\big)$$

$$c_L = \varsigma \cdot \xi_L(T_L(p)) > 0.$$

A similar way to case (2) shows the existence of an informative equilibrium.

If $p - \kappa \leq K$, as $T_f(p) = -\infty$. Then, the equilibrium becomes uninformative. ■

A.2. Proof of Proposition 2

Before we prove Proposition 2, we provide the following lemma, which immediately follows from the definition of $\overline{P}$.

Lemma 5. *$\mu > \overline{P}$ if and only if $\frac{\mu - \kappa}{2} > K$.*

Now, we proceed to the proof of Proposition 2. Suppose that $\frac{\mu - \kappa}{2} > \max\{K, 1 - \mu - \kappa\}$. This implies that $\mu > \frac{2 - \kappa}{3}$. Note that if $p = \mu$, as $\frac{\mu - \kappa}{2} > K$, $p > \overline{P}$ by Lemma 5. By Proposition 1, the equilibrium is uninformative and guarantees the profit $\frac{\mu - \kappa}{2} - K$.

Below, we examine the existence of an informative equilibrium that yields a profit larger than $\frac{\mu-\kappa}{2} - K$. As we consider an informative equilibrium, by Proposition 1, $p < \mu$. Note that the profit is at most $p - \kappa - K$. As we assume $\frac{\mu-\kappa}{2} > 1 - \mu - \kappa$, if an informative equilibrium yields a greater profit, the price needs to satisfy $p > 1 - \mu$. This implies that

$$\ln\left(\frac{1-p}{p}\frac{1-\mu}{\mu}\right) < 0.$$

Now we consider the limit $\sigma \to \infty$. Note that as $\lambda_0 - \lambda_1 = (2\mu - 1)(F(c_L) - F(c_H)) \in [0, 2\mu - 1] \subset [0, 1]$.[13] This shows that

$$\frac{T_L(p) - \lambda_1}{\sigma} < \frac{\sigma}{2}\ln\left(\frac{1-p}{p}\frac{1-\mu}{\mu}\right) \to -\infty,$$

$$\frac{T_L(p) - \lambda_0}{\sigma} < \frac{1}{2\sigma} + \frac{\sigma}{2}\ln\left(\frac{1-p}{p}\frac{1-\mu}{\mu}\right) \to -\infty.$$

Then, the firm's profit $\Pi(p)$ is at most $\left(\frac{1}{2}\right)(p - \kappa) - K$ at the limit. This is smaller than $\frac{\mu-\kappa}{2} - K$ as $p < \mu$.

Note that the above proof cannot apply when $p \approx \mu$, as we cannot exclude the case where the profit is larger than $\left(\frac{1}{2}\right)(p - \kappa) - K$ for any σ; it converges only at the limit. The following discussion is to verify that the profit is strictly less than $\left(\frac{1}{2}\right)(p - \kappa) - K$ when p is close to μ and σ is large enough.

Claim 2. *Suppose that $\mu > p > \max\left\{\frac{1}{2}, \overline{P}\right\}$. Then, $\Pi(p) < \left[\left(\frac{1}{2}\right)(p - \kappa) - K\right]$ for sufficiently large σ.*

Proof of Claim 2. Suppose that $p > \overline{P}$. In this case, as discussed in the proof of Proposition 1, the firm's cutoff point coincides with the H-type's cutoff point. Then, the profit is written as

[13]Note that the value of σ can be taken independent of w as the value of w affects only λ. Here, λ is bounded by values that are independent of w.

$$\Pi(p) = \Pr(v=1)\Big[\Big(\Phi\Big(\tfrac{T_H(p)-\lambda_1}{\sigma}\Big) - \Phi\Big(\tfrac{T_L(p)-\lambda_1}{\sigma}\Big)\Big)\cdot[\mu\cdot(p-\kappa)-K] + \Phi\Big(\tfrac{T_L(p)-\lambda_1}{\sigma}\Big)\cdot(p-\kappa-K)\Big]$$

$$+\Pr(v=0)\Big[\Big(\Phi\Big(\tfrac{T_H(p)-\lambda_0}{\sigma}\Big) - \Phi\Big(\tfrac{T_L(p)-\lambda_0}{\sigma}\Big)\Big)\cdot[(1-\mu)\cdot(p-\kappa)-K] + \Phi\Big(\tfrac{T_L(p)-\lambda_0}{\sigma}\Big)\cdot(p-\kappa-K)\Big].$$

Let $D(\sigma) = \Pi(p) - \Big[\Big(\frac{1}{2}\Big)(p-\kappa)-K\Big]$, which is calculated as

$$\Pr(v=1)\Big[\Big[\Phi\Big(\tfrac{T_H(p)-\lambda_1}{\sigma}\Big)-1\Big]\cdot[\mu\cdot(p-\kappa)-K] + \Phi\Big(\tfrac{T_L(p)-\lambda_1}{\sigma}\Big)\cdot(1-\mu)\cdot(p-\kappa)\Big]$$

$$+\Pr(v=0)\Big[\Big[\Phi\Big(\tfrac{T_H(p)-\lambda_0}{\sigma}\Big)-1\Big]\cdot[(1-\mu)\cdot(p-\kappa)-K] + \Phi\Big(\tfrac{T_L(p)-\lambda_0}{\sigma}\Big)\cdot\mu\cdot(p-\kappa)\Big].$$

Note that by $p \in (1-\mu,\mu)$ and $\lambda \in (0, 2\mu-1)$,

$$T_H(p)-\lambda_1 = \frac{\lambda}{2} + \frac{\sigma^2}{2\lambda}\Big[\ln\Big(\tfrac{\mu}{1-\mu}\Big) - \ln\Big(\tfrac{p}{1-p}\Big)\Big] > \frac{\sigma^2}{2}\Big[\ln\Big(\tfrac{\mu}{1-\mu}\Big) - \ln\Big(\tfrac{p}{1-p}\Big)\Big],$$

$$T_H(p)-\lambda_0 = -\frac{\lambda}{2} + \frac{\sigma^2}{2\lambda}\Big[\ln\Big(\tfrac{\mu}{1-\mu}\Big) - \ln\Big(\tfrac{p}{1-p}\Big)\Big] > \frac{\sigma^2}{2}\Big[\ln\Big(\tfrac{\mu}{1-\mu}\Big) - \ln\Big(\tfrac{p}{1-p}\Big)\Big] - \frac{1}{2},$$

$$T_L(p)-\lambda_1 = \frac{\lambda}{2} + \frac{\sigma^2}{2\lambda}\Big[\ln\Big(\tfrac{1-\mu}{\mu}\Big) - \ln\Big(\tfrac{p}{1-p}\Big)\Big] < \frac{\sigma^2}{2}\Big[\ln\Big(\tfrac{1-\mu}{\mu}\Big) - \ln\Big(\tfrac{p}{1-p}\Big)\Big] + \frac{1}{2},$$

$$T_L(p)-\lambda_0 = -\frac{\lambda}{2} + \frac{\sigma^2}{2\lambda}\Big[\ln\Big(\tfrac{1-\mu}{\mu}\Big) - \ln\Big(\tfrac{p}{1-p}\Big)\Big] < \frac{\sigma^2}{2}\Big[\ln\Big(\tfrac{1-\mu}{\mu}\Big) - \ln\Big(\tfrac{p}{1-p}\Big)\Big].$$

By these equations, we can verify that as $\sigma \to \infty$, $T_H(p)-\lambda_v \to \infty$ and $T_L(p)-\lambda_v \to -\infty$. Then, $\Phi\Big(\frac{T_H(p)-\lambda_v}{\sigma}\Big)-1 \to 0$ and $\Phi\Big(\frac{T_L(p)-\lambda_v}{\sigma}\Big) \to 0$.[14] Then, $\lim_{\sigma\to\infty} D(\sigma) = 0$. Now we verify the sign of D for sufficiently large σ. To this end, we calculate $D'(\sigma)$, which is

$$D'(\sigma) = \varphi(A_{H1})\cdot\left(\frac{1}{\lambda}B_H - \frac{\lambda}{2\sigma^2}\right)\cdot(\mu(p-\kappa)-K)$$

$$+\varphi(A_{L1})\cdot\left(\frac{1}{\lambda}B_L - \frac{\lambda}{2\sigma^2}\right)\cdot(1-\mu)\cdot(p-\kappa)$$

$$+\varphi(A_{H0})\cdot\left(\frac{1}{\lambda}B_H + \frac{\lambda}{2\sigma^2}\right)\cdot((1-\mu)\cdot(p-\kappa)-K)$$

$$+\varphi(A_{L0})\cdot\left(\frac{1}{\lambda}B_L + \frac{\lambda}{2\sigma^2}\right)\cdot\mu\cdot(p-\kappa),$$

[14]Note that $\mu > \frac{1}{2}$ and $p > \frac{1}{2}$ implies that $\Big|\ln\Big(\frac{1-\mu}{\mu}\Big) - \ln\Big(\frac{p}{1-p}\Big)\Big| > \Big|\ln\Big(\frac{\mu}{1-\mu}\Big) - \ln\Big(\frac{p}{1-p}\Big)\Big|$. This also implies that $\Phi\Big(\frac{T_H(p)-\lambda_v}{\sigma}\Big)$ converges more slowly than $\Phi\Big(\frac{T_L(p)-\lambda_v}{\sigma}\Big)$. This intuition works in the following analysis.

where for each $v \in \{0,1\}$ and $\theta \in \{H,L\}$

$$A_{\theta v} = \frac{T_\theta(p) - \lambda_v}{\sigma}, \quad B_\theta = \ln\left(\frac{\theta}{1-\theta}\right) - \ln\left(\frac{p}{1-p}\right),$$

and $B_H > 0 > B_L$ as $\mu > p > \frac{1}{2} > 1-\mu$.

Note that

$$\frac{\varphi(A_{H1})}{\varphi(A_{L0})} = \exp\left(\left[\frac{\lambda}{\sigma} - \frac{\sigma}{\lambda}\ln\left(\frac{1-\mu}{\mu}\right)\right]\sigma \ln\left(\frac{p}{1-p}\right)\right) > \exp\left(-\sigma^2 \ln\left(\frac{1-\mu}{\mu}\right)\ln\left(\frac{p}{1-p}\right)\right),$$

$$\frac{\varphi(A_{H0})}{\varphi(A_{L0})} = \exp\left(-\frac{\sigma}{\lambda}\ln\left(\frac{1-\mu}{\mu}\right)\left[\frac{\lambda}{\sigma} + \sigma \ln\left(\frac{p}{1-p}\right)\right]\right) > \exp\left(-\sigma^2 \ln\left(\frac{1-\mu}{\mu}\right)\ln\left(\frac{p}{1-p}\right)\right),$$

$$\frac{\varphi(A_{L1})}{\varphi(A_{L0})} = \exp\left(\ln\left(\frac{p}{1-p}\right) - \ln\left(\frac{1-\mu}{\mu}\right)\right).$$

Then, as $\mu > \frac{1}{2}$ and $p > \frac{1}{2}$, $\frac{\varphi(A_{H1})}{\varphi(A_{L0})} \to \infty$, and $\frac{\varphi(A_{H0})}{\varphi(A_{L0})} \to \infty$ as $\sigma \to \infty$. Further, $\frac{\varphi(A_{H0})}{\varphi(A_{H1})} \to 1$ as $\sigma \to \infty$. Note also that $(\mu(p-\kappa) - K) + ((1-\mu)(p-\kappa) - K) = p - \kappa - 2K > 0$. Using these facts, dividing $D'(\sigma)$ by $\frac{\varphi(A_{L0})}{\lambda}$ is positive for sufficiently large σ. As $\lim_{\sigma\to\infty} D(\sigma) = 0$ and $D'(\sigma) > 0$, $D(\sigma) < 0$ for sufficiently large σ. Therefore, $\Pi(p) < \left[\frac{1}{2}(p-\kappa) - K\right]$ for sufficiently large σ. ■

Then, we conclude that for each $p < \mu$, the profit is at most $\frac{1}{2}(p-\kappa) - K$ when σ is sufficiently large. Therefore, $p = \mu$ is the optimal price, and the equilibrium is uninformative. ■

A.3. Proof of Proposition 3

The proof consists of the following two claims, which hold under the assumption of Proposition 3.

Claim 3. *If the price induces an uninformative equilibrium, the expected profit is at most* 0.

Claim 4. *By setting $p = 1-\mu$, the equilibrium is informative. The expected profit is no less than $(1-\mu-\kappa-K)\frac{1}{2}$ in this case.*

Proof of Claim 3. By Proposition 1, the situation is either $p > \min\{\mu, \overline{P}\}$ or $p < \kappa + K$. If $p > \mu$, as $P_H(T) = \mu < p$ at the uninformative equilibrium, the profit is 0 as the H-type never purchases. If there is an uninformative equilibrium with $p \in \left(\overline{P}, \mu\right)$, as $P_H(T) = \mu > p > 1-\mu$, the profit is $\frac{\mu-\kappa}{2} - K < 0$ as only the H-type purchases.

At the uninformative equilibrium with $p < \kappa + K$, the profit is also 0 as the firm gives up selling. ∎

Proof of Claim 4. First, consider the case where $1-\mu \leq \underline{P}$. As $p = 1-\mu > K+\kappa$, $1-\mu < \mu$ and $1-\mu < \underline{P} < \overline{P}$, an equilibrium exists and it is necessarily informative by Proposition 1.

Note that $p = 1-\mu \leq \underline{P}$ implies $T = T_L(p)$. In this case, all consumers buy the product if $t < T$, and otherwise, never. Further, if $p = 1-\mu$, $T_L(p) = \frac{\lambda_1+\lambda_0}{\sigma}$. Then, the expected profit is

$$\left[\frac{1}{2}\Phi(x) + \frac{1}{2}\Phi(-x)\right] \cdot (1-\mu-\kappa-K) = \frac{1-\mu-\kappa-K}{2},$$

where $x = \frac{\lambda_0-\lambda_1}{2\sigma}$.

Next, consider the case that $1-\mu > \underline{P}$. In this case, at $t = T$, only H-type purchases the product.

By assumption, $K > \frac{\mu-\kappa}{2}$ implies that $\overline{P} > \mu > 1-\mu$, and as $1-\mu > K+\kappa$, Proposition 1 implies that an equilibrium exists and it is necessarily informative at $p = 1-\mu$. Further, the expected profit is

$$\begin{aligned} &\frac{1-\mu-\kappa}{2} \times \left[\mu \cdot \Phi\left(x+z'\right) + (1-\mu) \cdot \Phi(x) + (1-\mu) \cdot \Phi\left(-x+z'\right) + \mu \cdot \Phi(-x)\right] \\ &\qquad -\left[\mu \cdot \Phi\left(x+z'\right) + (1-\mu) \cdot \Phi\left(-x+z'\right)\right]K, \end{aligned} \tag{1}$$

where $z' = \frac{\sigma}{4x} \cdot \ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right)$.

As $T > T_L(p) = \frac{\lambda_0+\lambda_1}{2}$, the expected profit (1) is greater than $\frac{1-\mu-\kappa-K}{2} > 0$. ∎

Proof of Proposition 3. By Claims 3 and 4, and assumption $1-\mu-\kappa > K$, a price that leads to an informative equilibrium yields a positive profit, which is larger than the profit of any uninformative equilibrium. Then, the optimal price leads to an informative equilibrium. ■

A.4. Proofs in Section 5

Proof of Lemma 3. First note that for sufficiently small t, $\Pi_L^*(t) > \Pi_H^*(t)$. This is because for sufficiently small $t \approx -\infty$, $P_\theta(t) \approx 1$ and therefore $\Pi_L^*(t) \approx 1 > \mu \approx \Pi_H^*(t)$.

Let $a = \exp\left(\frac{\lambda_0 - \lambda_1}{\sigma^2} \cdot \left(t - \frac{\lambda_0+\lambda_1}{2}\right)\right)$. Then, $P_\theta(t)$ is written as

$$P_\theta(t) = \frac{\varphi\left(\frac{t-\lambda_1}{\sigma}\right) \cdot \theta}{\varphi\left(\frac{t-\lambda_1}{\sigma}\right) \cdot \theta + \varphi\left(\frac{t-\lambda_0}{\sigma}\right) \cdot (1-\theta)} = \frac{1}{1 + \frac{1-\theta}{\theta} \cdot a},$$

and also

$$\Pr(v = 1 \,|\, t) = \frac{1}{1+a}.$$

To simplify the notation, let denote $P(t) = \Pr(v = 1 \,|\, t)$. By differentiation, if $\lambda_0 > \lambda_1$, we have

$$P'_\theta(t) = -\frac{\lambda_0 - \lambda_1}{\sigma^2} \cdot \frac{1-\theta}{\theta} \cdot a \cdot (P_\theta(t))^2 < 0.$$

Note that we can write $P_H(t) = \frac{1}{1+\frac{1-\mu}{\mu}a}$ and $P_L(t) = \frac{1}{1+\frac{\mu}{1-\mu}a}$. This implies that $\frac{P_H(t)}{P_L(t)}$ and $\frac{P(t)}{P_L(t)}$ are increasing in t.

Now we calculate the ratio of the profits:

$$\begin{aligned} \frac{\Pi_H^*(t)}{\Pi_L^*(t)} &= (P(t)(2\mu - 1) + (1-\mu))\frac{P_H(t)}{P_L(t)} > 1 \Longleftrightarrow Z(a) > 0, \\ Z(a) &:= (1-\mu)\mu^2 a^2 - (2\mu-1)(1-(1-\mu)\mu)a - \mu(1-\mu)^2 > 0. \end{aligned} \tag{2}$$

Note that $Z(a)$ is a convex quadratic function of a, and $Z(0) < 0$. Also note that as $P_L(t) > K$, $a < \left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu}$. A calculation shows that $Z\left(\left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu}\right)$ is decreasing in μ, and let μ^{**} be the solution that $Z\left(\left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu}\right) = 0$. This implies that if $\mu < \mu^{**}$, $\Pi_H^*(t) < \Pi_L^*(t)$ for each t. On the contrary, if $\mu > \mu^{**}$, there is $\hat{t} < T_L(K)$ such that $\Pi_H^*(t) < \Pi_L^*(t)$ if and only if $t < \hat{t}$.

(c) As $T_L(K)$ is the solution of $P_L(t) = K$, That is,

$$T_L(K) = \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{\lambda_0 - \lambda_1} \cdot \ln\Big(\big(\tfrac{1}{K} - 1\big) \cdot \tfrac{1-\mu}{\mu}\Big).$$

Let μ^* be the solution to $\big(\frac{1}{K} - 1\big) \cdot \frac{1-\mu}{\mu} = 1$. This exists if $K < \frac{1}{2}$. Then, if $\mu < \mu^*$, $T_L(K) \to \infty$. In contrast, if $\mu > \mu^*$, $T_L(K) \to -\infty$. We can also verify that $Z\Big(\big(\frac{1}{K} - 1\big) \cdot \frac{1-\mu^*}{\mu^*}\Big) < 0$, that is, $Z(1) < 0$. This implies that $\mu^* < \mu^{**}$. ■

Proof of Proposition 4. Let $a = \exp\Big(\frac{\lambda_0 - \lambda_1}{\sigma^2} \cdot \big[t - \frac{\lambda_1 + \lambda_0}{2}\big]\Big)$. Note that

$$P_\theta = \frac{1}{1 + \frac{1-\theta}{\theta} a}$$

$$P_H(t) - P_L(t) = \frac{\frac{\mu}{1-\mu} - \frac{1-\mu}{\mu}}{\Big(1 + \frac{\mu}{1-\mu} a\Big) \cdot \Big(1 + \frac{1-\mu}{\mu} a\Big)} a,$$

$$-P'_\theta(t) = \frac{\lambda_0 - \lambda_1}{\sigma^2} \frac{1-\theta}{\theta} \cdot a \cdot (P_\theta(t))^2 > 0.$$

$$\begin{aligned}
\xi_L(t) - \xi_H(t) &= \frac{2\mu - 1}{\sigma}\Big[\varphi\Big(\tfrac{t-\lambda_0}{\sigma}\Big) - \varphi\Big(\tfrac{t-\lambda_1}{\sigma}\Big)\Big] \\
&= \frac{2\mu - 1}{\sigma} \cdot \varphi\Big(\tfrac{t-\lambda_0}{\sigma}\Big) \cdot \Big[1 - \exp\Big(-\tfrac{\lambda_0 - \lambda_1}{\sigma} \cdot \Big(t - \tfrac{\lambda_0 + \lambda_1}{2}\Big)\Big)\Big]
\end{aligned}$$

$$\xi_H(t) = \frac{1}{\sigma}\Big[\mu \cdot \varphi\Big(\tfrac{t-\lambda_1}{\sigma}\Big) + (1-\mu) \cdot \varphi\Big(\tfrac{t-\lambda_0}{\sigma}\Big)\Big].$$

As shown in the proof of Lemma 3, $\Pi^*_H(t) = \Pi^*_L(t)$ if and only if $Z(a) = 0$. (Z is defined in (2)). Therefore, when we write $\hat{a} = \exp\Big(\frac{\lambda_0 - \lambda_1}{\sigma^2} \cdot \big(\hat{t} - \frac{\lambda_1 + \lambda_0}{2}\big)\Big)$, $Z(\hat{a}) = 0$. Note also that the effect of t on Z is summarized in a. Then, even $\sigma \to \infty$, $\hat{a}$ is a finite value, which implies that $\hat{t} \in \Theta(\sigma^2)$.[15] Further,

$$\frac{\xi_H(t)}{\xi_H(\hat{t})} = \frac{\mu \cdot \exp\Big((\hat{t} - t) \cdot \frac{\hat{t} + t - 2\lambda_1}{2\sigma^2}\Big) + (1-\mu) \cdot \exp\Big((\hat{t} - t + \lambda_0 - \lambda_1) \cdot \frac{\hat{t} + t - (\lambda_0 + \lambda_1)}{2\sigma^2}\Big)}{\mu + (1-\mu) \cdot \exp\Big(\frac{1}{\sigma^2} \cdot (\lambda_0 - \lambda_1) \cdot \big(\hat{t} - \frac{\lambda_0 + \lambda_1}{2}\big)\Big)}.$$

[15] Θ is Landau symbol; that is $g(t) \in \Theta(f(t))$ implies that $\lim_{t \to \infty} \frac{g(t)}{f(t)} \in \mathbb{R} \setminus \{0\}$.

As $\frac{\hat{t}}{\sigma^2}$ converges to a real number, for each t with $|t| < |\hat{t}| + z$ for some constant z, $\frac{\xi_H(t)}{\xi_H(\hat{t})} \to \infty$. In contrast, if $t < \hat{t}$, $\frac{\xi_H(t)}{\xi_H(\hat{t})} \to 0$ as $\hat{t} \to -\infty$. By Lemma 3, $\hat{t} \to -\infty$ as $\sigma \to \infty$. Then, dividing D by $\xi_H(\hat{t})$ yields that

$$\int^{\hat{t}} [-P'_L(t)] \frac{\xi_L(t) - \xi_H(t)}{\xi_H(\hat{t})} \, dt - \int_{\hat{t}}^{T^*} [-P'_H(t)] \frac{\xi_H(t)}{\xi_H(\hat{t})} \, dt + [P_H(\hat{t}) - P_L(\hat{t})]. \qquad (3)$$

Note also that $\xi_L(t) < \xi_H(t)$ for sufficiently small t when $\lambda_0 > \lambda_1$. Also note that by $\frac{\mu}{2} > K$, $T^* \to \infty$ as $\sigma \to \infty$. Therefore, as $\sigma \to \infty$, the value of (3) diverges to $-\infty$. This shows that $D < 0$. ■

Proof of Proposition 5. Dividing $\tilde{D}$ by $\xi_H(T_*)$ yields that

$$\frac{\int^{T_*} [-P'_L(t)] (\xi_L(t) - \xi_H(t)) \, dt}{\xi_H(T_*)} + [P_H(T_*) - P_L(T_*)] \qquad (4)$$

As $\mu > \mu^*$, $\lim_{\sigma \to \infty} T_* = -\infty$ by Lemma 3. Therefore, the first term of (4) converges to

$$-P'_L(T_*) \frac{\xi_L(T_*) - \xi_H(T_*)}{\xi'_H(T_*)}.$$

Note that

$$\frac{\xi_L(T_*) - \xi_H(T_*)}{\xi'_H(T_*)} = \frac{-(2\mu - 1)\left(\exp\left(-\frac{1}{2}\left(\frac{T_* - \lambda_0}{\sigma}\right)^2\right) - \exp\left(-\frac{1}{2}\left(\frac{T_* - \lambda_1}{\sigma}\right)^2\right)\right)}{\mu \frac{T_* - \lambda_1}{\sigma^2} \exp\left(-\frac{1}{2}\left(\frac{T_* - \lambda_1}{\sigma}\right)^2\right) + (1 - \mu) \frac{T_* - \lambda_0}{\sigma^2} \exp\left(-\frac{1}{2}\left(\frac{T_* - \lambda_0}{\sigma}\right)^2\right)}$$

$$= \frac{(2\mu - 1)\left(1 - \exp\left(\frac{\lambda_0 - \lambda_1}{\sigma^2}\left(T_* - \frac{\lambda_1 + \lambda_0}{2}\right)\right)\right)}{\mu \frac{T_* - \lambda_1}{\sigma^2} + (1 - \mu) \frac{T_* - \lambda_0}{\sigma^2} \exp\left(\frac{\lambda_0 - \lambda_1}{\sigma^2}\left(T_* - \frac{\lambda_1 + \lambda_0}{2}\right)\right)}$$

As shown in Lemma 3, $\frac{T_*}{\sigma^2}$ converges to a finite value. Then, the above value converges to a finite value.

Note also that

$$-P_L'(t) = \frac{\lambda_0 - \lambda_1}{\sigma^2} \frac{\mu}{1-\mu} \times \frac{1}{\frac{1}{a} + 2\frac{\mu}{1-\mu} + \left(\frac{\mu}{1-\mu}\right)^2 a} > 0.$$

$$a = \exp\left(\frac{\lambda_0 - \lambda_1}{\sigma^2} \cdot \left(t - \frac{\lambda_1 + \lambda_0}{2}\right)\right)$$

As $\frac{T_*}{\sigma^2}$ converges to a finite value, a is also a finite value. Then, $-P_L'(T_*) \to 0$. Then, (4) reduced to $[P_H(T_*) - P_L(T_*)]$, which is calculated as

$$P_H(t) - P_L(t) = \frac{\frac{\mu}{1-\mu} - \frac{1-\mu}{\mu}}{\left(1 + \frac{\mu}{1-\mu}a\right)\left(1 + \frac{1-\mu}{\mu}a\right)} a,$$

As a is a finite value, $\lim_{\sigma^2 \to \infty} P_H(T_*) - P_L(T_*) > 0$. This concludes that $\tilde{D} > 0$. ■

B. Informative protest

This section considers the case where σ is small enough. First, the following proposition shows a sufficient condition for the existence of an informative equilibrium when σ is small enough.

Proposition 6. *For sufficiently small σ, an informative equilibrium exists if $p \in \left(\underline{P}, \max\left\{\mu, \overline{P}\right\}\right)$.*

Consider an informative equilibrium. By Proposition 1 (1), $p < \max\left\{\overline{P}, \mu\right\}$. Suppose that $\lim_{\sigma \to 0} \frac{\lambda}{\sigma} = \gamma < \infty$. Then, note that for each $T \in \left\{T_H, T_L, T_f\right\}$,

$$\frac{T - \lambda_1}{\sigma} = \frac{\lambda}{\sigma} + \frac{\sigma}{\lambda} \frac{X_T}{2},$$

$$\frac{T - \lambda_0}{\sigma} = -\frac{\lambda}{\sigma} + \frac{\sigma}{\lambda} \frac{X_T}{2},$$

for some X_T, which is independent of σ and λ. This shows that T is a finite value. Then, $c_L > c_H$ and thus $\lambda > 0$ in the limit. However, this also implies $\frac{\lambda}{\sigma} \to \infty$, which is a contradiction. Therefore, $\lim_{\sigma \to 0} \frac{\lambda}{\sigma} = \infty$.

This implies that $\Phi\left(\frac{T(p)-\lambda_1}{\sigma}\right) \to 1$, $\Phi\left(\frac{T_L(p)-\lambda_1}{\sigma}\right) \to 1$, $\Phi\left(\frac{T_f(p)-\lambda_0}{\sigma}\right) \to 0$, and $\Phi\left(\frac{T_L(p)-\lambda_0}{\sigma}\right) \to 1$. In this case, the profit of informative equilibrium converges to $\frac{p-\kappa-K}{2}$.

Note that the profits of uninformative equilibria are at most $\max\left\{\frac{\mu-\kappa}{2} - K, 0\right\}$. In contrast, the supremum of the profit of informative equilibrium is at least $\frac{\max\{\overline{P},\mu\}-\kappa-K}{2}$. Therefore, the optimal price is at least $\max\{\overline{P}, \mu\} - \varepsilon$ for small ε.

Note that when the protest is absent, the optimal price is $p = 1 - \mu$ when μ is small enough. We have the following observation.

Proposition 7. *Suppose that $1 - \mu - \kappa > \max\left\{0, \frac{\mu-\kappa}{2}\right\}$. Then, for sufficiently small σ, the price increases compared with the case when protest is absent.*

B.1. Proof of Proposition 6

We have two cases.

Case 1. Consider the case that $p \geq \overline{P}$, but $p < \mu$. As in the proof of Proposition 1,

$$c_L - c_H = \varsigma \cdot \frac{2\mu - 1}{\sigma} \cdot \left[\varphi\left(\frac{T_H(p)-\lambda_0}{\sigma}\right) - \varphi\left(\frac{T_H(p)-\lambda_1}{\sigma}\right)\right].$$

Let $z = \frac{\lambda}{\sigma}$. Then, for sufficiently small σ,

$$\frac{T_H(p) - \lambda_0}{\sigma} = -\frac{z}{2} + \frac{1}{2}\frac{1}{z} \cdot \ln\left(\frac{\mu}{1-\mu} - \frac{p}{1-\mu}\right)$$

$$\frac{T_H(p) - \lambda_1}{\sigma} = \frac{z}{2} + \frac{1}{2}\frac{1}{z} \cdot \ln\left(\frac{\mu}{1-\mu} - \frac{p}{1-\mu}\right).$$

Note that $p < \mu$, $\ln\left(\frac{\mu}{1-\mu} - \frac{p}{1-\mu}\right) > 0$, and then, by symmetry of φ, $\varphi\left(\frac{T_H(p)-\lambda_0}{\sigma}\right) > \varphi\left(\frac{T_H(p)-\lambda_1}{\sigma}\right)$.

Let $A = \ln\left(\frac{\mu}{1-\mu} - \frac{p}{1-\mu}\right)$. Then, we can write

$$B = \varphi\left(\frac{T_H(p)-\lambda_0}{\sigma}\right) - \varphi\left(\frac{T_H(p)-\lambda_1}{\sigma}\right) = \exp\left(-\frac{z^2}{8} - \frac{1}{z^2}\frac{A^2}{8}\right) \cdot \left(\exp\left(\frac{A}{4}\right) - \exp\left(-\frac{A}{4}\right)\right).$$

Now consider the following limit. Consider $\lambda = z\sigma$ with fixed z.

$$\lim_{z\to 0}\frac{(2\mu-1)(F(c_L)-F(c_H))}{\lambda} = \frac{1}{\sigma}\frac{\mathrm{d}(2\mu-1)(F(c_L)-F(c_H))}{\mathrm{d}z}$$

$$= \frac{1}{z}(2\mu-1)\cdot f(0)\cdot\varsigma\cdot\frac{2\mu-1}{\sigma^2}\cdot\frac{1}{4}\cdot\left(-z+\frac{A^2}{z}\right)\cdot B.$$

By considering $\sigma \propto (B)^{\frac{1}{2}}$, we can show that $\sigma\to 0$, $\left[\varphi\left(\frac{T_H(p)-\lambda_0}{\sigma}\right)-\varphi\left(\frac{T_H(p)-\lambda_1}{\sigma}\right)\right]\to 0$, and $\lim_{\sigma\to 0}\frac{(2\mu-1)\cdot(F(c_L)-F(c_H))}{\lambda}\to\infty$. Then, for such λ, $\frac{(2\mu-1)\cdot(F(c_L)-F(c_H))}{\lambda}>1$. In contrast, when $\lambda=2\mu-1$, $(2\mu-1)\cdot(F(c_L)-F(c_H))<2\mu-1$. Then, the intermediate value theorem implies the existence of λ such that $(2\mu-1)\cdot(F(c_L)-F(c_H))=\lambda>0$. This shows the existence of an informative equilibrium for small σ.

Case 2. Consider the case that $p\in\left(\underline{P},\overline{P}\right)$. As in the proof of Proposition 1,

$$c_L-c_H=\varsigma\left(\xi_L\left(T_f(p)\right)-\xi_H\left(T_f(p)\right)\right)+w\left[P_H\left(T_f(p)\right)-p\right]\xi_H\left(T_f(p)\right),$$

and

$$\varsigma\left(\xi_L\left(T_f(p)\right)-\xi_H\left(T_f(p)\right)\right)=\varsigma\cdot\frac{2\mu-1}{\sigma}\cdot\left[\varphi\left(\frac{T_H(p)-\lambda_0}{\sigma}\right)-\varphi\left(\frac{T_H(p)-\lambda_1}{\sigma}\right)\right].$$

As $w\left[P_H\left(T_f(p)\right)-p\right]\xi_H\left(T_f(p)\right)>0$ by $p<\overline{P}$, a similar discussion of case 1 proves the existence of an informative equilibrium. ■

References

Ali, S. N., & Lin, C. (2013). Why People Vote: Ethical Motives and Social Incentives. *American Economic Journal: Microeconomics*, *5*(2), 73–98.

Baron, D. P. (2001a). Private Politics, Corporate Social Responsibility, and Integrated Strategy. *Journal of Economics & Management Strategy*, *10*(1), 7–45.

Baron, D. P. (2003b). Private politics. *Journal of Economics & Management Strategy*, *12*(1), 31–66.

Battaglini, M. (2017). Public protests and policy making. *The Quarterly Journal of Economics*, *132*(1), 485–549.

Correa, S. (2024). Persistent protests. *American Economic Journal: Microeconomics*.

Delacote, P. (2009). On the sources of consumer boycotts ineffectiveness. *The Journal of Environment & Development*, *18*(3), 306–322.

Diermeier, D., & Van Mieghem, J. A. (2008). Voting with your pocketbook—a stochastic model of consumer boycotts. *Mathematical and Computer Modelling*, *48*(9–10), 1497–1509.

Egorov, G., & Harstad, B. (2017). Private politics and public regulation. *The Review of Economic Studies*, *84*(4), 1652–1682.

Ekmekci, M., & Lauermann, S. (2022). Informal elections with dispersed information. *Working paper: University of Bonn and University of Mannheim*.

Feddersen, T. J., & Gilligan, T. W. (2001). Saints and Markets: Activists and the Supply of Credence Goods. *Journal of Economics & Management Strategy*, *10*(1), 149–171.

Feddersen, T., & Sandroni, A. (2006). A Theory of Participation in Elections. *American Economic Review*, *96*(4), 1271–1282.

Glazer, A., Kanniainen, V., & Poutvaara, P. (2010). Firms' ethics, consumer boycotts, and signalling. *European Journal of Political Economy*, *26*(3), 340–350.

Hendel, I., Lach, S., & Spiegel, Y. (2017). Consumers' activism: the cottage cheese boycott. *The RAND Journal of Economics*, *48*(4), 972–1003.

Innes, R. (2006). A theory of consumer boycotts under symmetric information and imperfect competition. *The Economic Journal*, *116*(511), 355–381.

Liaukonytė, J., Tuchman, A., & Zhu, X. (2023). Frontiers: Spilling the Beans on Political Consumerism: Do Social Media Boycotts and Buycotts Translate to Real Sales Impact?. *Marketing Science*, *42*(1), 11–25.

Lohmann, S. (1993). A Signaling Model of Informative and Manipulative Political Action. *The American Political Science Review*, *87*(2), 319–333.

Lohmann, S. (1994). Information Aggregation Through Costly Political Action. *The American Economic Review*, *84*(3), 518–530.

Miyagiwa, K. (2009). Saving dolphins: Boycotts, trade sanctions, and unobservable technology. *International Economic Review*, *50*(3), 883–902.

Peck, J. (2017). Temporary boycotts as self-fulfilling disruptions of markets. *Journal of Economic Theory*, *169*, 1–12.