Informed Consumers Undermine Product Protests*

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Abstract

We model a protest against a firm aiming to remove a product that causes negative externalities. Both the firm and consumers are uncertain about the product's value, but consumers receive noisy signals. Price plays a key role in aggregating information. When prices are high, consumers with both good and bad signals derive almost the same utility from the product being sold, making protests uninformative. By endogenizing the price, we show that as consumer signals improve, protests become less informative, reducing social welfare. This suggests that consumer ignorance may play a role in protest success.

Keywords: Protest, boycotts, information aggregation, ethical voters, monopoly pricing

JEL Classification: D42, D72, D81, D82

1. Introduction

Firms often face protest campaigns calling for the removal of their products from the market due to concerns over negative externalities[1] (Egorov and Harstad, 2017).[2]

Although protest campaigns against firms' products are common, their effectiveness in influencing firms' decisions remains uncertain. Firms are not required to stop selling their products, even if protests attract large participation, since protesters are not nec-

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[1]Negative externalities include pollution, workplace conditions (e.g., the use of child labor and poor working conditions), and political statements (e.g., fostering discrimination against minorities).

[2]There are many protest activities aiming for product discontinuation, e.g., protests against Nike for labor violations (e.g., "Protesters call on NYU Bookstore to cut ties with Nike," *Washington Square News*, October 13, 2023), a threat against the publication of a book ("Japan firm nixes translation of U.S. book questioning trans surgery," *Japan Times*, December 6, 2023), and so on.

essarily their customers. Despite this, protests still impact firms' decisions by signaling consumer preferences. Consumers' participation in protests reflects their valuation of the product: lower participation signals higher perceived value, while higher participation suggests negative externalities outweigh its worth.

Thus, protests serve as an **information transmission mechanism**, revealing whether consumers perceive the product's value to be higher or lower than its externalities. However, the informativeness of protests is affected by firms' pricing strategies. Our results establish that higher prices significantly reduce the informativeness of protests, with the effect becoming more pronounced when consumers have precise information.

These findings suggest a need for price regulation to sustain protests as an effective information transmission mechanism.

In our model, a continuum of common-interest consumers decides whether to participate in protest campaigns to remove a firm's product with negative externalities. The firm and consumers observe public signals about the protest's turnout, which is the sum of the actual turnout and noise following a normal distribution. Observing the public signal, the firm decides whether to continue the sale. The firm's objective is to maximize its profit, deciding whether to discontinue the product based on the protest turnout. We assume the firm is uncertain about the product's value for consumers. The value includes the guilty feelings that purchasing a product causing negative externalities might induce. Further, continuing product sales is costly. Thus, if the protest is sufficiently informative and indicates the product is not popular, the firm discontinues the sales. Consumers are also uncertain about the product's value but receive noisy (and binary) signals about it.[3]

[3]We can interpret this as a model of endogenized social norms. That is, each consumer cares about the norm if a majority of consumers care about purchasing a product that causes negative externalities; otherwise, they do not. As a similar setting, for example, Fischer and Huddart (2008) consider players caring about average norms.

The protest aggregates consumer signals. Once the firm decides to sell, consumers choose to purchase. This affects protest value, as consumers hesitate to join if the purchase offers significant utility. Thus, product price is crucial in their decision to participate. We assume the firm sets a price before the protest campaign.

When the price is given, we show that the protest can be informative when the price is intermediate. Otherwise, the protest is no longer informative. The intuition is as follows: if the price is too low, the firm gives up continuing the product sale regardless of the protest turnout. Conversely, with a high price, consumers gain less value from purchasing the product if they receive a good signal, while consumers receiving a bad signal do not buy it. Thus, consumers' benefits from discontinuing product sales become similar regardless of whether the signals are good or bad. Consequently, the protest turnout becomes less informative about the consumers' received signals.

The above finding has implications for the firm's pricing strategy. Suppose the consumers' signals are sufficiently precise. In that case, the firm prefers to set a high price so that only consumers receiving good signals will purchase the product. However, this makes the protest uninformative. In summary, consumers' highly precise information makes the protest campaign uninformative. Conversely, suppose the consumers' signals are less accurate. In that case, even consumers receiving good signals are less willing to pay. Thus, the firm prefers to set a price so that both types (i.e., those receiving good and bad signals) of consumers purchase. In this case, consumers receiving good signals obtain an information rent, while those with bad signals do not. This discriminates the motivations for protest participation between consumers receiving good and bad signals, making the protest informative.

This impacts social welfare. Our analysis shows that increased protest informativeness enhances social welfare. Therefore, more precise consumer signals may reduce social welfare.

To underscore the novelty of our findings, we note that the informativeness of individual signals and that of the protest may have negative correlations, a departure from previous studies. For instance, while the model in Ekmekci and Lauermann (2022) shares similarities with ours, it focuses on a protest against a government where the price is not a variable. In their case, they demonstrate that the protest becomes more informative as individual signals become more precise.

One may consider that our results depend on the decision flow, in which the firm commits to a price *before* the consumers decide on participation. Hence, we also analyze the situation when the firm decides the price *after* the protest turnout is observed. The difference arises in the motivation of participation in the protest. If the firm believes that the protest is informative, much participation in a protest signals the unpopularity of the product to the firm, which leads to a price reduction,[4] which benefits the consumers who plan to buy the product. As consumers with a high willingness to pay are more likely to purchase the product, this incentive makes consumers receiving good signals participate in the protest more than those who receive bad signals. We show that when signal precision is high enough, this incentive overcomes the original incentive of participation, aiming for the discontinuation of the product. As a result, consumers receiving good signals participate more than those who receive bad signals. This, in turn, makes the protest uninformative. In contrast, when signal precision is low, the incentive of participation, aiming for the price reduction, is weak, so an informative equilibrium exists. This is because if signal precision is sufficiently high, the firm is more likely to target the good-signal receivers as customers, which motivates the price-reduction incentive more for them.

[4]Hendel et al. (2017) empirically demonstrated that price declines after a consumer boycott, organized in Israel on Facebook.

Therefore, our conclusion with the commit price case continues to hold for the uncommitted price case: With higher signal precision, the protest is more likely to be uninformative. Conversely, with intermediate signal precision, it becomes informative.

In conclusion, our result suggests that consumers' ignorance contributes to successful protest campaigns against a firm's product. Our analysis has a policy implication on price regulation: uninformative protest comes with the incentive of the firm to set a high price. Therefore, regulating high prices may improve the informativeness of product protests and social welfare.

The remainder of this study is organized as follows. The following subsection summarizes the related study. Section 2 explains our model. Section 3 analyzes the model with the fixed price, and Section 4 endogenizes the firm's optimal price. Section 5 considers the case where the price is decided after a product protest. Section 6 discusses some of our assumptions and draws a conclusion. All omitted proofs are relegated to the appendix.

1.1. Related Literature

Many studies have investigated the consumers' motivation to participate in protests, welfare consequences, and aggregation of dispersed information. Among the studies, our study contributes to the literature on information aggregation when the threshold of the voting outcome is endogenous. Battaglini (2017) studies a model of protest where the government's decision is made after observing protest turnout. The model of Ekmekci and Lauermann (2022) is closer to ours in the sense that voters are a continuum.[5]

Our study also related to studies of boycotts. Some studies (e.g., Diermeier and Van Mieghem, 2008, Delacote, 2009) model a boycott as a discrete public good game with a fixed threshold. In this modeling, as Delacote (2009) discusses, boycott campaigns

[5]Relatedly, Correa (2024) considers a dynamic model of protest with a continuum of people. Battaglini et al. (2020) experiment with a model similar to Battaglini (2017), and show that information sharing among the group enhances information aggregation.

have a weak point: While the participation of consumers who have a higher willingness to pay is more significant to boycott, they have less incentive to participate. In our study, however, this point is significant because the threshold for sales discontinuation is flexible. As a result, the positive correlation of consumers' willingness to pay and reluctance to participation makes the protest informative and signals to the firm that their product is unpopular.

There are many other models of boycotts. For example, Egorov and Harstad (2017) built a model of a boycott as a war of attrition. Baron (2001, 2003) considers an activist launching a boycott and analyzes the effects on strategic CSR behaviors. Several studies consider a market of credence goods and model a boycott as an instrument to make firms behave to benefit consumers. Feddersen and Gilligan (2001) and Innes (2006) consider a market with moral-concerned consumers and an activist. Feddersen and Gilligan (2001) consider a boycott as a signaling by the activist, while Innes (2006) shows that boycotts can arise under symmetric information. Miyagiwa (2009) considers consumers who purchase less as they become more suspicious of firms' bad behavior. Glazer et al. (2010), Heijnen and Made (2012), and Peck (2017) consider boycotts as signaling of their moral concerns by consumers. In our study, protest campaigns can be considered to signal moral concerns by interpreting the product's value as a degree of (not) concern for morality. The main difference is that our study assumes consumers are also uncertain about their moral concerns, and a problem of information aggregation arises.

Finally, our model is related to a study of turnout in a large election. Although it is well known that almost all electorates abstain from voting in costly voting models, Feddersen and Sandroni (2006) provide a model to explain high turnout by considering a group-wise utilitarian electorate. In their model, the electorate, referred to as an *ethical voter* decides whether to abstain from voting to maximize the total utilities of people who support the same candidates. Ali and Lin (2013) show that reputationconcerned electorates behave as ethical voters. Ekmekci and Lauermann (2022) also consider ethical voters in a model of an informal election, including protests. In this study, we also assume that consumers are ethical *consumers* when deciding whether to participate in protests.

2. Model

A firm sells a product to a continuum of consumers with unit demand. The population is normalized to 1. The utility of purchasing the product is $w \cdot v$, where $v \in \{0, 1\}$. We refer to v as *product value* and w as the scale parameter, the role of which is discussed later. Implications of v include a psychological factor, such as feeling guilty about purchasing a product having a negative externality. We assume that v is unknown to all players. For instance, this uncertainty about v captures situations where a negative externality of the product has just been revealed, and people are uncertain about how to respond collectively. The prior probability of $v = 1$ is assumed $\frac{1}{2}$ for notational simplicity.

When the product is sold, it imposes a negative externality on the consumers, leading to disutility $\varsigma > 0$. Consumers can engage in a protest campaign to stop the sale of the product. Participating in the campaign costs c_i for consumer i. c_i is identically and independently distributed according to function F over $\mathbb{R}_+$ among consumers. We assume that f is the density of F, and $f(0) > 0$. In summary, when the product is sold at price $w \cdot p$, the consumer's utility is as follows.[6]

$$u_i = w \cdot (v - p) \cdot \chi(\text{purchase}) - \varsigma - c_i \cdot \chi(\text{participate in protest}).$$

Before participating in the protest campaign, consumers receive a private signal about the product value v. $\theta \in \{\theta_H, \theta_L\}$ denotes the signal. θ_H suggests that $v = 1$ is more likely. We assume that $\Pr(\theta = \theta_H \,|\, v = 1) = \Pr(\theta = \theta_L \,|\, v = 0) = \mu \in \left(\frac{1}{2}, 1\right)$. μ is

[6]For each event E, $\chi(E)$ takes the value 1 if E occurs, and 0 otherwise."

referred to as *signal precision*. The parameter μ captures the extent to which individuals tend to receive signals that reinforce a shared perception of the product's value.

In particular, μ reflects the *endogenous nature of social norms*: if μ is close to 1, individuals are likely to receive signals that align with the prevailing belief about v, reinforcing a strong consensus. Conversely, if μ is closer to $\frac{1}{2}$, there is greater ambiguity, and social norms surrounding the product's value are weaker.

An alternative interpretation is that v is *not merely an intrinsic product characteristic* but rather reflects a prevailing social norm shaped by majority opinion. If a fraction $\mu > \frac{1}{2}$ of individuals receive θ_L, the prevailing norm dictates that the product has no value ($v = 0$). Conversely, if a majority receives θ_H, the product is perceived as valuable ($v = 1$). This feature captures the endogenous nature of product value as determined by collective sentiment rather than objective characteristics.

Given consumers' private signals, they decide whether to participate in the protest campaign. Following the framework of Ekmekci and Lauermann (2022) and Feddersen and Sandroni (2006), we assume that each consumer makes this decision to maximize overall consumer welfare, hence acting as an *ethical* consumer. In other words, a consumer participates if and only if the marginal effect of participation on expected utility exceeds the cost of participation.

One justification for this assumption is that a consumer's participation decision is observed by their social network, leading them to prefer being perceived as ethical. This, in turn, motivates them to act in a way that maximizes social welfare.[7]

However, the adequacy of this justification depends on what participation in the protest entails. On the one hand, if participation involves attending an event organized by activists, the cost may be too high for this motivation to hold. On the other hand,

[7] Ali and Lin (2013) explores this rationale in the context of voting decisions.

if participation means sharing or liking a post on social media that expresses regret or protest against a firm's product, this motivation seems more plausible.[8]

After the protest campaign, the firm and consumers observe a public signal, indicating the protest campaign's scale. Let the actual participation ratio be τ. The firm and consumers observe $t = \tau + \varepsilon$, where ε is a normal noise with 0 means and σ^2 variance. Let Φ be the standard normal distribution function and φ be its density. ε is interpreted as the noise created by activists.[9]

Observing the public signal t, the firm decides whether to discontinue the product. We assume that the firm incurs a cost of $wK > 0$ when continuing product sales. This includes the distribution cost and the reputation cost of selling a product that causes negative externalities. We also assume that the marginal cost of production is $w\kappa \geq 0$. Therefore, when the product is on sale, the firm's profit is

$$w \cdot [(\text{demand at } p) \cdot (p - \kappa) - K].$$

Finally, the decision flow is as follows.

1. The firm set a price.
2. Each consumer observes private signal θ_i and decides whether to participate in the protest.
3. Turnout τ realizes, and the firm and consumers observe public signal $t = \tau + \varepsilon$.
4. The firm decides whether to continue the product sale.
5. If the product is on sale, each consumer decides whether to buy it.

The solution concept is perfect Bayesian Nash equilibrium.

[8]Sharing and liking posts on social media can be enough to compel a firm to discontinue a product. For instance, Adidas withdrew the sale of shoes resembling shackles in response to criticism on Facebook (Hoffberger, 2012, "Adidas cancels shackle shoe after Facebook backlash," Daily Dot (https://www.dailydot.com/unclick/adidas-shackle-shoe-controversy/, accessed August 1, 2024)).

[9]This assumption is the same as Ekmekci and Lauermann (2022).

3. Equilibrium with a given price

3.1. After public signal observation

3.1.1. Consumer's choice

This section considers the condition of turnout when consumers purchase the product after public signal observation. Consider the consumer receiving signal θ. The consumer purchases if the expected product value exceeds the price, that is, $\Pr(v = 1 \mid \theta, t) \geq p$.[10] By abusing notation, we write $P_\theta(t) = \Pr(v = 1 \mid \theta, t)$. The expected value of product value is

$$P_\theta(t) = \frac{\Pr(t \mid v = 1)\theta}{\Pr(t \mid v = 1)\theta + \Pr(t \mid v = 0)(1 - \theta)}.$$

Let λ_v be the true turnout when the realized value is $v \in \{0, 1\}$. Then, as $t = \lambda_v + \varepsilon$ at state v, $P_\theta(t)$ is rewritten as follows (recall that φ is the pdf of the standard normal distribution).

$$P_\theta(t) = \frac{\varphi\left(\frac{t-\lambda_1}{\sigma}\right)\theta}{\varphi\left(\frac{t-\lambda_1}{\sigma}\right)\theta + \varphi\left(\frac{t-\lambda_0}{\sigma}\right)(1 - \theta)}.$$

By using this formula, when we assume $\lambda_0 > \lambda_1$, inequality $P_\theta(t) > p$ is rewritten as

$$t < \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\theta}{1-\theta}\right) - \ln\left(\frac{p}{1-p}\right)}{\lambda_0 - \lambda_1} =: T_\theta(p).$$

In other words, $T_\theta(p)$ is the supremum value of public signal t with which the type-θ consumers purchase the product. The following lemma shows that the type-H consumers are likelier to purchase the product than the type-L consumers.

Lemma 1. *If $\lambda_0 > \lambda_1$, $T_L(p) < T_H(p)$ for any p.*

[10]As this decision is in the last stage of the game, the purchase decision does not affect others' utilities. Thus, each consumer's purchase decision is based only on their utility, although the consumers have social welfare concerns.

3.1.2. The firm's choice

Given the consumer's choice observing public signal t, this section considers the condition when the firm discontinues the product sale. We focus on a cutoff equilibrium; the firm discontinues the product sale if and only if $t > T$. Now we calculate T.

Suppose that $T_L(p) < T < T_H(p)$. Then, at $t = T$, only the H-type consumer purchases the product. In this case, the firm's profit is

$$\Pi_H := w \cdot ([\Pr(v = 1 \mid T) \cdot \mu + \Pr(v = 0 \mid T) \cdot (1 - \mu)] \cdot (p - \kappa) - K).$$

This is because, when $v = 1$, the population is H-type is μ while that is $1 - \mu$ when $v = 0$. As the firm's decision is independent of w, we omit the notation w for the firm's profit hereafter.

Note that $\Pr(v = 1 \mid T) \cdot \mu + \Pr(v = 0 \mid T) \cdot (1 - \mu) \in [1 - \mu, \mu]$. Therefore, if $p - \kappa \geq \frac{K}{1-\mu}$, the expected profit is no less than 0. Then, the firm always keeps the sale. In contrast, if $\frac{K}{\mu} \geq p - \kappa$, the expected profit is no more than 0. Then, the firm discontinues the sale.

Otherwise $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$. In this case, the firm discontinues the sale if and only if

$$T \leq \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)} \right)}{\lambda_0 - \lambda_1} =: T_f(p).$$

The following lemma summarizes the discussion.

Lemma 2. *Let Π_H be the least turnout signal that the firm discontinues the sale when the H-type purchases, but the L-type does not. Then, if $p - \kappa > \frac{K}{1-\mu}$, $\Pi_H > 0$. If $\frac{K}{\mu} > p - \kappa$, $\Pi_H < 0$. If $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$, $\Pi_H \leq 0$ if and only if $t \leq T_f(p)$.*

Although this lemma focuses on only H-type purchases when the firm discontinues the product, the following sections discuss the remaining cases.

3.2. Before public signal observation

Before the public signal observation, the consumer chooses whether to participate in the protest. This section examines the condition and its informativeness regarding product value.

We focus on the cutoff strategy: each type-θ consumer participates in protest if and only if the marginal benefit of participation exceeds its cost. Following Ekmekci and Lauermann (2022), we consider *ethical* consumers: they maximize the aggregate utility of the same type of consumers written in the following formula.[11,12]

$$\int^{T-\mathrm{d}\tau} \Big(w \cdot [P_\theta(t) - p]_+ - \varsigma \Big) \cdot \xi_\theta(t)\,\mathrm{d}t,$$

where we define $[\cdot]_+ = \max\{\cdot, 0\}$ and $\xi_\theta(t) = \frac{1}{\sigma} \cdot \Big(\theta \cdot \varphi\Big(\frac{t-\lambda_1}{\sigma}\Big) + (1-\theta) \cdot \varphi\Big(\frac{t-\lambda_0}{\sigma}\Big)\Big)$. $\mathrm{d}\tau$ represents a marginal increase in the participation rate. In this formula, the firm applies a cutoff criterion: discontinuing the product if and only if $t > T$. A consumer's participation increases turnout by $\mathrm{d}\tau$, which also increases their costs by $c_i\,\mathrm{d}\tau$. Then, consumer i participates in the protest if and only if

$$\underbrace{\Big(\varsigma - w \cdot [P_\theta(T) - p]_+ \Big) \cdot \xi_\theta(T)}_{\text{marginal benefit of participation}} \geq c_i.$$

Now let $c_\theta = \Big(\varsigma - w[P_\theta(T) - p]_+ \Big) \cdot \xi_\theta(T)$ be the cutoff of type-θ consumer. Then, we have that

$$\lambda_1 = \mu \cdot F(c_H) + (1-\mu) \cdot F(c_L),$$

$$\lambda_0 = (1-\mu) \cdot F(c_H) + \mu \cdot F(c_L).$$

Therefore, $\lambda := \lambda_0 - \lambda_1 = (2\mu - 1) \cdot (F(c_L) - F(c_H))$. This value plays a critical role in showing the existence of an informative equilibrium.

[11] Note that assuming this utility does not affect the purchasing decision as it does not affect the other consumers' utility and the firm's decision.

[12] Our results continue to hold even when the consumer maximizes social welfare. See Section 6.

3.3. Equilibrium characterization with given price

This section characterizes the condition for the existence of an informative equilibrium. To this end, we define the equilibrium using the optimal actions discussed in earlier sections. Let denote $\lambda = \lambda_0 - \lambda_1$. Then, as $F(c) \in [0,1]$, $|\lambda| \leq 2\mu - 1$. Now, we define the equilibrium of the game where the price is given.

Definition 1. (T, c_H, c_L) is an *equilibrium* with given price p if

(i) $E[\text{profit} \,|\, t] < 0$ if and only if $t > T$.

(ii) $c_\theta = \left(\varsigma - w \cdot [P_\theta(T) - p]_+\right) \cdot \xi_\theta(T)$.

Note that if $c_H = c_L$, which implies $\lambda_0 = \lambda_1$. Then, the public signal t is independent of v. We call this case *uninformative*.

Definition 2. An equilibrium is *informative* if $c_L > c_H$, and is *uninformative* if $c_L = c_H$.

It's important to note that the uninformative equilibrium is a constant in our model. If the firm sets $T = \infty$ or $T = -\infty$, the protest does not affect whether the product sale continues. This leads to both types of consumers having indifferent incentives. Conversely, if the protest is uninformative, the firm's decision remains unchanged regardless of the turnout. This implies that $T = \infty$ or $T = -\infty$.

Therefore, it is crucial to reiterate that our focus is on when an informative equilibrium exists. Our discussion is guided by the following proposition, which characterizes the condition concerning p for the existence of an informative equilibrium. This proposition is a key tool in our analysis (the proof is relegated to the Appendix).

Proposition 1. *Let* $\overline{P} := \mu \frac{K + (1-\mu)\kappa}{\mu(1-\mu) + (2\mu - 1)K}$. *(a) If* $p \geq \max\{\mu, \overline{P}\}$ *or* $p \leq K + \kappa$, *no informative equilibrium exists. (b) Suppose that* $K + \kappa < p < \overline{P}, f(0) > 0$ *and* $F(0) > 0$. *Then, if* w *is sufficiently large, an informative equilibrium exists.*

Here, $\overline{P}$ is a threshold of p: $T_H(p) > T_f(p)$ if and only if $p < \overline{P}$. $T_H(p) > T = T_f(p)$ implies that at the threshold of the firm, on the condition that the firm gives up to continue the product sale, the H-type still has the incentive to purchase. This makes the H-type reluctant to participate in the protest, which is the source of the difference to the L-type consumer's behavior. In contrast, if $T_f(p) \geq T_H(p)$, the H-type never purchases the product, and the incentives of participating in the protest are indifferent as both H- and L-types consumers do not gain from the product. In this case, the protest is uninformative.

4. Firm's optimal pricing

This section focuses on the most informative equilibrium for each price the firm sets and investigates when informative equilibrium exists by endogenizing the price. All proofs of this section are relegated to the Appendix.

4.1. Benchmark

This section considers the model without protest as a benchmark. In the benchmark case, the H-type's expected product value is $E[v] = \mu$, and that of the L-type consumers is $E[v] = 1 - \mu$. If the firm keeps the product sale, the optimal price is $p = \mu$ or $p = 1 - \mu$. The expected profit when $p = \mu$ is $\frac{\mu-\kappa}{2} - K$, and that when $p = 1 - \mu$ is $1 - \mu - \kappa - K$.

4.2. Optimal price

Now, we consider the firm's optimal pricing with the protest. First, we define the firm's profit. As the firm keeps product sale unless $t < T_f(p)$, the firm's expected profit is

$$\Pi(p) = \sum_{v'=0}^{1} \Pr(v = v')\Big[\Big(\Phi\Big(\tfrac{T_f(p)-\lambda_{v'}}{\sigma}\Big) - \Phi\Big(\tfrac{T_L(p)-\lambda_{v'}}{\sigma}\Big)\Big) \cdot (\mu(p-\kappa) - K)$$

$$+\Phi\Big(\tfrac{T_L(p)-\lambda_{v'}}{\sigma}\Big) \cdot (p - \kappa - K)\Big].$$

The following proposition shows that the optimal price is $p = \mu$, in which case the protest is uninformative.

Proposition 2. *Suppose that $\frac{\mu-\kappa}{2} > \max\{K, 1-\mu-\kappa\}$. Then, for sufficiently large $\sigma > 0$, $p = \mu$ is the optimal price and protest is uninformative for any λ_0, λ_1, and w.*

Note that this result is irrelevant to the size of w as w affects the size of λ_1, λ_0, but they are bounded by $[0, 1]$. Therefore, as Proposition 1 shows, an informative equilibrium exists if $p < \left\{\mu, \overline{P}\right\}$. An informative equilibrium also benefits the firm as it discontinues the product and saves fixed cost K if it observes high t, which implies $v = 0$ is likely enough. On the contrary, informative equilibrium has a cost: H-type may not buy the product. Although both the cost and benefit of informative equilibrium are small when σ is sufficiently large, we show that if $\mu > \kappa + 2K$, this cost of informative equilibrium is more severe than the gain from informative equilibrium at a neighbor of $p = \mu$. Then, rather than gaining information by setting a lower price, $p = \mu$ yields a greater profit.

The above observation depends on whether an uninformative equilibrium with $p = \mu$ yields a sufficient profit. If not, an informative equilibrium leads to a higher profit, as the following proposition shows.

Proposition 3. *Suppose that $1 - \mu - \kappa > K > \frac{\mu - \kappa}{2}$. Then, for each σ, there is $\overline{w}$ such that for each $w > \overline{w}$, the protest is informative at the firm's optimal price.*

By Propositions 2 and 3, we observe that the resulting equilibrium is uninformative if μ is high, while it is informative when μ is small. Theorem 1 summarizes the result, and Figure 1 illustrates it.

Theorem 1. *Suppose that $\kappa < \frac{1}{2}$ and $K \in \left(\frac{1-2\kappa}{3}, \frac{1}{2} - \kappa\right)$. Take sufficiently large σ and w. Then, there exists $\mu^*, \overline{\mu}, \underline{\mu}$ with $\mu^* > \overline{\mu} > \underline{\mu}$ such that the equilibrium is uninformative if $\mu \in (\overline{\mu}, \mu^*)$,[13] and the equilibrium is informative if $\mu < \underline{\mu}$.*

This result implies that individual consumers' signal informativeness may negatively correlate with the protest's informativeness. This contrasts sharply with Ekmekci and Lauermann (2022), who shows that each person's signal informativeness positively correlates with the protest campaign's informativeness.

[13]The restriction $\mu < \mu^*$ is needed because Proposition 2 is the statement when μ is fixed.

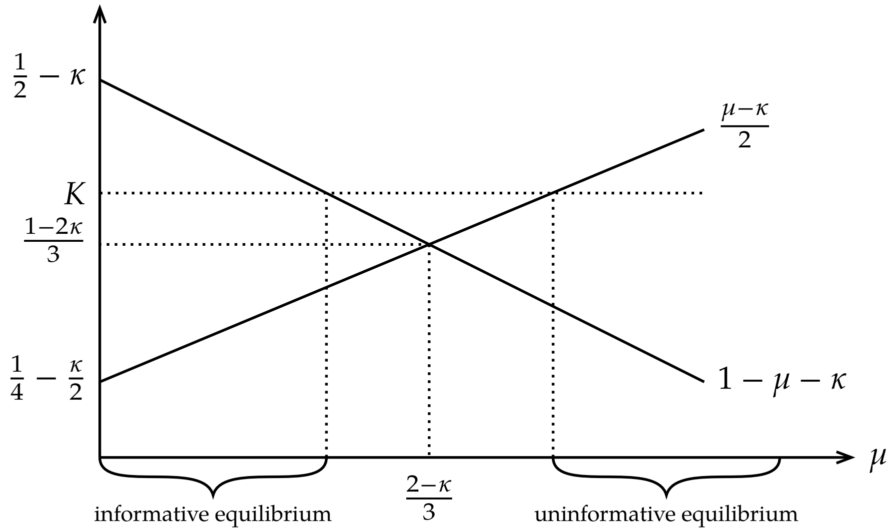

Figure 1: Classification of equilibria

The main difference between their and our models is the existence of a price-setting firm. In the models of Ekmekci and Lauermann (2022) and many protest studies, citizens campaign against the policy-setting government, which affects only the policy decision. However, the firm makes two-dimensional decisions in our model: whether to discontinue the product sale and price. With a higher informativeness of each individual's signal, consumers are more willing to pay, which leads to a greater profit for the firm by setting a higher price. This makes the protest uninformative by reducing the purchase incentive of H-type.

Remark 1. In this discussion, we examine social welfare, which is the sum of the firm's profit and consumer surplus. When $v = 1$, the maximized social welfare is $1 - \kappa - K - \zeta$, provided this value is positive. When $v = 0$, the maximized social welfare is 0. This welfare level is achieved when v is known at any $p \in (\kappa, 1)$. Specifically, if $v = 1$, the product is available for sale, and all consumers purchase it at the given price. On the other hand, if $v = 0$, since no consumers buy the product, the firm discontinues the sale.

[14]This definition is the same as Ekmekci and Lauermann (2022).

We refer to $\lambda_0 - \lambda_1$ as the **informativeness** of the protest.[14] As this value increases, the probability of product purchase decreases when $v = 0$, and increases when $v = 1$. Thus, the greater informativeness of the protest led to an improvement in social welfare.

Our findings suggest that as the precision of consumers' private signals increases, social welfare decreases.

5. Uncommitted price

In the previous section, the firm is supposed to commit to a price p before the public signal is observed. This section considers the case in which the firm decides the price after the public signal observation. Therefore, the new decision flow is as follows:

1. Each consumer observes private signal θ_i and decides whether to participate in the protest.
2. Turnout τ realizes, and the firm and consumers observe public signal $t = \tau + \varepsilon$.
3. The firm decides whether to continue the product sale and if the sale continues, the firm sets a price.
4. If the product is on sale, each consumer decides whether to buy it.

A crucial difference to the committed price case is that the turnout affects the price in the uncommitted case. With a greater turnout, if it is informative, it reduces the expected value of the product. Then, the firm sets a lower price. This makes the H-type consumers participate in the protest more than the L-type consumers, as the H-type consumers earn more information rents when purchasing. This incentive makes the equilibrium likelier to be uninformative.

Now, we investigate how our conclusion changes. The previous section shows that with higher μ, the equilibrium is likelier to be uninformative. Even in the uncommitted price case, a similar conclusion is drawn. The following is a formal analysis. All proofs of the following section are relegated to the Appendix.

5.1. Formal analysis of the uncommitted price case

To focus on the discussion on whether an informative equilibrium exists, we assume that $\lambda_0 > \lambda_1$. Then, we verify the consumers' incentives to participate in the protests. Note that even in the uncommitted price case, the consumers' decision criterion on the product purchase is unchanged. Thus, type-θ consumer purchases the product if and only if $t < T_\theta(p)$.

Given this strategy, the firm's profit is

$$\Pi = \begin{cases} -K & \text{if } \ t > T_H(p) \\ [\Pr(v = 1 \mid t) \cdot \mu + \Pr(v = 0 \mid t) \cdot (1 - \mu)] \cdot (p - \kappa) - K & \text{if } \ t \in (T_L(p), T_H(p)] \\ p - \kappa - K & \text{if } \ t \leq T_L(p). \end{cases}$$

This implies that the optimal price is either $p = P_H(t)$ or $P_L(t)$. Therefore, we need to compare the following equations.

$$\Pi_H^*(t) := [\Pr(v = 1 \mid t) \cdot (2\mu - 1) + 1 - \mu)] \cdot (P_H(t) - \kappa) - K$$

$$\Pi_L^*(t) := P_L(t) - \kappa - K.$$

$\Pi_H^*(t)$ represents the profit when the firm focuses solely on H-type consumers, while $\Pi_L^*(t)$ indicates the profit when targeting both consumer types. Also, the firm discontinues the product sales rather than selling it with $p = (T_L)^{-1}(t)$ if $p - \kappa < K$. In other words, $(T_L)^{-1}(t) < K + \kappa$. Equivalently, $t > T_L(K + \kappa)$.

To simplify the discussion, we assume $\kappa = 0$ hereafter. The following lemma identifies the condition under which one profit exceeds the other.

Lemma 3. *Assume that $\kappa = 0$. Then, there exist values μ^* and μ^{**} with $\mu^* < \mu^{**}$ such that: (a) If $\mu > \mu^{**}$, there is $\hat{t} < T_L(K)$ such that $\Pi_H^*(t) < \Pi_L^*(t)$ if and only if $t < \hat{t}$. (b) If $\mu < \mu^{**}$, $\Pi_H^*(t) < \Pi_L^*(t)$ for each $t < T_L(K)$. (c) $T_L(K) \to \infty$ if $\mu < \mu^*$, and $T_L(K) \to -\infty$ if $\mu > \mu^*$.*

Given this firm's behavior, we consider the consumers' incentive to participate in the protest. We first consider the case (a): there is $\hat{t} < T_L(K)$ such that $\Pi_L^*(t) > \Pi_H^*(t)$

if and only if $t < \hat{t}$. Let T^* be the solution to $\Pi_H^*(t) = 0$ with respect to t. This implies that at the optimal strategy, the firm sets price $P_L(t)$ when $t \in \left(-\infty, \hat{t}\right)$, sets price $P_H(t)$ when $t \in \left(\hat{t}, T^*\right)$, and when $t > T^*$, discontinues the product sale.

Under this firm's behavior, the utility of the H-type is

$$-\varsigma \cdot \int^{T^* - \mathrm{d}\tau} \xi_H(t)\,\mathrm{d}t + w \int_{\hat{t}-d\tau}^{T^* - \mathrm{d}\tau} [P_H(t) - P_H(t + \mathrm{d}\tau)]\xi_H(t)\,\mathrm{d}t$$

$$+w \cdot \int^{\hat{t}-d\tau} [P_H(t) - P_L(t + \mathrm{d}\tau)] \cdot \xi_H(t)\,\mathrm{d}t.$$

Here, $P_H(t)$ is the expected value of the product while $P_L(t)$ is the price. A marginal increase of turnout $\mathrm{d}\tau$ affects the price. As the firm set price $p = P_H(t)$ if $t > \hat{t}$, type-H consumers' the informational rent $P_H(t) - P_L(t)$ vanishes. Therefore, the decision to discontinue the product only affects the externality term ς.

The differentiation concerning $\mathrm{d}\tau$ is the marginal gain by participation. Therefore, consumer i participates in the protest if and only if $c_H \geq c_i$, where

$$c_H := \varsigma \cdot \xi_H(T^*)$$
$$+w \cdot \left[\int_{\hat{t}}^{T^*} \left[-P_H'(t)\right]\xi_H(t)\,\mathrm{d}t - \left[P_H(\hat{t}) - P_L(\hat{t})\right]\xi_H(\hat{t}) + \int^{\hat{t}} \left[-P_L'(t)\right]\xi_H(t)\,\mathrm{d}t\right].$$

Focus on the second line in the above equation. Note that $P_L'(t) < 0$ and $P_H'(t) < 0$. Then, the first and third terms are the gain from price manipulation by increasing turnout. In contrast, with a high turnout, the firm gives up on making all types of consumers purchase the product, and then, type-H's informational rent vanishes. This makes type H consumers reluctant to participate in the protest, captured by the second term.

Now we compare with L-type consumers' utility. The utility of the L-type is $-\varsigma \cdot \int^{T^* - \mathrm{d}\tau} \xi_L(t)\,\mathrm{d}t + w \cdot \int^{\hat{t}-d\tau} [P_L(t) - P_L(t + \mathrm{d}\tau)] \cdot \xi_L(t)\,\mathrm{d}t$ as the firm extracts type-L consumers' full surplus. Then, the type-L consumers participate in the protest if and only if

$$c_L := \varsigma \cdot \xi_L(T^*) + w \cdot \int^{\hat{t}} \left[-P_L'(t)\right]\xi_L(t)\,\mathrm{d}t \geq c_i.$$

We compare the relation between c_H and c_L with high μ cases. As we focus on a large w case, we consider the coefficients of w in $c_L - c_H$, that is

$$D := \int^{\hat{t}} \left[-P_L'(t)\right](\xi_L(t) - \xi_H(t))\,\mathrm{d}t - \int_{\hat{t}}^{T^*} \left[-P_H'(t)\right]\xi_H(t)\,\mathrm{d}t + \left[P_H(\hat{t}) - P_L(\hat{t})\right]\xi_H(\hat{t}).$$

Proposition 4. *Suppose that $\mu > \max\{\mu^{**}, 2K\}$. Then, for sufficiently large σ, $D < 0$.*

Proposition 4 implies that the protest becomes uninformative with high μ. This is because this inequality $c_H > c_L$ is gained by the supposition that $\lambda_0 > \lambda_1$, which means that $c_L > c_H$. This is a contradiction. So, the equilibrium should be uninformative. This is a similar feature to the committed price case. The intuition is the following. With a high μ, the firm continues to sell even when t is sufficiently high, in which case, while the L-type gives up to purchase, the H-type purchases. Then, the H-type's incentive for price manipulation is much higher than that of the L-type. In contrast, the effect on the informational rent is small as the firm is likely to focus on selling only to the H-types when μ is high enough. This motivates the H-type to participate in the protest more than the L-type.

Next, we consider the case (b): for each $t < T_L(\kappa + K)$, $\Pi_L^*(t) > \Pi_H^*(t)$. This implies that for any $t < T_L(\kappa + K)$, the firm set price $P_L(t)$ and discontinues the product sale if $t > T_L(\kappa + K)$. To simplify the notation, we denote $T_* = T_L(\kappa + K)$.

In this case, The utility of the H-type is

$$-\varsigma \int^{T_* - \mathrm{d}\tau} \xi_H(t)\,\mathrm{d}t + w \int^{T_* - d\tau} [P_H(t) - P_L(t + \mathrm{d}\tau)]\xi_H(t)\,\mathrm{d}t.$$

The differentiation concerning $\mathrm{d}\tau$ is the marginal gain by participation. Therefore, they participate in the protest if and only if

$$c_H := \varsigma\xi_H(T_*) - w[P_H(T_*) - P_L(T_*)]\xi_H(T_*) + w \int^{T_*} \left[-P_L'(t)\right]\xi_H(t)\,\mathrm{d}t \geq c_i.$$

Similarly, the utility of the L-type is $-\varsigma \int^{T_*-\mathrm{d}\tau} \xi_L(t)\,\mathrm{d}t + w \int^{T_*-d\tau} [P_L(t) - P_L(t + \mathrm{d}\tau)]\xi_L(t)\,\mathrm{d}t$. Then, they participate in the protest if and only if

$$c_L := \varsigma\xi_L(T_*) + w \int^{T_*} [-P_L'(t)]\xi_L(t)\,\mathrm{d}t \geq c_i.$$

We also compare c_H with c_L and thus, focus on the coefficient of w in $c_L - c_H$,

$$\tilde{D} := \int^{T_*} [-P_L'(t)](\xi_L(t) - \xi_H(t))\,\mathrm{d}t + [P_H(T_*) - P_L(T_*)]\xi_H(T_*).$$

Proposition 5. *Suppose that $\mu \in (\mu^*, \mu^{**})$. Then, for sufficiently large σ, $\tilde{D} > 0$.*

$c_L > c_H$ implies that the H-type is likelier to participate in the protest. Then, the equilibrium becomes informative with a large w. Intuition is the following. With lower μ, the firm gives up selling even with a small t. Then, the incentive for price manipulation becomes smaller, and the effect of losing informational rent dominates that incentive.

In conclusion, the informative equilibrium is more likely with intermediate μ cases, although not in high μ cases. Our conclusion gained in the committed price case remains: precise information consumers gain does not contribute to the protest's success.

6. Conclusion and Discussions

This study examines when a protest campaign successfully aggregates information about consumers' values for a product, which determines the firm's decision. We show that consumers' individual signal precision does not necessarily contribute to the protest's informativeness. One implication of our result is that consumers' ignorance may contribute to the success of protest campaigns.

In the remainder of this study, we discuss the assumptions in our model.

Objective of participation in protest: We assumed that consumers consider their type's aggregate utility when deciding whether to participate in the protest. However, as in Ali and Lin (2013), if this motivation stems from a reputation concern, it is more plausible to assume that consumers consider the expected social welfare.

Consider a scenario where each consumer's decision to participate in the protest is based on maximizing the expected social welfare. The consumer's belief, represented by $\theta \in \{\theta_H, \theta_L\}$, influences their expected social welfare as follows.

$$\theta \int^{T-\mathrm{d}\tau} \Big(w \cdot [P_\theta(t) - p]_+ - \varsigma\Big)\xi_\theta(t)\,\mathrm{d}t + (1-\theta)\int^{T-\mathrm{d}\tau} \Big(w \cdot [P_{\theta'}(t) - p]_+ - \varsigma\Big)\xi_{\theta'}(t)\,\mathrm{d}t,$$

where $\theta' \in \{\theta_H, \theta_L\}$ with $\theta' \neq \theta$. In this case, the H-type participates in the protest if and only if $c_i \leq c_H^* := \mu c_H + (1-\mu)c_L$ while the L-type does if and only if $c_i \leq c_L^* := (1-\mu)c_H + \mu c_L$. As long as $c_L > c_H$, the cutoff point is lower for the L-type consumers. We can apply a similar discussion to obtain the same conclusions.

Concerns for negative externalities: In our model, concern for the product value plays an important role, but concern for the negative externality itself does not. If the concern regarding the negative externalities is sufficiently large, the participation rates of H- and L-type consumers converge, making the protest uninformative. This observation holds even when concerns for the externality vary among consumers since the firm does not care about the externality itself. If the product value includes guilty feelings about consuming a product causing negative externalities, it would be correlated with the level of concern for the externality. In this case, our conclusion remains true even if concerns for negative externality are high.

Noisy campaign: We have assumed that noise follows a normal distribution with sufficiently high variance. Considering a sufficiently high noise is crucial for the committed price case. The firm benefits from an informative equilibrium if the noise variance is small enough. The firm can save the fixed cost K when $v = 0$. In this case, if $v = 0$, the firm discontinues the sale. If $v = 1$, the optimal price can be high but less than μ for the existence of an informative equilibrium. An interesting observation is that if μ is small enough, the protest can make the price higher. To see this, consider the case where the protest cannot occur for some external reason. Then, the price is set so that both H- and L-type consumers buy when μ is small enough. In contrast, when a protest is informative about the popularity, the firm continues the sale if and only if τ is small enough. When the noise is small enough, this signals that the value of the product is high. Then, the firm sets a higher price when the protest can occur. A formal analysis is relegated to Section B in the appendix.

Another observation is that although the protest revealed almost complete information, the price is bounded above by $\max\left\{\overline{P}, \mu\right\}$, to make the protest informative, by Proposition 1. This observation is important because it seems optimal for the firm to set a price close to 1 as the protest reveals whether $v = 1$ or $v = 0$. In fact, if doing so, the H-type's motivation becomes close to the L-type's, and the protest becomes uninformative. Note also that $\overline{P}$ is increasing in μ when κ is small enough. Then, even if the individual signal precision is high, it does not necessarily benefit the consumers, as it only increases the price.

Appendix

A. Proofs

A.1. Proof of Proposition 1

Before the proof of Proposition 1, we prepare the following lemma.

Lemma 4. *Define $\underline{P}$ be the larger solution to $\frac{1-\mu}{\mu}\frac{1-p}{p} = \frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa}-(1-\mu)}$ with respect to p. Then, $T_H(p) < T_f(p)$ if and only if $p > \overline{P}$ and $T_L(p) < T_f(p)$ if and only if $p > \underline{P}$. Further, $\frac{K}{1-\mu} + \kappa > \overline{P} > \underline{P} > \frac{K}{\mu} + \kappa$.*

Proof of Lemma 4. (a) If $p \geq \frac{K}{1-\mu} + \kappa$, $T_f(p) = \infty$ by Lemma 2. If $p < \frac{K}{1-\mu} + \kappa$, $T_f(p) < T_H(p)$ if and only if $\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa}-(1-\mu)} < \frac{\mu}{1-\mu}\frac{1-p}{p}$, which is equivalently $p > \overline{P}$. Further, a simple calculation shows $\frac{K}{1-\mu} + \kappa > \overline{P}$.

(b) If $p < \frac{K}{\mu}$, by Lemma 2, $T_f(p) = -\infty$. Consider the case where $p \geq \frac{K}{\mu}$. Note that $T_L(p) < T_f(p)$ if and only if $\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa}-(1-\mu)} > \frac{1-\mu}{\mu}\frac{1-p}{p}$, equivalently, $g(p) > 0$, where g is a convex quadratic function (illustrated in Figure 2). Then, $T_L(p) < T_f(p)$ if and only if $p > \underline{P}$. The relation $\underline{P} > \frac{K}{\mu} + \kappa$ is shown by the fact that $\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa}-(1-\mu)} > \frac{1-\mu}{\mu}\frac{1-p}{p}$ fails if $p = \frac{K}{\mu} + \kappa$ and larger p satisfies $T_f(p) > T_H(p) > T_L(p)$. ∎

Proof of Proposition 1. (1) Suppose that $p \geq \max\{\overline{P}, \mu\}$.

If $p \geq \overline{P}$, as long as $t \leq T_H(p)$, $t \geq T_f(p)$. At $t > T_H(P)$, H type does not purchase the product. Then, the firm gives up to sell the product. This implies that the firm's cutoff point is $T = T_H(p)$. In this case, the equilibrium condition is

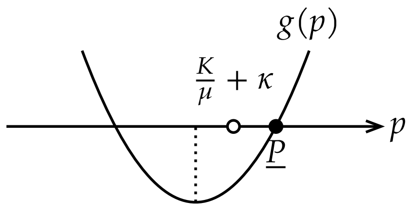

Figure 2: Illustration of g

$$\lambda_0 - \lambda_1 = (2\mu - 1) \cdot (F(c_L) - F(c_H)),$$

$$c_\theta = \varsigma \cdot \xi_\theta(T_H(p)),$$

$$T_H(p) = \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{p}{1-p}\right)}{\lambda_0 - \lambda_1}$$

$$\xi_\theta(T_H(p)) = \frac{1}{\sigma} \cdot \left[\theta \cdot \varphi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right) + (1 - \theta) \cdot \varphi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right)\right].$$

By $p \geq \mu$, $\frac{\sigma^2}{2} \cdot \frac{\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{p}{1-p}\right)}{\lambda_0 - \lambda_1} \leq 0$ when $\lambda_0 \geq \lambda_1$. As φ is symmetric and single-peaked around 0, we show that $\varphi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right) \leq \varphi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right)$. Note also that

$$c_L - c_H = \varsigma \cdot \frac{2\mu - 1}{\sigma} \cdot \left[\varphi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right) - \varphi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right)\right] \leq 0.$$

Therefore, $\lambda_0 \geq \lambda_1$ implies $c_L \leq c_H$, which shows that no informative equilibrium exists.

(2) Consider the case $p \in \left(\underline{P}, \overline{P}\right)$. Then, if $\lambda_0 > \lambda_1$, $T_L(p) < T_f(p) < T_H(p)$ for any c_H, c_L.

Recall that the following equations characterize the equilibriums:

$$\lambda := \lambda_0 - \lambda_1 = (2\mu - 1) \cdot (F(c_L) - F(c_H)),$$

$$c_\theta = \left(\varsigma - w \cdot \left[P_\theta\left(T_f(p)\right) - p\right]_+\right) \cdot \xi_\theta\left(T_f(p)\right),$$

$$T_f(p) - \lambda_1 = \frac{\lambda}{2} + \frac{\sigma^2}{2} \cdot \frac{1}{\lambda} \cdot \ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right),$$

$$T_f(p) - \lambda_0 = -\frac{\lambda}{2} + \frac{\sigma^2}{2} \cdot \frac{1}{\lambda} \cdot \ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right),$$

$$P_\theta\left(T_f(p)\right) = \frac{\varphi\left(\frac{T_f(p) - \lambda_1}{\sigma}\right) \cdot \theta}{\varphi\left(\frac{T_f(p) - \lambda_1}{\sigma}\right) \cdot \theta + \varphi\left(\frac{T_f(p) - \lambda_0}{\sigma}\right) \cdot (1 - \theta)},$$

$$\xi_\theta\left(T_f(p)\right) = \frac{1}{\sigma} \cdot \left[\theta \cdot \varphi\left(\frac{T_f(p) - \lambda_1}{\sigma}\right) + (1 - \theta) \cdot \varphi\left(\frac{T_f(p) - \lambda_0}{\sigma}\right)\right].$$

This shows that c_L and c_H are depend on λ, but not on λ_0, λ_1 individually. Therefore, the equilibrium is a fixed point of a self-map $\zeta : \lambda \mapsto (2\mu - 1)(F(c_L) - F(c_H))$. We can easily verify that ζ is continuous.

Below, we show that $\zeta(\lambda) > \lambda$ for some $\lambda > 0$ when w is large enough. By the definition of $\overline{P}$ and $\underline{P}$, $P_H\big(T_f(p)\big) > p$, and $p > P_L\big(T_f(p)\big)$. This implies that

$$c_H = \big(\varsigma - w\big[P_H\big(T_f(p)\big) - p\big]\big) \cdot \xi_H\big(T_f(p)\big)$$

$$c_L = \varsigma \cdot \xi_L\big(T_f(p)\big) > 0.$$

Then,

$$c_L - c_H = \varsigma \cdot \Big(\xi_L\big(T_f(p)\big) - \xi_H\big(T_f(p)\big)\Big) + w \cdot \Big[P_H\big(T_f(p)\big) - p\Big] \cdot \xi_H\big(T_f(p)\big),$$

which is positive for large w. Note that as $\lambda \to 0$, $T_f(p) \to \infty$, which implies that $c_L \to 0$ and $c_H \to 0$. By the Taylor expansion, we can take sufficiently small $\lambda > 0$ so that $(2\mu - 1)(F(c_L) - F(c_H)) \approx (2\mu - 1) \cdot f(0) \cdot (c_L - c_H)$, and then,

$$(2\mu - 1)(F(c_L) - F(c_H)) > (2\mu - 1) \cdot f(0) \cdot w \cdot \Big[P_H\big(T_f(p)\big) - p\Big] \cdot \xi_H\big(T_f(p)\big)$$

We can take large w that is proportional to $\frac{\lambda}{\left[P_H\left(T_f(p)\right) - p\right] \cdot \xi_H\left(T_f(p)\right)}$. Then, while both c_H and c_L is close to 0, $\zeta(\lambda) = (2\mu - 1)(F(c_L) - F(c_H)) > \lambda$ for small $\lambda > 0$. As $\zeta(2\mu - 1) < 2\mu - 1$, the intermediate value theorem implies the existence of a fixed point of ζ, which is in $(0, 2\mu - 1)$. This implies that the fixed point is a positive value. This shows the existence of an informative equilibrium.

(3) Consider the case $p < \underline{P}$. This implies that the firm prefers to give up selling if $t > T_L(p)$. If $t \leq T_L(p)$, both H and L types purchase the product. This implies the firm's profit is $p - \kappa$. If $p - \kappa > K$, the give up point is $T = T_L(p)$. Then, by the definition of $T_L(p)$,

$$c_H = (\varsigma - w[P_H(T_L(p)) - p]) \cdot \xi_H\big(T_f(p)\big)$$

$$c_L = \varsigma \cdot \xi_L(T_L(p)) > 0.$$

A similar way to case (2) shows the existence of informative equilibrium.

If $p - \kappa \leq K$, the firm also prefers to give up at $t \leq T_L(p)$ and therefore, $T = -\infty$. Then, the equilibrium becomes uninformative. ■

A.2. Proof of Proposition 2

Before we prove Proposition 2, we provide the following lemma, which immediately follows from the definition of $\overline{P}$.

Lemma 5. *$\mu > \overline{P}$ if and only if $\frac{\mu-\kappa}{2} > K$.*

Now, we proceed to the proof of Proposition 2. Suppose that $\frac{\mu-\kappa}{2} > \max\{K, 1-\mu-\kappa\}$. This implies that $\mu > \frac{2-\kappa}{3}$. Note that if $p = \mu$, as $\frac{\mu-\kappa}{2} > K$, $p > \overline{P}$ by Lemma 5. By Proposition 1, the equilibrium is uninformative and guarantees the profit $\frac{\mu-\kappa}{2} - K$.

Below, we examine the existence of an informative equilibrium that yields a profit larger than $\frac{\mu-\kappa}{2} - K$. As we consider an informative equilibrium, by Proposition 1, $p < \mu$. Note that the profit is at most $p - \kappa - K$. As we assume $\frac{\mu-\kappa}{2} > 1 - \mu - \kappa$, if an informative equilibrium yields a greater profit, the price needs to satisfy $p > 1 - \mu$. This implies that

$$\ln\left(\frac{1-p}{p}\frac{1-\mu}{\mu}\right) < 0.$$

Now we consider the limit $\sigma \to \infty$. Note that as $\lambda_0 - \lambda_1 = (2\mu - 1)(F(c_L) - F(c_H)) \in [0, 2\mu - 1] \subset [0, 1]$.[15] This shows that

$$\frac{T_L(p) - \lambda_1}{\sigma} < \frac{\sigma}{2}\ln\left(\frac{1-p}{p}\frac{1-\mu}{\mu}\right) \to -\infty,$$

$$\frac{T_L(p) - \lambda_0}{\sigma} < \frac{1}{2\sigma} + \frac{\sigma}{2}\ln\left(\frac{1-p}{p}\frac{1-\mu}{\mu}\right) \to -\infty.$$

Then, the firm's profit $\Pi(p)$ is at most $\left(\frac{1}{2}\right)(p-\kappa) - K$ at the limit. This is smaller than $\frac{\mu-\kappa}{2} - K$ as $p < \mu$.

[15]Note that the value of σ can be taken independent of w as the value of w affects only λ. Here, λ is bounded by values that are independent of w.

Note that the above proof cannot apply when $p \approx \mu$, as we cannot exclude the case where the profit is larger than $\left(\frac{1}{2}\right)(p-\kappa)-K$ for any σ; it converges only at the limit. The following discussion is to verify that the profit is strictly less than $\left(\frac{1}{2}\right)(p-\kappa)-K$ when p is close to μ and σ is large enough.

Claim 1. *Suppose that $\mu > p > \max\left\{\frac{1}{2}, \overline{P}\right\}$. Then, $\Pi(p) < \left[\left(\frac{1}{2}\right)(p-\kappa)-K\right]$ for sufficiently large σ.*

Proof of Claim 1. Suppose that $p > \overline{P}$. In this case, as discussed in the proof of Proposition 1, the firm's cutoff point coincides with the H-type's cutoff point. Then, the profit is written as

$$\Pi(p) = \Pr(v=1)\left[\left(\Phi\left(\tfrac{T_H(p)-\lambda_1}{\sigma}\right)-\Phi\left(\tfrac{T_L(p)-\lambda_1}{\sigma}\right)\right)\cdot[\mu\cdot(p-\kappa)-K]+\Phi\left(\tfrac{T_L(p)-\lambda_1}{\sigma}\right)\cdot(p-\kappa-K)\right]$$

$$+\Pr(v=0)\left[\left(\Phi\left(\tfrac{T_H(p)-\lambda_0}{\sigma}\right)-\Phi\left(\tfrac{T_L(p)-\lambda_0}{\sigma}\right)\right)\cdot[(1-\mu)\cdot(p-\kappa)-K]+\Phi\left(\tfrac{T_L(p)-\lambda_0}{\sigma}\right)\cdot(p-\kappa-K)\right].$$

Let $D(\sigma) = \Pi(p) - \left[\left(\frac{1}{2}\right)(p-\kappa)-K\right]$, which is calculated as

$$\Pr(v=1)\left[\left[\Phi\left(\tfrac{T_H(p)-\lambda_1}{\sigma}\right)-1\right]\cdot[\mu\cdot(p-\kappa)-K]+\Phi\left(\tfrac{T_L(p)-\lambda_1}{\sigma}\right)\cdot(1-\mu)\cdot(p-\kappa)\right]$$

$$+\Pr(v=0)\left[\left[\Phi\left(\tfrac{T_H(p)-\lambda_0}{\sigma}\right)-1\right]\cdot[(1-\mu)\cdot(p-\kappa)-K]+\Phi\left(\tfrac{T_L(p)-\lambda_0}{\sigma}\right)\cdot\mu\cdot(p-\kappa)\right].$$

Note that by $p \in (1-\mu, \mu)$ and $\lambda \in (0, 2\mu-1)$,

$$T_H(p)-\lambda_1 = \frac{\lambda}{2}+\frac{\sigma^2}{2\lambda}\left[\ln\left(\tfrac{\mu}{1-\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right] > \frac{\sigma^2}{2}\left[\ln\left(\tfrac{\mu}{1-\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right],$$

$$T_H(p)-\lambda_0 = -\frac{\lambda}{2}+\frac{\sigma^2}{2\lambda}\left[\ln\left(\tfrac{\mu}{1-\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right] > \frac{\sigma^2}{2}\left[\ln\left(\tfrac{\mu}{1-\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right]-\frac{1}{2},$$

$$T_L(p)-\lambda_1 = \frac{\lambda}{2}+\frac{\sigma^2}{2\lambda}\left[\ln\left(\tfrac{1-\mu}{\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right] < \frac{\sigma^2}{2}\left[\ln\left(\tfrac{1-\mu}{\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right]+\frac{1}{2},$$

$$T_L(p)-\lambda_0 = -\frac{\lambda}{2}+\frac{\sigma^2}{2\lambda}\left[\ln\left(\tfrac{1-\mu}{\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right] < \frac{\sigma^2}{2}\left[\ln\left(\tfrac{1-\mu}{\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right].$$

By these equations, we can verify that as $\sigma \to \infty$, $T_H(p)-\lambda_v \to \infty$ and $T_L(p)-\lambda_v \to -\infty$. Then, $\Phi\left(\frac{T_H(p)-\lambda_v}{\sigma}\right)-1 \to 0$ and $\Phi\left(\frac{T_L(p)-\lambda_v}{\sigma}\right) \to 0$.[16] Then, $\lim_{\sigma\to\infty} D(\sigma) = 0$.

Now we verify the sign of D for sufficiently large σ. To this end, we calculate $D'(\sigma)$, which is

$$D'(\sigma) = \varphi(A_{H1}) \cdot \left(\frac{1}{\lambda}B_H - \frac{\lambda}{2\sigma^2}\right) \cdot (\mu(p-\kappa) - K)$$

$$+\varphi(A_{L1}) \cdot \left(\frac{1}{\lambda}B_L - \frac{\lambda}{2\sigma^2}\right) \cdot (1-\mu) \cdot (p-\kappa)$$

$$+\varphi(A_{H0}) \cdot \left(\frac{1}{\lambda}B_H + \frac{\lambda}{2\sigma^2}\right) \cdot ((1-\mu) \cdot (p-\kappa) - K)$$

$$+\varphi(A_{L0}) \cdot \left(\frac{1}{\lambda}B_L + \frac{\lambda}{2\sigma^2}\right) \cdot \mu \cdot (p-\kappa),$$

where

$$A_{\theta v} = \frac{T_\theta(p) - \lambda_v}{\sigma}, \quad v \in \{0,1\},$$

$$B_\theta = \ln\left(\frac{\theta}{1-\theta}\right) - \ln\left(\frac{p}{1-p}\right),$$

and $B_H > 0 > B_L$ as $\mu > p > \frac{1}{2} > 1-\mu$.

Note that

$$\frac{\varphi(A_{H1})}{\varphi(A_{L0})} = \exp\left(\left[\tfrac{\lambda}{\sigma} - \tfrac{\sigma}{\lambda}\ln\left(\tfrac{1-\mu}{\mu}\right)\right]\sigma\ln\left(\tfrac{p}{1-p}\right)\right) > \exp\left(-\sigma^2\ln\left(\tfrac{1-\mu}{\mu}\right)\ln\left(\tfrac{p}{1-p}\right)\right),$$

$$\frac{\varphi(A_{H0})}{\varphi(A_{L0})} = \exp\left(-\tfrac{\sigma}{\lambda}\ln\left(\tfrac{1-\mu}{\mu}\right)\left[\tfrac{\lambda}{\sigma} + \sigma\ln\left(\tfrac{p}{1-p}\right)\right]\right) > \exp\left(-\sigma^2\ln\left(\tfrac{1-\mu}{\mu}\right)\ln\left(\tfrac{p}{1-p}\right)\right),$$

$$\frac{\varphi(A_{L1})}{\varphi(A_{L0})} = \exp\left(\ln\left(\tfrac{p}{1-p}\right) - \ln\left(\tfrac{1-\mu}{\mu}\right)\right).$$

Then, as $\mu > \frac{1}{2}$ and $p > \frac{1}{2}$, $\frac{\varphi(A_{H1})}{\varphi(A_{L0})} \to \infty$, and $\frac{\varphi(A_{H0})}{\varphi(A_{L0})} \to \infty$ as $\sigma \to \infty$. Further, $\frac{\varphi(A_{H0})}{\varphi(A_{H1})} \to 1$ as $\sigma \to \infty$. Note also that $(\mu(p-\kappa) - K) + ((1-\mu)(p-\kappa) - K) = p - \kappa - 2K > 0$. Using these facts, dividing $D'(\sigma)$ by $\frac{\varphi(A_{L0})}{\lambda}$ is positive for sufficiently large σ. As

[16]Note that $\mu > \frac{1}{2}$ and $p > \frac{1}{2}$ implies that $\left|\ln\left(\frac{1-\mu}{\mu}\right) - \ln\left(\frac{p}{1-p}\right)\right| > \left|\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{p}{1-p}\right)\right|$. This also implies that $\Phi\left(\frac{T_H(p)-\lambda_v}{\sigma}\right)$ converges more slowly than $\Phi\left(\frac{T_L(p)-\lambda_v}{\sigma}\right)$. This intuition works in the following analysis.

$\lim_{\sigma \to \infty} D(\sigma) = 0$ and $D'(\sigma) > 0$, $D(\sigma) < 0$ for sufficiently large σ. Therefore, $\Pi(p) < \left[\frac{1}{2}(p - \kappa) - K\right]$ for sufficiently large σ. ■

Then, we conclude that for each $p < \mu$, the profit is at most $\frac{1}{2}(p - \kappa) - K$ when σ is sufficiently large. Therefore, $p = \mu$ is the optimal price, and the equilibrium is uninformative. ■

A.3. Proof of Proposition 3

The proof consists of the following two claims, which hold under the assumption of Proposition 3.

Claim 2. *If informative equilibrium does not exist, the expected profit is at most* 0.

Claim 3. *By setting $p = 1 - \mu$, the equilibrium is informative. The expected profit is no less than $(1 - \mu - \kappa - K)\frac{1}{2}$ in this case.*

Proof of Claim 2. By Proposition 1, the situation is either $p > \min\{\mu, \overline{P}\}$ or $p < \kappa + K$. If $p > \mu$, as $P_H(T) = \mu < p$ at the uninformative equilibrium, the profit is 0 as the H type never purchases. If there is an uninformative equilibrium with $p \in (\overline{P}, \mu)$, as $P_H(T) = \mu > p > 1 - \mu$, the profit is $\frac{\mu - \kappa}{2} - K < 0$ as only the H type purchases.

At the uninformative equilibrium with $p < \kappa + K$, the profit is also 0 as the firm gives up selling. ■

Proof of Claim 3. First, consider the case where $1 - \mu \le \underline{P}$. As $p = 1 - \mu > K + \kappa$, $1 - \mu < \mu$ and $1 - \mu < \underline{P} < \overline{P}$, an equilibrium exists and it is necessarily informative by Proposition 1.

Note that $p = 1 - \mu \le \underline{P}$ implies $T = T_L(p)$. In this case, all consumer buys the product if $t < T$, and otherwise, never. Further, if $p = 1 - \mu$, $T_L(p) = \frac{\lambda_1 + \lambda_0}{\sigma}$. Then, the expected profit is

$$\left[\frac{1}{2}\Phi(x) + \frac{1}{2}\Phi(-x)\right] \cdot (1 - \mu - \kappa - K) = \frac{1 - \mu - \kappa - K}{2},$$

where $x = \frac{\lambda_0 - \lambda_1}{2\sigma}$.

Next, consider the case that $1-\mu > \underline{P}$. In this case, at $t = T$, only H type purchases the product.

By assumption, $K > \frac{\mu-\kappa}{2}$ implies that $\overline{P} > \mu > 1-\mu$, and as $1-\mu > K+\kappa$, Proposition 1 implies that an equilibrium exists and it is necessarily informative at $p = 1-\mu$. Further, the expected profit is

$$\begin{aligned} &\frac{1-\mu-\kappa}{2} \times \left[\mu \cdot \Phi\left(x+z'\right) + (1-\mu)\cdot\Phi(x) + (1-\mu)\cdot\Phi\left(-x+z'\right) + \mu\cdot\Phi(-x)\right] \\ &\qquad\qquad -\left[\mu\cdot\Phi\left(x+z'\right) + (1-\mu)\cdot\Phi\left(-x+z'\right)\right]K, \end{aligned} \tag{1}$$

where $z' = \frac{\sigma}{4x}\cdot\ln\left(\frac{\mu-\frac{K}{p-\kappa}}{\frac{K}{p-\kappa}-(1-\mu)}\right)$.

As $T > T_L(p) = \frac{\lambda_0+\lambda_1}{2}$, the expected profit (1) is greater than $\frac{1-\mu-\kappa-K}{2} > 0$. ∎

Proof of Proposition 3. By Claims 2 and 3, and assumption $1-\mu-\kappa > K$, a price that leads to an informative equilibrium yields a positive profit, which is larger than the profit of any uninformative equilibrium. Then, the optimal price leads to an informative equilibrium. ∎

A.4. Proofs in Section 5

Proof of Lemma 3. First note that for sufficiently small t, $\Pi_L^*(t) > \Pi_H^*(t)$. This is because for sufficiently small $t \approx -\infty$, $P_\theta(t) \approx 1$ and therefore $\Pi_L^*(t) \approx 1 > \mu \approx \Pi_H^*(t)$.

Let $a = \exp\left(\frac{\lambda_0-\lambda_1}{\sigma^2}\cdot\left(t-\frac{\lambda_0+\lambda_1}{2}\right)\right)$. Then, $P_\theta(t)$ is written as

$$P_\theta(t) = \frac{\varphi\left(\frac{t-\lambda_1}{\sigma}\right)\cdot\theta}{\varphi\left(\frac{t-\lambda_1}{\sigma}\right)\cdot\theta + \varphi\left(\frac{t-\lambda_0}{\sigma}\right)\cdot(1-\theta)} = \frac{1}{1+\frac{1-\theta}{\theta}\cdot a},$$

and also

$$\Pr(v=1\,|\,t) = \frac{1}{1+a}.$$

To simplify the notation, let denote $P(t) = \Pr(v=1\,|\,t)$. By differentiation, if $\lambda_0 > \lambda_1$, we have

$$P'_\theta(t) = -\frac{\lambda_0 - \lambda_1}{\sigma^2} \cdot \frac{1-\theta}{\theta} \cdot a \cdot (P_\theta(t))^2 < 0.$$

Note that we can write $P_H(t) = \frac{1}{1+\frac{1-\mu}{\mu}a}$ and $P_L(t) = \frac{1}{1+\frac{\mu}{1-\mu}a}$. This implies that $\frac{P_H(t)}{P_L(t)}$ and $\frac{P(t)}{P_L(t)}$ are increasing in t.

Now we calculate the ratio of the profits:

$$\begin{aligned} \frac{\Pi_H^*(t)}{\Pi_L^*(t)} &= (P(t)(2\mu-1) + (1-\mu))\frac{P_H(t)}{P_L(t)} > 1 \iff Z(a) > 0, \\ Z(a) &:= (1-\mu)\mu^2 a^2 - (2\mu-1)(1-(1-\mu)\mu)a - \mu(1-\mu)^2 > 0. \end{aligned} \tag{2}$$

Note that $Z(a)$ is a convex quadratic function of a, and $Z(0) < 0$. Also note that as $P_L(t) > K, a < \left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu}$. A calculation shows that $Z\Big(\left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu}\Big)$ is decreasing in μ, and let μ^{**} be the solution that $Z\Big(\left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu}\Big) = 0$. This implies that if $\mu < \mu^{**}$, $\Pi_H^*(t) < \Pi_L^*(t)$ for each t. On the contrary, if $\mu > \mu^{**}$, there is $\hat{t} < T_L(K)$ such that $\Pi_H^*(t) < \Pi_L^*(t)$ if and only if $t < \hat{t}$.

(c) As $T_L(K)$ is the solution of $P_L(t) = K$, That is,

$$T_L(K) = \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{\lambda_0 - \lambda_1} \cdot \ln\Big(\left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu}\Big).$$

Let μ^* be the solution to $\left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu} = 1$. This exists if $K < \frac{1}{2}$. Then, if $\mu < \mu^*$, $T_L(K) \to \infty$. In contrast, if $\mu > \mu^*$, $T_L(K) \to -\infty$. We can also verify that $Z\Big(\left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu^*}{\mu^*}\Big) < 0$, that is, $Z(1) < 0$. This implies that $\mu^* < \mu^{**}$. ■

Proof of Proposition 4. Let $a = \exp\Big(\frac{\lambda_0-\lambda_1}{\sigma^2}\cdot\Big[t-\frac{\lambda_1+\lambda_0}{2}\Big]\Big)$. Note that

$$P_\theta = \frac{1}{1+\frac{1-\theta}{\theta}a}$$

$$P_H(t)-P_L(t) = \frac{\frac{\mu}{1-\mu}-\frac{1-\mu}{\mu}}{\Big(1+\frac{\mu}{1-\mu}a\Big)\cdot\Big(1+\frac{1-\mu}{\mu}a\Big)}a,$$

$$-P'_\theta(t) = \frac{\lambda_0-\lambda_1}{\sigma^2}\frac{1-\theta}{\theta}\cdot a\cdot(P_\theta(t))^2 > 0.$$

$$\begin{aligned}\xi_L(t)-\xi_H(t) &= \frac{2\mu-1}{\sigma}\Big[\varphi\Big(\frac{t-\lambda_0}{\sigma}\Big)-\varphi\Big(\frac{t-\lambda_1}{\sigma}\Big)\Big]\\ &= \frac{2\mu-1}{\sigma}\cdot\varphi\Big(\frac{t-\lambda_0}{\sigma}\Big)\cdot\Big[1-\exp\Big(-\frac{\lambda_0-\lambda_1}{\sigma}\cdot\Big(t-\frac{\lambda_0+\lambda_1}{2}\Big)\Big)\Big]\end{aligned}$$

$$\xi_H(t) = \frac{1}{\sigma}\Big[\mu\cdot\varphi\Big(\frac{t-\lambda_1}{\sigma}\Big)+(1-\mu)\cdot\varphi\Big(\frac{t-\lambda_0}{\sigma}\Big)\Big].$$

As shown in the proof of Lemma 3, $\Pi^*_H(t)=\Pi^*_L(t)$ if and only if $Z(a)=0$. (Z is defined in (2)). Therefore, when we write $\hat{a}=\exp\Big(\frac{\lambda_0-\lambda_1}{\sigma^2}\cdot\Big(\hat{t}-\frac{\lambda_1+\lambda_0}{2}\Big)\Big)$, $Z(\hat{a})=0$. Note also that the effect of t on Z is summarized in a. Then, even $\sigma\to\infty$, $\hat{a}$ is a finite value, which implies that $\hat{t}\in\Theta(\sigma^2)$.[17] Further,

$$\frac{\xi_H(t)}{\xi_H(\hat{t})} = \frac{\mu\cdot\exp\Big((\hat{t}-t)\cdot\frac{\hat{t}+t-2\lambda_1}{2\sigma^2}\Big)+(1-\mu)\cdot\exp\Big((\hat{t}-t+\lambda_0-\lambda_1)\cdot\frac{\hat{t}+t-(\lambda_0+\lambda_1)}{2\sigma^2}\Big)}{\mu+(1-\mu)\cdot\exp\Big(\frac{1}{\sigma^2}\cdot(\lambda_0-\lambda_1)\cdot\Big(\hat{t}-\frac{\lambda_0+\lambda_1}{2}\Big)\Big)}.$$

As $\frac{\hat{t}}{\sigma^2}$ converges to a real number, for each t with $|t|<|\hat{t}|+z$ for some constant z, $\frac{\xi_H(t)}{\xi_H(\hat{t})}\to\infty$. In contrast, if $t<\hat{t}$, $\frac{\xi_H(t)}{\xi_H(\hat{t})}\to 0$ as $\hat{t}\to-\infty$. By Lemma 3, $\hat{t}\to-\infty$ as $\sigma\to\infty$. Then, dividing D by $\xi_H(\hat{t})$ yields that

$$\int^{\hat{t}}[-P'_L(t)]\frac{\xi_L(t)-\xi_H(t)}{\xi_H(\hat{t})}\,\mathrm{d}t-\int_{\hat{t}}^{T^*}[-P'_H(t)]\frac{\xi_H(t)}{\xi_H(\hat{t})}\,\mathrm{d}t+\big[P_H(\hat{t})-P_L(\hat{t})\big]. \tag{3}$$

[17] Θ is Landau symbol; that is $g(t)\in\Theta(f(t))$ implies that $\lim_{t\to\infty}\frac{g(t)}{f(t)}\in\mathbb{R}\setminus\{0\}$.

Note also that $\xi_L(t) < \xi_H(t)$ for sufficiently small t when $\lambda_0 > \lambda_1$. Also note that by $\frac{\mu}{2} > K$, $T^* \to \infty$ as $\sigma \to \infty$. Therefore, as $\sigma \to \infty$, the value of (3) diverges to $-\infty$. This shows that $D < 0$. ■

Proof of Proposition 5. Dividing $\tilde{D}$ by $\xi_H(T_*)$ yields that

$$\frac{\int^{T_*}[-P_L'(t)](\xi_L(t) - \xi_H(t))\,\mathrm{d}t}{\xi_H(T_*)} + [P_H(T_*) - P_L(T_*)] \tag{4}$$

As $\mu > \mu^*$, $\lim_{\sigma\to\infty} T_* = -\infty$ by Lemma 3. Therefore, the first term of (4) converges to

$$-P_L'(T_*)\frac{\xi_L(T_*) - \xi_H(T_*)}{\xi_H'(T_*)}.$$

Note that

$$\begin{aligned}\frac{\xi_L(T_*) - \xi_H(T_*)}{\xi_H'(T_*)} &= \frac{-(2\mu-1)\left(\exp\left(-\frac{1}{2}\left(\frac{T_*-\lambda_0}{\sigma}\right)^2\right) - \exp\left(-\frac{1}{2}\left(\frac{T_*-\lambda_1}{\sigma}\right)^2\right)\right)}{\mu\frac{T_*-\lambda_1}{\sigma^2}\exp\left(-\frac{1}{2}\left(\frac{T_*-\lambda_1}{\sigma}\right)^2\right) + (1-\mu)\frac{T_*-\lambda_0}{\sigma^2}\exp\left(-\frac{1}{2}\left(\frac{T_*-\lambda_0}{\sigma}\right)^2\right)} \\ &= \frac{(2\mu-1)\left(1-\exp\left(\frac{\lambda_0-\lambda_1}{\sigma^2}\left(T_* - \frac{\lambda_1+\lambda_0}{2}\right)\right)\right)}{\mu\frac{T_*-\lambda_1}{\sigma^2} + (1-\mu)\frac{T_*-\lambda_0}{\sigma^2}\exp\left(\frac{\lambda_0-\lambda_1}{\sigma^2}\left(T_* - \frac{\lambda_1+\lambda_0}{2}\right)\right)}\end{aligned}$$

As shown in Lemma 3, $\frac{T_*}{\sigma^2}$ converges to a finite value. Then, the above value converges to a finite value.

Note also that

$$-P_L'(t) = \frac{\lambda_0-\lambda_1}{\sigma^2}\frac{\mu}{1-\mu} \times \frac{1}{\frac{1}{a} + 2\frac{\mu}{1-\mu} + \left(\frac{\mu}{1-\mu}\right)^2 a} > 0.$$

$$a = \exp\left(\frac{\lambda_0-\lambda_1}{\sigma^2}\cdot\left(t - \frac{\lambda_1+\lambda_0}{2}\right)\right)$$

As $\frac{T_*}{\sigma^2}$ converges to a finite value, a is also a finite value. Then, $-P_L'(T_*) \to 0$. Then, (4) reduced to $[P_H(T_*) - P_L(T_*)]$, which is calculated as

$$P_H(t) - P_L(t) = \frac{\frac{\mu}{1-\mu} - \frac{1-\mu}{\mu}}{\left(1+\frac{\mu}{1-\mu}a\right)\left(1+\frac{1-\mu}{\mu}a\right)}a,$$

As a is a finite value, $\lim_{\sigma^2 \to \infty} P_H(T_*) - P_L(T_*) > 0$. This concludes that $\tilde{D} > 0$. ■

B. Informative protest

This section considers the case where σ is small enough. First, the following proposition shows a sufficient condition for the existence of an informative equilibrium when σ is small enough.

Proposition 6. *For sufficiently small σ, an informative equilibrium exists if $p \in \left(\underline{P}, \max\left\{\mu, \overline{P}\right\}\right)$.*

Consider an informative equilibrium. By Proposition 1 (1), $p < \max\left\{\overline{P}, \mu\right\}$. Suppose that $\lim_{\sigma \to 0} \frac{\lambda}{\sigma} = \gamma < \infty$. Then, note that for each $T \in \left\{T_H, T_L, T_f\right\}$,

$$\frac{T - \lambda_1}{\sigma} = \frac{\lambda}{\sigma} + \frac{\sigma}{\lambda}\frac{X}{2},$$

$$\frac{T - \lambda_0}{\sigma} = -\frac{\lambda}{\sigma} + \frac{\sigma}{\lambda}\frac{X}{2},$$

for some X, which is independent of σ and λ. This shows that T is a finite value. Then, $c_L > c_H$ and thus $\lambda > 0$ in the limit. However, this also implies $\frac{\lambda}{\sigma} \to \infty$, which is a contradiction. Therefore, $\lim_{\sigma \to 0} \frac{\lambda}{\sigma} = \infty$.

This implies that $\Phi\left(\frac{T(p) - \lambda_1}{\sigma}\right) \to 1$, $\Phi\left(\frac{T_L(p) - \lambda_1}{\sigma}\right) \to 1$, $\Phi\left(\frac{T_f(p) - \lambda_0}{\sigma}\right) \to 0$, and $\Phi\left(\frac{T_L(p) - \lambda_0}{\sigma}\right) \to 1$. In this case, the profit of informative equilibrium converges to $\frac{p - \kappa - K}{2}$.

Note that the profits of uninformative equilibria are at most $\max\left\{\frac{\mu - \kappa}{2} - K, 0\right\}$. In contrast, the supremum of the profit of informative equilibrium is at least $\frac{\max\left\{\overline{P}, \mu\right\} - \kappa - K}{2}$. Therefore, the optimal price is at least $\max\left\{\overline{P}, \mu\right\} - \varepsilon$ for small ε.

Note that when the protest is absent, the optimal price is $p = 1 - \mu$ when μ is small enough. We have the following observation.

Proposition 7. *Suppose that $1 - \mu - \kappa > \max\left\{0, \frac{\mu - \kappa}{2}\right\}$. Then, for sufficiently small σ, the price increases compared with the case when protest is absent.*

B.1. Proof of Proposition 6

We have two cases.

Case 1. Consider the case that $p \geq \overline{P}$, but $p < \mu$. As in the proof of Proposition 1,

$$c_L - c_H = \varsigma \cdot \frac{2\mu - 1}{\sigma} \cdot \left[\varphi\left(\tfrac{T_H(p)-\lambda_0}{\sigma}\right) - \varphi\left(\tfrac{T_H(p)-\lambda_1}{\sigma}\right)\right].$$

Let $z = \frac{\lambda}{\sigma}$. Then, for sufficiently small σ,

$$\frac{T_H(p) - \lambda_0}{\sigma} = -\frac{z}{2} + \frac{1}{2}\frac{1}{z} \cdot \ln\left(\frac{\mu}{1-\mu} - \frac{p}{1-\mu}\right)$$

$$\frac{T_H(p) - \lambda_1}{\sigma} = \frac{z}{2} + \frac{1}{2}\frac{1}{z} \cdot \ln\left(\frac{\mu}{1-\mu} - \frac{p}{1-\mu}\right).$$

Note that $p < \mu$, $\ln\left(\frac{\mu}{1-\mu} - \frac{p}{1-\mu}\right) > 0$, and then, by symmetry of φ, $\varphi\left(\frac{T_H(p)-\lambda_0}{\sigma}\right) > \varphi\left(\frac{T_H(p)-\lambda_1}{\sigma}\right)$.

Let $A = \ln\left(\frac{\mu}{1-\mu} - \frac{p}{1-\mu}\right)$. Then, we can write

$$B = \varphi\left(\tfrac{T_H(p)-\lambda_0}{\sigma}\right) - \varphi\left(\tfrac{T_H(p)-\lambda_1}{\sigma}\right) = \exp\left(-\tfrac{z^2}{8} - \tfrac{1}{z^2}\tfrac{A^2}{8}\right) \cdot \left(\exp\left(\tfrac{A}{4}\right) - \exp\left(-\tfrac{A}{4}\right)\right).$$

Now consider the following limit. Consider $\lambda = z\sigma$ with fixed z.

$$\begin{aligned}\lim_{z\to 0} \frac{(2\mu-1)(F(c_L) - F(c_H))}{\lambda} &= \frac{1}{\sigma}\frac{\mathrm{d}(2\mu-1)(F(c_L) - F(c_H))}{\mathrm{d}z} \\ &= \frac{1}{z}(2\mu - 1) \cdot f(0) \cdot \varsigma \cdot \frac{2\mu-1}{\sigma^2} \cdot \frac{1}{4} \cdot \left(-z + \frac{A^2}{z}\right) \cdot B.\end{aligned}$$

By considering $\sigma \propto (B)^{\frac{1}{2}}$, we can show that $\sigma \to 0$, $\left[\varphi\left(\frac{T_H(p)-\lambda_0}{\sigma}\right) - \varphi\left(\frac{T_H(p)-\lambda_1}{\sigma}\right)\right] \to 0$, and $\lim_{\sigma\to 0} \frac{(2\mu-1)\cdot(F(c_L)-F(c_H))}{\lambda} \to \infty$. Then, for such λ, $\frac{(2\mu-1)\cdot(F(c_L)-F(c_H))}{\lambda} > 1$. In contrast, when $\lambda = 2\mu - 1$, $(2\mu-1)\cdot(F(c_L) - F(c_H)) < 2\mu - 1$. Then, the intermediate value theorem implies the existence of λ such that $(2\mu - 1)\cdot(F(c_L) - F(c_H)) = \lambda > 0$. This shows the existence of an informative equilibrium for small σ.

Case 2. Consider the case that $p \in \left(\underline{P}, \overline{P}\right)$. As in the proof of Proposition 1,

$$c_L - c_H = \varsigma\left(\xi_L\left(T_f(p)\right) - \xi_H\left(T_f(p)\right)\right) + w\left[P_H\left(T_f(p)\right) - p\right]\xi_H\left(T_f(p)\right),$$

and

$$\varsigma\Big(\xi_L\big(T_f(p)\big) - \xi_H\big(T_f(p)\big)\Big) = \varsigma \cdot \frac{2\mu - 1}{\sigma} \cdot \Big[\varphi\Big(\tfrac{T_H(p)-\lambda_0}{\sigma}\Big) - \varphi\Big(\tfrac{T_H(p)-\lambda_1}{\sigma}\Big)\Big].$$

As $w\big[P_H\big(T_f(p)\big) - p\big]\xi_H\big(T_f(p)\big) > 0$ by $p < \overline{P}$, a similar discussion of case 1 proves the existence of an informative equilibrium. ■

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