

The Breakdown of Protest-Based Information Aggregation under Strategic Pricing and Precise Individual Signals*

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ABSTRACT

Consumer protests can serve as a mechanism for aggregating dispersed information about product-related externalities. This paper shows that protest turnout may become uninformative when signal precision increases and firms set prices strategically. In our model, a monopolistic firm sells a product with uncertain social cost. Consumers receive private binary signals and may protest to influence the firm's decision to continue or discontinue the product. Participation in a protest incurs a cost, and both the firm and consumers observe turnout with noise. The firm sets the price either before or after observing the turnout. We show that higher signal precision induces the firm to set a higher price, which screens out less-informed consumers but also homogenizes protest incentives among buyers. As a result, turnout fails to reveal meaningful information. This breakdown of information aggregation arises under both committed and uncommitted pricing. Our findings highlight a paradox: more informed consumers may undermine the informativeness of protest, limiting its effectiveness as a disciplinary device.

Keywords: Protest, information aggregation, ethical agents, monopoly pricing

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1. Introduction

Consumers frequently engage in protest campaigns against firms whose products are associated with negative externalities, such as labor abuses, environmental damage, or controversial political stances.¹ Despite their prevalence, the effectiveness of these campaigns in influencing firm behavior remains uncertain. While firms are not obliged to discontinue products based on protest participation, protests can still impact firm decisions by conveying information about consumer preferences and ethical concerns.

This paper develops a theoretical model of product protests and highlights a surprising result: when consumers receive highly precise private signals about a product's value, the informativeness of protests—as a mechanism for aggregating dispersed information—declines. As a result, more informed consumers can undermine the effectiveness of protest campaigns. This stands in contrast to the conventional wisdom in the information aggregation literature, where more precise individual signals are typically associated with more informative collective outcomes.

Our model features a monopolistic firm selling a product that may impose negative externalities. Consumers privately observe binary signals about the product's intrinsic value, which includes both its consumption utility and a moral disutility. Before purchasing, they may participate in a protest campaign to influence the firm's decision to discontinue the product. Participation incurs a cost, and both the firm and consumers observe protest turnout with noise. The firm then decides whether to continue selling the product. Crucially, the firm sets the product price either before or after observing protest turnout, and this decision critically shapes the protest's informativeness.

We show that when prices are high, consumers with both positive and negative signals derive similar marginal utility from discontinuing the product, leading to con-

¹In the real world, there are many protest activities aiming for product discontinuation, e.g., protests against Nike for labor violations (e.g., "Protesters call on NYU Bookstore to cut ties with Nike," *Washington Square News*, October 13, 2023), a threat against the publication of a book ("Japan firm nixes translation of U.S. book questioning trans surgery," *Japan Times*, December 6, 2023), and so on.

vergence in their protest incentives. This convergence undermines the informativeness of protest turnout.

The firm's incentive to set a high price when consumer signals are precise is also central to the mechanism. With more precise private signals, consumers with a favorable signal are more confident that the product is valuable and are thus willing to pay more. Anticipating this, the firm can profitably set a higher price that effectively screens out consumers with unfavorable signals. However, this pricing strategy flattens the marginal value of product discontinuation across signal types—since only high-signal consumers end up purchasing, their incentive to protest diminishes, and low-signal consumers have already self-selected out. As a result, protest turnout fails to reflect meaningful variation in underlying beliefs, rendering it uninformative. Nevertheless, an informative equilibrium exists when consumers have less precise signals.

We obtain this result under both committed and uncommitted pricing regimes. In the committed case, the firm sets the price before observing protest turnout, while in the uncommitted case, the price is chosen *ex post*. In both scenarios, increased signal precision reduces the informativeness of protests, although the underlying mechanisms differ.

Our main result is that signal precision and protest informativeness are negatively correlated, under both committed and uncommitted pricing regimes. This result differs sharply from Ekmekci and Lauermann (2022), who find that protests become more informative as signals become more precise in the context of political regimes without strategic price setting. In our setting, price serves as a selection mechanism that reduces heterogeneity in protest participation, thereby dampening information aggregation.

Our work adds to the growing literature on protest and information design. Direct evidence for our mechanism is still absent, yet Goya Foods' boycott in 2020 offers

suggestive support: prices remained unchanged or even rose because the firm could profit from a smaller pool of highly committed buyers, thereby limiting the campaign’s impact (see Liaukonytė et al., 2023). The parallel is clear: when the customer base consists mainly of high-WTP consumers, a firm can “price-screen” the market and neutralize the informational power of protest, exactly the channel our model formalizes.

From a policy perspective, our results suggest that unregulated pricing may undermine protests as an information-transmitting device. Regulating prices in ethically sensitive markets could help preserve the social value of consumer protest, especially in contexts where firm behavior depends on perceived consumer sentiment.

The rest of the paper is organized as follows: Section 2 presents the model. Section 3 analyzes equilibrium under fixed prices. Section 4 endogenizes the price-setting behavior. Section 5 considers an uncommitted pricing regime. Section 6 discusses model assumptions and concludes.

1.1. Related Literature

This paper contributes to three strands of literature: the theory of protest and information aggregation, economic models of consumer boycotts, and the modeling of ethical behavior in economic decision-making.

First, we relate to the literature on protest turnout as a signal of latent preferences or information. Lohmann (1993, 1994), Battaglini (2017), and Ekmekci and Lauermann (2022) analyze how collective action can aggregate dispersed information. However, these studies typically consider settings in which individual signals are independent of the market structure, as they consider situations in political contexts. In contrast, we show that in a private market with endogenous pricing, increased signal precision can reduce the informativeness of protest turnout. This arises because the firm uses pricing to screen consumers, which homogenizes participation incentives.

Second, our work contributes to the literature on consumer boycotts and protests (e.g., Baron, 2001, Feddersen and Gilligan, 2001, Baron, 2003, Innes, 2006, Miyagiwa, 2009, Glazer et al., 2010, Egorov and Harstad, 2017, Peck, 2017, and Correa, 2024). For example, Egorov and Harstad (2017) model boycotts as dynamic wars of attrition, while Diermeier and Van Mieghem (2008) and Delacote (2009) study boycotts as public goods games with fixed thresholds. Our approach differs in that it emphasizes boycotts as information transmission mechanisms and considers how pricing behavior endogenously interacts with the informativeness of protests.

Finally, we build on studies that incorporate ethical concerns and social norms into decision-making. Feddersen and Sandroni (2006) and Ali and Lin (2013) model ethical voting based on group utilitarianism and reputational concerns. Our model incorporates a similar structure by assuming that consumers behave based on group utilitarian, but with signal-dependent motivations that lead to strategic protest participation.

2. Model

A firm sells a product to a continuum of consumers with unit demand. The population is normalized to 1. The utility of purchasing the product is $w \cdot v$, where $v \in \{0, 1\}$. We refer to v as *product value* and w as the scale parameter, the role of which is discussed later. Implications of v include a psychological factor, such as feeling guilty about purchasing a product having a negative externality. We assume that v is unknown to all players. For instance, this uncertainty about v captures situations where a negative externality of the product has just been revealed, and people are uncertain about how to respond collectively. The prior probability of $v = 1$ is assumed to be $\frac{1}{2}$ for notational simplicity.

When the product is sold, it imposes a negative externality on consumers, leading to disutility $\varsigma > 0$. Consumers can engage in a protest campaign to stop the sale of

the product. Participating in the campaign costs c_i for consumer i . c_i is identically and independently distributed according to function F over \mathbb{R}_+ among consumers. We assume that f is the density of F , and $f(0) > 0$. In summary, when the product is sold at price $w \cdot p$, the consumer's utility is as follows.²

$$u_i = [w \cdot (v - p) \cdot \chi(\text{purchase}) - \zeta] \cdot \chi(\text{on sale}) - c_i \cdot \chi(\text{participate in protest}).$$

Before participating in the protest campaign, consumers receive a private signal about the product value v . $\theta \in \{\theta_H, \theta_L\}$ denotes the signal. θ_H suggests that $v = 1$ is more likely. We assume that $\Pr(\theta = \theta_H | v = 1) = \Pr(\theta = \theta_L | v = 0) = \mu \in (\frac{1}{2}, 1)$. μ is referred to as *signal precision*. The parameter μ captures the extent to which individuals tend to receive signals that reinforce a shared perception of the product's value.

In particular, μ reflects the *endogenous nature of social norms*: if μ is close to 1, individuals are likely to receive signals that align with the prevailing belief about v , reinforcing a strong consensus. Conversely, if μ is closer to $\frac{1}{2}$, there is greater ambiguity, and social norms surrounding the product's value are weaker.³

Given consumers' private signals, they decide whether to participate in the protest campaign. We assume that consumers act as **ethical agents**. Specifically, they behave according to a **rule-utilitarian** (or **group-utilitarian**) principle, choosing to protest if their participation increases the aggregate welfare of their group. This framework, which follows Feddersen and Sandroni (2006) and Ekmekci and Lauermann (2022), allows us to directly model the pivotal incentives central to collective action. The precise objective function is given in Section 3.2. A further discussion about assuming ethical agents is given in Section 6.

²For each event E , $\chi(E)$ takes the value 1 if E occurs, and 0 otherwise.

³An alternative interpretation is that v is *not merely an intrinsic product characteristic* but rather reflects a prevailing social norm shaped by majority opinion. If a fraction $\mu > \frac{1}{2}$ of individuals receive θ_L , the prevailing norm dictates that the product has no value ($v = 0$). Conversely, if a majority receives θ_H , the product is perceived as valuable ($v = 1$). This feature captures the endogenous nature of product value as determined by collective sentiment rather than objective characteristics.

One justification for this assumption is that a consumer's participation decision is observed by their social network, leading them to prefer being perceived as ethical. This, in turn, motivates them to act in a way that maximizes consumers' welfare.⁴

However, the adequacy of this justification depends on what participation in the protest entails. On the one hand, if participation involves attending an event organized by activists, the cost may be too high for this motivation to hold. On the other hand, if participation means sharing or liking a post on social media that expresses regret or protest against a firm's product, this motivation seems more plausible.⁵

After the protest campaign, the firm and consumers observe a public signal, indicating the protest campaign's scale. Let the actual participation ratio be τ . The firm and consumers observe $t = \tau + \varepsilon$, where ε is a normal noise with 0 mean and σ^2 variance. Let Φ be the standard normal distribution function and φ be its density. ε is interpreted as the noise created by activists.⁶

Observing the public signal t , the firm decides whether to discontinue the product. We assume that the firm incurs a cost of $wK > 0$ when continuing product sales. This includes the distribution cost and the reputation cost of selling a product that causes negative externalities. We also assume that the marginal cost of production is $w\kappa \geq 0$. Therefore, when the product is on sale, the firm's profit is

$$w \cdot [(\text{demand at } p) \cdot (p - \kappa) - K].$$

Finally, the decision flow is as follows.

1. The firm set a price.
2. Each consumer observes a private signal θ_i and decides whether to participate in the protest.

⁴Ali and Lin (2013) explores this rationale in the context of voting decisions.

⁵Sharing and liking posts on social media can be enough to compel a firm to discontinue a product. For instance, Adidas withdrew the sale of shoes resembling shackles in response to criticism on Facebook (Hoffberger, 2012, "Adidas cancels shackle shoe after Facebook backlash," Daily Dot (<https://www.dailymail.com/unclick/adidas-shackle-shoe-controversy/>), accessed August 1, 2024)).

⁶This assumption is the same as Ekmekci and Lauermaun (2022).

3. Turnout τ realizes, and the firm and consumers observe public signal $t = \tau + \varepsilon$.
4. The firm decides whether to continue the product sale.
5. If the product is on sale, each consumer decides whether to buy it.

The solution concept is the perfect Bayesian Nash equilibrium.

3. Equilibrium with a given price

3.1. After public signal observation

3.1.1. Consumer's choice

This section considers consumers' purchase decisions after public signal observation.

Consider the consumer receiving signal θ . The consumer purchases if the expected product value exceeds the price, that is, $\Pr(v = 1 | \theta, t) \geq p$.⁷ By abusing notation, we write $P_\theta(t) = \Pr(v = 1 | \theta, t)$. The expected value of the product value is

$$P_\theta(t) = \frac{\Pr(t | v = 1)\theta}{\Pr(t | v = 1)\theta + \Pr(t | v = 0)(1 - \theta)}.$$

Let λ_v be the true turnout when the realized value is $v \in \{0, 1\}$. Then, as $t = \lambda_v + \varepsilon$ at state v , $P_\theta(t)$ is rewritten as follows (recall that φ is the pdf of the standard normal distribution).

$$P_\theta(t) = \frac{\varphi\left(\frac{t - \lambda_1}{\sigma}\right)\theta}{\varphi\left(\frac{t - \lambda_1}{\sigma}\right)\theta + \varphi\left(\frac{t - \lambda_0}{\sigma}\right)(1 - \theta)}.$$

By using this formula, when we assume $\lambda_0 > \lambda_1$, inequality $P_\theta(t) > p$ is rewritten as

$$t < \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2 \ln\left(\frac{\theta}{1 - \theta}\right) - \ln\left(\frac{p}{1 - p}\right)}{\lambda_0 - \lambda_1} =: T_\theta(p).$$

In other words, $T_\theta(p)$ is the supremum value of public signal t with which the type- θ consumers purchase the product. The following lemma shows that the type- H consumers are more likely to purchase the product than the type- L consumers.

⁷As this decision is in the last stage of the game, the purchase decision does not affect others' utilities. Thus, each consumer's purchase decision is based only on their utility, although the consumers have welfare concerns.

Lemma 1. *If $\lambda_0 > \lambda_1$, $T_L(p) < T_H(p)$ for any p .*

3.1.2. The firm's choice

Given the consumer's choice observing public signal t , this section considers the condition under which the firm discontinues the product sale. We focus on a cutoff equilibrium; there is a cutoff $T_f(p)$ such that the firm discontinues the product sale if and only if $t > T_f(p)$. The cutoff is given by the following lemma (the proof is in the appendix).

Lemma 2. *Suppose that $T_L(p) < T_H(p)$, and let*

$$\tilde{T} := \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right)}{\lambda_0 - \lambda_1}.$$

Then, the cutoffs are classified as follows.

- (i) *If $p - \kappa \geq \frac{K}{1-\mu}$, $T_f(p) = T_H(p)$.*
- (ii) *If $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$, $T_f(p) = \max\{\min\{T_H(p), \tilde{T}\}, T_L(p)\}$.*
- (iii) *If $\frac{K}{\mu} \geq p - \kappa \geq K$, $T_f(p) = T_L(p)$.*
- (iv) *If $K > p - \kappa$, $T_f(p) = -\infty$.*

Let us see the intuition of this result. When p is higher, the firm is more likely to continue the sale as long as H -type consumers buy the product. However, when p or t is high so that H -type does not buy, the firm never continues the sale, and therefore, the cutoff of the firm is capped by H -type's cutoff. In contrast, if $p < \kappa + K$, the firm's profit is negative even if all consumers buy the product. Then, regardless of the realized public signal, the firm discontinues the sale (i.e., $T_f(p) = -\infty$).

3.2. Before public signal observation

Before the public signal observation, the consumer chooses whether to participate in the protest. This section examines the condition and its informativeness regarding product value.

We focus on the cutoff strategy: each type- θ consumer participates in protest if and only if the marginal benefit of participation exceeds its cost. Following Ekmekci and Lauermann (2022), we consider *ethical* agents: they maximize the aggregate utility of the same type of consumers written in the following formula.^{8,9}

$$\int^{T_f(p)-d\tau} \left(w \cdot [P_\theta(t) - p]_+ - \varsigma \right) \cdot \xi_\theta(t) dt,$$

where we define $[\cdot]_+ = \max\{\cdot, 0\}$ and $\xi_\theta(t) = \frac{1}{\sigma} \cdot \left(\theta \cdot \varphi\left(\frac{t-\lambda_1}{\sigma}\right) + (1-\theta) \cdot \varphi\left(\frac{t-\lambda_0}{\sigma}\right) \right) \cdot d\tau$ represents a marginal increase in the participation rate. In this formula, the firm applies a cutoff criterion: discontinuing the product if and only if $t > T_f(p)$. A consumer's participation increases turnout by $d\tau$, which also increases their costs by $c_i d\tau$. Then, consumer i participates in the protest if and only if

$$\underbrace{\left(\varsigma - w \cdot [P_\theta(T_f(p)) - p]_+ \right) \cdot \xi_\theta(T_f(p))}_{\text{marginal benefit of participation}} \geq c_i.$$

Now let $c_\theta = \left(\varsigma - w [P_\theta(T_f(p)) - p]_+ \right) \cdot \xi_\theta(T_f(p))$ be the cutoff of type- θ consumer. Then, we have that

$$\lambda_1 = \mu \cdot F(c_H) + (1 - \mu) \cdot F(c_L),$$

$$\lambda_0 = (1 - \mu) \cdot F(c_H) + \mu \cdot F(c_L).$$

Therefore, $\lambda := \lambda_0 - \lambda_1 = (2\mu - 1) \cdot (F(c_L) - F(c_H))$. This value plays a critical role in showing the existence of an informative equilibrium.

3.3. Equilibrium of the continuation game after the price is given

This section defines the equilibrium of the continuation game after the price is given and provides a characterization.

Definition 1. (T, c_H, c_L) is an *equilibrium* with given price p if

⁸Note that assuming this utility does not affect the purchasing decision, as it does not affect the other consumers' utility and the firm's decision.

⁹Our results continue to hold even when each consumer maximizes overall consumer welfare. See Section 6.

$$(i) \quad T = T_f(p)$$

$$(ii) \quad c_\theta = (\zeta - w \cdot [P_\theta(T) - p]_+) \cdot \xi_\theta(T).$$

Note that if $c_H = c_L$, which implies $\lambda_0 = \lambda_1$. Then, the public signal t is independent of v . We call this case *uninformative*. In other words, in an uninformative equilibrium, both types of consumers participate in the same cutoff, and therefore, the turnout does not reflect the true state of the world v .

Definition 2. An equilibrium is *informative* if $c_L > c_H$, and is *uninformative* if $c_L = c_H$.

It's important to note that the uninformative equilibrium exists for any parameters. If the firm sets $T = \infty$ or $T = -\infty$, the protest does not affect whether the product sale continues. This leads to both types of consumers having indifferent incentives. Conversely, if the protest is uninformative, the firm's decision remains unchanged regardless of the turnout. This implies that $T = \infty$ or $T = -\infty$.

Therefore, it is crucial to reiterate that our focus is on when an informative equilibrium exists. Our discussion is guided by the following proposition, which characterizes the condition concerning p for the existence of an informative equilibrium: An informative equilibrium exists only when the price is an intermediate value. This proposition is a key tool in our analysis (the proof is relegated to the Appendix).

Proposition 1. Let $\bar{P} := \mu \frac{K+(1-\mu)\kappa}{\mu(1-\mu)+(2\mu-1)K}$. (a) If $p \geq \max\{\mu, \bar{P}\}$ or $p \leq K + \kappa$, no informative equilibrium exists. (b) Suppose that $K + \kappa < p < \bar{P}$, $f(0) > 0$ and $F(0) > 0$. Then, if w is sufficiently large, an informative equilibrium exists.

Here, \bar{P} is a threshold of p : $T_H(p) > T_f(p)$ if and only if $p < \bar{P}$. Note that the consumers' participation decision is determined by the event that they are pivotal, that is, at the firm's cutoff point $T_f(p)$. $T_H(p) > T = T_f(p)$ implies that at the threshold of the firm, the H -type still has the incentive to purchase, and the utility is positive. This creates the difference in participation incentive between H - and L -type consumers. In

other words, the H -type consumers are more reluctant to participate in the protest. In contrast, if p is large enough, as Lemma 2 shows, $T_f(p) = T_H(p)$. This implies that the H -type's utility is 0 at the firm's cutoff point. Then, when the consumer is pivotal on the decision for the sale discontinuation, both H - and L -type consumers gain 0 utility from the product sale. This implies that the incentives of participating in the protest are the same for both H - and L -type consumers. In this case, turnout has no information about the type distribution, and therefore, the protest is uninformative.

For part (b), an informative equilibrium exists when an intermediate price creates a divergence in incentives. At such a price, an H -type consumer, when pivotal, still expects to gain positive surplus from purchasing the product, making them reluctant to protest. An L -type consumer expects no such surplus. This difference in expected utility sustains different protest cutoffs ($c_L > c_H$), allowing for information aggregation. When the scaling parameter w is sufficiently large, this utility difference becomes significant enough to guarantee the existence of an informative equilibrium.

4. Firm's optimal pricing

This section focuses on the most informative equilibrium for each price the firm sets and investigates when an informative equilibrium exists by endogenizing the price. All proofs of this section are relegated to the Appendix.

4.1. Benchmark

This section considers the model without protest as a benchmark. In the benchmark case, the H -type's expected product value is $E[v] = \mu$, and that of the L -type consumers is $E[v] = 1 - \mu$. If the firm keeps the product sale, the optimal price is $p = \mu$ or $p = 1 - \mu$. The expected profit when $p = \mu$ is $\frac{\mu - \kappa}{2} - K$, and that when $p = 1 - \mu$ is $1 - \mu - \kappa - K$.

4.2. Optimal price

Now, we consider the firm's optimal pricing with the protest. First, we define the firm's profit. As the firm keeps product sale unless $t < T_f(p)$, the firm's expected profit is

$$\begin{aligned}\Pi(p) = \sum_{v' \in \{0,1\}} \Pr(v = v') & \left[\left(\Phi\left(\frac{T_f(p) - \lambda_{v'}}{\sigma}\right) - \Phi\left(\frac{T_L(p) - \lambda_{v'}}{\sigma}\right) \right) \cdot (\mu(p - \kappa) - K) \right. \\ & \left. + \Phi\left(\frac{T_L(p) - \lambda_{v'}}{\sigma}\right) \cdot (p - \kappa - K) \right].\end{aligned}$$

The following proposition shows that the optimal price is $p = \mu$, in which case the protest is uninformative.

Proposition 2. *Suppose that $\frac{\mu - \kappa}{2} > \max\{K, 1 - \mu - \kappa\}$. Then, for sufficiently large $\sigma > 0$, $p = \mu$ is the optimal price and the protest is uninformative for any λ_0, λ_1 , and w .*

The intuition is as follows. As observed in the benchmark, the firm is incentivised to set a high price to target only H-type when μ is high. Then, as Proposition 1 (a) shows, the equilibrium is uninformative for such high p .

To see more detail, note that this result is irrelevant to the size of w as w affects the size of λ_1, λ_0 , but they are bounded by $[0, 1]$. Therefore, as Proposition 1 shows, an informative equilibrium exists if $p < \{\mu, \bar{P}\}$. An informative equilibrium also benefits the firm as it discontinues the product and saves fixed cost K if it observes high t , which implies $v = 0$ is likely enough. On the contrary, informative equilibrium has a cost: H-type may not buy the product. Although both the cost and benefit of informative equilibrium are small when σ is sufficiently large, we show that if $\mu > \kappa + 2K$, the cost of informative equilibrium is more severe than the gain from informative equilibrium at a neighbor of $p = \mu$. Then, rather than gaining information by setting a lower price, $p = \mu$ yields a greater profit.

The above observation depends on whether an uninformative equilibrium with $p = \mu$ yields a sufficient profit. If not, an informative equilibrium leads to a higher profit, as the following proposition shows.

Proposition 3. *Suppose that $1 - \mu - \kappa > K > \frac{\mu - \kappa}{2}$. Then, for each σ , there is \bar{w} such that for each $w > \bar{w}$, the protest is informative at the firm's optimal price.*

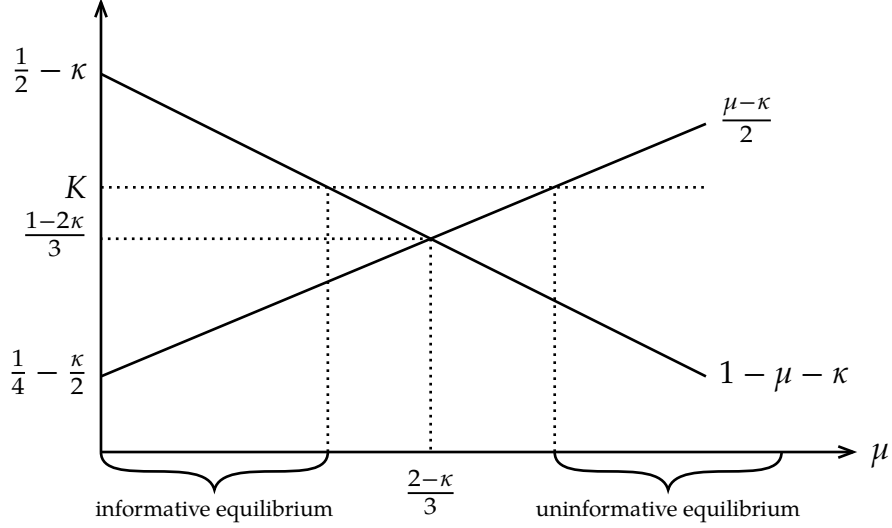


Figure 1: Classification of equilibria

With the condition of Proposition 3, while setting the price to $p = \mu$ yields a deficit, setting a low price inducing an informative equilibrium yields a positive profit. Therefore, the firm prefers to set a low price.

By Propositions 2 and 3, we observe that the resulting equilibrium is uninformative if μ is high, while it is informative when μ is small. Theorem 1 summarizes the result, and Figure 1 illustrates it.

Theorem 1. Suppose that $\kappa < \frac{1}{2}$ and $K \in (\frac{1-2\kappa}{3}, \frac{1}{2} - \kappa)$. Take sufficiently large σ and w . Then, there exists $\mu^*, \bar{\mu}, \underline{\mu}$ with $\mu^* > \bar{\mu} > \underline{\mu}$ such that the equilibrium is uninformative if $\mu \in (\bar{\mu}, \mu^*)$,¹⁰ and the equilibrium is informative if $\mu < \underline{\mu}$.

This result implies that the informativeness of individual consumers' signals may negatively correlate with the protest's informativeness. This contrasts sharply with Ekmekci and Lauermann (2022), who shows that each person's signal informativeness positively correlates with the protest campaign's informativeness.

The main difference between their and our models is the existence of a price-setting firm. In the models of Ekmekci and Lauermann (2022) and many protest studies, citizens campaign against the policy-setting government, which affects only the policy

¹⁰The restriction $\mu < \mu^*$ is needed because Proposition 2 is the statement when μ is fixed.

decision. However, the firm makes two-dimensional decisions in our model: whether to discontinue the product sale and price. With a higher informativeness of each individual's signal, consumers are more willing to pay, which leads to a greater profit for the firm by setting a higher price. This makes the protest uninformative by reducing the purchase incentive of H -type.

4.3. Welfare Analysis

In this discussion, we examine social welfare, which is the sum of the firm's profit and consumer surplus. When $v = 1$, the maximized social welfare is $1 - \kappa - K - \zeta$, provided this value is positive. When $v = 0$, the maximized social welfare is 0. This welfare level is achieved when v is known at any $p \in (\kappa, 1)$. Specifically, if $v = 1$, the product is available for sale, and all consumers purchase it at the given price. On the other hand, if $v = 0$, since no consumers buy the product, the firm discontinues the sale.

We refer to $\lambda_0 - \lambda_1$ as the **informativeness** of the protest.¹¹ As this value increases, the probability of product purchase decreases when $v = 0$, and increases when $v = 1$. Thus, the greater informativeness of the protest led to an improvement in social welfare.

Our findings suggest that as the consumers' private signals become precise, social welfare can decrease. The following corollary is a summary.

Corollary 1. *Suppose that the same assumption as Theorem 1 holds. Then, social welfare is higher when $\mu < \underline{\mu}$ than when $\mu \in (\bar{\mu}, \mu^*)$ for sufficiently large w and σ .*

5. Uncommitted price

In the previous section, the firm is supposed to commit to a price p before the public signal is observed. This section considers the case in which the firm decides the price after observing the public signal. Therefore, the new decision flow is as follows:

¹¹This definition is the same as Ekmekci and Lauermann (2022).

1. Each consumer observes a private signal θ_i and decides whether to participate in the protest.
2. Turnout τ realizes, and the firm and consumers observe public signal $t = \tau + \varepsilon$.
3. The firm decides whether to continue selling the product, and if so, sets a price.
4. If the product is on sale, each consumer decides whether to buy it.

A crucial difference to the committed price case is that the turnout affects the price in the uncommitted case. With a greater turnout, if it is informative, it reduces the expected value of the product. Then, the firm sets a lower price.¹² Then, protest participation has two effects on type- H consumers' utility. On one hand, a protest threatens to eliminate their potential informational rents, discouraging participation. On the other hand, a larger protest can signal a lower product value to the firm, inducing it to set a lower price. This "price manipulation" motive encourages H -type participation. The overall effect on equilibrium informativeness depends on the relative strength of these competing incentives.

Now, we investigate how our conclusion changes. The previous section shows that with higher μ , the equilibrium is less likely to be informative. We show that even in the uncommitted price case, the same conclusion is drawn. The following is a formal analysis. All proofs of the following section are relegated to the Appendix.

5.1. Formal analysis of the uncommitted price case

To focus on the discussion on whether an informative equilibrium exists, we assume that $\lambda_0 > \lambda_1$. Then, we verify the consumers' incentives to participate in the protests. Note that even in the uncommitted price case, after the price is set, the consumers' decision criterion for purchasing the product is unchanged. Thus, type- θ consumer purchases the product if and only if $t < T_\theta(p)$.

Given this strategy, the firm's profit is

¹²Indeed, protests tend to decrease prices. Hendel et al. (2017) empirically demonstrated that price declines after a consumer boycott, organized in Israel on Facebook.

$$\Pi = \begin{cases} -K & \text{if } t > T_H(p) \\ [\Pr(v = 1 | t) \cdot \mu + \Pr(v = 0 | t) \cdot (1 - \mu)] \cdot (p - \kappa) - K & \text{if } t \in (T_L(p), T_H(p)] \\ p - \kappa - K & \text{if } t \leq T_L(p). \end{cases}$$

This implies that the optimal price is either $p = P_H(t)$ or $P_L(t)$. Therefore, we need to compare the following equations.

$$\Pi_H^*(t) := [\Pr(v = 1 | t) \cdot (2\mu - 1) + 1 - \mu] \cdot (P_H(t) - \kappa) - K$$

$$\Pi_L^*(t) := P_L(t) - \kappa - K.$$

$\Pi_H^*(t)$ represents the profit when the firm focuses solely on H-type consumers, while $\Pi_L^*(t)$ indicates the profit when targeting both consumer types. Also, the firm discontinues the product sales rather than selling it with $p = (T_L)^{-1}(t)$ if $p - \kappa < K$. In other words, $(T_L)^{-1}(t) < K + \kappa$. Equivalently, $t > T_L(K + \kappa)$.

To simplify the discussion, we assume $\kappa = 0$ hereafter. The following lemma identifies the condition under which one profit exceeds the other.

Lemma 3. Assume that $\kappa = 0$. Then, there exist values μ^* and μ^{**} with $\mu^* < \mu^{**}$ such that:

(a) If $\mu > \mu^{**}$, there is $\hat{t} < T_L(K)$ such that $\Pi_H^*(t) < \Pi_L^*(t)$ if and only if $t < \hat{t}$. (b) If $\mu < \mu^{**}$, $\Pi_H^*(t) < \Pi_L^*(t)$ for each $t < T_L(K)$. (c) $T_L(K) \rightarrow \infty$ if $\mu < \mu^*$, and $T_L(K) \rightarrow -\infty$ if $\mu > \mu^*$.

Given this firm's behavior, we consider the consumers' incentive to participate in the protest. We first consider the case (a): there is $\hat{t} < T_L(K)$ such that $\Pi_L^*(t) > \Pi_H^*(t)$ if and only if $t < \hat{t}$. Let T^* be the solution to $\Pi_H^*(t) = 0$ with respect to t . This implies that at the optimal strategy, the firm sets price $P_L(t)$ when $t \in (-\infty, \hat{t})$, sets price $P_H(t)$ when $t \in (\hat{t}, T^*)$, and when $t > T^*$, discontinues the product sale.

Under this firm's behavior, the expected utility of the H-type is

$$\begin{aligned} & -\varsigma \cdot \int^{T^*-d\tau} \zeta_H(t) dt + w \int_{\hat{t}-d\tau}^{T^*-d\tau} [P_H(t) - P_H(t + d\tau)] \zeta_H(t) dt \\ & + w \cdot \int^{\hat{t}-d\tau} [P_H(t) - P_L(t + d\tau)] \cdot \zeta_H(t) dt. \end{aligned}$$

Here, $P_H(t)$ is the expected value of the product while $P_L(t)$ is the price. A marginal increase of turnout $d\tau$ affects the price. As the firm set price $p = P_H(t)$ if $t > \hat{t}$, type- H consumers' the informational rent $P_H(t) - P_L(t)$ vanishes. Therefore, the decision to discontinue the product only affects the externality term ς .

The differentiation concerning $d\tau$ is the marginal gain by participation. Therefore, consumer i participates in the protest if and only if $c_H \geq c_i$, where

$$c_H := \varsigma \cdot \xi_H(T^*) + w \cdot \left[\int_{\hat{t}}^{T^*} [-P'_H(t)] \xi_H(t) dt - [P_H(\hat{t}) - P_L(\hat{t})] \xi_H(\hat{t}) + \int^{\hat{t}} [-P'_L(t)] \xi_H(t) dt \right].$$

Focus on the second line in the above equation. Note that $P'_L(t) < 0$ and $P'_H(t) < 0$. Then, the first and third terms are the gain from price manipulation by increasing turnout. In contrast, with a high turnout, the firm gives up on making all types of consumers purchase the product, and then, type- H 's informational rent vanishes. This makes type H consumers reluctant to participate in the protest, captured by the second term.

Now we compare with L -type consumers' utility. The utility of the L -type is $-\varsigma \cdot \int^{T^*-d\tau} \xi_L(t) dt + w \cdot \int^{\hat{t}-d\tau} [P_L(t) - P_L(t+d\tau)] \cdot \xi_L(t) dt$ as the firm extracts type- L consumers' full surplus. Then, the type- L consumers participate in the protest if and only if

$$c_L := \varsigma \cdot \xi_L(T^*) + w \cdot \int^{\hat{t}} [-P'_L(t)] \xi_L(t) dt \geq c_i.$$

We compare the relation between c_H and c_L with high μ cases. As we focus on a large w case, we consider the coefficients of w in $c_L - c_H$, that is

$$D := \int^{\hat{t}} [-P'_L(t)] (\xi_L(t) - \xi_H(t)) dt - \int_{\hat{t}}^{T^*} [-P'_H(t)] \xi_H(t) dt + [P_H(\hat{t}) - P_L(\hat{t})] \xi_H(\hat{t}).$$

Proposition 4. Suppose that $\mu > \max\{\mu^{**}, 2K\}$. Then, for sufficiently large σ , $D < 0$.

Proposition 4 implies that the protest becomes uninformative with high μ . This is because this inequality $c_H > c_L$ is gained by the supposition that $\lambda_0 > \lambda_1$, which means that $c_L > c_H$. This is a contradiction. So, the equilibrium should be uninformative. This is a similar feature to the committed price case. The intuition is the following. With a high μ , the firm continues to sell even when t is sufficiently high, in which case, while the L-type gives up to purchase, the H-type purchases. Then, the H-type's incentive for price manipulation is much higher than that of the L-type. In contrast, the effect on the informational rent is small as the firm is likely to focus on selling only to the H-types when μ is high enough. This motivates the H-type to participate in the protest more than the L-type.

Next, we consider the case (b): for each $t < T_L(\kappa + K)$, $\Pi_L^*(t) > \Pi_H^*(t)$. This implies that for any $t < T_L(\kappa + K)$, the firm set price $P_L(t)$ and discontinues the product sale if $t > T_L(\kappa + K)$. To simplify the notation, we denote $T_* = T_L(\kappa + K)$.

In this case, the utility of the H-type is

$$-\varsigma \int^{T_* - d\tau} \xi_H(t) dt + w \int^{T_* - d\tau} [P_H(t) - P_L(t + d\tau)] \xi_H(t) dt.$$

The differentiation concerning $d\tau$ is the marginal gain by participation. Therefore, they participate in the protest if and only if

$$c_H := \varsigma \xi_H(T_*) - w[P_H(T_*) - P_L(T_*)] \xi_H(T_*) + w \int^{T_*} [-P'_L(t)] \xi_H(t) dt \geq c_i.$$

Similarly, the utility of the L-type is $-\varsigma \int^{T_* - d\tau} \xi_L(t) dt + w \int^{T_* - d\tau} [P_L(t) - P_L(t + d\tau)] \xi_L(t) dt$. Then, they participate in the protest if and only if

$$c_L := \varsigma \xi_L(T_*) + w \int^{T_*} [-P'_L(t)] \xi_L(t) dt \geq c_i.$$

We also compare c_H with c_L and thus, focus on the coefficient of w in $c_L - c_H$,

$$\tilde{D} := \int^{T_*} [-P'_L(t)] (\xi_L(t) - \xi_H(t)) dt + [P_H(T_*) - P_L(T_*)] \xi_H(T_*).$$

Proposition 5. *Suppose that $\mu \in (\mu^*, \mu^{**})$. Then, for sufficiently large σ , $\tilde{D} > 0$.*

$c_L > c_H$ implies that the H-type is likelier to participate in the protest. Then, the equilibrium becomes informative with a large w . Intuition is the following. With a lower μ , the firm gives up selling even with a small t . Then, the incentive for price manipulation becomes smaller, and the effect of losing informational rent dominates that incentive.

In conclusion, the informative equilibrium is more likely with intermediate μ cases, although not in high μ cases. Our conclusion gained in the committed price case remains: precise information that consumers gain does not contribute to the success of the protest.

6. Conclusion and Discussions

This study examines when a protest campaign successfully aggregates information about consumers' values for a product, which determines the firm's decision. We show that consumers' individual signal precision does not necessarily contribute to the protest's informativeness. One implication of our result is that consumers' ignorance may contribute to the success of protest campaigns.

In the remainder of this study, we discuss the assumptions in our model.

Objective of participation in protest: We assumed that consumers consider their type's aggregate utility when deciding whether to participate in the protest. However, as in Ali and Lin (2013), if this motivation stems from a reputation concern, it is more plausible to assume that consumers consider the expected overall consumer welfare.

Consider a scenario where each consumer's decision to participate in the protest is based on maximizing the expected consumer welfare. The consumer's belief, represented by $\theta \in \{\theta_H, \theta_L\}$, influences their expected consumer welfare as follows.

$$\int^{T_f(p) - d\tau} \left[\theta (w \cdot [P_\theta(t) - p]_+ - \varsigma) \xi_\theta(t) + (1 - \theta) (w \cdot [P_{\theta'}(t) - p]_+ - \varsigma) \xi_{\theta'}(t) \right] dt,$$

where $\theta' \in \{\theta_H, \theta_L\}$ with $\theta' \neq \theta$. In this case, the H-type participates in the protest if and only if $c_i \leq c_H^* := \mu c_H + (1 - \mu) c_L$ while the L-type does if and only if $c_i \leq c_L^* :=$

$(1 - \mu)c_H + \mu c_L$. As long as $c_L > c_H$, the cutoff point is lower for the L-type consumers. We can apply a similar discussion to obtain the same conclusions.

The “ethical” agent assumption: A potential concern is our assumption that consumers act as ‘ethical agents’ when deciding whether to protest. We argue that this assumption, while a simplification, allows for a clear analysis of our main contribution without loss of generality for the core economic trade-off.

First, this framework is a standard and convenient method for directly modeling pivotal incentives, which are central to any model of collective action. The main driver of our results is not the ethical motivation itself, but how the firm’s pricing strategy alters the consumers’ expected utility of being pivotal. By homogenizing the incentives of high-signal and low-signal consumers at the firm’s decision threshold, the firm’s pricing, not the specific form of consumer motivation, renders the protest uninformative.

Second, we conjecture that our results are robust and would hold in a setting with purely self-interested consumers. Following the approach in Battaglini (2017) and Ekmekci and Lauermann (2022), one could model a finite n -player game where each agent strategically calculates their probability of being pivotal. However, the key distinction and contribution of our paper is the introduction of a strategic price-setting firm. In such a model, consumers’ calculations would have to incorporate the firm’s pricing rule, which depends on the informativeness of the protest. The central mechanism we identify—that high signal precision incentivizes the firm to set a high price, which in turn screens the market and dampens information aggregation—would therefore remain. Consequently, the fundamental paradox highlighted in this paper is expected to persist.

Concerns regarding negative externalities: In our model, utility from the product purchase plays an important role, but concern regarding the negative externality itself

does not. If the concern regarding the negative externalities is sufficiently large, the participation rates of H- and L-type consumers converge, making the protest uninformative. This observation holds even when concerns about the externality vary among consumers since the firm does not care about the externality itself. However, if the product value includes guilty feelings about consuming a product causing negative externalities, it would be correlated with the level of concern for the externality. In this case, our conclusion remains true even if concerns about negative externality are high.

Noisy campaign: We have assumed that noise follows a normal distribution with sufficiently high variance. Considering a sufficiently high noise is crucial for the committed price case. The firm benefits from an informative equilibrium if the noise variance is small enough. The firm can save the fixed cost K when $v = 0$. In this case, if $v = 0$, the firm discontinues the sale. If $v = 1$, the optimal price can be high but less than μ for the existence of an informative equilibrium. An interesting observation is that if μ is small enough, the protest can make the price higher. To see this, consider the case where the protest cannot occur for some external reason. Then, the price is set so that both H- and L-type consumers buy when μ is small enough. In contrast, when a protest is informative about the popularity, the firm continues the sale if and only if τ is small enough. When the noise is small enough, this signals that the value of the product is high. Then, the firm sets a higher price when the protest can occur. A formal analysis is relegated to Section B in the appendix.

Another observation is that although the protest revealed almost complete information, the price is bounded above by $\max\{\bar{P}, \mu\}$, to make the protest informative, by Proposition 1. This observation is important because it seems optimal for the firm to set a price close to 1 as the protest reveals whether $v = 1$ or $v = 0$. In fact, if doing so, the H-type's motivation becomes close to the L-type's, and the protest becomes uninformative. Note also that \bar{P} is increasing in μ when κ is small enough. Then, even

if the individual signal precision is high, it does not necessarily benefit the consumers, as it only increases the price.

Modeling of a Protest: Our model focuses on protest as an expressive act, distinct from the purchase decision itself (a boycott). This focus is motivated by the many contemporary consumer campaigns where public expression on platforms like social media is a central feature. Our contribution is to show how information aggregation can fail at this expressive stage. A richer model could integrate both channels—expression and purchasing—to study their dynamic interplay.

Mechanism Design by the Firm: In our model, the firm’s pricing strategy is limited to setting a price. While a firm can indeed use its price-posting mechanism to elicit information, our analysis reveals that the two mechanisms of information —protest and pricing— are in direct conflict. The firm’s profit-maximizing use of its pricing power to screen consumers is precisely what destroys the informational value of the protest. This tension between the private incentive to extract surplus and the collective goal of information aggregation lies at the heart of our findings and remains a fruitful area for further exploration.

Furthermore, there is a fundamental limitation to using a price-posting mechanism for information elicitation related to the timing of information. In our model, the firm must incur a cost K to continue selling the product. Therefore, any strategy to learn from realized demand requires the firm to take a risk for this cost. If the product value turns out to be low ($v = 0$), the information arrives ‘too late,’ only after the cost K has been sunk.

In contrast, the protest mechanism provides a signal before this irreversible cost is paid. An informative protest, therefore, serves as a valuable, ex-ante signal that allows the firm to avoid incurring a loss, a benefit that a post-entry ‘price experiment’ cannot

offer. This highlights the unique economic importance of protests as a pre-commitment signaling device, justifying our focus on their information aggregation properties.

Appendix

A. Proofs

Proof of Lemma 2. To simplify the notation, in this proof, we use T as the firm's cutoff. Note that the cutoff depends on the consumers' behavior. Therefore, we consider the cases (i) $t \geq T_H(p)$, (ii) $t \in (T_L(p), T_H(p))$, and (iii) $t \leq T_L(p)$.

(i) Suppose that $t \geq T_H(p)$. Note that if $t > T_H(p)$, the firm discontinues the sale as no consumer purchases the product, which leads to a deficit. Consider the case $t = T_H(p)$. Then, as only type H consumers purchase the product, the firm's profit is

$$\Pi_H := w \cdot ([\Pr(v = 1 | T) \cdot \mu + \Pr(v = 0 | T) \cdot (1 - \mu)] \cdot (p - \kappa) - K).$$

We can show the following claim:

Claim 1. (i) If $p - \kappa > \frac{K}{1-\mu}$, $\Pi_H > 0$. (ii) If $\frac{K}{\mu} > p - \kappa$, $\Pi_H < 0$. (iii) If $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$, $\Pi_H > 0$ if and only if $t < \tilde{T}$, where

$$\tilde{T} := \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right)}{\lambda_0 - \lambda_1}.$$

Therefore, $T = T_H(p)$ if either (a) $p - \kappa > \frac{K}{1-\mu}$, or (b) $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$ and $\tilde{T} \geq T_H(p)$. Otherwise, at $t = T_H(p)$, the profit is negative, and therefore, the cutoff is below $T_H(p)$.

(ii) Suppose that $T_L(p) < t < T_H(p)$. Then, at $t = T$, only the H-type consumer purchases the product. In this case, the firm's profit is Π_H . Therefore, if $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$ and $\tilde{T} \in (T_L(p), T_H(p))$, $T = \tilde{T}$. Otherwise, there is no t such that $\Pi_H = 0$.

(iii) Suppose that $t \leq T_L(p)$. Then, the firm's profit is

$$P_L := w[(p - \kappa) - K].$$

Therefore, $P_L \geq 0$ if and only if $p - \kappa \geq K$.

The remaining cases to consider the cutoff point are the following.

(a) $p - \kappa \geq \frac{K}{\mu}$ and $T_L(p) \geq \tilde{T}$. Then, the profit is positive if and only if $t < T_L(p)$, the cutoff is $T_L(p)$.

(b) $\frac{K}{\mu} > p - \kappa \geq K$. Then, the profit is positive if and only if $t < T_L(p)$, the cutoff is also $T_L(p)$.

(c) $K > p - \kappa$. Then, the profit is negative for any t . Therefore, the firm never continues the sale; in other words, the cutoff is $-\infty$.

In summary, we have the following:

(i) If $p - \kappa > \frac{K}{1-\mu}$, $T = T_H(p)$.

(ii) If $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$, $T = \max\{\min\{T_H(p), \tilde{T}\}, T_L(p)\}$.

(iii) If $\frac{K}{\mu} > p - \kappa \geq K$, $T = T_L(p)$.

(iv) If $K > p - \kappa$, $T = -\infty$. ■

A.1. Proof of Proposition 1

Before the proof of Proposition 1, we prepare the following lemma.

Lemma 4. Define \underline{P} be the largest solution to $\frac{1-\mu}{\mu} \frac{1-p}{p} = \frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}$ with respect to p . Then, $T_f(p) = T_H(p)$ if and only if $p > \bar{P}$ and $T_f(p) > T_L(p)$ if and only if $p > \underline{P}$. Further, $\frac{K}{1-\mu} + \kappa > \bar{P} > \underline{P} > \frac{K}{\mu} + \kappa$.

Proof of Lemma 4. (a) If $p \geq \frac{K}{1-\mu} + \kappa$, $T_f(p) = T_H(p)$ by Lemma 2. If $p < \frac{K}{1-\mu} + \kappa$, $\tilde{T} < T_H(p)$ if and only if $\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)} < \frac{\mu}{1-\mu} \frac{1-p}{p}$, which is equivalently $p > \bar{P}$. Further, a simple calculation shows $\frac{K}{1-\mu} + \kappa > \bar{P}$.

(b) If $p < \frac{K}{\mu} + \kappa$, by Lemma 2, $T_f(p) = -\infty$. Consider the case where $p \geq \frac{K}{\mu} + \kappa$. Then, $T_f(p) = \max\{T_L(p), \tilde{P}\}$. Note that $T_L(p) < \tilde{T}$ if and only if $\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)} > \frac{1-\mu}{\mu} \frac{1-p}{p}$, equivalently, $g(p) > 0$, where g is a convex quadratic function (illustrated in Figure 2). Note that $g(p) < 0$ at $p = \frac{K}{\mu} + \kappa$. Then, $T_L(p) < T_f(p)$ if and only if $p > \underline{P}$. ■

Proof of Proposition 1. (1) Suppose that $p \geq \max\{\bar{P}, \mu\}$.

If $p \geq \bar{P}$, by Lemma 4, $T_f(p) = T_H(p)$. In this case, the equilibrium condition is

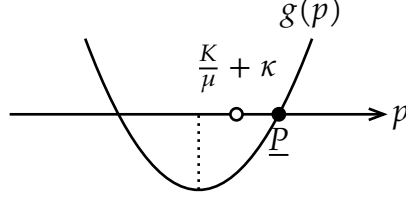


Figure 2: Illustration of g

$$\lambda_0 - \lambda_1 = (2\mu - 1) \cdot (F(c_L) - F(c_H)),$$

$$c_\theta = \varsigma \cdot \xi_\theta(T_H(p)),$$

$$T_H(p) = \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2 \ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{p}{1-p}\right)}{\lambda_0 - \lambda_1}$$

$$\xi_\theta(T_H(p)) = \frac{1}{\sigma} \cdot \left[\theta \cdot \varphi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right) + (1 - \theta) \cdot \varphi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right) \right].$$

By $p \geq \mu$, $\frac{\sigma^2}{2} \cdot \frac{\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{p}{1-p}\right)}{\lambda_0 - \lambda_1} \leq 0$ when $\lambda_0 \geq \lambda_1$. As φ is symmetric and single-peaked around 0, we show that $\varphi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right) \leq \varphi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right)$. Note also that

$$c_L - c_H = \varsigma \cdot \frac{2\mu - 1}{\sigma} \cdot \left[\varphi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right) - \varphi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right) \right] \leq 0.$$

Therefore, $\lambda_0 \geq \lambda_1$ implies $c_L \leq c_H$. In contrast, $c_L \leq c_H$ implies that $\lambda_0 \leq \lambda_1$. This shows that $c_H = c_L$ holds at any equilibrium. In other words, no informative equilibrium exists.

(2) Consider the case $p \in (\underline{P}, \bar{P})$. Then, if $\lambda_0 > \lambda_1$, $T_L(p) < T_f(p) < T_H(p)$ for any c_H, c_L .

Recall that the following equations characterize the equilibria:

$$\lambda := \lambda_0 - \lambda_1 = (2\mu - 1) \cdot (F(c_L) - F(c_H)),$$

$$c_\theta = \left(\varsigma - w \cdot [P_\theta(T_f(p)) - p]_+ \right) \cdot \xi_\theta(T_f(p)),$$

$$T_f(p) - \lambda_1 = \frac{\lambda}{2} + \frac{\sigma^2}{2} \cdot \frac{1}{\lambda} \cdot \ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right),$$

$$T_f(p) - \lambda_0 = -\frac{\lambda}{2} + \frac{\sigma^2}{2} \cdot \frac{1}{\lambda} \cdot \ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right),$$

$$P_\theta(T_f(p)) = \frac{\varphi\left(\frac{T_f(p)-\lambda_1}{\sigma}\right) \cdot \theta}{\varphi\left(\frac{T_f(p)-\lambda_1}{\sigma}\right) \cdot \theta + \varphi\left(\frac{T_f(p)-\lambda_0}{\sigma}\right) \cdot (1-\theta)},$$

$$\xi_\theta(T_f(p)) = \frac{1}{\sigma} \cdot \left[\theta \cdot \varphi\left(\frac{T_f(p)-\lambda_1}{\sigma}\right) + (1-\theta) \cdot \varphi\left(\frac{T_f(p)-\lambda_0}{\sigma}\right) \right].$$

This shows that c_L and c_H depend on λ , but not on λ_0, λ_1 individually. Therefore, the equilibrium is a fixed point of a self-map $\zeta : \lambda \mapsto (2\mu - 1)(F(c_L) - F(c_H))$. We can easily verify that ζ is continuous.

Below, we show that $\zeta(\lambda) > \lambda$ for some $\lambda > 0$ when w is large enough. By the definition of \bar{P} and \underline{P} , $P_H(T_f(p)) > p$, and $p > P_L(T_f(p))$. This implies that

$$c_H = (\varsigma - w[P_H(T_f(p)) - p]) \cdot \xi_H(T_f(p))$$

$$c_L = \varsigma \cdot \xi_L(T_f(p)) > 0.$$

Then,

$$c_L - c_H = \varsigma \cdot (\xi_L(T_f(p)) - \xi_H(T_f(p))) + w \cdot [P_H(T_f(p)) - p] \cdot \xi_H(T_f(p)),$$

which is positive for large w . Note that as $\lambda \rightarrow 0$, $T_f(p) \rightarrow \infty$, which implies that $c_L \rightarrow 0$ and $c_H \rightarrow 0$. By the Taylor expansion, we can take sufficiently small $\lambda > 0$ so that $(2\mu - 1)(F(c_L) - F(c_H)) \approx (2\mu - 1) \cdot f(0) \cdot (c_L - c_H)$, and then,

$$(2\mu - 1)(F(c_L) - F(c_H)) > (2\mu - 1) \cdot f(0) \cdot w \cdot [P_H(T_f(p)) - p] \cdot \xi_H(T_f(p))$$

We can take large w that is proportional to $\frac{\lambda}{[P_H(T_f(p)) - p] \cdot \xi_H(T_f(p))}$. Then, while both c_H and c_L is close to 0, $\zeta(\lambda) = (2\mu - 1)(F(c_L) - F(c_H)) > \lambda$ for small $\lambda > 0$. As $\zeta(2\mu - 1) < 2\mu - 1$, the intermediate value theorem implies the existence of a fixed point of ζ , which is in $(0, 2\mu - 1)$. This implies that the fixed point is a positive value. This shows the existence of an informative equilibrium.

(3) Consider the case $p < \underline{P}$. Then, by Lemma 2 and Lemma 4, if $p - \kappa > K$, $T_f(p) = T_L(p)$, and otherwise, $T_f(p) = -\infty$.

Suppose that $p - \kappa > K$. Then, by the definition of $T_L(p)$,

$$c_H = (\varsigma - w[P_H(T_L(p)) - p]) \cdot \xi_H(T_f(p))$$

$$c_L = \varsigma \cdot \xi_L(T_L(p)) > 0.$$

A similar way to case (2) shows the existence of an informative equilibrium.

If $p - \kappa \leq K$, as $T_f(p) = -\infty$. Then, the equilibrium becomes uninformative. ■

A.2. Proof of Proposition 2

Before we prove Proposition 2, we provide the following lemma, which immediately follows from the definition of \bar{P} .

Lemma 5. $\mu > \bar{P}$ if and only if $\frac{\mu - \kappa}{2} > K$.

Now, we proceed to the proof of Proposition 2. Suppose that $\frac{\mu - \kappa}{2} > \max\{K, 1 - \mu - \kappa\}$. This implies that $\mu > \frac{2 - \kappa}{3}$. Note that if $p = \mu$, as $\frac{\mu - \kappa}{2} > K$, $p > \bar{P}$ by Lemma 5. By Proposition 1, the equilibrium is uninformative and guarantees the profit $\frac{\mu - \kappa}{2} - K$.

Below, we examine the existence of an informative equilibrium that yields a profit larger than $\frac{\mu - \kappa}{2} - K$. As we consider an informative equilibrium, by Proposition 1, $p < \mu$. Note that the profit is at most $p - \kappa - K$. As we assume $\frac{\mu - \kappa}{2} > 1 - \mu - \kappa$, if an informative equilibrium yields a greater profit, the price needs to satisfy $p > 1 - \mu$. This implies that

$$\ln\left(\frac{1-p}{p} \frac{1-\mu}{\mu}\right) < 0.$$

Now we consider the limit $\sigma \rightarrow \infty$. Note that as $\lambda_0 - \lambda_1 = (2\mu - 1)(F(c_L) - F(c_H)) \in [0, 2\mu - 1] \subset [0, 1]$.¹³ This shows that

$$\frac{T_L(p) - \lambda_1}{\sigma} < \frac{\sigma}{2} \ln\left(\frac{1-p}{p} \frac{1-\mu}{\mu}\right) \rightarrow -\infty,$$

$$\frac{T_L(p) - \lambda_0}{\sigma} < \frac{1}{2\sigma} + \frac{\sigma}{2} \ln\left(\frac{1-p}{p} \frac{1-\mu}{\mu}\right) \rightarrow -\infty.$$

¹³Note that the value of σ can be taken independent of w as the value of w affects only λ . Here, λ is bounded by values that are independent of w .

Then, the firm's profit $\Pi(p)$ is at most $\left(\frac{1}{2}\right)(p - \kappa) - K$ at the limit. This is smaller than $\frac{\mu - \kappa}{2} - K$ as $p < \mu$.

Note that the above proof cannot apply when $p \approx \mu$, as we cannot exclude the case where the profit is larger than $\left(\frac{1}{2}\right)(p - \kappa) - K$ for any σ ; it converges only at the limit. The following discussion is to verify that the profit is strictly less than $\left(\frac{1}{2}\right)(p - \kappa) - K$ when p is close to μ and σ is large enough.

Claim 2. Suppose that $\mu > p > \max\left\{\frac{1}{2}, \bar{P}\right\}$. Then, $\Pi(p) < \left[\left(\frac{1}{2}\right)(p - \kappa) - K\right]$ for sufficiently large σ .

Proof of Claim 2. Suppose that $p > \bar{P}$. In this case, as discussed in the proof of Proposition 1, the firm's cutoff point coincides with the H-type's cutoff point. Then, the profit is written as

$$\begin{aligned} \Pi(p) = & \Pr(v = 1) \left[\left(\Phi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right) - \Phi\left(\frac{T_L(p) - \lambda_1}{\sigma}\right) \right) \cdot [\mu \cdot (p - \kappa) - K] + \Phi\left(\frac{T_L(p) - \lambda_1}{\sigma}\right) \cdot (p - \kappa - K) \right] \\ & + \Pr(v = 0) \left[\left(\Phi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right) - \Phi\left(\frac{T_L(p) - \lambda_0}{\sigma}\right) \right) \cdot [(1 - \mu) \cdot (p - \kappa) - K] + \Phi\left(\frac{T_L(p) - \lambda_0}{\sigma}\right) \cdot (p - \kappa - K) \right]. \end{aligned}$$

Let $D(\sigma) = \Pi(p) - \left[\left(\frac{1}{2}\right)(p - \kappa) - K\right]$, which is calculated as

$$\begin{aligned} \Pr(v = 1) & \left[\left(\Phi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right) - 1 \right) \cdot [\mu \cdot (p - \kappa) - K] + \Phi\left(\frac{T_L(p) - \lambda_1}{\sigma}\right) \cdot (1 - \mu) \cdot (p - \kappa) \right] \\ & + \Pr(v = 0) \left[\left(\Phi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right) - 1 \right) \cdot [(1 - \mu) \cdot (p - \kappa) - K] + \Phi\left(\frac{T_L(p) - \lambda_0}{\sigma}\right) \cdot \mu \cdot (p - \kappa) \right]. \end{aligned}$$

Note that by $p \in (1 - \mu, \mu)$ and $\lambda \in (0, 2\mu - 1)$,

$$T_H(p) - \lambda_1 = \frac{\lambda}{2} + \frac{\sigma^2}{2\lambda} \left[\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{p}{1-p}\right) \right] > \frac{\sigma^2}{2} \left[\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{p}{1-p}\right) \right],$$

$$T_H(p) - \lambda_0 = -\frac{\lambda}{2} + \frac{\sigma^2}{2\lambda} \left[\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{p}{1-p}\right) \right] > \frac{\sigma^2}{2} \left[\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{p}{1-p}\right) \right] - \frac{1}{2},$$

$$T_L(p) - \lambda_1 = \frac{\lambda}{2} + \frac{\sigma^2}{2\lambda} \left[\ln\left(\frac{1-\mu}{\mu}\right) - \ln\left(\frac{p}{1-p}\right) \right] < \frac{\sigma^2}{2} \left[\ln\left(\frac{1-\mu}{\mu}\right) - \ln\left(\frac{p}{1-p}\right) \right] + \frac{1}{2},$$

$$T_L(p) - \lambda_0 = -\frac{\lambda}{2} + \frac{\sigma^2}{2\lambda} \left[\ln\left(\frac{1-\mu}{\mu}\right) - \ln\left(\frac{p}{1-p}\right) \right] < \frac{\sigma^2}{2} \left[\ln\left(\frac{1-\mu}{\mu}\right) - \ln\left(\frac{p}{1-p}\right) \right].$$

By these equations, we can verify that as $\sigma \rightarrow \infty$, $T_H(p) - \lambda_v \rightarrow \infty$ and $T_L(p) - \lambda_v \rightarrow -\infty$. Then, $\Phi\left(\frac{T_H(p) - \lambda_v}{\sigma}\right) - 1 \rightarrow 0$ and $\Phi\left(\frac{T_L(p) - \lambda_v}{\sigma}\right) \rightarrow 0$.¹⁴ Then, $\lim_{\sigma \rightarrow \infty} D(\sigma) = 0$. Now we verify the sign of D for sufficiently large σ . To this end, we calculate $D'(\sigma)$, which is

$$\begin{aligned} D'(\sigma) = & \varphi(A_{H1}) \cdot \left(\frac{1}{\lambda} B_H - \frac{\lambda}{2\sigma^2} \right) \cdot (\mu(p - \kappa) - K) \\ & + \varphi(A_{L1}) \cdot \left(\frac{1}{\lambda} B_L - \frac{\lambda}{2\sigma^2} \right) \cdot (1 - \mu) \cdot (p - \kappa) \\ & + \varphi(A_{H0}) \cdot \left(\frac{1}{\lambda} B_H + \frac{\lambda}{2\sigma^2} \right) \cdot ((1 - \mu) \cdot (p - \kappa) - K) \\ & + \varphi(A_{L0}) \cdot \left(\frac{1}{\lambda} B_L + \frac{\lambda}{2\sigma^2} \right) \cdot \mu \cdot (p - \kappa), \end{aligned}$$

where

$$A_{\theta v} = \frac{T_{\theta}(p) - \lambda_v}{\sigma}, \quad v \in \{0, 1\},$$

$$B_{\theta} = \ln\left(\frac{\theta}{1 - \theta}\right) - \ln\left(\frac{p}{1 - p}\right),$$

and $B_H > 0 > B_L$ as $\mu > p > \frac{1}{2} > 1 - \mu$.

Note that

$$\frac{\varphi(A_{H1})}{\varphi(A_{L0})} = \exp\left(\left[\frac{\lambda}{\sigma} - \frac{\sigma}{\lambda} \ln\left(\frac{1-\mu}{\mu}\right)\right] \sigma \ln\left(\frac{p}{1-p}\right)\right) > \exp\left(-\sigma^2 \ln\left(\frac{1-\mu}{\mu}\right) \ln\left(\frac{p}{1-p}\right)\right),$$

$$\frac{\varphi(A_{H0})}{\varphi(A_{L0})} = \exp\left(-\frac{\sigma}{\lambda} \ln\left(\frac{1-\mu}{\mu}\right) \left[\frac{\lambda}{\sigma} + \sigma \ln\left(\frac{p}{1-p}\right)\right]\right) > \exp\left(-\sigma^2 \ln\left(\frac{1-\mu}{\mu}\right) \ln\left(\frac{p}{1-p}\right)\right),$$

$$\frac{\varphi(A_{L1})}{\varphi(A_{L0})} = \exp\left(\ln\left(\frac{p}{1-p}\right) - \ln\left(\frac{1-\mu}{\mu}\right)\right).$$

¹⁴Note that $\mu > \frac{1}{2}$ and $p > \frac{1}{2}$ implies that $\left|\ln\left(\frac{1-\mu}{\mu}\right) - \ln\left(\frac{p}{1-p}\right)\right| > \left|\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{p}{1-p}\right)\right|$. This also implies that $\Phi\left(\frac{T_H(p) - \lambda_v}{\sigma}\right)$ converges more slowly than $\Phi\left(\frac{T_L(p) - \lambda_v}{\sigma}\right)$. This intuition works in the following analysis.

Then, as $\mu > \frac{1}{2}$ and $p > \frac{1}{2}$, $\frac{\varphi(A_{H1})}{\varphi(A_{L0})} \rightarrow \infty$, and $\frac{\varphi(A_{H0})}{\varphi(A_{L0})} \rightarrow \infty$ as $\sigma \rightarrow \infty$. Further, $\frac{\varphi(A_{H0})}{\varphi(A_{H1})} \rightarrow 1$ as $\sigma \rightarrow \infty$. Note also that $(\mu(p - \kappa) - K) + ((1 - \mu)(p - \kappa) - K) = p - \kappa - 2K > 0$. Using these facts, dividing $D'(\sigma)$ by $\frac{\varphi(A_{L0})}{\lambda}$ is positive for sufficiently large σ . As $\lim_{\sigma \rightarrow \infty} D(\sigma) = 0$ and $D'(\sigma) > 0$, $D(\sigma) < 0$ for sufficiently large σ . Therefore, $\Pi(p) < \left[\frac{1}{2}(p - \kappa) - K\right]$ for sufficiently large σ . ■

Then, we conclude that for each $p < \mu$, the profit is at most $\frac{1}{2}(p - \kappa) - K$ when σ is sufficiently large. Therefore, $p = \mu$ is the optimal price, and the equilibrium is uninformative. ■

A.3. Proof of Proposition 3

The proof consists of the following two claims, which hold under the assumption of Proposition 3.

Claim 3. *If informative equilibrium does not exist, the expected profit is at most 0.*

Claim 4. *By setting $p = 1 - \mu$, the equilibrium is informative. The expected profit is no less than $(1 - \mu - \kappa - K)\frac{1}{2}$ in this case.*

Proof of Claim 3. By Proposition 1, the situation is either $p > \min\{\mu, \bar{P}\}$ or $p < \kappa + K$. If $p > \mu$, as $P_H(T) = \mu < p$ at the uninformative equilibrium, the profit is 0 as the H-type never purchases. If there is an uninformative equilibrium with $p \in (\bar{P}, \mu)$, as $P_H(T) = \mu > p > 1 - \mu$, the profit is $\frac{\mu - \kappa}{2} - K < 0$ as only the H-type purchases.

At the uninformative equilibrium with $p < \kappa + K$, the profit is also 0 as the firm gives up selling. ■

Proof of Claim 4. First, consider the case where $1 - \mu \leq \underline{P}$. As $p = 1 - \mu > K + \kappa$, $1 - \mu < \mu$ and $1 - \mu < \underline{P} < \bar{P}$, an equilibrium exists and it is necessarily informative by Proposition 1.

Note that $p = 1 - \mu \leq \underline{P}$ implies $T = T_L(p)$. In this case, all consumers buy the product if $t < T$, and otherwise, never. Further, if $p = 1 - \mu$, $T_L(p) = \frac{\lambda_1 + \lambda_0}{\sigma}$. Then, the expected profit is

$$\left[\frac{1}{2}\Phi(x) + \frac{1}{2}\Phi(-x) \right] \cdot (1 - \mu - \kappa - K) = \frac{1 - \mu - \kappa - K}{2},$$

where $x = \frac{\lambda_0 - \lambda_1}{2\sigma}$.

Next, consider the case that $1 - \mu > \underline{p}$. In this case, at $t = T$, only H -type purchases the product.

By assumption, $K > \frac{\mu - \kappa}{2}$ implies that $\bar{p} > \mu > 1 - \mu$, and as $1 - \mu > K + \kappa$, Proposition 1 implies that an equilibrium exists and it is necessarily informative at $p = 1 - \mu$. Further, the expected profit is

$$\begin{aligned} \frac{1 - \mu - \kappa}{2} \times \left[\mu \cdot \Phi(x + z') + (1 - \mu) \cdot \Phi(x) + (1 - \mu) \cdot \Phi(-x + z') + \mu \cdot \Phi(-x) \right] \\ - \left[\mu \cdot \Phi(x + z') + (1 - \mu) \cdot \Phi(-x + z') \right] K, \end{aligned} \quad (1)$$

where $z' = \frac{\sigma}{4x} \cdot \ln\left(\frac{\mu - \frac{K}{p - \kappa}}{\frac{K}{p - \kappa} - (1 - \mu)}\right)$.

As $T > T_L(p) = \frac{\lambda_0 + \lambda_1}{2}$, the expected profit (1) is greater than $\frac{1 - \mu - \kappa - K}{2} > 0$. ■

Proof of Proposition 3. By Claims 3 and 4, and assumption $1 - \mu - \kappa > K$, a price that leads to an informative equilibrium yields a positive profit, which is larger than the profit of any uninformative equilibrium. Then, the optimal price leads to an informative equilibrium. ■

A.4. Proofs in Section 5

Proof of Lemma 3. First note that for sufficiently small t , $\Pi_L^*(t) > \Pi_H^*(t)$. This is because for sufficiently small $t \approx -\infty$, $P_\theta(t) \approx 1$ and therefore $\Pi_L^*(t) \approx 1 > \mu \approx \Pi_H^*(t)$.

Let $a = \exp\left(\frac{\lambda_0 - \lambda_1}{\sigma^2} \cdot \left(t - \frac{\lambda_0 + \lambda_1}{2}\right)\right)$. Then, $P_\theta(t)$ is written as

$$P_\theta(t) = \frac{\varphi\left(\frac{t - \lambda_1}{\sigma}\right) \cdot \theta}{\varphi\left(\frac{t - \lambda_1}{\sigma}\right) \cdot \theta + \varphi\left(\frac{t - \lambda_0}{\sigma}\right) \cdot (1 - \theta)} = \frac{1}{1 + \frac{1 - \theta}{\theta} \cdot a},$$

and also

$$\Pr(v = 1 | t) = \frac{1}{1 + a}.$$

To simplify the notation, let denote $P(t) = \Pr(v = 1 | t)$. By differentiation, if $\lambda_0 > \lambda_1$, we have

$$P'_\theta(t) = -\frac{\lambda_0 - \lambda_1}{\sigma^2} \cdot \frac{1 - \theta}{\theta} \cdot a \cdot (P_\theta(t))^2 < 0.$$

Note that we can write $P_H(t) = \frac{1}{1 + \frac{1-\mu}{\mu}a}$ and $P_L(t) = \frac{1}{1 + \frac{\mu}{1-\mu}a}$. This implies that $\frac{P_H(t)}{P_L(t)}$ and $\frac{P(t)}{P_L(t)}$ are increasing in t .

Now we calculate the ratio of the profits:

$$\frac{\Pi_H^*(t)}{\Pi_L^*(t)} = (P(t)(2\mu - 1) + (1 - \mu)) \frac{P_H(t)}{P_L(t)} > 1 \iff Z(a) > 0, \quad (2)$$

$$Z(a) := (1 - \mu)\mu^2 a^2 - (2\mu - 1)(1 - (1 - \mu)\mu)a - \mu(1 - \mu)^2 > 0.$$

Note that $Z(a)$ is a convex quadratic function of a , and $Z(0) < 0$. Also note that as $P_L(t) > K, a < \left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu}$. A calculation shows that $Z\left(\left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu}\right)$ is decreasing in μ , and let μ^{**} be the solution that $Z\left(\left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu}\right) = 0$. This implies that if $\mu < \mu^{**}$, $\Pi_H^*(t) < \Pi_L^*(t)$ for each t . On the contrary, if $\mu > \mu^{**}$, there is $\hat{t} < T_L(K)$ such that $\Pi_H^*(t) < \Pi_L^*(t)$ if and only if $t < \hat{t}$.

(c) As $T_L(K)$ is the solution of $P_L(t) = K$, That is,

$$T_L(K) = \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{\lambda_0 - \lambda_1} \cdot \ln\left(\left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu}\right).$$

Let μ^* be the solution to $\left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu}{\mu} = 1$. This exists if $K < \frac{1}{2}$. Then, if $\mu < \mu^*$, $T_L(K) \rightarrow \infty$. In contrast, if $\mu > \mu^*$, $T_L(K) \rightarrow -\infty$. We can also verify that $Z\left(\left(\frac{1}{K} - 1\right) \cdot \frac{1-\mu^*}{\mu^*}\right) < 0$, that is, $Z(1) < 0$. This implies that $\mu^* < \mu^{**}$. ■

Proof of Proposition 4. Let $a = \exp\left(\frac{\lambda_0 - \lambda_1}{\sigma^2} \cdot \left[t - \frac{\lambda_1 + \lambda_0}{2}\right]\right)$. Note that

$$\begin{aligned}
P_\theta &= \frac{1}{1 + \frac{1-\theta}{\theta}a} \\
P_H(t) - P_L(t) &= \frac{\frac{\mu}{1-\mu} - \frac{1-\mu}{\mu}}{\left(1 + \frac{\mu}{1-\mu}a\right) \cdot \left(1 + \frac{1-\mu}{\mu}a\right)}a, \\
-P'_\theta(t) &= \frac{\lambda_0 - \lambda_1}{\sigma^2} \frac{1-\theta}{\theta} \cdot a \cdot (P_\theta(t))^2 > 0. \\
\zeta_L(t) - \zeta_H(t) &= \frac{2\mu - 1}{\sigma} \left[\varphi\left(\frac{t-\lambda_0}{\sigma}\right) - \varphi\left(\frac{t-\lambda_1}{\sigma}\right) \right] \\
&= \frac{2\mu - 1}{\sigma} \cdot \varphi\left(\frac{t-\lambda_0}{\sigma}\right) \cdot \left[1 - \exp\left(-\frac{\lambda_0 - \lambda_1}{\sigma} \cdot \left(t - \frac{\lambda_0 + \lambda_1}{2}\right)\right) \right] \\
\zeta_H(t) &= \frac{1}{\sigma} \left[\mu \cdot \varphi\left(\frac{t-\lambda_1}{\sigma}\right) + (1-\mu) \cdot \varphi\left(\frac{t-\lambda_0}{\sigma}\right) \right].
\end{aligned}$$

As shown in the proof of Lemma 3, $\Pi_H^*(t) = \Pi_L^*(t)$ if and only if $Z(a) = 0$. (Z is defined in (2)). Therefore, when we write $\hat{a} = \exp\left(\frac{\lambda_0 - \lambda_1}{\sigma^2} \cdot \left(\hat{t} - \frac{\lambda_1 + \lambda_0}{2}\right)\right)$, $Z(\hat{a}) = 0$. Note also that the effect of t on Z is summarized in a . Then, even $\sigma \rightarrow \infty$, \hat{a} is a finite value, which implies that $\hat{t} \in \Theta(\sigma^2)$.¹⁵ Further,

$$\frac{\zeta_H(t)}{\zeta_H(\hat{t})} = \frac{\mu \cdot \exp\left(\left(\hat{t} - t\right) \cdot \frac{\hat{t} + t - 2\lambda_1}{2\sigma^2}\right) + (1-\mu) \cdot \exp\left(\left(\hat{t} - t + \lambda_0 - \lambda_1\right) \cdot \frac{\hat{t} + t - (\lambda_0 + \lambda_1)}{2\sigma^2}\right)}{\mu + (1-\mu) \cdot \exp\left(\frac{1}{\sigma^2} \cdot (\lambda_0 - \lambda_1) \cdot \left(\hat{t} - \frac{\lambda_0 + \lambda_1}{2}\right)\right)}.$$

As $\frac{\hat{t}}{\sigma^2}$ converges to a real number, for each t with $|t| < |\hat{t}| + z$ for some constant z , $\frac{\zeta_H(t)}{\zeta_H(\hat{t})} \rightarrow \infty$. In contrast, if $t < \hat{t}$, $\frac{\zeta_H(t)}{\zeta_H(\hat{t})} \rightarrow 0$ as $\hat{t} \rightarrow -\infty$. By Lemma 3, $\hat{t} \rightarrow -\infty$ as $\sigma \rightarrow \infty$. Then, dividing D by $\zeta_H(\hat{t})$ yields that

$$\int_{\hat{t}}^{\hat{t}} [-P'_L(t)] \frac{\zeta_L(t) - \zeta_H(t)}{\zeta_H(\hat{t})} dt - \int_{\hat{t}}^{T^*} [-P'_H(t)] \frac{\zeta_H(t)}{\zeta_H(\hat{t})} dt + [P_H(\hat{t}) - P_L(\hat{t})]. \quad (3)$$

¹⁵ Θ is Landau symbol; that is $g(t) \in \Theta(f(t))$ implies that $\lim_{t \rightarrow \infty} \frac{g(t)}{f(t)} \in \mathbb{R} \setminus \{0\}$.

Note also that $\xi_L(t) < \xi_H(t)$ for sufficiently small t when $\lambda_0 > \lambda_1$. Also note that by $\frac{\mu}{2} > K$, $T^* \rightarrow \infty$ as $\sigma \rightarrow \infty$. Therefore, as $\sigma \rightarrow \infty$, the value of (3) diverges to $-\infty$. This shows that $D < 0$. ■

Proof of Proposition 5. Dividing \tilde{D} by $\xi_H(T_*)$ yields that

$$\frac{\int^{T_*} [-P'_L(t)] (\xi_L(t) - \xi_H(t)) dt}{\xi_H(T_*)} + [P_H(T_*) - P_L(T_*)] \quad (4)$$

As $\mu > \mu^*$, $\lim_{\sigma \rightarrow \infty} T_* = -\infty$ by Lemma 3. Therefore, the first term of (4) converges to

$$-P'_L(T_*) \frac{\xi_L(T_*) - \xi_H(T_*)}{\xi'_H(T_*)}.$$

Note that

$$\begin{aligned} \frac{\xi_L(T_*) - \xi_H(T_*)}{\xi'_H(T_*)} &= \frac{-(2\mu - 1) \left(\exp\left(-\frac{1}{2} \left(\frac{T_* - \lambda_0}{\sigma}\right)^2\right) - \exp\left(-\frac{1}{2} \left(\frac{T_* - \lambda_1}{\sigma}\right)^2\right) \right)}{\mu \frac{T_* - \lambda_1}{\sigma^2} \exp\left(-\frac{1}{2} \left(\frac{T_* - \lambda_1}{\sigma}\right)^2\right) + (1 - \mu) \frac{T_* - \lambda_0}{\sigma^2} \exp\left(-\frac{1}{2} \left(\frac{T_* - \lambda_0}{\sigma}\right)^2\right)} \\ &= \frac{(2\mu - 1) \left(1 - \exp\left(\frac{\lambda_0 - \lambda_1}{\sigma^2} \left(T_* - \frac{\lambda_1 + \lambda_0}{2}\right)\right) \right)}{\mu \frac{T_* - \lambda_1}{\sigma^2} + (1 - \mu) \frac{T_* - \lambda_0}{\sigma^2} \exp\left(\frac{\lambda_0 - \lambda_1}{\sigma^2} \left(T_* - \frac{\lambda_1 + \lambda_0}{2}\right)\right)} \end{aligned}$$

As shown in Lemma 3, $\frac{T_*}{\sigma^2}$ converges to a finite value. Then, the above value converges to a finite value.

Note also that

$$\begin{aligned} -P'_L(t) &= \frac{\lambda_0 - \lambda_1}{\sigma^2} \frac{\mu}{1 - \mu} \times \frac{1}{\frac{1}{a} + 2 \frac{\mu}{1 - \mu} + \left(\frac{\mu}{1 - \mu}\right)^2 a} > 0. \\ a &= \exp\left(\frac{\lambda_0 - \lambda_1}{\sigma^2} \cdot \left(t - \frac{\lambda_1 + \lambda_0}{2}\right)\right) \end{aligned}$$

As $\frac{T_*}{\sigma^2}$ converges to a finite value, a is also a finite value. Then, $-P'_L(T_*) \rightarrow 0$. Then, (4) reduced to $[P_H(T_*) - P_L(T_*)]$, which is calculated as

$$P_H(t) - P_L(t) = \frac{\frac{\mu}{1 - \mu} - \frac{1 - \mu}{\mu}}{\left(1 + \frac{\mu}{1 - \mu} a\right) \left(1 + \frac{1 - \mu}{\mu} a\right)} a,$$

As a is a finite value, $\lim_{\sigma \rightarrow \infty} P_H(T_*) - P_L(T_*) > 0$. This concludes that $\tilde{D} > 0$. ■

B. Informative protest

This section considers the case where σ is small enough. First, the following proposition shows a sufficient condition for the existence of an informative equilibrium when σ is small enough.

Proposition 6. *For sufficiently small σ , an informative equilibrium exists if $p \in (\underline{P}, \max\{\mu, \bar{P}\})$.*

Consider an informative equilibrium. By Proposition 1 (1), $p < \max\{\bar{P}, \mu\}$. Suppose that $\lim_{\sigma \rightarrow 0} \frac{\lambda}{\sigma} = \gamma < \infty$. Then, note that for each $T \in \{T_H, T_L, T_f\}$,

$$\frac{T - \lambda_1}{\sigma} = \frac{\lambda}{\sigma} + \frac{\sigma X}{\lambda 2},$$

$$\frac{T - \lambda_0}{\sigma} = -\frac{\lambda}{\sigma} + \frac{\sigma X}{\lambda 2},$$

for some X , which is independent of σ and λ . This shows that T is a finite value. Then, $c_L > c_H$ and thus $\lambda > 0$ in the limit. However, this also implies $\frac{\lambda}{\sigma} \rightarrow \infty$, which is a contradiction. Therefore, $\lim_{\sigma \rightarrow 0} \frac{\lambda}{\sigma} = \infty$.

This implies that $\Phi\left(\frac{T(p) - \lambda_1}{\sigma}\right) \rightarrow 1$, $\Phi\left(\frac{T_L(p) - \lambda_1}{\sigma}\right) \rightarrow 1$, $\Phi\left(\frac{T_f(p) - \lambda_0}{\sigma}\right) \rightarrow 0$, and $\Phi\left(\frac{T_L(p) - \lambda_0}{\sigma}\right) \rightarrow 1$. In this case, the profit of informative equilibrium converges to $\frac{p - \kappa - K}{2}$.

Note that the profits of uninformative equilibria are at most $\max\left\{\frac{\mu - \kappa}{2} - K, 0\right\}$. In contrast, the supremum of the profit of informative equilibrium is at least $\frac{\max\{\bar{P}, \mu\} - \kappa - K}{2}$. Therefore, the optimal price is at least $\max\{\bar{P}, \mu\} - \varepsilon$ for small ε .

Note that when the protest is absent, the optimal price is $p = 1 - \mu$ when μ is small enough. We have the following observation.

Proposition 7. *Suppose that $1 - \mu - \kappa > \max\left\{0, \frac{\mu - \kappa}{2}\right\}$. Then, for sufficiently small σ , the price increases compared with the case when protest is absent.*

B.1. Proof of Proposition 6

We have two cases.

Case 1. Consider the case that $p \geq \bar{P}$, but $p < \mu$. As in the proof of Proposition 1,

$$c_L - c_H = \varsigma \cdot \frac{2\mu - 1}{\sigma} \cdot \left[\varphi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right) - \varphi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right) \right].$$

Let $z = \frac{\lambda}{\sigma}$. Then, for sufficiently small σ ,

$$\frac{T_H(p) - \lambda_0}{\sigma} = -\frac{z}{2} + \frac{1}{2} \frac{1}{z} \cdot \ln\left(\frac{\mu}{1-\mu} - \frac{p}{1-\mu}\right)$$

$$\frac{T_H(p) - \lambda_1}{\sigma} = \frac{z}{2} + \frac{1}{2} \frac{1}{z} \cdot \ln\left(\frac{\mu}{1-\mu} - \frac{p}{1-\mu}\right).$$

Note that $p < \mu$, $\ln\left(\frac{\mu}{1-\mu} - \frac{p}{1-\mu}\right) > 0$, and then, by symmetry of φ , $\varphi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right) > \varphi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right)$.

Let $A = \ln\left(\frac{\mu}{1-\mu} - \frac{p}{1-\mu}\right)$. Then, we can write

$$B = \varphi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right) - \varphi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right) = \exp\left(-\frac{z^2}{8} - \frac{1}{z^2} \frac{A^2}{8}\right) \cdot \left(\exp\left(\frac{A}{4}\right) - \exp\left(-\frac{A}{4}\right)\right).$$

Now consider the following limit. Consider $\lambda = z\sigma$ with fixed z .

$$\begin{aligned} \lim_{z \rightarrow 0} \frac{(2\mu - 1)(F(c_L) - F(c_H))}{\lambda} &= \frac{1}{\sigma} \frac{d(2\mu - 1)(F(c_L) - F(c_H))}{dz} \\ &= \frac{1}{z} (2\mu - 1) \cdot f(0) \cdot \varsigma \cdot \frac{2\mu - 1}{\sigma^2} \cdot \frac{1}{4} \cdot \left(-z + \frac{A^2}{z}\right) \cdot B. \end{aligned}$$

By considering $\sigma \propto (B)^{\frac{1}{2}}$, we can show that $\sigma \rightarrow 0$, $\left[\varphi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right) - \varphi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right)\right] \rightarrow 0$, and $\lim_{\sigma \rightarrow 0} \frac{(2\mu - 1) \cdot (F(c_L) - F(c_H))}{\lambda} \rightarrow \infty$. Then, for such λ , $\frac{(2\mu - 1) \cdot (F(c_L) - F(c_H))}{\lambda} > 1$. In contrast,

when $\lambda = 2\mu - 1$, $(2\mu - 1) \cdot (F(c_L) - F(c_H)) < 2\mu - 1$. Then, the intermediate value theorem implies the existence of λ such that $(2\mu - 1) \cdot (F(c_L) - F(c_H)) = \lambda > 0$.

This shows the existence of an informative equilibrium for small σ .

Case 2. Consider the case that $p \in (\underline{P}, \bar{P})$. As in the proof of Proposition 1,

$$c_L - c_H = \varsigma \left(\xi_L(T_f(p)) - \xi_H(T_f(p)) \right) + w \left[P_H(T_f(p)) - p \right] \xi_H(T_f(p)),$$

and

$$\varsigma(\xi_L(T_f(p)) - \xi_H(T_f(p))) = \varsigma \cdot \frac{2\mu - 1}{\sigma} \cdot \left[\varphi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right) - \varphi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right) \right].$$

As $w[P_H(T_f(p)) - p]\xi_H(T_f(p)) > 0$ by $p < \bar{P}$, a similar discussion of case 1 proves the existence of an informative equilibrium. ■

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