

Question 1)

Constant time:

$$10$$

$$n + 10$$

logarithmic:

$$10 \log n$$

Log-linear time:

$$10 n \log n$$

$$n \log n^{20}$$

Quadratic:

$$n^2 \log n + n^3$$

Cubic time:

$$n^3 \log n + n^3$$

$$n^3 \log n$$

Question 2)

1 - For each iteration of the outer loop, loop 2 runs for $x - y$ iterations, running linearly with respect to x . However, each iteration of loop 1 shrinks the space that the inner loop needs to iterate over. loop 2 runs in $O(x)$ time per outer loop, with a best case performance of $O(1)$ if y is 1 greater than x .

2 - Loop 1 runs from 1 to n , for a total of $n-1$ iterations. This is a linear execution with respect to n . The outer loop has a worst case of $O(n)$ and a best case of $O(1)$ if $n == 1$, as due to the comparison, only a single iteration will occur.

3 - The asymptotic complexity of the two_loops algorithm overall is $O(n^2)$. The outer loop runs in $O(n)$ linear time, leaving the inner loop with a smaller number of iterations each time, but still running linearly. For a given iteration of the outer loop, we see the following:

when $x = 1$, the inner loop runs $n - 1$ times

when $x = 2$, the inner loop runs $n - 2$ times

The inner loop's two assignment statements both run in $O(1)$ time, as well as incrementing the value of x after the inner loop, and are always considered constant time operations. These are too small to affect the asymptotic complexity of the overall function.

Question 3)

iteration 1: left = 1, right = 64, mid = $1+63/2 = 1+31 = 32$, mid_squared = 1024, Left = 1, right = 31

iteration 2: left = 1, right = 31, mid = $1+30/2 = 1+15 = 16$, mid_squared = 256, Left = 1, right = 15

iteration 3: left = 1, right = 15, mid = $1+14/2 = 1+7 = 8$, mid_squared = 64, Left = 1, right = 7

mid_squared == n, return

The asymptotic complexity should be $O(\log n)$ or logarithmic time, as for every iteration, we're shrinking the space between left and right by half, similarly to a binary search through the space. This results in very few iterations needed to find our exit statement.

Question 4)

Lets start by going line by line through the main_algorithm and calculating each line's runtime

To start, line 1 calls `process_a()`, which we know is $O(n^2)$

Line 2 starts an $O(n)$ loop

Line 3 calls `process_b()`, which we know runs in $O(n \log n)$

Multiplying out the loop, we know the cumulative calls to `process_b()` add up to $O(n^2 \log n)$, and then adding on the first call to `process_a()`, we get a total of $O(n^2 (n^2 \log n))$.

We then drop the smallest part, here the `log n`, resulting in $O(n^2 * n^2)$, or $O(n^4)$. This is our final runtime complexity.