CSCI 3104: Algorithms Spring 2022

Recitation #2 - Greedy

Problem 1

Consider the problem of "making change." We have an infinite supply of pennies (worth 1 cent), nickels (worth 5 cents), dimes (worth 10 cents), and quarters (worth 25 cents). We take as input an integer $n \geq 0$. Our goal is to make n cents in coins, using the fewest number of coins possible.

The greedy algorithm chooses as many quarters as possible, followed by as many dimes as possible, then as many nickels as possible. Finally, the greedy algorithm uses pennies to finish making change.

We will use a greedy exchange argument to prove that this greedy solution is optimal.

a.	For any n ,	how many	coins of	each	type	can	appear	in	an	optimal	solution
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• What is the maximum number of pennies (1 cent)?

That is the maximum number of pennics can be replaced by 1 nickels.

That is the maximum number of nickels (5 cents)?

Description:

Descripti

• What is the maximum number of nickels (5 cents)?

1. otherwise 2 nickels

• What is the maximum number of dimes (10 cents)?

2, otherwise 3 dimes

by I quarters and I nickels

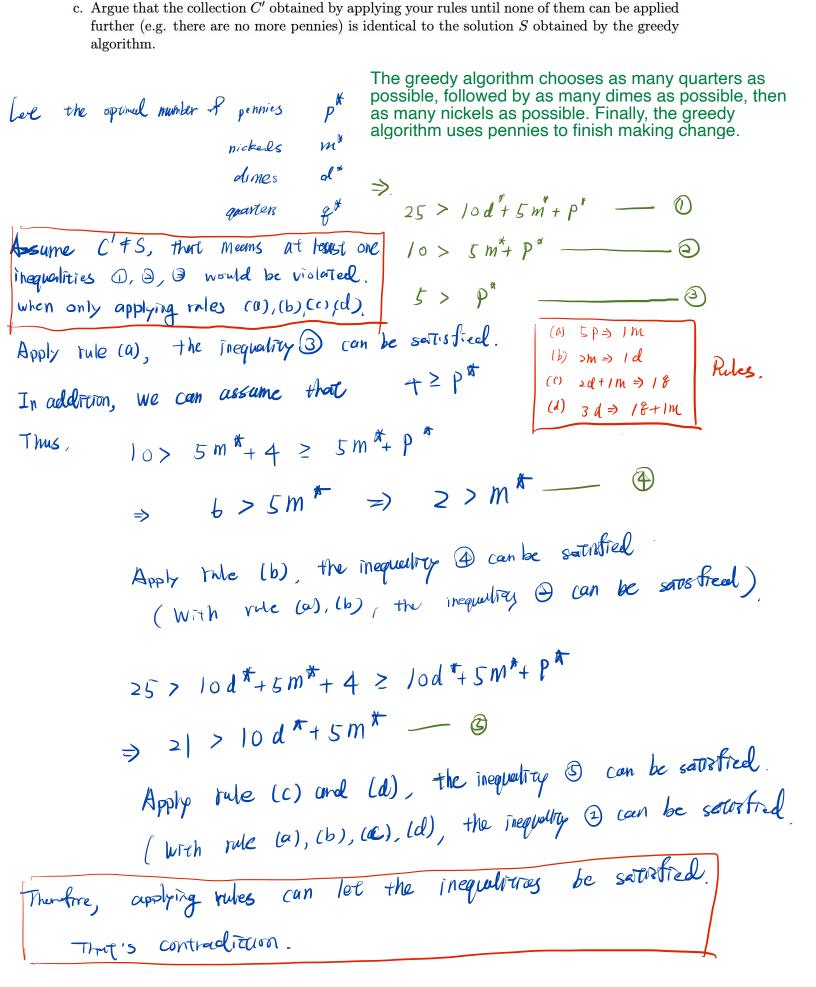
- b. Given some collection of coins C worth n cents, devise some rules by which coins in C can be exchanged to obtain a smaller collection C' also worth n cents (e.g. "if there are 5 pennies in C, replace them with 1 nickel"). Will you always be able to apply these rules?
- c. Argue that the collection C' obtained by applying your rules until none of them can be applied further (e.g. there are no more pennies) is identical to the solution S obtained by the greedy algorithm.
- d. Why does c. imply that the greedy algorithm produces an optimal solution?

- b. Given some collection of coins C worth n cents, devise some rules by which coins in C can be exchanged to obtain a smaller collection C' also worth n cents (e.g. "if there are 5 pennies in C, replace them with 1 nickel"). Will you always be able to apply these rules?
 - (a) 5 pennies -> 1 nictel
 - (b) 2 nickels -> 1 dime
 - (c) 2 dimes and Inickels -> 1 quarters
- (d) 3 dimes -> Iquarters and Inickels

We can apply rules to reduce the number of coins until

there is no more than 5 pennics / 2 nickels / 2 dimes and I nickel

/ 3 dimes.



d. Why does c. imply that the greedy algorithm produces an optimal solution?

Consider starting from an optimal solution C. Since Cis optimal, it has the minimum size and so we control of statement apply any rules to shrink it anymore. Since none of rules can be applied, Comust be identical to S, which is the solution applying the greedy algorithm. In other words, the greedy algorithm opplying the greedy algorithm. In other words, the greedy algorithm