TA FCQ's. Please take take a few minutes to fill out TA FCQs! They help us improve our teaching and help the department track courses. You can fill them out here: colorado.campuslabs.com/courseeval

Problem 1

Reformulate the following optimization problems as decision problems. For each, argue that the decision version is in P.

a. *Sequence Alignment.* Given strings (A, B), find the minimum value of edit operations to convert string A into string B.

b. *Minimum Spanning Tree.* Given a weighted graph $G = (V, E, w)$ with weights $w(e) \geq 0$ for all $e \in E$, find a spanning tree of minimal cost.

c. *Interval Scheduling.* Given a set of intervals $L = \{(s_1, e_1), \ldots, (s_n, e_n)\}$, find a maximal-size subset of intervals which do not overlap.

Problem 2

A *Hamiltonian cycle* on a directed graph $G = (V, E)$ is a cycle which visits each vertex in V exactly once. Recall that a *cycle* is a path with the same start and end vertices.

a. The Hamiltonian Cycle problem is: given a directed graph $G = (V, E)$, does G contain a Hamiltonian cycle?

   Show that Hamiltonian Cycle is NP-hard via a reduction from 3SAT.

b. The Hamiltonian Path problem is, similarly, does there exist a *path* which visits every vertex in the graph? (recall that a path need not start and end at the same place, while a cycle does)

   Show that Hamiltonian Cycle $\leq_p$ Hamiltonian Path, that is, Hamiltonian Path is NP-hard by a reduction to Hamiltonian Cycle.

Problem 3 (Bonus)

The pandemic has ended, and you're having a big group of friends over for a celebratory dinner! Unfortunately, each of your m friends has very restrictive dietary needs, many of which are incompatible.

You have a large recipe book R with n recipes in it, and friend i can eat a subset $R_i \subseteq R$ of the foods in your recipe book. You've been trying to come up with a set of dishes $M \subseteq R$ to cook such that every guest can eat at least one dish ($M \cap R_i \neq \emptyset$ for all i), but you have the time to make at most k dishes before your friends arrive.

Show that the problem Meal Planning of determining whether there exists a set M of recipes you can cook such that $|M| \leq k$ and every guest can eat at least one dish is NP-complete.

(*Hint:* try reducing from Set Cover or 3SAT.)