CSCI 3104: Algorithms
Spring 2022
Recitation #12 - Midterm 2 Review I

SOLUTIONS

Problem 1

Analyze the runtime of the following code snippets and prove an asymptotic bound.

a.

```
procedure loopsA(integer n):
  for  i = 1; i <= n:
     i = i + 2;
     print i;
     for j = 5; j <= n:
        j = j * 2;
        print j;
end procedure
```

b.

```
procedure loopsB(integer n):
  for  i = 1; i <= n:
     i = i * 2;
     print i;
     for j = 1; j <= i:
        j = j + 2;
        print j;
end procedure
```

Problem 2

Consider a modified mergesort algorithm which, on alternating levels of the recursion, partitions the input into either an $(4, n-4)$ or $(n/4, 3n/4)$ split. Assume the first partition is $(n/4, 3n/4)$.
(*hint:* it may help to draw out the recurrence tree.)

a. Write down a recurrence for the modified mergesort algorithm.

b. Solve the recurrence relation using the tree method. How does this tree compare to the recursion tree from the previous problem?

Asymptotics Cheat Sheet

(Limit Comparison Test) If the limit $L := \lim_{x\to\infty} \frac{f(x)}{g(x)}$ exists, then:

- if $0 < L < \infty$, then $f(x) \in \Theta(g(x))$ ("they grow at the same rate")
- if $L = 0$, then $f(x) \in O(g(x))$, but $f(x) \notin \Theta(g(x))$ ("g grows faster than f")
- if $L = \infty$, then $g(x) \in O(f(x))$, but $g(x) \notin \Theta(f(x))$ ("f grows faster than g")

(L'Hôpital's rule) Suppose g and f are both differentiable functions, with either

a. $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to\infty} g(x) = 0$; or

b. $\lim_{x\to\infty} f(x) = \pm\infty$ and $\lim_{x\to\infty} g(x) = \pm\infty$

If $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ exists, then $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$.

Some Helpful Identities

$$a\sum_{i=0}^{n} r^i = a\left(\tfrac{1-r^{n+1}}{1-r}\right) \qquad \text{(Finite Geometric Series)}$$

$$a\sum_{i=0}^{n} i = \frac{an(n+1)}{2} \qquad \text{(Sum of first } n \text{ integers/special case of Finite Arithmetic Series)}$$

$$\sum_{i=0}^{n} ix^i = \frac{x(nx^{n+1} - (n+1)x^n + 1)}{(x-1)^2}$$

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$