CSCI 3104: Algorithms
Spring 2022
Recitation #11 - Hashing & Balanced Binary Trees

SOLUTIONS

Problem 1

For this problem, we consider using *balanced binary trees* as dictionaries.

a. What makes a tree a balanced binary tree?

A tree is a balanced binary tree if...

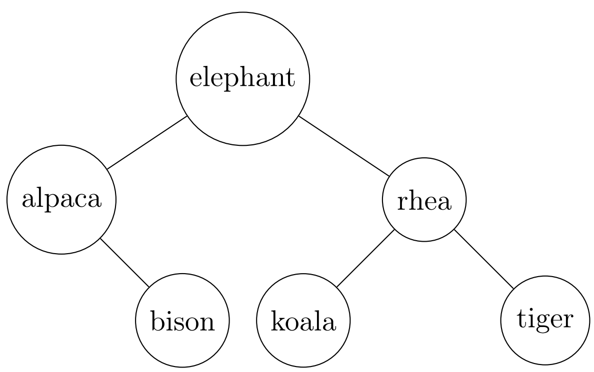
- each vertex has at most 2 children
- each vertex's children are all balanced binary trees
- the difference in height between any vertex's two child subtrees is at most 1

(source: JournalDev)

b. Arrange the following word list into an (alphabetically) sorted balanced binary tree (There are several possibilities, see how many you can find):

bison, tiger, elephant, alpaca, rhea, koala

One possible balanced binary tree is



c. List some of the benefits of using a balanced binary tree. List some of the drawbacks. (with regard to a hash-based dictionary, or in general.)

Benefits:

Intro to Alg, Chapter 11.2 Hash tables

Any given element is equally likely to hash into any of the m slots, independently of where any other element has hashed to. We call this the assumption of simple uniform hashing.

Given a hash table T with m slots that stores n elements, we define the ***load factor*** α for T as n/m, that is, the average number of elements stored in a chain. Our analysis will be in terms of α, which can be less than, equal to, or greater than 1.

CHAINED-HASH-INSERT(T, x)

insert x at the head of list $T[h(x.key)]$

CHAINED-HASH-SEARCH(T, k)

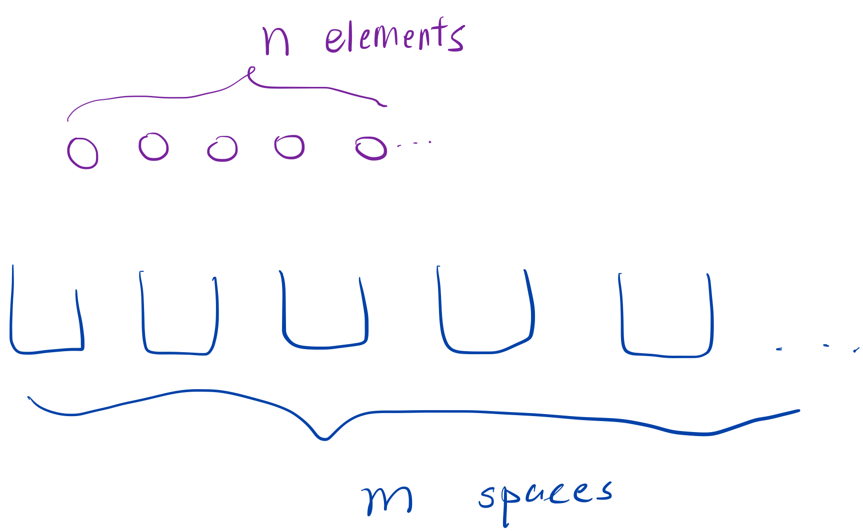
search for an element with key k in list $T[h(k)]$

CHAINED-HASH-DELETE(T, x)

delete x from the list $T[h(x.key)]$

Theorem 11.1
In a hash table in which collisions are resolved by chaining, an unsuccessful search takes average-case time $\Theta(1+\alpha)$, under the assumption of simple uniform hashing.

Theorem 11.2
In a hash table in which collisions are resolved by chaining, a successful search takes average-case time $\Theta(1+\alpha)$, under the assumption of simple uniform hashing.



With simple uniform assumption, if we solve the conflict by chaining,

$$E[\text{\# of elements in a certain space}] = \frac{n}{m}.$$

$= \alpha$

the load factor

- Guaranteed $O(\log n)$ lookup time
- Easy to produce a sorted list of elements in the dictionary

Drawbacks:

- High cost for insertions and deletions ($O(\log n)$, even when it seems like it should be simple like deleting a leaf)
- $O(\log n)$ lookup time

Problem 2

Find the average-case insertion, deletion, and lookup times for a hash table under the Simple Uniform Hashing Assumption, where the table has m buckets:

a. $m = \Theta(n^2)$ buckets.

We first compute the load factor $\alpha = n/m = \Theta(1/n)$, and use the fact that lookup and deletion time are $\mathcal{O}(1 + \alpha)$.

Insertion. $\Theta(1)$, assuming we prepend the current element to our linked list at every collision.

Lookup & Deletion. $\mathcal{O}(1 + 1/n)$.

(source: Levet Notes)

b. $m = \Theta(\sqrt{n})$ buckets.

We first compute the load factor $\alpha = n/m = \Theta(\sqrt{n})$, and use the fact that lookup and deletion time are $\mathcal{O}(1 + \alpha)$.

Insertion. $\Theta(1)$, assuming we prepend the current element to our linked list at every collision.

Lookup & Deletion. $\mathcal{O}(1 + \sqrt{n})$.

(source: Levet Notes)

c. $m = \Theta(2^n)$ buckets.

We first compute the load factor $\alpha = n/m = \Theta(n/(2^n))$, and use the fact that lookup and deletion time are $\mathcal{O}(1 + \alpha)$.

Insertion. $\Theta(1)$, assuming we prepend the current element to our linked list at every collision.

Lookup & Deletion. $\mathcal{O}(1 + n2^{-n})$.

(source: Levet Notes)