

Problem 1

A mean wizard has turned you into a goose. Your new flock is preparing to migrate south for the winter, and you've been tasked with using your algorithm design skills to plan the best route.

Your migration path can be described as n ponds, with pond i having some amount $a_i > 0$ of yummy plants to eat. Each day, you can fly south from your current pond to any of the next m ponds, where you will stop for the night and eat all a_i plants. Each day of flying consumes some amount of energy C . Assume we start at pond 0, with $a_0 = 0$.

Your goal is to design a route that maximizes plants consumed less energy used flying. That is, you want to choose a set P of ponds to rest at such that each resting place is at most m ponds apart, the last resting place is pond n , and the energy surplus $\sum_{i \in P} a_i - C|P|$ is maximized.

- a. Identify a subproblem for your flock.
- b. Define a recurrence for the maximum energy surplus you can achieve if you land at pond i .
- c. Use your recurrence to find the optimal energy surplus for your migration route for the following ponds, when $m = 3$ and $C = 5$. What route would you take?

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
0	14	10	4	11	5	3	6	14	7

- d. **(Bonus.)** How might you modify your recurrence if your flock also insisted the migration take at most d days?